Modeling of optimal phytosanitary policies in crops of economic importance in the state of Sonora.

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Contents

Tomato Leaf Curl Virus

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mm 40 60 80 100 120

Others Controls

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Cultural Control

- physical barriers,
- planting dates,
- replanting of infected plants,
- host plant resistance.

80

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- Parasitoids,
- predators
- fungi.

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- Pymetrozine,
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R. E. Shun-xiang, W. A. Zhen-zhong, Q. I. Bao-li, X. I. Yuan.
The pest status of bemisia tabaci in china and non-chemical control strategies*.
Insect Science, 8(3):279–288, 2001.

H. A. Smith and M. C. Giurcanu.
New Insecticides for Management of Tomato Yellow Leaf Curl, a Virus Vectored by the Silverleaf Whitefly, Bemisia tabaci.
Journal of Insect Science, 14(1):4–7, jan 2014.

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Objetive

Model optimal phytosanitary policies for diseases in farm crops using ODE, PDE, SDE.

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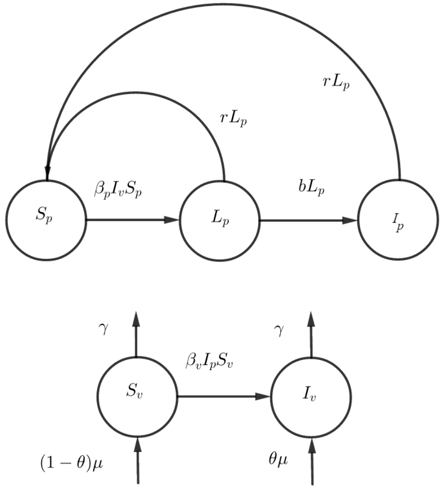
J. Holt, J. Colvin, and V. Muniyappa.
Identifying control strategies for tomato leaf curl virus disease using an epidemiological model.
Journal of Applied Ecology, 36(5):625–633, oct 1999.

Plant Model without control

Tomato Leaf Curl Virus Disease Using an Epidemiological Model

Hypothesis:

- Remove from latent and infected plants,
- plants become latent by infected vectors,
- latent plants become infectious plants,
- vectors become infected by infected plants,
- vectors die or depart per day,
- immigration from alternative hosts.



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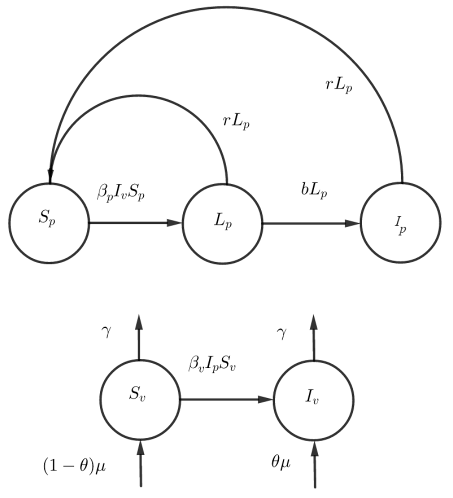

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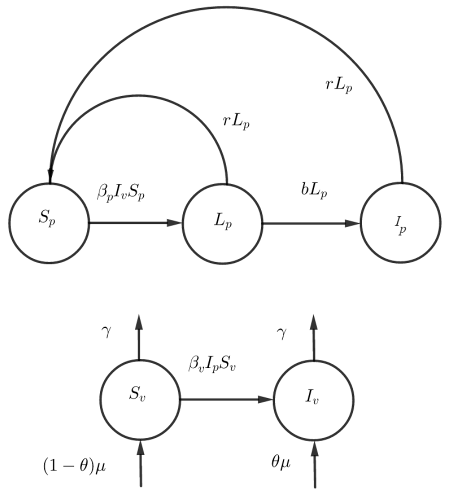

