① Duraciones $\Rightarrow P(d_k|d_{k-1}) = \int (p(d_k|\theta_k)(p(\theta_k|d_{k-1})$ ①

~~$P(\ldots\theta) = P(t|\delta, d)\, P(d|\theta)$~~ $P(d_k|d_{k-1}) = \int p(d_k|\theta_k)\, \underbrace{p(\theta_k|d_{k-1})}_{\text{posterior}}$

$\rightarrow P(d|\theta) \sim \text{Gamma}(d|\alpha_d, \theta)$

$p(\theta) \sim \text{Gamma}(\theta|\alpha_\theta, \beta_\theta)$ $\quad f_{\theta|d}(\theta|d) \propto f_{d|\theta}(d|\theta) f(\theta)$

$$p(d|\theta) = \frac{\theta^{\alpha_d}}{\Gamma(\alpha_d)} d^{\alpha_d - 1} e^{-\theta d}$$

$$p(\theta) = \frac{\beta_\theta^{\alpha_\theta}}{\Gamma(\alpha_\theta)} \theta^{\alpha_\theta - 1} e^{-\beta_\theta \theta}$$

$$\Rightarrow f_{\theta|d}(\theta|d) \propto \frac{\theta^{\alpha_d}}{\Gamma(\alpha_d)} d^{\alpha_d-1} e^{-\theta d} \frac{\beta_\theta^{\alpha_\theta}}{\Gamma(\alpha_\theta)} \theta^{\alpha_\theta - 1} e^{-\beta_\theta \theta} = \frac{\beta_\theta^{\alpha_\theta}}{\Gamma(\alpha_d)\Gamma(\alpha_\theta)} d^{\alpha_d - 1} \theta^{\alpha_d + \alpha_\theta - 1} e^{-\theta(d+\beta_\theta)}$$

$$\Rightarrow f_{\theta|d}(\theta|d) \propto \theta^{\alpha_d + \alpha_\theta - 1} e^{-\theta(d+\beta_\theta)} \quad \Rightarrow \underline{\theta|d \sim \text{Gamma}(\alpha_d + \alpha_\theta, d+\beta_\theta)}$$

$$\Rightarrow P(d_k|d_{k-1}) = \int p(d_k|\theta_k)\, p(\theta_k|d_{k-1}) = \int \frac{\theta_k^{\alpha_d}}{\Gamma(\alpha_d)} d_k^{\alpha_d - 1} e^{-\theta_k d_k} \frac{(d_{k-1}+\beta_\theta)^{\alpha_d+\alpha_\theta}}{\Gamma(\alpha_d+\alpha_\theta)} \theta_k^{\alpha_d+\alpha_\theta-1} e^{-\theta_k(d_{k-1}+\beta_\theta)}$$

$$= \int \frac{d_k^{\alpha_d-1} (d_{k-1}+\beta_\theta)^{\alpha_d+\alpha_\theta}}{\Gamma(\alpha_d)\Gamma(\alpha_d+\alpha_\theta)} \theta_k^{2\alpha_d+\alpha_\theta-1} e^{-\theta_k(d_k+d_{k-1}+\beta_\theta)}$$

$$= \frac{d_k^{\alpha_d-1}(d_{k-1}+\beta_\theta)^{\alpha_d+\alpha_\theta}}{\Gamma(\alpha_d)\Gamma(\alpha_d+\alpha_\theta)} \cdot \frac{\Gamma(2\alpha_d+\alpha_\theta)}{(d_k+d_{k-1}+\beta_\theta)^{2\alpha_d+\alpha_\theta}} \cancelto{1}{\int \frac{(d_k+d_{k-1}+\beta_\theta)^{2\alpha_d+\alpha_\theta}}{\Gamma(2\alpha_d+\alpha_\theta)} \theta_k^{2\alpha_d+\alpha_\theta-1} e^{-\theta_k(d_k+d_{k-1}+\beta_\theta)} d\theta}$$

$$\boxed{\Rightarrow P(d_k|d_{k-1}) = \frac{d_k^{\alpha_d-1}(d_{k-1}+\beta_\theta)^{\alpha_d+\alpha_\theta}}{\Gamma(\alpha_d)\Gamma(\alpha_d+\alpha_\theta)} \frac{\Gamma(2\alpha_d+\alpha_\theta)}{(d_k+d_{k-1}+\beta_\theta)^{2\alpha_d+\alpha_\theta}}}$$

