# Modeling of optimal phytosanitary policies in crops of economic importance in the state of Sonora.

Gabriel Adrián Salcedo Varela

Departamento de matemáticas, División de Ciencas Exactas y Naturales
Universidad de Sonora

November 29, 2019

# Contents

# Tomato Leaf Curl Virus

mm 40 60 80 100 120

mm 40 60 80 100 120

40

## Objetive

Model optimal phytosanitary policies for diseases in agricultural crops.

60

80

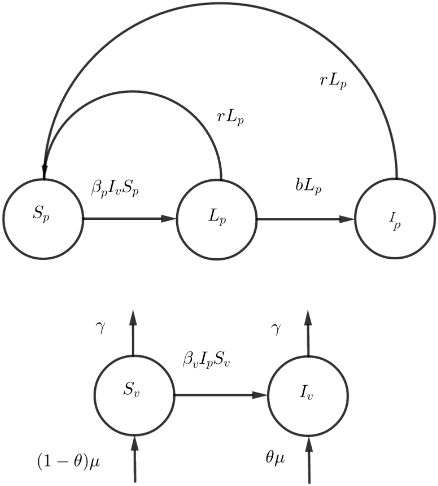
Holt, J. Colvin, and V. Muniyappa.
Identifying control strategies for tomato leaf curl virus disease using an epidemiological model.
*Journal of Applied Ecology*, 36(5):625–633, oct 1999.

# Plant Model without control

Tomato Leaf Curl Virus Disease Using an Epidemiological Model

**Hypothesis:**

- infection by infeted plants and vectors,
- output and input for plants and vectors.



# Others Controls

mm 40 60 80 100 120

**Cultural Control**

- physical barriers,
- planting dates,
- removal of infested plants,
- host plant resistance.

**Biological control**

- Parasitoids,
- predators
- fungi.

**Insecticide**

- Pymetrozine,
- flupyradifurone,
- cyazypyr.

R. E. Shun-xiang, W. A. Zhen-zhong, Q. I. Bao-li, X. I. Yuan.
The pest status of bemisia tabaci in china and non-chemical control strategies*.
*Insect Science*, 8(3):279–288, 2001.

H. A. Smith and M. C. Giurcanu.
New Insecticides for Management of Tomato Yellow Leaf Curl, a Virus Vectored by the Silverleaf Whitefly, Bemisia tabaci.
*Journal of Insect Science*, 14(1):4–7, jan 2014.

80

# Others Controls

mm 40 60 80 100 120

80

## Cultural Control

- physical barriers,
- planting dates,
- removal of infested plants,
- host plant resistance.

## Biological control

- Parasitoids,
- predators
- fungi.

## Insecticide

- Pymetrozine,
- flupyradifurone,
- cyazypyr.

# Others Controls

mm 40 60 80 100 120

## Cultural Control

- physical barriers,
- planting dates,
- removal of infested plants,
- host plant resistance.

## Biological control

- Parasitoids,
- predators
- fungi.

## Insecticide

- Pymetrozine,
- flupyradifurone,
- cyazypyr.

80

# Others Controls

mm 40 60 80 100 120

## Cultural Control

- physical barriers,
- planting dates,
- removal of infested plants,
- host plant resistance.

## Biological control

- Parasitoids,
- predators
- fungi.

## Insecticide

- Pymetrozine,
- flupyradifurone,
- cyazypyr.

80

# Others Controls

mm 40 60 80 100 120

## Cultural Control

- physical barriers,
- planting dates,
- removal of infested plants,
- host plant resistance.

## Biological control

- Parasitoids,
- predators
- fungi.

## Insecticide

- Pymetrozine,
- flupyradifurone,
- cyazypyr.

80

# Others Controls

mm 40 60 80 100 120

80

## Cultural Control

- physical barriers,
- planting dates,
- removal of infested plants,
- host plant resistance.

## Biological control

- Parasitoids,
- predators
- fungi.

## Insecticide

- Pymetrozine,
- flupyradifurone,
- cyazypyr.

