

$$\pi(\alpha_d | \alpha_\theta, \beta_\theta, d_{j-1}, \theta_{ij}, \alpha_{01}, \beta_0) \propto \prod_{i=1}^I \prod_{j=2}^{n_i} G(\theta_{ij} | \alpha_d + \alpha_\theta, d_{ij-1} + \beta_0) \times \text{Gamma}(\alpha_d | \alpha_0, \beta_0) \\ \times \prod_{i=1}^I G(\theta_{i1} | \alpha_d + \alpha_\theta, d_{i1} + \beta_0) \times G(\alpha_d | \alpha_0, \beta_0) \\ \propto \prod_{i=1}^I \prod_{j=2}^{n_i} \frac{(d_{ij} + d_{ij-1} + \beta_0)^{\alpha_d + \alpha_\theta}}{\Gamma(\alpha_d + \alpha_\theta)} \theta_{ij}^{\alpha_d + \alpha_\theta - 1} e^{-\theta_{ij}(d_{ij-1} + \beta_0)} \\ \times \frac{(d_{i1} + \beta_0)^{\alpha_d + \alpha_\theta}}{\Gamma(\alpha_d + \alpha_\theta)} \theta_{i1}^{\alpha_d + \alpha_\theta - 1} e^{-\theta_{i1}(d_{i1} + \beta_0)} \times \frac{\beta_0^{\alpha_0}}{\Gamma(\alpha_0)} \alpha_d^{\alpha_0 - 1} e^{-\alpha_d \beta_0}$$

$$\pi(\alpha_d | (\theta_{ij})_{i=1, j=1}^{n_i-1}) \propto \left[\prod_{i=1}^I \prod_{j=2}^{n_i} G(\theta_{ij} | 2\alpha_d + \alpha_\theta, d_{ij} + d_{ij-1} + \beta_0) \times \prod_{i=1}^I G(\theta_{i1} | \alpha_d + \alpha_\theta, d_{i1} + \beta_0) \right] \times G(\alpha_d | \alpha_0, \beta_0)$$

$$\left[\prod_{i=1}^I \prod_{j=2}^{n_i} \frac{(d_{ij} + d_{ij-1} + \beta_0)^{2\alpha_d + \alpha_\theta}}{\Gamma(2\alpha_d + \alpha_\theta)} \theta_{ij}^{2\alpha_d + \alpha_\theta - 1} e^{-\theta_{ij}(d_{ij} + d_{ij-1} + \beta_0)} \times \frac{(d_{i1} + \beta_0)^{\alpha_d + \alpha_\theta}}{\Gamma(\alpha_d + \alpha_\theta)} \theta_{i1}^{\alpha_d + \alpha_\theta - 1} e^{-\theta_{i1}(d_{i1} + \beta_0)} \right]$$

$$\times \frac{\beta_0^{\alpha_0}}{\Gamma(\alpha_0)} \alpha_d^{\alpha_0 - 1} e^{-\alpha_d \beta_0}$$

$$\propto \left[\prod_{i=1}^I \prod_{j=2}^{n_i} \frac{(d_{ij} + d_{ij-1} + \beta_0)^{2\alpha_d + \alpha_\theta}}{\Gamma(2\alpha_d + \alpha_\theta)} \frac{(d_{i1} + \beta_0)^{\alpha_d + \alpha_\theta}}{\Gamma(\alpha_d + \alpha_\theta)} \theta_{ij}^{2\alpha_d + \alpha_\theta - 1} \theta_{i1}^{\alpha_d + \alpha_\theta - 1} \right] \alpha_d^{\alpha_0 - 1} e^{-\alpha_d \beta_0}$$

$$\propto \left(\frac{1}{\Gamma(\alpha_d + \alpha_\theta)} \right)^I \left(\frac{1}{\Gamma(2\alpha_d + \alpha_\theta)} \right)^{\sum n_i} \alpha_d^{\alpha_0 - 1} e^{-\alpha_d \beta_0} \left[\left(\prod_{i=1}^I (d_{i1} + \beta_0)^{\alpha_d + \alpha_\theta} \theta_{i1}^{\alpha_d + \alpha_\theta - 1} \right) \prod_{j=2}^{n_i} (d_{ij} + d_{ij-1} + \beta_0)^{2\alpha_d + \alpha_\theta} \theta_{ij}^{2\alpha_d + \alpha_\theta - 1} \right]$$

