Modeling of optimal phytosanitary policies in crops of economic importance in the state of Sonora.

Gabriel Adrián Salcedo Varela

Departamento de matemáticas, División de Ciencas Exactas y Naturales
Universidad de Sonora

November 30, 2019

Contents

Tomato Leaf Curl Virus

Tomato Leaf Curl Virus

mm 40 60 80 100 120

mm 40 60 80 100 120

40

Objetive

Model optimal phytosanitary policies for diseases in agricultural crops.

60

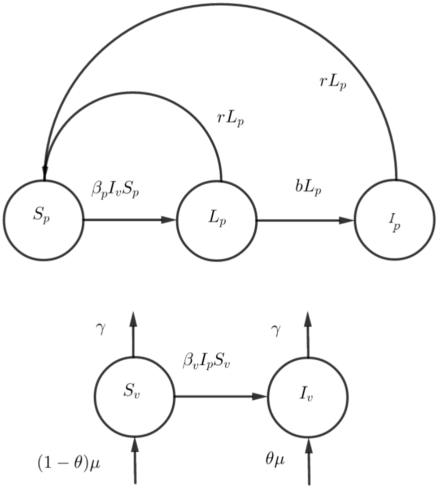
80

Plant Model without control

Tomato Leaf Curl Virus Disease Using an Epidemiological Model

Hypothesis:

- Remove from latent and infected plants,
- plants become latent plants by infected vectors,
- latent plants become infectious plants,
- vectors become infected vectors by infected plants,
- vectors die per day,
- immigration from alternative hosts.



Others Controls

mm 40 60 80 100 120

Cultural Control

- physical barriers,
- planting dates,
- removal of infested plants,
- host plant resistance.

Biological control

- Parasitoids,
- predators
- fungi.

Insecticide

- Pymetrozine,
- flupyradifurone,
- cyazypyr.

80

Others Controls

mm 40 60 80 100 120

Cultural Control

- physical barriers,
- planting dates,
- removal of infested plants,
- host plant resistance.

Biological control

- Parasitoids,
- predators
- fungi.

Insecticide

- Pymetrozine,
- flupyradifurone,
- cyazypyr.

80

Others Controls

mm 40 60 80 100 120 80

Cultural Control

- physical barriers,
- planting dates,
- removal of infested plants,
- host plant resistance.

Biological control

- Parasitoids,
- predators
- fungi.

Insecticide

- Pymetrozine,
- flupyradifurone,
- cyazypyr.

Others Controls

mm 40 60 80 100 120

Cultural Control

- physical barriers,
- planting dates,
- removal of infested plants,
- host plant resistance.

Biological control

- Parasitoids,
- predators
- fungi.

Insecticide

- Pymetrozine,
- flupyradifurone,
- cyazypyr.

80

Others Controls

mm 40 60 80 100 120

Cultural Control

- physical barriers,
- planting dates,
- removal of infested plants,
- host plant resistance.

Biological control

- Parasitoids,
- predators
- fungi.

Insecticide

- Pymetrozine,
- flupyradifurone,
- cyazypyr.

80

Others Controls

mm 40 60 80 100 120 80

Cultural Control

- physical barriers,
- planting dates,
- removal of infested plants,
- host plant resistance.

Biological control

- Parasitoids,
- predators
- fungi.

Insecticide

- Pymetrozine,
- flupyradifurone,
- cyazypyr.

Others Controls

mm 40 60 80 100 120

Cultural Control

- physical barriers,
- planting dates,
- removal of infested plants,
- host plant resistance.

Biological control

- Parasitoids,
- predators
- fungi.

Insecticide

- Pymetrozine,
- flupyradifurone,
- cyazypyr.

80

Others Controls

mm 40 60 80 100 120

Cultural Control

- physical barriers,
- planting dates,
- removal of infested plants,
- host plant resistance.

Biological control

- Parasitoids,
- predators
- fungi.

Insecticide

- Pymetrozine,
- flupyradifurone,
- cyazypyr.

80

Others Controls

mm 40 60 80 100 120

Cultural Control

- physical barriers,
- planting dates,
- removal of infested plants,
- host plant resistance.

Biological control

- Parasitoids,
- predators
- fungi.

Insecticide

- Pymetrozine,
- flupyradifurone,
- cyazypyr.

80

Others Controls

mm 40 60 80 100 120

Cultural Control

- physical barriers,
- planting dates,
- removal of infested plants,
- host plant resistance.