# Others Controls

mm 40 60 80 100 120

## Cultural Control

- physical barriers,
- planting dates,
- removal of infested plants,
- host plant resistance.

## Biological control

- Parasitoids,
- predators
- fungi.

## Insecticide

- Pymetrozine,
- flupyradifurone,
- cyazypyr.

80

# Others Controls

mm 40 60 80 100 120

## Cultural Control

- physical barriers,
- planting dates,
- removal of infested plants,
- host plant resistance.

## Biological control

- Parasitoids,
- predators
- fungi.

## Insecticide

- Pymetrozine,
- flupyradifurone,
- cyazypyr.

80

# Others Controls

mm 40 60 80 100 120

## Cultural Control

- physical barriers,
- planting dates,
- removal of infested plants,
- host plant resistance.

## Biological control

- Parasitoids,
- predators
- fungi.

## Insecticide

- Pymetrozine,
- flupyradifurone,
- cyazypyr.

80

# Others Controls

mm 40 60 80 100 120

## Cultural Control

- physical barriers,
- planting dates,
- removal of infested plants,
- host plant resistance.

## Biological control

- Parasitoids,
- predators
- fungi.

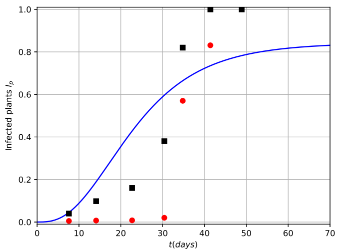
## Insecticide

- Pymetrozine,
- flupyradifurone,
- cyazypyr.

80

$$
\begin{aligned}
\frac{dS_p}{dt} &= -\beta_p S_p I_v + r(L_p + I_p),\\
\frac{dL_p}{dt} &= \beta_p S_p I_v - bL_p - rL_p,\\
\frac{dI_p}{dt} &= bL_p - rI_p,\\
\frac{dS_v}{dt} &= -\beta_v S_v I_p - \gamma S_v - (1-\theta)\mu,\\
\frac{dI_v}{dt} &= \beta_v S_v I_p - \gamma I_v - \theta\mu,\\
S_p(0) &= S_{p_0}, L_p(0) = L_{p_0}, I_p(0) = I_{p_0},\\
S_v(0) &= S_{v_0}, I_v(0) = I_{v_0}.
\end{aligned}
$$

| Par. | Value | Descrip. |
|---|---|---|
| $\beta_p$ | 0.1 | plant latent rate |
| $r$ | 0.01 | plant remove rate |
| $b$ | 0.075 | plant infectious rate |
| $\gamma$ | 0.06 | vector die or depar rate |
| $\mu$ | 0.3 | immigration rate |
| $\theta$ | 0.2 | infected vectors arrival |
| $\beta_v$ | 0.003 | vector infected rate |



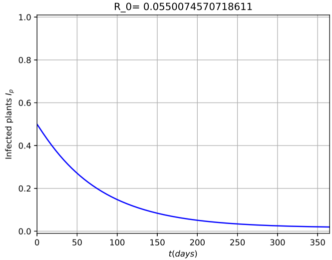
$$R_0 = \sqrt{\frac{\beta_v \mu b \beta_p}{r^2 (r+b)\gamma}}.$$

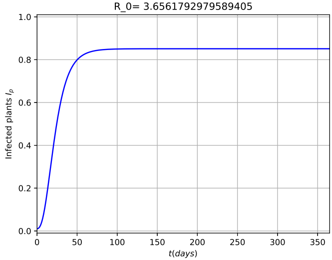
If $R_0 < 1$,

$$\lim_{t \to \infty} (S_p, L_p, I_p, S_v, I_v) = (N_p, 0, 0, \frac{\mu}{\gamma}, 0).$$

If $R_0 < 1$,

$$\lim_{t \to \infty} (S_p, L_p, I_p, S_v, I_v) = (S_p^*, L_p^*, I_p^*, S_v^*, I_v^*).$$



