Modeling of optimal phytosanitary policies in crops of economic importance in the state of Sonora.

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Contents

Tomato Leaf Curl Virus

mm 40 60 80 100 120

mm 40 60 80 100 120

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Objetive

Model optimal phytosanitary policies for diseases in agricultural crops.

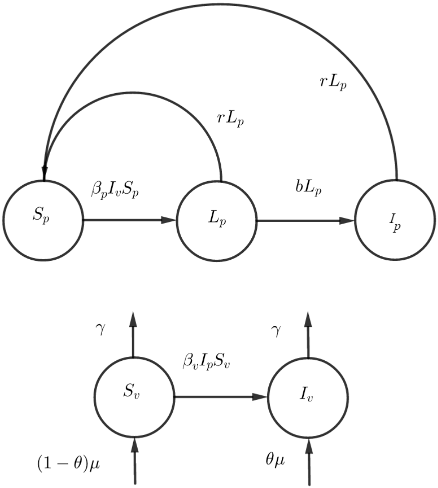
J. Holt, J. Colvin, and V. Muniyappa.
Identifying control strategies for tomato leaf curl virus disease using an epidemiological model.
Journal of Applied Ecology, 36(5):625–633, oct 1999.

Plant Model without control

Tomato Leaf Curl Virus Disease Using an Epidemiological Model

Hypothesis:

- Susceptible plants will become infected when an infected whitefly feeds on it, both latent and infected we will remove and re-plant susceptible plants
- Whiteflies become infected when they feed on an infected plant,
- there will be entry of whiteflies that come from alternative hosts.



Others Controls

- Cultural Control.
- Physical barriers.
- Planting dates.
- Removal of infested plants.
- Host plant resistance.
- Biological control.

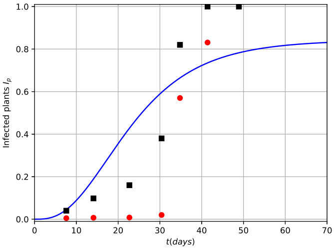
Consider the following ordinary differential equations:

$$
\begin{aligned}
\frac{dS_p}{dt} &= -\beta_p S_p I_v + r(L_p + I_p),\\
\frac{dL_p}{dt} &= \beta_p S_p I_v - bL_p - rL_p,\\
\frac{dI_p}{dt} &= bL_p - rI_p,\\
\frac{dS_v}{dt} &= -\beta_v S_v I_p - \gamma S_v - (1-\theta)\mu,\\
\frac{dI_v}{dt} &= \beta_v S_v I_p - \gamma I_v - \theta\mu,
\end{aligned}
$$

$$S_p(0) = S_{p0}, L_p(0) = L_{p0}, I_p(0) = I_{p0},$$

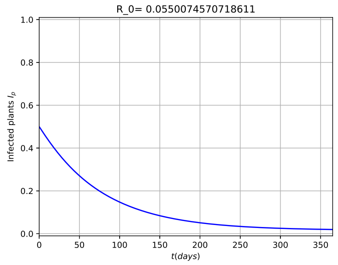
$$S_v(0) = S_{v0}, I_v(0) = I_{v0}.$$

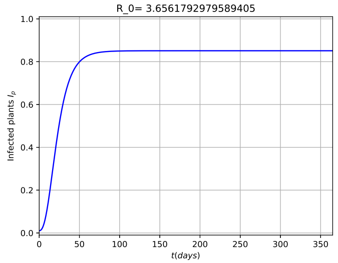
Par.	Value	Descrip.
β_p	0.1	plant latent rate
r	0.01	plant remove rate
b	0.075	plant infectious rate
γ	0.06	vector die or depar rate
μ	0.3	immigration rate
θ	0.2	infected vectors arrival
β_v	0.003	vector infected rate



Computing the R_0 we have,

$$R_0 = \sqrt{\frac{\beta_v \mu b \beta_p}{r^2(r+b)\gamma}}.$$





Plant Model with control

Tomato Leaf Curl Virus Disease Using an Epidemiological Model

The controlled system is the following:

$$\begin{aligned}
\frac{dS_p}{dt} &= -\beta_p S_p I_v + (r + u_1) L_p + (r + u_2) I_p, \\
\frac{dL_p}{dt} &= \beta_p S_p I_v - b L_p - (r + u_1) L_p, \\
\frac{dI_p}{dt} &= b L_p - (r + u_2) I_p, \\
\frac{dS_v}{dt} &= -\beta_v S_v I_p - (\gamma + u_3) S_v - (1 - \theta)\mu, \\
\frac{dI_v}{dt} &= \beta_v S_v I_p - (\gamma + u_3) I_v - \theta\mu,
\end{aligned}$$

With the cost functional:

$$\int_0^T \left[A_1 I_p(t) + A_2 L_p(t) + A_3 I_v(t) + c_1 u_1(t)^2 + c_2 u_2(t)^2 + c_3 u_3(t)^2 \right] dt.$$