Biological control

- Parasitoids,
- predators
- fungi.

Insecticide

- Pymetrozine,
- flupyradifurone,
- cyazypyr.

80

Others Controls

mm 40 60 80 100 120

Cultural Control

- physical barriers,
- planting dates,
- removal of infested plants,
- host plant resistance.

Biological control

- Parasitoids,
- predators
- fungi.

Insecticide

- Pymetrozine,
- flupyradifurone,
- cyazypyr.

R. E. Shun-xiang, W. A. Zhen-zhong, Q. I. Bao-li, X. I. Yuan.
The pest status of bemisia tabaci in china and non-chemical control strategies*.
Insect Science, 8(3):279–288, 2001.

H. A. Smith and M. C. Giurcanu.
New Insecticides for Management of Tomato Yellow Leaf Curl, a Virus Vectored by the Silverleaf Whitefly, Bemisia tabaci.
Journal of Insect Science, 14(1):4–7, jan 2014.

80

mm 40 60 80 100 120

40
60
80

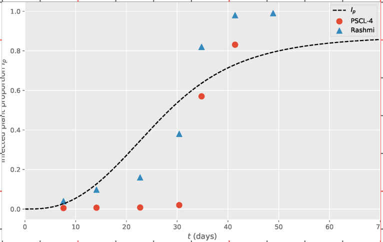
J. Holt, J. Colvin, and V. Muniyappa.
Identifying control strategies for tomato leaf curl virus disease using an epidemiological model.
Journal of Applied Ecology, 36(5):625–633, oct 1999.

$$
\begin{aligned}
\frac{dS_p}{dt} &= -\beta_p S_p I_v + r(L_p + I_p),\\
\frac{dL_p}{dt} &= \beta_p S_p I_v - bL_p - rL_p,\\
\frac{dI_p}{dt} &= bL_p - rI_p,\\
\frac{dS_v}{dt} &= -\beta_v S_v I_p - \gamma S_v + (1-\theta)\mu,\\
\frac{dI_v}{dt} &= \beta_v S_v I_p - \gamma I_v + \theta\mu,\\
S_p(0) &= S_{p_0}, L_p(0) = L_{p_0}, I_p(0) = I_{p_0},\\
S_v(0) &= S_{v_0}, I_v(0) = I_{v_0}.
\end{aligned}
$$

Par.	Unit	Descrip.
β_p	vector^{-1}day^{-1}	plants became latently infected rate
r	day^{-1}	replanting rate
b	day^{-1}	latent class to the infectious class rate
γ	day^{-1}	vector die or depart rate
μ	plant^{-1}day^{-1}	immigration rate
θ	proportion	proportion of infected vector arrival to crop
β_v	plant^{-1}day^{-1}	vector infectious acquisition rate

$$
\begin{aligned}
\frac{dS_p}{dt} &= -\beta_p S_p I_v + r(L_p + I_p),\\
\frac{dL_p}{dt} &= \beta_p S_p I_v - bL_p - rL_p,\\
\frac{dI_p}{dt} &= bL_p - rI_p,\\
\frac{dS_v}{dt} &= -\beta_v S_v I_p - \gamma S_v + (1-\theta)\mu,\\
\frac{dI_v}{dt} &= \beta_v S_v I_p - \gamma I_v + \theta\mu,
\end{aligned}
$$

$$S_p(0) = S_{p_0},\, L_p(0) = L_{p_0},\, I_p(0) = I_{p_0},$$
$$S_v(0) = S_{v_0},\, I_v(0) = I_{v_0}.$$

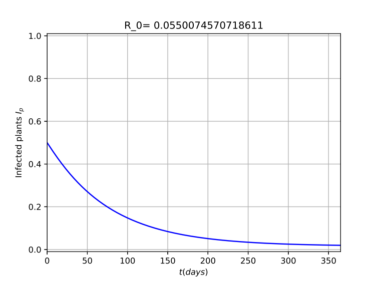


$$R_0 = \sqrt{\frac{\beta_v \mu b \beta_p}{r^2 (r+b)\gamma}}.$$

$$R_0 = \sqrt{\frac{\beta_v \mu b \beta_p}{r^2(r+b)\gamma}}.$$

If $R_0 < 1$,

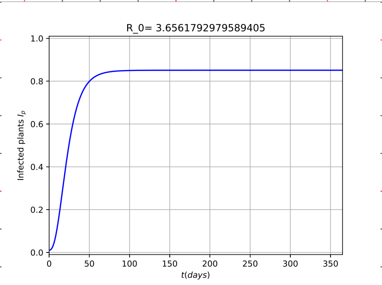
$$\lim_{t\to\infty} (S_p, L_p, I_p, S_v, I_v) = (N_p, 0, 0, \frac{\mu}{\gamma}, 0).$$