# Plant Model with control

## Tomato Leaf Curl Virus Disease Using an Epidemiological Model

$$
\begin{aligned}
\frac{dS_p}{dt} &= -\beta_p S_p I_v + (r + u_1) L_p + (r + u_2) I_p,\\
\frac{dL_p}{dt} &= \beta_p S_p I_v - b L_p - (r + u_1) L_p,\\
\frac{dI_p}{dt} &= b L_p - (r + u_2) I_p,\\
\frac{dS_v}{dt} &= -\beta_v S_v I_p - (\gamma + u_3) S_v - (1 - \theta)\mu,\\
\frac{dI_v}{dt} &= \beta_v S_v I_p - (\gamma + u_3) I_v - \theta\mu,
\end{aligned}
$$

Minimize

$$J(u_1, u_2, u_3) = \int_0^T A_1 I_p(t) + A_2 L_p(t) + A_3 I_v(t) + c_1 u_1(t)^2 + c_2 u_2(t)^2 + c_3 u_3(t)^2 dt,$$

subject to

$$\begin{cases} \frac{dS_p}{dt} = & -\beta_p S_p I_v + (r+u_1)L_p + (r+u_2)I_p, \\ \frac{dL_p}{dt} = & \beta_p S_p I_v - bL_p - (r+u_1)L_p, \\ \frac{dI_p}{dt} = & bL_p - (r+u_2)I_p, \\ \frac{dS_v}{dt} = & -\beta_v S_v I_p - (\gamma+u_3)S_v - (1-\theta)\mu, \\ \frac{dI_v}{dt} = & \beta_v S_v I_p - (\gamma+u_3)I_v - \theta\mu, \\ S_p(0) & = S_{p_0}, L_p(0) = L_{p_0}, I_p(0) = I_{p_0}, S_v(0) = S_{v_0}, I_v(0) = I_{v_0}. \end{cases}$$

$T \in (0, \infty)$ be fixed. Consider the control system:

$$\begin{cases} \dot{x}(s) = f(s, u(s), x(s)) \; s \in [t_0, T], \\ x(t_0) = x_0, \end{cases}$$

with terminal state constraint

$$x(T; t_0, x_0, u(\cdot)) \in M,$$

where $M \subseteq \mathbb{R}^n$ is fixed.

$M : \mathbb{R}_+ \to 2^{\mathbb{R}^n}$ is a moving target in $\mathbb{R}^n$ if for any $t \in \mathbb{R}_+$, $M(t)$ is a measurable.

80

$$J(t_0, x_0; u(\cdot)) = \int_{t_0}^{T} g(s, u(s), x(s))ds + h(T, x(T)) \equiv J^T(t_0, x_0, u(\cdot)).$$

**Problem** $(OC)^T$

Given $(t_0, x_0) \in \mathbb{R}_+ \times \mathbb{R}^n$ with $\tilde{\mathcal{U}}^M_{x_0}[t_0, T] \neq \emptyset$, find a $\bar{u}(\cdot) \in \tilde{\mathcal{U}}^M_{x_0}[t_0, T]$ such that

$$J^T(t_0, x_0; \bar{u}(\cdot)) = \inf_{u(\cdot) \in \tilde{\mathcal{U}}^M_{x_0}[t_0, T]} J^T(t_0, x_0; u(\cdot)).$$

**(C1)**

$f : \mathbb{R}_+ \times U \times \mathbb{R}^n \to \mathbb{R}^n$ is measurable, satisfies a lipchitz condition in $x$, and $|f(t,u,0)| \leq L$, for every $(t,u) \in \mathbb{R}_+ \times U$.

$\omega : \mathbb{R}_+ \times \mathbb{R}_+ \to \mathbb{R}_+$, increasing, and $\omega(r,0) = 0$ for every $r \geq 0$.

**(C2)**

$g : \mathbb{R}_+ \times U \times \mathbb{R}^n \to \mathbb{R}$ and $h : \mathbb{R}^n \to \mathbb{R}$ are measurable, and

$$|g(s,u,x_1) - g(s,u,x_2)| + |h(x_1) - h(x_2)| \leq \omega(|x_1| \vee |x_2|, |x_1 - x_2|)$$

for every $(s,u) \in \mathbb{R}_+ \times U, x_1, x_2 \in \mathbb{R}^n$.