$$\pi(\alpha_\theta | (\theta_{ij})_{i=1, j=1}^{n_i-1}) \propto \left[\prod_{i=1}^I \prod_{j=2}^{n_i} G(\theta_{ij} | 2\alpha_d + \alpha_\theta, d_{ij} + d_{ij-1} + \beta_0) \times \prod_{i=1}^I G(\theta_{i1} | \alpha_d + \alpha_\theta, d_{i1} + \beta_0) \right] \times G(\alpha_\theta | \alpha_0, \beta_0)$$

$$= \left(\prod_{i=1}^I \frac{(d_{i1} + \beta_0)^{\alpha_d + \alpha_\theta}}{\Gamma(\alpha_d + \alpha_\theta)} \theta_{i1}^{\alpha_d + \alpha_\theta - 1} e^{-\theta_{i1}(d_{i1} + \beta_0)} \right) \prod_{j=2}^{n_i} \frac{(d_{ij} + d_{ij-1} + \beta_0)^{2\alpha_d + \alpha_\theta}}{\Gamma(2\alpha_d + \alpha_\theta)} \theta_{ij}^{2\alpha_d + \alpha_\theta - 1} e^{-\theta_{ij}(d_{ij} + d_{ij-1} + \beta_0)} \times \frac{\beta_0^{\alpha_0}}{\Gamma(\alpha_0)} \alpha_\theta^{\alpha_0 - 1} e^{-\alpha_\theta \beta_0}$$

$$\propto \left(\prod_{i=1}^I \frac{(d_{i1} + \beta_0)^{\alpha_d + \alpha_\theta}}{\Gamma(\alpha_d + \alpha_\theta)} \theta_{i1}^{\alpha_d + \alpha_\theta - 1} \right) \prod_{j=2}^{n_i} \frac{(d_{ij} + d_{ij-1} + \beta_0)^{2\alpha_d + \alpha_\theta}}{\Gamma(2\alpha_d + \alpha_\theta)} \theta_{ij}^{2\alpha_d + \alpha_\theta - 1} \alpha_\theta^{\alpha_0 - 1} e^{-\alpha_\theta \beta_0}$$

$$= \left(\frac{1}{\Gamma(\alpha_d + \alpha_\theta)} \right)^I \left(\frac{1}{\Gamma(2\alpha_d + \alpha_\theta)} \right)^{\sum n_i} \alpha_\theta^{\alpha_0 - 1} e^{-\alpha_\theta \beta_0} \left[\left(\prod_{i=1}^I (d_{i1} + \beta_0)^{\alpha_d + \alpha_\theta} \theta_{i1}^{\alpha_d + \alpha_\theta - 1} \right) \prod_{j=2}^{n_i} (d_{ij} + d_{ij-1} + \beta_0)^{2\alpha_d + \alpha_\theta} \theta_{ij}^{2\alpha_d + \alpha_\theta - 1} \right] \alpha_\theta^{\alpha_0 - 1} e^{-\alpha_\theta \beta_0}$$

$$\pi(\beta_0 | (\theta_{ij})_{i=1}^I, j=1}^{n_i}) \propto \left[\prod_{i=1}^I \Gamma(\theta_{i1} | \alpha_d + \alpha_0, d_{i1} + \beta_0) \prod_{j=2}^{n_i} \Gamma(\theta_{ij} | 2\alpha_d + \alpha_0, d_{ij} + d_{ij-1} + \beta_0) \right] \times \Gamma(\beta_0 | \alpha_0, \beta_0)$$

$$= \left[\prod_{i=1}^I \frac{(d_{i1} + \beta_0)^{\alpha_d + \alpha_0}}{\Gamma(\alpha_d + \alpha_0)} \theta_{i1}^{\alpha_d + \alpha_0 - 1} e^{-\theta_{i1}(d_{i1} + \beta_0)} \prod_{j=2}^{n_i} \frac{(d_{ij} + d_{ij-1} + \beta_0)^{2\alpha_d + \alpha_0}}{\Gamma(2\alpha_d + \alpha_0)} \theta_{ij}^{2\alpha_d + \alpha_0 - 1} e^{-\theta_{ij}(d_{ij} + d_{ij-1} + \beta_0)} \right] \frac{\beta_0^{\alpha_0}}{\Gamma(\alpha_0)} \beta_0^{\alpha_0 - 1} e^{-\beta_0 \beta_0}$$

