New System

Gabriel A. Salcedo Varela
Universidad de Sonora, México

Abstract

This is a sample document which shows the most important features of the Standard LaTeX Journal Article class.

1 Tomato Model

$$\dot{S_p^y} = -\beta_p^y S_p^y I_v + r_y^1 L_p^y + r_y^2 I_p^y + r_a I_p^a - \alpha S_p^y \tag{1}$$
$$\dot{S_p^a} = -\beta_p^a S_p^a I_v + \alpha S_p^y \tag{2}$$
$$\dot{L_p^y} = \beta_p^y S_p^y I_v - b_y L_p^y - r_y^1 L_p^y \tag{3}$$
$$\dot{L_p^a} = \beta_p^a S_p^a I_v - b_a L_p^a \tag{4}$$
$$\dot{I_p^y} = b_y L_p^J - r_y^2 I_p^y \tag{5}$$
$$\dot{I_p^a} = b_a L_p^a - r_a I_p^a \tag{6}$$
$$\dot{S_v} = -\beta_v^y S_v I_p^y - \beta_v^a S_v I_p^a - \gamma S_v + (1-\theta)\mu \tag{7}$$
$$\dot{I_v} = \beta_v^y S_v I_p^y + \beta_v^a S_v I_p^a - \gamma I_v + \theta\mu \tag{8}$$

2 Free disease equilibrium and R_0

$x_0 = (N_p, 0, 0, 0, 0, 0, \frac{\mu}{\gamma}, 0)$.

In this case we can cumpute the R_0 from the next generation matrix, then

$$F(S_p^y, S_p^a, L_p^y, L_p^a, I_p^y, I_p^a, S_v, I_v) = \begin{bmatrix} 0 & 0 & 0 & 0 & \beta_p^Y S_p^y \\ 0 & 0 & 0 & 0 & \beta_p^a S_p^a \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \beta_v^y S_v & \beta_v^a S_v & 0 \end{bmatrix} \tag{9}$$

and

$$V(S_p^y, S_p^a, L_p^y, L_p^a, I_p^y, I_p^a, S_v, I_v) = \begin{bmatrix} b_y + r_y^1 & 0 & 0 & 0 & 0 \\ 0 & b_a & 0 & 0 & 0 \\ -b_y & 0 & r_y^2 & 0 & 0 \\ 0 & -b_a & 0 & r_a & 0 \\ 0 & 0 & 0 & 0 & \gamma \end{bmatrix} \tag{10}$$

And the next generation matrix is defined by $K = F(x_0)V(x_0)^{-1}$

$$K = \begin{bmatrix} 0 & 0 & 0 & 0 & \frac{\beta_p^y N_p}{\gamma} \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ \frac{\beta_v^y \mu b_y}{\gamma(b_y r_y^2 + r_y^1 r_y^2)} & \frac{\beta_v^a \mu}{\gamma b_a} & \frac{\beta_v^y \mu}{\gamma r_y^2} & \frac{\beta_v^a \mu}{\gamma r_a} & 0 \end{bmatrix} \tag{11}$$

And the R_0 is the eigenvalue of maximum norm,

$$R_0 = \sqrt{\frac{\beta_p^y N_p}{\gamma}} \sqrt{\frac{\beta_v^y \mu b_y}{\gamma(b_y r_y^2 + r_y^1 r_y^2)}} \tag{12}$$

3 Controlled Model

$$\dot{S_p^y} = -\beta_p^y S_p^y I_v + (r_y^1 + u_1(t))L_p^y + (r_y^2 + u_2(t))I_p^y + (r_a + u_3(t))I_p^a - \alpha S_p^y \tag{13}$$
$$\dot{S_p^a} = -\beta_p^a S_p^a I_v + \alpha S_p^y \tag{14}$$
$$\dot{L_p^y} = \beta_p^y S_p^y I_v - b_y L_p^y - (r_y^1 + u_1(t))L_p^y \tag{15}$$
$$\dot{L_p^a} = \beta_p^a S_p^a I_v - b_a L_p^a \tag{16}$$
$$\dot{I_p^y} = b_y L_p^J - (r_y^2 + u_2(t))I_p^y \tag{17}$$
$$\dot{I_p^a} = b_a L_p^a - (r_a + u_3(t))I_p^a \tag{18}$$
$$\dot{S_v} = -\beta_v^y S_v I_p^y - \beta_v^a S_v I_p^a - (\gamma + u_4(t))S_v + (1-\theta)\mu \tag{19}$$
$$\dot{I_v} = \beta_v^y S_v I_p^y + \beta_v^a S_v I_p^a - (\gamma + u_4(t))I_v + \theta\mu \tag{20}$$