For any $(t, x) \in [0, T] \times \mathbb{R}^n$,

$$\mathbb{E}(t, x) = \{(z^0, z) \in \mathbb{R} \times \mathbb{R}_+ | z^0 \geq g(t, u, x), z = f(t, u, x), u \in U\}.$$

Cesari property:

$$\bigcap_{\delta>0} \bar{co}\mathbb{E}(t, B_\delta(x)) = \mathbb{E}(t, x).$$

**(C3)**

For almost all $t \in [0, T]$, Cesari property holds for any $x \in \mathbb{R}^n$.

## Existence Theorem

Let (C1)-(C3) hold. Let $M \subseteq \mathbb{R}^n$ be a non-empty closed set. Let $(t_0, x_0) \in [0, T] \times \mathbb{R}^n$ be given and $\tilde{\mathcal{U}}_x^M[t_0, T] \neq \emptyset$. Then problem $(OC)^T$ admits at least one optimal pair.

40

60

80

## Pontryagin's Maximum Principle

If $u^*(t)$ and $x^*(t)$ are optimal for the problem $(OC)^T$, then there exists a piecewise differentiable adjoint variable $\lambda(t)$ such that

$$H(t, x^*(t), u(t), \lambda(t)) \leq H(t, x^*(t), u^*(t), \lambda(t))$$

for all controls $u$ at each time $t$, where the Hamiltonian $H$ is

$$H = g(t, x(t), u(t)) + \lambda(t) f(t, x(t), u(t)),$$

and

$$\begin{aligned} \lambda'(t) &= -\frac{\partial H(t, x^*(t), u^*(t), \lambda(t))}{\partial x}, \\ \lambda(T) &= 0. \end{aligned}$$

**Step 1.** Make an initial guess for $\vec{u}$ over the interval.

**Step 2.** Using the initial condition $x_1 = x(t_0) = a$ and the values for $\vec{u}$, solve $\vec{x}$ forward in time according to its differential equation in the optimality system.

**Step 3.** Using the transversality condition $\lambda_{N+1} = \lambda(t_1) = 0$ and the values for $\vec{u}$ and $\vec{x}$, solve $\vec{\lambda}$ backward in time according to its differential equation in the optimality system.

**Step 4.** Update $\vec{u}$ by entering the new $\vec{x}$ and $\vec{\lambda}$ values into the characterization of the optimal control.

**Step 5.** Check convergence. If the values of the variables in this iteration and the last iteration and the last iteration are negligibly close, output the current values as solutions. If values are not close, return to Step 2.

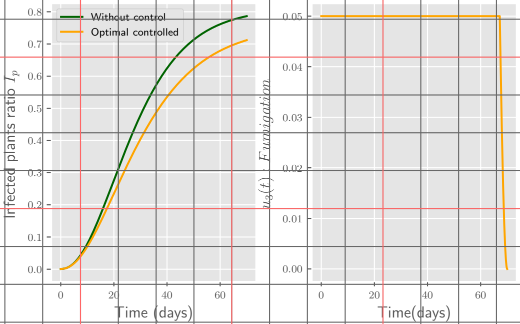
mm 40 60 80 100 120

40

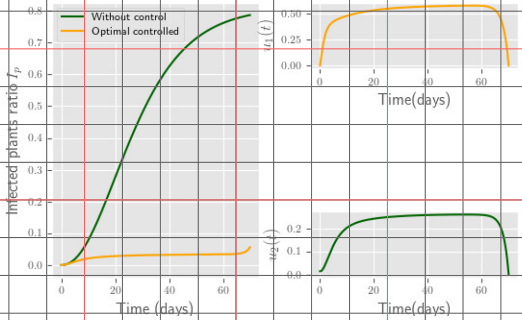
60

80

# Case with one controls



# Case with two controls



# Case with three controls