$$R_0 = \sqrt{\frac{\beta_v \mu b \beta_p}{r^2(r+b)\gamma}}.$$

If $R_0 > 1$,

$$\lim_{t\to\infty} (S_p, L_p, I_p, S_v, I_v) = (S_p^*, L_p^*, I_p^*, S_v^*, I_v^*).$$



Plant Model with control

Tomato Leaf Curl Virus Disease Using an Epidemiological Model

$$
\begin{aligned}
\frac{dS_p}{dt} &= -\beta_p S_p I_v + (r + u_1)L_p + (r + u_2)I_p,\\
\frac{dL_p}{dt} &= \beta_p S_p I_v - bL_p - (r + u_1)L_p,\\
\frac{dI_p}{dt} &= bL_p - (r + u_2)I_p,\\
\frac{dS_v}{dt} &= -\beta_v S_v I_p - (\gamma + u_3)S_v + (1-\theta)\mu,\\
\frac{dI_v}{dt} &= \beta_v S_v I_p - (\gamma + u_3)I_v + \theta\mu,
\end{aligned}
$$

Controls:

- u_1: replanting latent plant,
- u_2: replanting infected plants,
- u_3: fumigation.

OC

$$\min_{\bar{u}(\cdot)\in\tilde{\mathcal{U}}_{x_0}[t_0,T]} J(u_1,u_2,u_3) = \int_0^T (A_1 I_p(t) + A_2 L_p(t) + A_3 I_v(t) + c_1 u_1(t)^2 + c_2 u_2(t)^2 + c_3 u_3(t)^2)dt,$$

s.t.

$$\begin{aligned}
\frac{dS_p}{dt} &= -\beta_p S_p I_v + (r+u_1)L_p + (r+u_2)I_p,\\
\frac{dL_p}{dt} &= \beta_p S_p I_v - bL_p - (r+u_1)L_p,\\
\frac{dI_p}{dt} &= bL_p - (r+u_2)I_p,\\
\frac{dS_v}{dt} &= -\beta_v S_v I_p - (\gamma+u_3)S_v + (1-\theta)\mu,\\
\frac{dI_v}{dt} &= \beta_v S_v I_p - (\gamma+u_3)I_v + \theta\mu,\\
& S_p(0) = S_{p_0}, L_p(0) = L_{p_0},\\
& I_p(0) = I_{p_0}, S_v(0) = S_{v_0}, I_v(0) = I_{v_0}.
\end{aligned}$$

Existence Theory

mm 40 60 80 100 120

40

60

$$\begin{cases} \dot{x}(s) = f(s, u(s), x(s)) \; s \in [t_0, T], \\ x(t_0) = x_0, \end{cases}$$

with terminal state constraint

$$x(T; t_0, x_0, u(\cdot)) \in M, M \subseteq \mathbb{R}^n.$$

$$J(t_0, x_0; u(\cdot)) = \int_{t_0}^{T} g(s, u(s), x(s))ds + h(T, x(T)).$$

mm 40 60 80 100 120

Problem (OC)

$(t_0, x_0) \in \mathbb{R}_+ \times \mathbb{R}^n$ with $\tilde{\mathcal{U}}_{x_0}[t_0, T] \neq \emptyset$, find a $\bar{u}(\cdot) \in \tilde{\mathcal{U}}_{x_0}[t_0, T]$ s.t.

$$J(t_0, x_0; \bar{u}(\cdot)) = \inf_{u(\cdot) \in \tilde{\mathcal{U}}_{x_0}[t_0, T]} J(t_0, x_0; u(\cdot)).$$

60

80

Hypothesis:

(C-1) $f : \mathbb{R}_+ \times U \times \mathbb{R}^n \to \mathbb{R}^n$ is measurable, satisfies a lipchitz condition in x, $|f(t, u, 0)| \leq L$, $\forall\, (t, u) \in \mathbb{R}_+ \times U$.