$$\propto \left[\prod_{i=1}^I (d_{i1} + \beta_0)^{\alpha_d + \alpha_0} e^{-\theta_{i1} \beta_0} \prod_{j=2}^{n_i} (d_{ij} + d_{ij-1} + \beta_0)^{2\alpha_d + \alpha_0} e^{-\theta_{ij} \beta_0} \right] \beta_0^{\alpha_0 - 1} e^{-\beta_0 \beta_0} = \prod_{i=1}^I \prod_{j=1}^{n_i} (d_{ij} + \beta_0)^{\alpha_d + \alpha_0} e^{-\theta_{ij} \beta_0}$$

$$\pi(\alpha_\gamma | (\gamma_{ij})_{i=1}^I, j=1}^{n_i}) \propto \left[\prod_{i=1}^I \left(\frac{1}{\gamma_{i1}} \right)^{d_{i1} + \alpha_\gamma + 1} e^{-\left(\frac{\beta_\gamma}{\gamma_{i1}} + \left(\frac{C_{i1}}{\gamma_{i1}} \right)^{d_{i1}} \right)} \prod_{j=2}^{n_i} \left(\frac{1}{\gamma_{ij}} \right)^{2d_{ij} + \alpha_\gamma + 1} e^{-\left(\left(\frac{\beta_\gamma}{\gamma_{ij}} + \left(\frac{C_{ij-1} + C_{ij}}{\gamma_{ij}} \right)^{d_{ij}} \right)} \right]$$

$$\times \Gamma(\alpha_\gamma | \alpha_0, \beta_0)$$

$$= \left[\prod_{i=1}^I \left(\frac{1}{\gamma_{i1}} \right)^{d_{i1}} \left(\frac{1}{\gamma_{i1}} \right)^{\alpha_\gamma} \left(\frac{1}{\gamma_{i1}} \right) e^{-\left(\frac{\beta_\gamma}{\gamma_{i1}} \right)} e^{-\left(\frac{C_{i1}}{\gamma_{i1}} \right)^{d_{i1}}} \prod_{j=2}^{n_i} \left(\frac{1}{\gamma_{ij}} \right)^{2d_{ij}} \left(\frac{1}{\gamma_{ij}} \right)^{\alpha_\gamma} \left(\frac{1}{\gamma_{ij}} \right) e^{-\left(\frac{\beta_\gamma}{\gamma_{ij}} \right)} e^{-\left(\frac{C_{ij-1} + C_{ij}}{\gamma_{ij}} \right)^{d_{ij}}} \right] \frac{\beta_0^{\alpha_0}}{\Gamma(\alpha_0)} \alpha_\gamma^{\alpha_0 - 1} e^{-\alpha_\gamma \beta_0}$$

$$\left[\prod_{i=1}^I \left(\frac{1}{\gamma_{i1}} \right)^{\alpha_\gamma} \prod_{j=2}^{n_i} \left(\frac{1}{\gamma_{ij}} \right)^{\alpha_\gamma} \right] \alpha_\gamma^{\alpha_0 - 1} e^{-\alpha_\gamma \beta_0} = \left[\prod_{i=1}^I \prod_{j=1}^{n_i} \left(\frac{1}{\gamma_{ij}} \right)^{\alpha_\gamma} \right] \alpha_\gamma^{\alpha_0 - 1} e^{-\alpha_\gamma \beta_0}$$

$$\pi(\beta_\gamma | (\gamma_{ij})_{i=1}^I, j=1}^{n_i}) \propto \left[\prod_{i=1}^I \left(\frac{1}{\gamma_{i1}} \right)^{d_{i1} + \alpha_\gamma + 1} e^{-\left(\left(\frac{\beta_\gamma}{\gamma_{i1}} \right) + \left(\frac{C_{i1}}{\gamma_{i1}} \right)^{d_{i1}} \right)} \prod_{j=2}^{n_i} \left(\frac{1}{\gamma_{ij}} \right)^{2d_{ij} + \alpha_\gamma + 1} e^{-\left(\left(\frac{\beta_\gamma}{\gamma_{ij}} \right) + \left(\frac{C_{ij-1} + C_{ij}}{\gamma_{ij}} \right)^{d_{ij}} \right)} \right]$$