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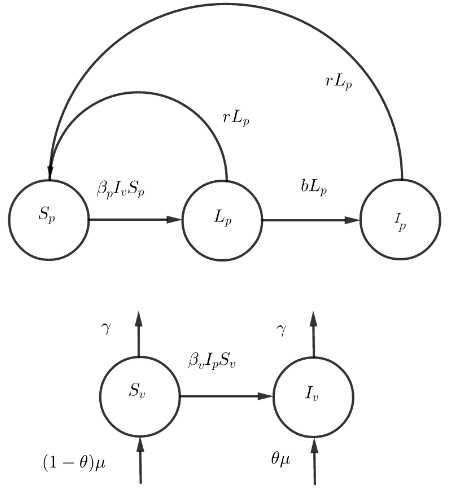

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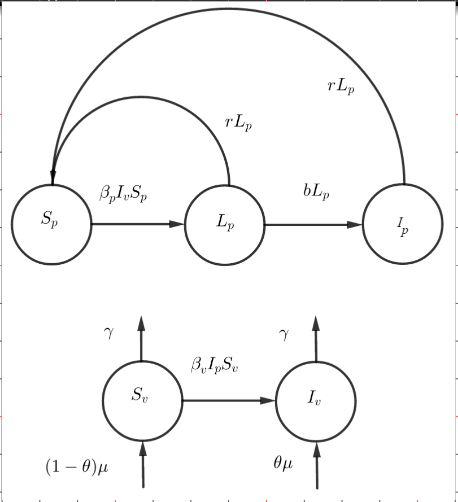

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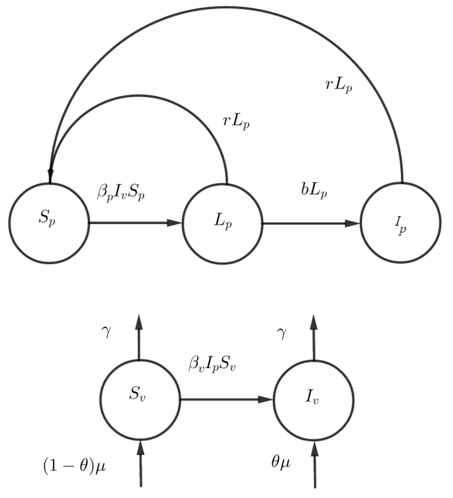

$$
\begin{aligned}
\frac{dS_p}{dt} &= -\beta_p S_p I_v + r(L_p + I_p),\\
\frac{dL_p}{dt} &= \beta_p S_p I_v - bL_p - rL_p,\\
\frac{dI_p}{dt} &= bL_p - rI_p,\\
\frac{dS_v}{dt} &= -\beta_v S_v I_p - \gamma S_v + (1-\theta)\mu,\\
\frac{dI_v}{dt} &= \beta_v S_v I_p - \gamma I_v + \theta\mu,\\
S_p(0) &= S_{p_0}, L_p(0) = L_{p_0}, I_p(0) = I_{p_0},\\
S_v(0) &= S_{v_0}, I_v(0) = I_{v_0}.
\end{aligned}
$$

Par.	Unit	Descrip.
β_p	vector^{-1}day^{-1}	plants became latently infected rate
r	day^{-1}	replanting rate
b	day^{-1}	latent class to the infectious class rate
γ	day^{-1}	vector die or depart rate
μ	plant^{-1}day^{-1}	immigration rate
θ	proportion	proportion of infected vector arrival to crop
β_v	plant^{-1}day^{-1}	vector infectious acquisition rate

80 100 120

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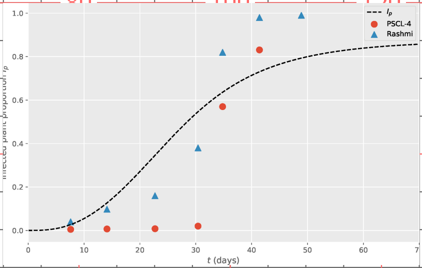

$$R_0 = \sqrt{\frac{\beta_v \mu b \beta_p}{r^2(r+b)\gamma}}.$$

60 80 100 120

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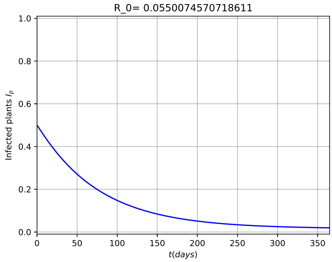
If $R_0 < 1$,

$$\lim_{t\to\infty}(S_p, L_p, I_p, S_v, I_v) = (N_p, 0, 0, \frac{\mu}{\gamma}, 0).$$