(C-2) $g : \mathbb{R}_+ \times U \times \mathbb{R}^n \to \mathbb{R}$, $h : \mathbb{R}^n \to \mathbb{R}$ are measurable, and

$$|g(s, u, x_1) - g(s, u, x_2)| + |h(x_1) - h(x_2)| \leq \omega(|x_1| \vee |x_2|, |x_1 - x_2|)$$

$\forall\, (s, u) \in \mathbb{R}_+ \times U, x_1, x_2 \in \mathbb{R}^n$.

(C-3) For a.a. $t \in [0, T]$, Cesari property holds $\forall\, x \in \mathbb{R}^n$.

80 100 120

Hypothesis:

(**C**-1) $f : \mathbb{R}_+ \times U \times \mathbb{R}^n \to \mathbb{R}^n$ is measurable, satisfies a lipchitz condition in x, $|f(t,u,0)| \leq L$, $\forall (t,u) \in \mathbb{R}_+ \times U$.

(**C**-2) $g : \mathbb{R}_+ \times U \times \mathbb{R}^n \to \mathbb{R}$, $h : \mathbb{R}^n \to \mathbb{R}$ are measurable, and

$$\begin{aligned}&|g(s,u,x_1) - g(s,u,x_2)| + \\ &|h(x_1) - h(x_2)| \\ &\leq \omega(|x_1| \vee |x_2|, |x_1 - x_2|)\end{aligned}$$

$\forall (s,u) \in \mathbb{R}_+ \times U, x_1, x_2 \in \mathbb{R}^n$.

(**C**-3) For a.a. $t \in [0,T]$, Cesari property holds $\forall\, x \in \mathbb{R}^n$.

$\omega : \mathbb{R}_+ \times \mathbb{R}_+ \to \mathbb{R}_+$, increasing, $\omega(r,0) = 0\ \forall r \geq 0$.

Hypothesis:

(C-1) $f : \mathbb{R}_+ \times U \times \mathbb{R}^n \to \mathbb{R}^n$ is measurable, satisfies a lipchitz condition in x, $|f(t,u,0)| \leq L$, $\forall (t,u) \in \mathbb{R}_+ \times U$.

(C-2) $g : \mathbb{R}_+ \times U \times \mathbb{R}^n \to \mathbb{R}$, $h : \mathbb{R}^n \to \mathbb{R}$ are measurable, and

$$|g(s,u,x_1) - g(s,u,x_2)| + |h(x_1) - h(x_2)| \leq \omega(|x_1| \vee |x_2|, |x_1 - x_2|)$$

$\forall (s,u) \in \mathbb{R}_+ \times U, x_1, x_2 \in \mathbb{R}^n$.

(C-3) For a.a. $t \in [0,T]$, Cesari property holds $\forall\, x \in \mathbb{R}^n$.

80 100 120

$$\mathbb{E}(t,x) = \{(z^0, z) \in \mathbb{R} \times \mathbb{R}^n | \; z^0 \geq g(t,u,x), \; z = f(t,u,x), \; u \in U\}.$$

Cesari property

$$\bigcap_{\delta} \bar{co}\mathbb{E}(t, B_\delta(x)) = \mathbb{E}(t,x)$$

Hypothesis:

(C-1) $f : \mathbb{R}_+ \times U \times \mathbb{R}^n \to \mathbb{R}^n$ is measurable, satisfies a lipchitz condition in x, $|f(t, u, 0)| \le L$, $\forall\,(t, u) \in \mathbb{R}_+ \times U$.

(C-2) $g : \mathbb{R}_+ \times U \times \mathbb{R}^n \to \mathbb{R}$, $h : \mathbb{R}^n \to \mathbb{R}$ are measurable, and

$$
\begin{aligned}
&|g(s, u, x_1) - g(s, u, x_2)| + \\
&|h(x_1) - h(x_2)| \\
&\le \omega(|x_1| \vee |x_2|, |x_1 - x_2|)
\end{aligned}
$$

$\forall\,(s, u) \in \mathbb{R}_+ \times U, x_1, x_2 \in \mathbb{R}^n$.

(C-3) For a.a. $t \in [0, T]$, Cesari property holds $\forall\ x \in \mathbb{R}^n$.