$$\times \Gamma(\beta_\gamma | \alpha_0, \beta_0)$$

$$= \left[\prod_{i=1}^I \left(\frac{1}{\gamma_{i1}} \right)^{d_{i1} + \alpha_\gamma + 1} e^{-\left(\frac{\beta_\gamma}{\gamma_{i1}} \right)} e^{-\left(\frac{C_{i1}}{\gamma_{i1}} \right)^{d_{i1}}} \prod_{j=2}^{n_i} \left(\frac{1}{\gamma_{ij}} \right)^{2d_{ij} + \alpha_\gamma + 1} e^{-\left(\frac{\beta_\gamma}{\gamma_{ij}} \right)} e^{-\left(\frac{C_{ij-1} + C_{ij}}{\gamma_{ij}} \right)^{d_{ij}}} \right] \frac{\beta_0^{\alpha_0}}{\Gamma(\alpha_0)} \beta_\gamma^{\alpha_0 - 1} e^{-\beta_\gamma \beta_0}$$

$$\propto \left[\prod_{i=1}^I e^{-\left(\frac{\beta_\gamma}{\gamma_{i1}} \right)} \times \prod_{j=2}^{n_i} e^{-\left(\frac{\beta_\gamma}{\gamma_{ij}} \right)} \right] \beta_\gamma^{\alpha_0 - 1} e^{-\beta_\gamma \beta_0} = e^{-\sum_{i=1}^I \sum_{j=1}^{n_i} \left(\frac{\beta_\gamma}{\gamma_{ij}} \right)} \beta_\gamma^{\alpha_0 - 1} e^{-\beta_\gamma \beta_0} = \beta_\gamma^{\alpha_0 - 1} e^{-\beta_\gamma (\beta_0 + \sum_{i=1}^I \sum_{j=1}^{n_i} \left(\frac{1}{\gamma_{ij}} \right))}$$

$$\alpha \beta_\gamma \sim \text{Gamma}(\alpha_0, \beta_0 + \sum_{i=1}^I \sum_{j=1}^{n_i} \frac{1}{\gamma_{ij}})$$

$$\bullet \ln(\pi(\alpha_d | (\theta_{ij})_{i=1, j=1}^{\pm n_i}))$$

$$\ln\left(\left(\frac{1}{\Gamma(\alpha_d + \alpha_\theta)}\right)^I \left(\frac{1}{\Gamma(2\alpha_d + \alpha_\theta)}\right)^{\sum_{i=1}^{\pm} n_i} \alpha_d^{\alpha_\theta - 1} e^{-\alpha_d \beta_\theta} \left[\prod_{i=1}^I (d_{i1} + \beta_\theta)^{\alpha_d + \alpha_\theta} \theta_{i1}^{(\alpha_d + \alpha_\theta)} \prod_{j=2}^{n_i} (d_{ij} + d_{ij-1} + \beta_\theta)^{2\alpha_d + \alpha_\theta} \theta_{ij}^{2\alpha_d + \alpha_\theta}\right]\right)$$

$$= -I \ln(\Gamma(\alpha_d + \alpha_\theta)) - \sum_{i=1}^{\pm} n_i \ln(\Gamma(2\alpha_d + \alpha_\theta)) + (\alpha_\theta - 1) \ln(\alpha_d) - \alpha_d \beta_\theta + \left(\sum_{i=1}^I (\alpha_d + \alpha_\theta) \ln(d_{i1} + \beta_\theta) + (\alpha_d + \alpha_\theta) \ln(\theta_{i1}) + \sum_{i=1}^{\pm} \sum_{j=2}^{n_i} (2\alpha_d + \alpha_\theta) \ln(d_{ij} + d_{ij-1} + \beta_\theta) + (2\alpha_d + \alpha_\theta - 1) \ln(\theta_{ij})\right)$$