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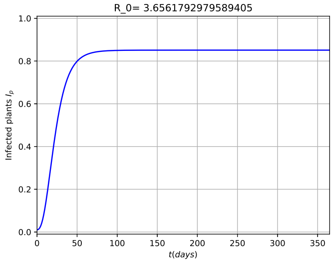
$$R_0 = \sqrt{\frac{\beta_v \mu b \beta_p}{r^2(r+b)\gamma}}.$$

If $R_0 > 1$,

$$\lim_{t\to\infty}(S_p, L_p, I_p, S_v, I_v) = (S_p^*, L_p^*, I_p^*, S_v^*, I_v^*).$$



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\frac{dS_v}{dt} &= -\beta_v S_v I_p - (\gamma + u_3)S_v + (1 - \theta)\mu,\\
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Controls:

- u_1: replanting latent plant,
- u_2: replanting infected plants,
- u_3: fumigation.

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\end{aligned}
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- u_1: replanting latent plant,
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Cost Functional

$$
\int_0^T (A_1 I_p(t) + A_2 L_p(t) + A_3 I_v(t) + c_1 u_1(t)^2 + c_2 u_2(t)^2 + c_3 u_3(t)^2)dt,
$$

OC

$$\min_{\bar{u}(\cdot)\in\tilde{\mathcal{U}}_{x_0}[t_0,T]} J(u_1, u_2, u_3)$$

s.t.

$$
\begin{aligned}
\frac{dS_p}{dt} &= -\beta_p S_p I_v + (r+u_1)L_p + (r+u_2)I_p,\\
\frac{dL_p}{dt} &= \beta_p S_p I_v - bL_p - (r+u_1)L_p,\\
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&u_i(t) \in [0, u_{i_{max}}]
\end{aligned}
$$

Existence Theory

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$$\begin{cases} \dot{x}(s) = f(s, u(s), x(s)) \; s \in [t_0, T], \\ x(t_0) = x_0, \end{cases}$$

with terminal state constraint

$$x(T; t_0, x_0, u(\cdot)) \in M, M \subseteq \mathbb{R}^n.$$

$$J(t_0, x_0; u(\cdot)) = \int_{t_0}^{T} g(s, u(s), x(s))ds + h(x(T)).$$

mm 40 60 80 100 120

Problem (OC)

$(t_0, x_0) \in \mathbb{R}_+ \times \mathbb{R}^n$ with $\tilde{\mathcal{U}}_{x_0}[t_0, T] \neq \emptyset$, find a $\bar{u}(\cdot) \in \tilde{\mathcal{U}}_{x_0}[t_0, T]$ s.t.

$$J(t_0, x_0; \bar{u}(\cdot)) = \inf_{u(\cdot) \in \tilde{\mathcal{U}}_{x_0}[t_0, T]} J(t_0, x_0; u(\cdot)).$$

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Hypothesis:

(C-1) $f : \mathbb{R}_+ \times U \times \mathbb{R}^n \to \mathbb{R}^n$ is measurable, satisfies a lipchitz condition in x, $|f(t,u,0)| \leq L$, $\forall\,(t,u) \in \mathbb{R}_+ \times U$.

(C-2) $g : \mathbb{R}_+ \times U \times \mathbb{R}^n \to \mathbb{R}$, $h : \mathbb{R}^n \to \mathbb{R}$ are measurable, and

$$|g(s,u,x_1) - g(s,u,x_2)| + |h(x_1) - h(x_2)| \leq \omega(|x_1| \vee |x_2|, |x_1 - x_2|)$$

$\forall\,(s,u) \in \mathbb{R}_+ \times U, x_1, x_2 \in \mathbb{R}^n$.

(C-3) For a.a. $t \in [0,T]$, Cesari property holds $\forall\, x \in \mathbb{R}^n$.

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$$|g(s,u,x_1) - g(s,u,x_2)| + |h(x_1) - h(x_2)| \leq \omega(|x_1| \vee |x_2|, |x_1 - x_2|)$$

$\forall\,(s,u) \in \mathbb{R}_+ \times U, x_1, x_2 \in \mathbb{R}^n$.

