

$$\propto \prod_{i=1}^I \prod_{j=2}^{N(t_i)} \frac{(d_{ij-1} + \beta_0)^{\alpha_d + \alpha_\theta}}{\Gamma(\alpha_d + \alpha_\theta)} \theta_{ij}^{\alpha_d} \alpha_d^{\alpha_\theta - 1} e^{-\alpha_d \beta_0}$$

$$\propto \left(\frac{\alpha_d^{\alpha_\theta - 1} e^{-\alpha_d \beta_0}}{\Gamma(\alpha_d + \alpha_\theta)} \right)^{\sum_{i=1}^I N(t_i)} \prod_{i=1}^I \prod_{j=2}^{N(t_i)} (d_{ij-1} + \beta_0)^{\alpha_d + \alpha_\theta} \theta_{ij}^{\alpha_d}$$

$$\Rightarrow \pi(\alpha_\theta | (\theta_{ij})_{i=1, j=1}^{I, N(t_i)}) \propto \prod_{i=1}^I \prod_{j=2}^{N(t_i)} \pi(\alpha_\theta | (\theta_{ij})_{i=1, j=1}^{I, N(t_i)})$$

$$\propto \prod_{i=1}^I \prod_{j=2}^{N(t_i)} \frac{(d_{ij-1} + \beta_0)^{\alpha_d + \alpha_\theta}}{\Gamma(\alpha_d + \alpha_\theta)} \theta_{ij}^{\alpha_\theta} \alpha_\theta^{\alpha_\theta - 1} e^{-\alpha_\theta \beta_0}$$

$$\propto \left(\frac{\alpha_\theta^{\alpha_\theta - 1} e^{-\alpha_\theta \beta_0}}{\Gamma(\alpha_d + \alpha_\theta)} \right)^{\sum_{i=1}^I N(t_i)} \prod_{i=1}^I \prod_{j=2}^{N(t_i)} (d_{ij-1} + \beta_0)^{\alpha_d + \alpha_\theta} \theta_{ij}^{\alpha_\theta}$$

$$\pi(\beta_0 | (\theta_{ij})_{i=1, j=1}^{I, N(t_i)}) \propto \prod_{i=1}^I \prod_{j=2}^{N(t_i)} \pi(\beta_0 | (\theta_{ij})_{i=1, j=1}^{I, N(t_i)})$$

$$\propto \prod_{i=1}^I \prod_{j=2}^{N(t_i)} (d_{ij-1} + \beta_0)^{\alpha_d + \alpha_\theta} \beta_0^{\alpha_\theta - 1} e^{-\beta_0 (\theta_{ij} + \beta_0)}$$

$$\propto (\beta_0^{\alpha_\theta - 1})^{\sum_{i=1}^I N(t_i)} e^{-\beta_0 (\sum_{i=1}^I \sum_{j=1}^{N(t_i)} (\theta_{ij} + \beta_0))} \prod_{i=1}^I \prod_{j=2}^{N(t_i)} (d_{ij-1} + \beta_0)^{\alpha_d + \alpha_\theta}$$

$$\Rightarrow \pi(\alpha_x | (\chi_{ij})_{i=1, j=1}^{I, N(t_i)}) \propto \prod_{i=1}^I \prod_{j=2}^{N(t_i)} \pi(\alpha_x | (\chi_{ij})_{i=1, j=1}^{I, N(t_i)})$$

$$\propto \prod_{i=1}^I \prod_{j=2}^{N(t_i)} \left(\frac{1}{\chi_{ij}} \right)^{\alpha_x} \alpha_x^{\alpha_x - 1} e^{-\alpha_x \beta_0}$$

$$\propto (\alpha_x^{\alpha_x - 1} e^{-\alpha_x \beta_0})^{\sum_{i=1}^I N(t_i)} \prod_{i=1}^I \prod_{j=1}^{N(t_i)} \left(\frac{1}{\chi_{ij}} \right)^{\alpha_x}$$

$$\Rightarrow \pi(\beta_x | (\chi_{ij})_{i=1, j=1}^{I, N(t_i)}) \propto \prod_{i=1}^I \prod_{j=1}^{N(t_i)} \pi(\beta_x | (\chi_{ij})_{i=1, j=1}^{I, N(t_i)})$$

$$\propto \prod_{i=1}^I \prod_{j=1}^{N(t_i)} \beta_x^{\alpha_x - 1} e^{-\beta_x (\frac{1}{\chi_{ij}} + \beta_0)}$$

$$\propto (\beta_x^{\alpha_x - 1})^{\sum_{i=1}^I N(t_i)} e^{-\beta_x (\sum_{i=1}^I \sum_{j=1}^{N(t_i)} (\frac{1}{\chi_{ij}} + \beta_0))}$$

