Perfect Graph Recognition and Coloring

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Perfect Graphs are interesting

In all perfect graphs, the *graph coloring problem*, *maximum clique problem*, and *maximum independent set problem* can all be solved in polynomial time. (Grötschel, Lovász, Schrijver 1988)

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Families of graphs that are perfect

- Bipartite graphs
- Line graphs of bipartite graphs
- Chordal graphs
- Comparability graphs
- ...

Strong Perfect Graph Theorem

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Berge Graphs

A graph is *Berge* if no induced subgraph of G is an odd cycle of length at least five or the complement of one.

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Strong Perfect Graph Theorem

A graph G is Perfect if and only if it is Berge.

Recognizing Berge Graphs - an overview of an overview

An odd hole

An odd hole in G is an induced subgraph of G that is a cycle of odd length at least five.

A graph G is Berge if G and its complement both have no odd hole.

Algorithm

The idea of our algorithm is to decompose the input graph G into a polynomial number of simpler graphs $G_1, \ldots, G_m$ so that:

- G is odd-hole-free if and only if every G_i is odd-hole-free.
- G it is easy to check if G_i is odd-hole-free.

Decompositions

2-join

A graph G has a *2-join* $V_1|V_2$ with special sets (A_1, A_2, B_1, B_2) if $A_i, B_i \subset V_i$, every vertex of A_1 is adjacent to every vertex of A_2, every vertex of B_1 is adjacent to every vertex of B_2 and there are no other adjacencies between V_1 and V_2.

Double Star

A set S of vertices is a double star if S contains two adjacent vertices u and v such that $S \subseteq N(u) \cup N(v)$.

Double-star cutsets pose a problem.

Perfect Graph Colouring

Skew Partitions

A *skew partition* in G is a partition (A, B) of $V(G)$, such that $G[A]$ is not connected and $\overline{G}[B]$ is not connected.

Decomposition Theorem

Every Berge graph either admits a balanced skew partition, or admits one of two other decompositions, or it belongs to one of five well-understood classes.

Goals

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- Implement Berge Graph recognition algorithm.
  - Compare with existing *Java* implementation.
  - Are there any other implementations to compare to?
- Implement Perfect Graph colouring algorithm.
  - Compare with existing linear programming solutions.