(C-3) For a.a. $t \in [0,T]$, Cesari property holds $\forall\, x \in \mathbb{R}^n$.

Modulus of continuity

$\omega : \mathbb{R}_+ \times \mathbb{R}_+ \to \mathbb{R}_+$, increasing, $\omega(r,0) = 0$ $\forall r \geq 0$.

Hypothesis:

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$\forall (s,u) \in \mathbb{R}_+ \times U, x_1, x_2 \in \mathbb{R}^n$.

(C-3) For a.a. $t \in [0,T]$, Cesari property holds $\forall\, x \in \mathbb{R}^n$.

$\bar{co}(E)$: closed convex hull of E,

$$\mathbb{E}(t,x) = \{(z^0, z) \in \mathbb{R} \times \mathbb{R}^n | z^0 \geq g(t,u,x), z = f(t,u,x),\ u \in U\}.$$

Cesari property

$$\bigcap_{\delta} \bar{co}\mathbb{E}(t, B_\delta(x)) = \mathbb{E}(t,x)$$

Hypothesis:

(**C**-1) $f : \mathbb{R}_+ \times U \times \mathbb{R}^n \to \mathbb{R}^n$ is measurable, satisfies a lipchitz condition in x, $|f(t, u, 0)| \leq L$, $\forall\, (t, u) \in \mathbb{R}_+ \times U$.

(**C**-2) $g : \mathbb{R}_+ \times U \times \mathbb{R}^n \to \mathbb{R}$, $h : \mathbb{R}^n \to \mathbb{R}$ are measurable, and

$$|g(s, u, x_1) - g(s, u, x_2)| + |h(x_1) - h(x_2)| \leq \omega(|x_1| \vee |x_2|, |x_1 - x_2|)$$

$\forall\, (s, u) \in \mathbb{R}_+ \times U, x_1, x_2 \in \mathbb{R}^n$.

(**C**-3) For a.a. $t \in [0, T]$, Cesari property holds $\forall\ x \in \mathbb{R}^n$.

Existence Theorem

Let (C1)-(C3) hold. Then problem (OC) admits at least one optimal pair.

mm 40 60 80 100 120

$$\begin{cases} \dot{x}(s) = f(s, u(s), x(s)) \; s \in [t_0, T], \\ x(t_0) = x_0, \end{cases}$$

$$J(t_0, x_0; u(\cdot)) = \int_{t_0}^{T} g(s, u(s), x(s)) ds + h(x(T)).$$

Hamiltonian:

$$H = g(t, x(t), u(t)) + \lambda(t) f(t, x(t), u(t)),$$

Optimallity Constraints:

$$\frac{\partial H}{\partial u_i} = 0.$$

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Pontryagin's Maximum Principle

If $u^*(t)$ and $x^*(t)$ are optimal for the problem (OC), then there exists a piecewise differentiable adjoint variable $\lambda(t)$ s.t.

$$H(t, x^*(t), u(t), \lambda(t)) \leq H(t, x^*(t), u^*(t), \lambda(t))$$

$\forall$ u at t,

$$\lambda'(t) = -\frac{\partial H(t, x^*(t), u^*(t), \lambda(t))}{\partial x},$$
$$\lambda(T) = 0.$$

mm 40 60 80 100 120

$$
\begin{aligned}
H = {} & A_1 I_v + A_2 L_p + A_3 I_v + \sum_{i=1}^{3} c_i u_i^2 \\
& + \lambda_1(-\beta_p S_p I_v + (r + u_1)L_p + (r + u_2)I_p) \\
& + \lambda_2(\beta_p S_p I_v - bL_p - (r + u_1)L_p) \\
& + \lambda_3(bL_p - (r + u_2)I_p) \\
& + \lambda_4(-\beta_v S_v I_p - (\gamma + u_3)S_v + (1 - \theta)\mu) \\
& + \lambda_5(\beta_v S_v I_p - (\gamma + u_3)I_v + \theta\mu).
\end{aligned}
$$