Log

$$\begin{aligned}
 \textcircled{1} \ln(\pi(\alpha_d | (\theta_{ij})_{i=1, j=1}^{\sum N(t_i)})) &= \ln\left(\left(\frac{\alpha_d^{\alpha_d-1} e^{-\alpha_d \beta_0}}{\Gamma(\alpha_d + \alpha_0)}\right)^{\sum N(t_i)} \prod_{i=1}^{\sum N(t_i)} \prod_{j=2}^{\sum N(t_i)} (d_{ij-1} + \beta_0)^{\alpha_d + \alpha_0} \theta_{ij}^{\alpha_d}\right) \\
 &= \sum_{i=1}^{\sum N(t_i)} \left[\ln(\alpha_d^{\alpha_d-1} e^{-\alpha_d \beta_0}) - \ln(\Gamma(\alpha_d + \alpha_0)) \right] + \ln\left(\prod_{i=1}^{\sum N(t_i)} \prod_{j=2}^{\sum N(t_i)} (d_{ij-1} + \beta_0)^{\alpha_d + \alpha_0} \theta_{ij}^{\alpha_d}\right) \\
 &= \sum_{i=1}^{\sum N(t_i)} \left[(\alpha_d - 1) \ln(\alpha_d) - \alpha_d \beta_0 - \ln(\Gamma(\alpha_d + \alpha_0)) \right] + \sum_{i=1}^{\sum N(t_i)} \sum_{j=2}^{\sum N(t_i)} \left[(\alpha_d + \alpha_0) \ln(d_{ij-1} + \beta_0) + \alpha_d \ln(\theta_{ij}) \right]
 \end{aligned}$$

$$\begin{aligned}
 \textcircled{2} \ln(\pi(\alpha_0 | (\theta_{ij})_{i=1, j=1}^{\sum N(t_i)})) &= \ln\left(\left(\frac{\alpha_0^{\alpha_0-1} e^{-\alpha_0 \beta_0}}{\Gamma(\alpha_d + \alpha_0)}\right)^{\sum N(t_i)} \prod_{i=1}^{\sum N(t_i)} \prod_{j=2}^{\sum N(t_i)} (d_{ij-1} + \beta_0)^{\alpha_d + \alpha_0} \theta_{ij}^{\alpha_0}\right) \\
 &= \sum_{i=1}^{\sum N(t_i)} \left[(\alpha_0 - 1) \ln(\alpha_0) - \alpha_0 \beta_0 - \ln(\Gamma(\alpha_d + \alpha_0)) \right] + \sum_{i=1}^{\sum N(t_i)} \sum_{j=2}^{\sum N(t_i)} \left[(\alpha_d + \alpha_0) \ln(d_{ij-1} + \beta_0) + \alpha_0 \ln(\theta_{ij}) \right]
 \end{aligned}$$

$$\begin{aligned}
 \textcircled{3} \ln(\pi(\beta_0 | (\theta_{ij})_{i=1, j=1}^{\sum N(t_i)})) &= \ln\left((\beta_0^{\alpha_0-1})^{\sum N(t_i)} e^{-\beta_0 (\sum_{i=1}^{\sum N(t_i)} \sum_{j=1}^{\sum N(t_i)} (\theta_{ij} + \beta_0))} \prod_{i=1}^{\sum N(t_i)} \prod_{j=2}^{\sum N(t_i)} (d_{ij-1} + \beta_0)^{\alpha_d + \alpha_0}\right) \\
 &= \left[\sum_{i=1}^{\sum N(t_i)} (\alpha_0 - 1) \ln(\beta_0) \right] - \beta_0 \left(\sum_{i=1}^{\sum N(t_i)} \sum_{j=1}^{\sum N(t_i)} (\theta_{ij} + \beta_0) \right) + \sum_{i=1}^{\sum N(t_i)} \sum_{j=2}^{\sum N(t_i)} \left[(\alpha_d + \alpha_0) \ln(d_{ij-1} + \beta_0) \right]
 \end{aligned}$$

$$\begin{aligned}
 \textcircled{4} \ln(\pi(\alpha_x | (\gamma_{ij})_{i=1, j=1}^{\sum N(t_i)})) &= \ln\left((\alpha_x^{\alpha_x-1} e^{-\alpha_x \beta_0})^{\sum N(t_i)} \prod_{i=1}^{\sum N(t_i)} \prod_{j=1}^{\sum N(t_i)} \left(\frac{1}{\gamma_{ij}}\right)^{\alpha_x}\right) \\
 &= \sum_{i=1}^{\sum N(t_i)} \left[\ln(\alpha_x^{\alpha_x-1} e^{-\alpha_x \beta_0}) \right] + \ln\left(\prod_{i=1}^{\sum N(t_i)} \prod_{j=1}^{\sum N(t_i)} \left(\frac{1}{\gamma_{ij}}\right)^{\alpha_x}\right) \\
 &= \sum_{i=1}^{\sum N(t_i)} \left[(\alpha_x - 1) \ln(\alpha_x) - \alpha_x \beta_0 \right] + \sum_{i=1}^{\sum N(t_i)} \sum_{j=1}^{\sum N(t_i)} \left(\alpha_x \cdot -\ln(\gamma_{ij}) \right)
 \end{aligned}$$

$$\begin{aligned}
 \textcircled{5} \ln(\pi(\beta_x | (\gamma_{ij})_{i=1, j=1}^{\sum N(t_i)})) &= \ln\left((\beta_x^{\alpha_x-1})^{\sum N(t_i)} e^{-\beta_x (\sum_{i=1}^{\sum N(t_i)} \sum_{j=1}^{\sum N(t_i)} (\frac{1}{\gamma_{ij}} + \beta_0))}\right) \\
 &= \sum_{i=1}^{\sum N(t_i)} \left[(\alpha_x - 1) \ln(\beta_x) - \beta_x \left(\sum_{i=1}^{\sum N(t_i)} \sum_{j=1}^{\sum N(t_i)} \left(\frac{1}{\gamma_{ij}} + \beta_0\right) \right) \right]
 \end{aligned}$$