Existence Theorem

Let (C1)-(C3) hold. Then problem (OC) admits at least one optimal pair.

mm 40 60 80 100 120

40

$$H = g(t, x(t), u(t)) + \lambda(t) f(t, x(t), u(t)),$$

60

80

mm 40 60 80 100 120

40

$$H = g(t, x(t), u(t)) + \lambda(t) f(t, x(t), u(t)),$$

$$\begin{cases} \dot{x}(s) = f(s, u(s), x(s)) \ s \in [t_0, T], \\ x(t_0) = x_0, \end{cases}$$

$$J(t_0, x_0; u(\cdot)) = \int_{t_0}^{T} g(s, u(s), x(s))ds + h(x(T)).$$

80

Pontryagin's Maximum Principle

If $u^*(t)$ and $x^*(t)$ are optimal for the problem (OC), then there exists a piecewise differentiable adjoint variable $\lambda(t)$ s.t.

$$H(t, x^*(t), u(t), \lambda(t)) \leq H(t, x^*(t), u^*(t), \lambda(t))$$

$\forall$ u at t,

$$\begin{aligned}\lambda'(t) &= -\frac{\partial H(t, x^*(t), u^*(t), \lambda(t))}{\partial x},\\ \lambda(T) &= 0.\end{aligned}$$

mm 40 60 80 100 120

40
60
80

$$
\begin{aligned}
H = {} & A_1 I_v + A_2 L_p + A_3 I_v + \sum_{i=1}^{3} c_i u_i^2 \\
& + \lambda_1(-\beta_p S_p I_v + (r + u_1)L_p + (r + u_2)I_p) \\
& + \lambda_2(\beta_p S_p I_v - bL_p - (r + u_1)L_p) \\
& + \lambda_3(bL_p - (r + u_2)I_p) \\
& + \lambda_4(-\beta_v S_v I_p - (\gamma + u_3)S_v + (1 - \theta)\mu) \\
& + \lambda_5(\beta_v S_v I_p - (\gamma + u_3)I_v + \theta\mu).
\end{aligned}
$$

$$
\begin{aligned}
\frac{d\lambda_1}{dt} &= \beta_p(\lambda_1 - \lambda_2), \\
\frac{d\lambda_2}{dt} &= -A_2 + (r + u_1)(\lambda_2 - \lambda_1) + b(\lambda_2 - \lambda_3), \\
\frac{d\lambda_3}{dt} &- A_1 + (r + u_2)(\lambda_3 - \lambda_1) + \beta_v S_v(\lambda_4 - \lambda_5), \\
\frac{d\lambda_4}{dt} &= \beta_v I_p(\lambda_4 - \lambda_5) + (\gamma + u_3)\lambda_4, \\
\frac{d\lambda_5}{dt} &= -A_3 + \beta_p S_p(\lambda_1 - \lambda_2) + (\gamma + u_3)\lambda_5,
\end{aligned}
$$

$$
u_1^* = \min\left(\max\left(0, \frac{L_p(\lambda_2 - \lambda_1)}{2c_1}\right), u_{1_{max}}\right)
$$

$$
u_2^* = \min\left(\max\left(0, \frac{I_p(\lambda_3 - \lambda_1)}{2c_2}\right), u_{2_{max}}\right)
$$

$$
u_3^* = \min\left(\max\left(0, \frac{S_v\lambda_4 + I_v\lambda_5}{2c_3}\right), u_{3_{max}}\right)
$$

