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Technological level: Intermediate
Programming level: Junior +
Technologies used: Python 3.10 | PyCharm | Linux | OpenOffice

Description of the problem: The single-stage space rocket is propelled by liquid hydrogen and oxygen, while the propellants of the two two-stage rockets are LH-LOX (liquid oxygen and hydrogen) and kerosene, respectively. The masses of the rockets should be compared.
1. total mass the same for both rockets;
2. the specific impulse is the same for both rockets;
3. design factors the same for both rockets;
4. I Space velocity: $V_I = 7785\, m/s$
5. Loss of speed: $\Delta V_{loss} = 2056\, m/s$
6. Payload weight: $m_{pw} = 1285\, kg$
7. Construction factor: $\varepsilon = 0.08$

<u>Solving the task</u>:

Single-stage rocket$_{\text{LH2-LO2}}$
Exhaust gas velocity: $w = 4500\, m/s$
Total speed increase: $\Delta V = V_I + \Delta V_{loss}$

weight of construction: $m_w = m_{pw} \dfrac{(x-1)}{1+\dfrac{1}{\varepsilon}-x}$, $x = e^{(\Delta v/w)}$

fuel weight: $m_f = \dfrac{m_w}{\varepsilon}$

gross weight: $m_{g1} = m_w + m_f$

Two-stage rocket$_{\text{LH2-LO2}}$
Exhaust gas velocity: $w = 4500\, m/s$
Total speed increase: $\Delta V_I = \dfrac{\Delta V}{2}$, $\Delta V_{II} = \dfrac{\Delta V}{2}$

<u>Second stage</u>:

$$x_{II2} = e^{(\Delta v_{II}/w)} \quad , \quad m_{II2} = m_{pw} \quad , \quad m_{KII} = m_{II2} \frac{(x_{II2}-1)}{1+\frac{1}{\varepsilon}-x_{II2}} \quad , \quad m_{fII} = \frac{m_{KII}}{\varepsilon} \quad , \quad m_{gII} = m_{KII} + m_{fII}$$

<u>First stage</u>:

$$x_{I1} = e^{(\Delta v_I/w)} \quad , \quad m_{I1} = m_{pw} + m_{qII} \quad , \quad m_{KI} = m_{I1} \frac{(x_{I1}-1)}{1+\frac{1}{\varepsilon}-x_{I1}} \quad , \quad m_{fI} = \frac{m_{KI}}{\varepsilon} \quad , \quad m_{gI} = m_{KI} + m_{fI}$$

Two-stage rocket$_{\text{kerosene_LO2}}$
Exhaust gas velocity: $w = 3200\, m/s$
Total speed increase: $\Delta V_I = \dfrac{\Delta V}{2}$, $\Delta V_{II} = \dfrac{\Delta V}{2}$

Second stage:

$$x_{II2}=e^{(\Delta v_{II}/w)} \quad , \quad m_{II2}=m_{pw} \quad , \quad m_{KII}=m_{II2}\frac{(x_{II2}-1)}{1+\frac{1}{\varepsilon}-x_{II2}} \quad , \quad m_{fII}=\frac{m_{KII}}{\varepsilon} \quad , \quad m_{gII}=m_{KII}+m_{fII}$$

First stage:

$$x_{I1}=e^{(\Delta v_{I}/w)} \quad , \quad m_{I1}=m_{pw}+m_{qII} \quad , \quad m_{KI}=m_{I1}\frac{(x_{I1}-1)}{1+\frac{1}{\varepsilon}-x_{I1}} \quad , \quad m_{fI}=\frac{m_{KI}}{\varepsilon} \quad , \quad m_{gI}=m_{KI}+m_{fI}$$

Summary:

```
A = single_stage_rocket mass:  28912.5 [kg]
B = twoVER1_stage_rocket:  15917.645 [kg]
C = twoVER2_stage_rocket:  54097.812 [kg]
ratio A / B =  1.816
ratio C / B =  3.399
```

Conclusions:

1. A two-stage rocket has less mass for the same speed gain;
2. A rocket powered by liquid hydrogen and oxygen is lighter than the same rocket powered by kerosene and liquid oxygen

Code:

```
''' Adrian Szklarski, 08.06.2022

    The single-stage space rocket is propelled by liquid
    hydrogen and oxygen, while the propellants of the two
    two-stage rockets are LH-LOX (liquid oxygen and hydrogen)
    and kerosene, respectively. T
    he masses of the rockets should be compared. '''


class Missile:

    def __init__(self, vI, dv_loss, m_pw, epsilon):
        self.vI = vI
        self.dv_loss = dv_loss
        self.m_pw = m_pw
        self.epsilon = epsilon

    def single_stage_rocket(self):
        self.DV = self.vI + self.dv_loss
        self.mw = m_pw * ((((1 / 0.128) + 1) - 1) / (1 - ((1 / 0.128) + 1) + (1 / self.epsilon)))
        self.mf = self.mw / self.epsilon
        self.q1 = self.mw + self.mf
        return round(self.q1, 3)

    def twoVER1_stage_rocket(self):
        # Second stage
```

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        self.DV1 = (self.vI + self.dv_loss) / 2
        self.kII = m_pw * ((1.966 / (1 - 2.966 + (1 / self.epsilon))))
        self.mfII = self.kII / self.epsilon
        self.qII = self.kII + self.mfII
        # First stage
        self.kI = (m_pw + self.qII) * ((1.966 / (1 - 2.966 + (1 / self.epsilon))))
        self.mfI = self.kI / self.epsilon
        self.qI = self.kI + self.mfI
        # sumary mass
        self.ms = m_pw + self.qI + self.qII
        return round(self.ms, 3)

    def twoVER2_stage_rocket(self):
        # Second stage
        self.DV1 = (self.vI + self.dv_loss) / 2
        self.kIIk = m_pw * ((3.613 / (1 - 4.613 + (1 / self.epsilon))))
        self.mfIIk = self.kIIk / self.epsilon
        self.qIIk = self.kIIk + self.mfIIk
        # First stage
        self.kIk = (m_pw + self.qIIk) * ((3.613 / (1 - 4.613 + (1 / self.epsilon))))
        self.mfIk = self.kIk / self.epsilon
        self.qIk = self.kIk + self.mfIk
        # sumary mass
        self.msk = m_pw + self.qIk + self.qIIk
        return round(self.msk, 3)

    def __str__(self):
        return f'{self.q1}, {self.ms}, {self.msk}'


if __name__ == '__main__':
    vI = 7785  # m/s
    dv_loss = 2056  # m/s
    m_pw = 1285  # kg
    epsilon = 0.08

    M = Missile(vI, dv_loss, m_pw, epsilon)
    print(' A = single_stage_rocket mass: ', M.single_stage_rocket(), '[kg]\n',
          'B = twoVER1_stage_rocket: ', M.twoVER1_stage_rocket(), '[kg]\n',
          'C = twoVER2_stage_rocket: ', M.twoVER2_stage_rocket(), '[kg]\n',
          'ratio A / B = ', round(M.single_stage_rocket() / M.twoVER1_stage_rocket(), 3), '\n',
          'ratio C / B = ', round(M.twoVER2_stage_rocket() / M.twoVER1_stage_rocket(), 3))
```