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$$
\begin{aligned}
\frac{d\lambda_1}{dt} &= \beta_p(\lambda_1 - \lambda_2), \\
\frac{d\lambda_2}{dt} &= -A_2 + (r + u_1)(\lambda_2 - \lambda_1) + b(\lambda_2 - \lambda_3), \\
\frac{d\lambda_3}{dt} &- A_1 + (r + u_2)(\lambda_3 - \lambda_1) + \beta_v S_v(\lambda_4 - \lambda_5), \\
\frac{d\lambda_4}{dt} &= \beta_v I_p(\lambda_4 - \lambda_5) + (\gamma + u_3)\lambda_4, \\
\frac{d\lambda_5}{dt} &= -A_3 + \beta_p S_p(\lambda_1 - \lambda_2) + (\gamma + u_3)\lambda_5,
\end{aligned}
$$

$$
u_1^* = \min\left(\max\left(0, \frac{L_p(\lambda_2 - \lambda_1)}{2c_1}\right), u_{1_{max}}\right)
$$

$$
u_2^* = \min\left(\max\left(0, \frac{I_p(\lambda_3 - \lambda_1)}{2c_2}\right), u_{2_{max}}\right)
$$

$$
u_3^* = \min\left(\max\left(0, \frac{S_v\lambda_4 + I_v\lambda_5}{2c_3}\right), u_{3_{max}}\right)
$$

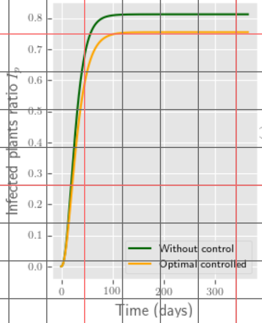
The most popular

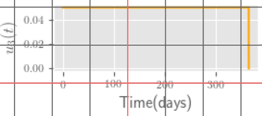
Algorithm 2 Forward Backward Sweep

Input: $t_0, t_f, x_0, h, \text{tol}, \lambda_f$
Output: x^*, u^*, λ

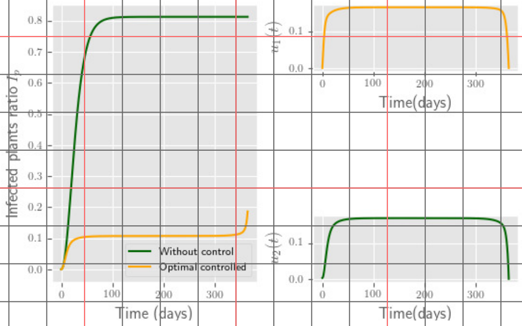
procedure FORWARD_BACKWARD_SWEEP$(g, \lambda_{\text{function}}, u, x_0, \lambda_f, h, n_{max})$
  while $\epsilon > \text{tol}$ **do**
    $u_{\text{old}} \leftarrow u$
    $x_{\text{old}} \leftarrow x$
    $x \leftarrow$ RUNGE_KUTTA_FORWARD(g, u, x_0, h)
    $\lambda_{\text{old}} \leftarrow \lambda$
    $\lambda \leftarrow$ RUNGE_KUTTA_BACKWARD$(\lambda_{\text{function}}, x, \lambda_f, h)$
    $u_1 \leftarrow$ OPTIMALITY_CONDITION(u, x, λ)
    $u \leftarrow \alpha u_1 + (1-\alpha) u_{old}, \qquad \alpha \in [0, 1]$ ▷ convex combination
    $\epsilon_u \leftarrow \dfrac{||u - u_{\text{old}}||}{||u||}$
    $\epsilon_x \leftarrow \dfrac{||x - x_{\text{old}}||}{||x||}$ ▷ relative error
    $\epsilon_\lambda \leftarrow \dfrac{||\lambda - \lambda_{\text{old}}||}{||\lambda||}$
    $\epsilon \leftarrow \max\{\epsilon_u, \epsilon_x, \epsilon_\lambda\}$
  end while
  return x^*, u^*, λ ▷ Optimal pair
end procedure