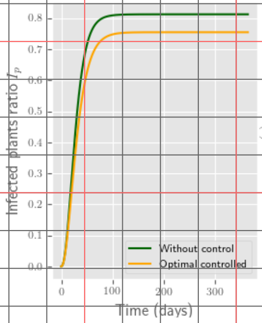
The most popular

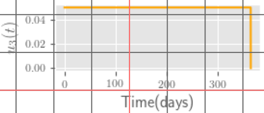
Algorithm 2 Forward Backward Sweep

Input: $t_0, t_f, x_0, h, \text{tol}, \lambda_f$
Output: x^*, u^*, λ

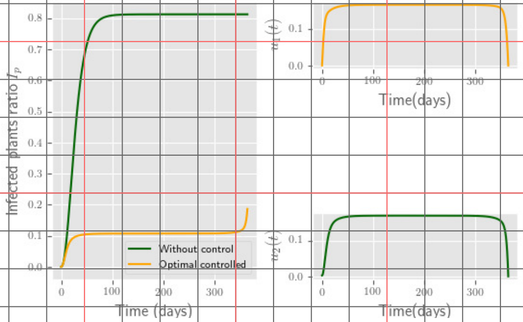
procedure FORWARD_BACKWARD_SWEEP$(g, \lambda_{\text{function}}, u, x_0, \lambda_f, h, n_{max})$
  while $\epsilon > \text{tol}$ **do**
    $u_{\text{old}} \leftarrow u$
    $x_{\text{old}} \leftarrow x$
    $x \leftarrow$ RUNGE_KUTTA_FORWARD(g, u, x_0, h)
    $\lambda_{\text{old}} \leftarrow \lambda$
    $\lambda \leftarrow$ RUNGE_KUTTA_BACKWARD$(\lambda_{\text{function}}, x, \lambda_f, h)$
    $u_1 \leftarrow$ OPTIMALITY_CONDITION(u, x, λ)
    $u \leftarrow \alpha u_1 + (1-\alpha) u_{old}, \qquad \alpha \in [0,1]$ ▷ convex combination
    $\epsilon_u \leftarrow \dfrac{||u - u_{\text{old}}||}{||u||}$
    $\epsilon_x \leftarrow \dfrac{||x - x_{\text{old}}||}{||x||}$ ▷ relative error
    $\epsilon_\lambda \leftarrow \dfrac{||\lambda - \lambda_{\text{old}}||}{||\lambda||}$
    $\epsilon \leftarrow \max\{\epsilon_u, \epsilon_x, \epsilon_\lambda\}$
  end while
  return x^*, u^*, λ ▷ Optimal pair
end procedure

Case with one controls

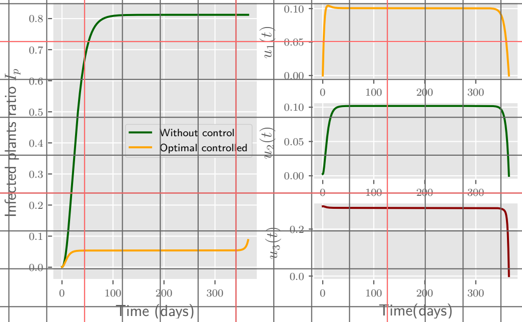




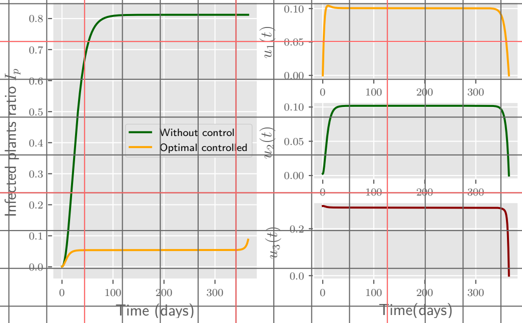
Case with two controls



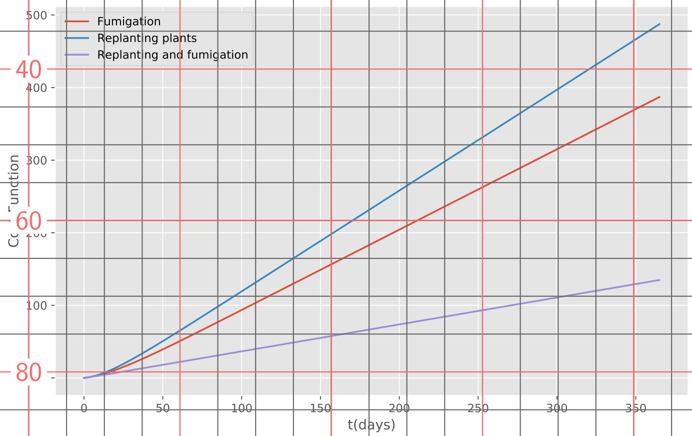
Case with three controls



Case with three controls



Cost Function Comparation



F. M. Hilker, L. J. S. Allen, V. A. Bokil, C. J. Briggs, Z. Feng, K. A. Garrett, L. J. Gross, F. M. Hamelin, M. J. Jeger, C. A. Manore, A. G. Power, M. G. Redinbaugh, M. A. Rúa, and N. J. Cunniffe.
Modeling Virus Coinfection to Inform Management of Maize Lethal Necrosis in Kenya.
Phytopathology, 107(10):PHYTO–03–17–008, aug 2017.

P. Grandits, R. M. Kovacevic, and V. M. Veliov.
Optimal control and the value of information for a stochastic epidemiological SIS-model.
Journal of Mathematical Analysis and Applications, 476(2):665–695, aug 2019.

J. Yong and X. Y. Zhou.
Stochastic controls: Hamiltonian systems and HJB equations, volume 43.
Springer Science & Business Media, 1999.