$$\bullet \ln(\pi(\alpha_\theta | (\theta_{ij})_{i=1, j=1}^{\pm n_i}))$$

$$\ln\left(\left(\frac{1}{\Gamma(\alpha_d + \alpha_\theta)}\right)^I \left(\frac{1}{\Gamma(2\alpha_d + \alpha_\theta)}\right)^{\sum_{i=1}^{\pm} n_i} \left(\prod_{i=1}^I (d_{i1} + \beta_\theta)^{\alpha_d + \alpha_\theta} \theta_{i1}^{\alpha_d + \alpha_\theta - 1} \prod_{j=2}^{n_i} (d_{ij} + d_{ij-1} + \beta_\theta)^{2\alpha_d + \alpha_\theta} \theta_{ij}^{2\alpha_d + \alpha_\theta - 1}\right) \alpha_\theta^{\alpha_\theta - 1} e^{-\alpha_\theta \beta_\theta}\right)$$

$$= -I \ln(\Gamma(\alpha_d + \alpha_\theta)) - \sum_{i=1}^{\pm} n_i \ln(\Gamma(2\alpha_d + \alpha_\theta)) + (\alpha_\theta - 1) \ln(\alpha_\theta) - \alpha_\theta \beta_\theta + \left(\sum_{i=1}^I (\alpha_d + \alpha_\theta) \ln(d_{i1} + \beta_\theta) + (\alpha_d + \alpha_\theta - 1) \ln(\theta_{i1}) + \sum_{i=1}^{\pm} \sum_{j=2}^{n_i} (2\alpha_d + \alpha_\theta) \ln(d_{ij} + d_{ij-1} + \beta_\theta) + (2\alpha_d + \alpha_\theta - 1) \ln(\theta_{ij})\right)$$

$$+ \sum_{i=1}^{\pm} \sum_{j=2}^{n_i} (2\alpha_d + \alpha_\theta) \ln(d_{ij} + d_{ij-1} + \beta_\theta) + (2\alpha_d + \alpha_\theta - 1) \ln(\theta_{ij})$$

$$\bullet \ln(\pi(\beta_\theta | (\theta_{ij})_{i=1, j=1}^{\pm n_i}))$$

$$\ln\left(\prod_{i=1}^I (d_{i1} + \beta_\theta)^{\alpha_d + \alpha_\theta} \prod_{j=2}^{n_i} (d_{ij} + d_{ij-1} + \beta_\theta)^{2\alpha_d + \alpha_\theta} \beta_\theta^{\alpha_\theta - 1} e^{-\beta_\theta (\beta_\theta + \sum_{i=1}^I \sum_{j=1}^{n_i} \theta_{ij})}\right)$$

$$= \sum_{i=1}^I (\alpha_d + \alpha_\theta) \ln(d_{i1} + \beta_\theta) + \sum_{i=1}^{\pm} \sum_{j=2}^{n_i} (2\alpha_d + \alpha_\theta) \ln(d_{ij} + d_{ij-1} + \beta_\theta) + (\alpha_\theta - 1) \ln(\beta_\theta) - \beta_\theta (\beta_\theta + \sum_{i=1}^I \sum_{j=1}^{n_i} \theta_{ij})$$

$$\bullet \ln(\pi(\alpha_\gamma | (\gamma_{ij})_{i=1, j=1}^{\pm n_i}))$$

$$\ln\left(\left(\prod_{i=1}^{\pm} \prod_{j=1}^{n_i} \left(\frac{1}{\gamma_{ij}}\right)^{\alpha_\gamma}\right) \alpha_\gamma^{\alpha_\gamma - 1} e^{-\alpha_\gamma \beta_\gamma}\right) = \sum_{i=1}^{\pm} \sum_{j=1}^{n_i} (\alpha_\gamma) \ln(\gamma_{ij}) + (\alpha_\gamma - 1) \ln(\alpha_\gamma) - \alpha_\gamma \beta_\gamma$$

$$\bullet \ln(\pi(\beta_\gamma | (\gamma_{ij})_{i=1, j=1}^{\pm n_i}))$$

$$\ln\left(\beta_\gamma^{\alpha_\gamma - 1} e^{-\beta_\gamma (\beta_\gamma + \sum_{i=1}^{\pm} \sum_{j=1}^{n_i} (\frac{1}{\gamma_{ij}}))}\right) = (\alpha_\gamma - 1) \ln(\beta_\gamma) - \beta_\gamma (\beta_\gamma + \sum_{i=1}^{\pm} \sum_{j=1}^{n_i} (\frac{1}{\gamma_{ij}}))$$