Case with one controls

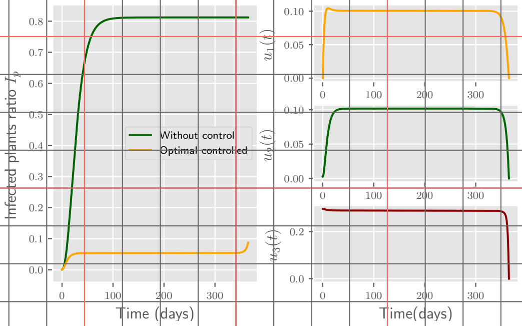




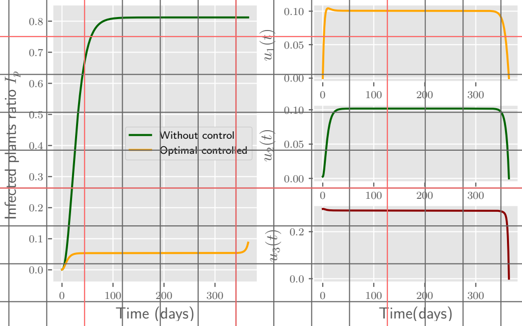
Case with two controls



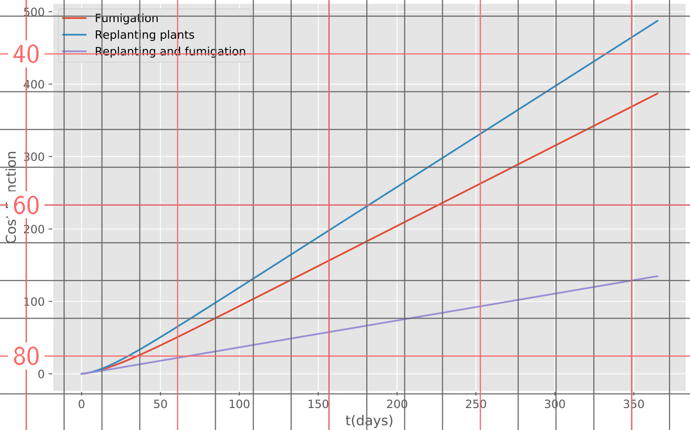
Case with three controls



Case with three controls



Cost Function Comparation



Perspective

$(\Omega, \mathscr{F}, \{\mathscr{F}_t\}_{t\geq 0}, \mathbb{P})$,
$W(t)$: m-dimensional Brownian motion.

$$\begin{aligned} dx(t) &= f(t, u(t), x(t))dt + \sigma(t, u(t), x(t))dW(t) \\ x(0) &= x_0, \end{aligned}$$

$f : [0, T] \times U \times \mathbb{R}^n \to \mathbb{R}^n, \sigma : [0, T] \times U \times \mathbb{R}^n \to \mathbb{R}^{n+m}$,

U : separable metric space.

60

$$\mathcal{U}[0, T] := \{u : [0, T] \times \Omega \to U | u(\cdot) \text{ is } \{\mathscr{F}_t\}_{t\geq 0} - adapted\}$$

80

120

mm 40 60 80 100 120

4

6

80

Definition

$(\Omega, \mathscr{F}, \{\mathscr{F}_t\}_{t\geq 0}, \mathbb{P})$, $W(t)$:m-dimensional standard $\{\mathscr{F}_t\}_{t\geq 0}$ Brownian motion, $u(\cdot)$ is and s-admissible control, $(u(\cdot), x(\cdot))$ is an s-admissible pair, if

- $u(\cdot) \in \mathcal{U}[0, T]$,
- $x(\cdot)$ is unique solution,
- some prescribed state constraints are satisfied,
- $g(\cdot, u(\cdot), x(\cdot)) \in L^1_{\mathscr{F}}(0, T; \mathbb{R})$ and $h(x(T)) \in L^1_{\mathscr{F}_T}(\Omega; \mathbb{R})$

mm 40 60 80 100 120

40

$$J(u(\cdot)) = \mathbb{E}\left\{ \int_0^T g(t, u(t), x(t))dt + h(x(T)) \right\}$$

6

8

$(OC)^s$

$$\min_{\mathcal{U}_{ad}^s[0,T]} J(u(\cdot))$$

s.t.

$$\begin{aligned} dx(t) &= f(t, u(t), x(t))dt + \sigma(t, u(t), x(t))dW(t) \\ x(0) &= x_0, \end{aligned}$$

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