We need minimize the following functional cost

$$\begin{aligned} J(x,u) = \int_0^T &(A_1 L_p^y + A_2 I_p^y + A_3 I_p^a + A_4 I_v + c_1 u_1^2(t) \\ &+ c_2 u_2^2(t) + c_3 u_3^2(t) + c_4 u_4^2(t))dt \end{aligned} \tag{21}$$

We need define the following Hamiltonian to apply the Pontryagain's principle theorem.

$$\begin{aligned} &H(L_p^y, I_p^y, I_p^a, I_v, u_1, u_2, u_3, u_4) = A_1 L_p^y + A_2 I_p^y + A_3 I_p^a + A_4 I_v \\ &+ c_1 u_1^2(t) + c_2 u_2^2(t) + c_3 u_3^2(t) + c_4 u_4^2(t) + \lambda_1[-\beta_p^y S_p^y I_v + (r_y^1 + u_1(t))L_p^y \\ &+ (r_y^2 + u_2(t))I_p^y + (r_a + u_3(t))I_p^a - \alpha S_p^y] + \lambda_2[-\beta_p^a S_p^a I_v + \alpha S_p^y] \\ &+ \lambda_3[\beta_p^y S_p^y I_v - b_y L_p^y - (r_y^1 + u_1(t))L_p^y] + \lambda_4[\beta_p^a S_p^a I_v - b_a L_p^a] \\ &+ \lambda_5[b_y L_p^J - (r_y^2 + u_2(t))I_p^y] + \lambda_6[b_a L_p^a - (r_a + u_3(t))I_p^a] \\ &+ \lambda_7[-\beta_v^y S_v I_p^y - \beta_v^a S_v I_p^a - (\gamma + u_4(t))S_v + (1-\theta)\mu] \\ &+ \lambda_8[\beta_v^y S_v I_p^y + \beta_v^a S_v I_p^a - (\gamma + u_4(t))I_v + \theta\mu] \end{aligned} \tag{22}$$

By the Pontryagain's principle theorem we have the following adjoint system

$$\dot{\lambda_1} = \alpha(\lambda_2 - \lambda_1) + \beta_p^y I_v(\lambda_3 - \lambda_1) \tag{23}$$

$$\dot{\lambda_2} = \beta_p^a I_v(\lambda_4 - \lambda_2) \tag{24}$$

$$\dot{\lambda_3} = A_1 + r_y^1(\lambda_1 - \lambda_3) + u_1(\lambda_1 - \lambda_3) + b_y(\lambda_5 - \lambda_3) \tag{25}$$

$$\dot{\lambda_4} = b_a(\lambda_6 - \lambda_4) \tag{26}$$

$$\dot{\lambda_5} = A_2 + (r_y^2 + u_2)(\lambda_1 - \lambda_5) + \beta_v^y S_v(\lambda_8 - \lambda_7) \tag{27}$$

$$\dot{\lambda_6} = A_3 + (r_a + u_3)(\lambda_1 - \lambda_6) + \beta_v^a S_v(\lambda_8 - \lambda_7) \tag{28}$$

$$\dot{\lambda_7} = (\beta_v^y I_p^y + \beta_v^a I_p^a)(\lambda_8 - \lambda_7) - (\gamma + u_4)\lambda_7 \tag{29}$$

$$\dot{\lambda_8} = A_4 + \beta_p^y S_p^y(\lambda_3 - \lambda_1) + \beta_p^a S_p^a(\lambda_4 - \lambda_2) - (\gamma + u_4)\lambda_8 \tag{30}$$

and the optimallity condition is given by :

$$\bar{u_1} = \min\left\{0, \max\left\{u_1^{max}, \frac{\bar{L_p^y}(\lambda_3 - \lambda_1)}{2c_1}\right\}\right\} \tag{31}$$

$$\bar{u_2} = \min\left\{0, \max\left\{u_2^{max}, \frac{\bar{I_p^y}(\lambda_5 - \lambda_1)}{2c_2}\right\}\right\} \tag{32}$$

$$\bar{u_3} = \min\left\{0, \max\left\{u_3^{max}, \frac{\bar{I_p^a}(\lambda_6 - \lambda_1)}{2c_3}\right\}\right\} \tag{33}$$

$$\bar{u_4} = \min\left\{0, \max\left\{u_4^{max}, \frac{\bar{S_v}\lambda_7 + \bar{I_v}\lambda_8}{2c_4}\right\}\right\} \tag{34}$$

References

A The First Appendix