Case with one controls

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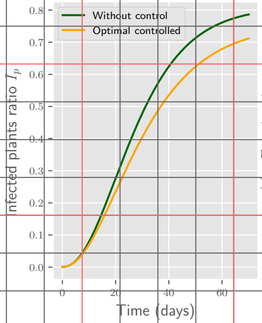
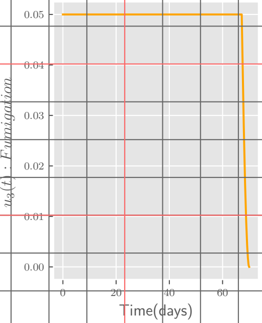
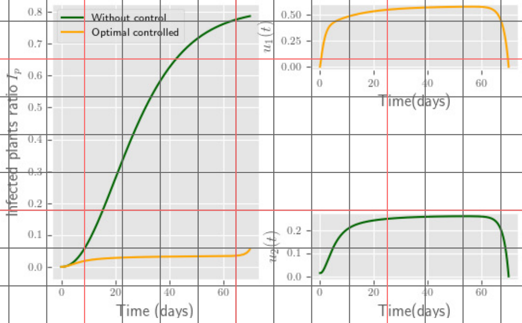
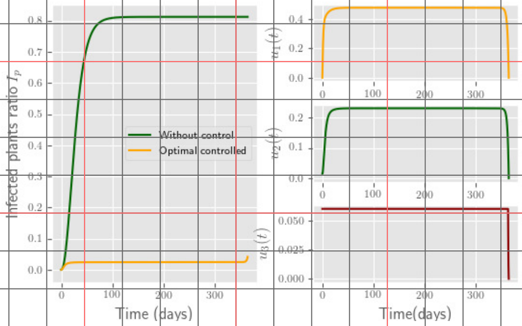




Case with two controls



Case with three controls



Let $T \in (0, \infty)$ be fixed. Consider the control system:

$$\begin{cases} \dot{X}(s) = f(s, u(s), X(s)) \ s \in [t, T], \\ X(t) = x, \end{cases}$$

with terminal state constraint

$$X(T; t, x, u(\cdot)) \in M,$$

where $M \subseteq \mathbb{R}^n$ is fixed.

Any map $M : \mathbb{R}_+ \to 2^{\mathbb{R}^n}$ is called a moving target in $\mathbb{R}^n$ if for any $t \in \mathbb{R}_+$, $M(t)$ is a measurable set in $\mathbb{R}^n$.

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Problem (C)

Let $M(\cdot)$ be a moving target set in $\mathbb{R}^n$. For given $(t,x) \in \mathbb{R}_+ \times \mathbb{R}^n$, find a control $u(\cdot) \in \mathcal{U}[t,\infty)$ such that for some $\tau \geq t$,

$$X(\tau; t, x, u(\cdot)) \in M(\tau).$$

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(C1)

The map $f : \mathbb{R}_+ \times U \times \mathbb{R}^n \to \mathbb{R}^n$ is measurable and there exists a constant $L > 0$ such that

$$\begin{cases} |f(t,u,x_1) - f(t,u,x_2)| \leq L|x_1 - x_2|, \; (t,u) \in \mathbb{R}_+ \times U, \; x_1, x_2 \in \mathbb{R}^n, \\ |f(t,u,0)| \leq L, \text{ for every } (t,u) \in \mathbb{R}_+ \times U. \end{cases}$$

(C2)

The maps $g : \mathbb{R}_+ \times U \times \mathbb{R}^n \to \mathbb{R}$ and $h.\mathbb{R}^n \to \mathbb{R}$ are measurable and there exists a continuous function $\omega : \mathbb{R}_+ \times \mathbb{R}_+ \to \mathbb{R}_+$, called a local modulus of continuity, which is increasing in each argument, and $\omega(r,0) = 0$ for every $r \geq 0$, such that

$$|g(s,u,x_1) - g(s,u,x_2)| + |h(x_1) - h(x_2)| \leq \omega(|x_1| \vee |x_2|, |x_1 - x_2|)$$

for every $(s,u) \in \mathbb{R}_+ \times U, x_1, x_2 \in \mathbb{R}^n$, where $|x_1| \vee |x_2| = \max\{|x_1|, |x_2|\}$.

For any $(t, x) \in [0, T] \times \mathbb{R}^n$, let us introduce the following set:

$$\mathbb{E}(t, x) = \{(z^0, z) \in \mathbb{R} \times \mathbb{R}_+ | z^0 \geq g(t, u, x), z = f(t, u, x), u \in U\}.$$

The following assumption gives some compatibility between the control system and the cost functional.

(C3)

For almost all $t \in [0, T]$, the following Cesari property holds for any $x \in \mathbb{R}^n$,

$$\bigcap_{\delta > 0} \bar{co}\mathbb{E}(t, B_\delta(x)) = \mathbb{E}(t, x),$$

where, we recall that $B_\delta(x)$ is the open ball centered at x with radius $\delta > 0$, and $\bar{co}(E)$ stands for the closed convex hull of the set E (the smallest closed convex set containing E).

Existence Theorem

Let (C1)-(C3) hold. Let $M \subseteq \mathbb{R}^n$ be a non-empty closed set. Let $(t, x) \in [0, T] \times \mathbb{R}^n$ be given and $\tilde{\mathcal{U}}_x^M[t, T] \neq \emptyset$. Then problem $(OC)^T$ admits at least one optimal pair.

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