② Costos,

$$p(c_k \mid d_k, \gamma_k, c_{k-1}) = \int p(c_k \mid \gamma_k, d_k) \underbrace{p(\gamma_k \mid d_k, c_{k-1})}_{\text{falta}}$$

$$\left.\begin{array}{l} \rightarrow p(c \mid \gamma, d) \sim \text{Weibull}(c \mid d, \gamma) \\ \rightarrow p(\gamma) \sim \text{Inv. Gamma}(\gamma \mid \alpha_\gamma, \beta_\gamma) \end{array}\right\} f_{\gamma \mid d,c}(\gamma \mid d, c) \propto f_{c \mid d,\gamma}(c \mid d, \gamma) f(\gamma)$$

forma

$$\rightarrow f(c \mid d, \gamma) = \frac{d}{\gamma^d} c^{d-1} e^{-\left(\frac{c}{\gamma}\right)^d}$$

escala

$$\rightarrow f(\gamma \mid \alpha_\gamma, \beta_\gamma) = \frac{\beta_\gamma^{\alpha_\gamma}}{\Gamma(\alpha_\gamma)} \left(\frac{1}{\gamma}\right)^{\alpha_\gamma+1} e^{-\left(\frac{\beta_\gamma}{\gamma}\right)} \quad \Big| \quad \text{gamma inversa}$$

$$\Rightarrow f_{\gamma \mid d,c}(\gamma \mid d, c) \propto \frac{d}{\gamma^d} c^{d-1} e^{-\left(\frac{c}{\gamma}\right)^d} \frac{\beta_\gamma^{\alpha_\gamma}}{\Gamma(\alpha_\gamma)} \left(\frac{1}{\gamma}\right)^{\alpha_\gamma+1} e^{-\left(\frac{\beta_\gamma}{\gamma}\right)}$$

$$= \frac{d \beta_\gamma^{\alpha_\gamma} c^{d-1}}{\Gamma(\alpha_\gamma)} \gamma^{-d} \gamma^{-(\alpha_\gamma+1)} e^{-\frac{\beta_\gamma}{\gamma}} e^{-\left(\frac{c}{\gamma}\right)^d}$$

$$= \frac{d \beta_\gamma^{\alpha_\gamma} c^{d-1}}{\Gamma(\alpha_\gamma)} \left(\frac{1}{\gamma}\right)^{d+\alpha_\gamma+1} e^{-\frac{\beta_\gamma}{\gamma}} e^{-\left(\frac{c}{\gamma}\right)^d} \propto \underline{\left(\frac{1}{\gamma}\right)^{d+\alpha_\gamma+1} e^{-\frac{\beta_\gamma}{\gamma}} e^{-\left(\frac{c}{\gamma}\right)^d}}$$

$= P(\gamma \mid c, d, \alpha_\gamma, \beta_\gamma)$ No tiene forma analítica cerrada como distribución, pero su kernel se utiliza en el slice sampler.

$$\ln(p(\gamma \mid c, d, \alpha_\gamma, \beta_\gamma)) = -(d+\alpha_\gamma+1)\ln(\gamma) - \frac{\beta_\gamma}{\gamma} - \left(\frac{c}{\gamma}\right)^d$$

Slice sampler: (Neal, 2001)
→ R – función (unidimensional.)
* $f(x \mid \theta) \propto \underbrace{g(x \mid \theta)}_{\text{kernel}}$ $\quad - c(\theta) = ?$

aplica slice sampler.

Verosimilitud.

$$V(\{\alpha_d,\alpha_\theta,\beta_\theta,\alpha_\gamma,\beta_\gamma\}\{\theta_k,\gamma_k\}_{k=1}^{N(T)} \mid (c_k,d_k)_{k=1}^{N(T)}) =$$

$$= \prod_{i=1}^{N(T)} \text{Gamma}(d_i|\alpha_d,\theta_i)\cdot \text{Gamma}(\theta_i|\alpha_d+\alpha_\theta, d_{i-1}+\beta_\theta)\cdot$$

$$\text{Weibull}(c_i|d_i,\gamma_i)\cdot gW(\gamma_i|\alpha_\gamma+d_i,\beta_\gamma,c_{i-1}^{-d_i},-d_i)$$

$$\pi(\alpha_d)\cdot\pi(\alpha_\theta)\cdot\pi(\beta_\theta)\cdot\pi(\alpha_\gamma)\pi(\beta_\gamma).$$

$$\Rightarrow \pi(\theta_i|\alpha_d,\alpha_\theta,\beta_\theta) \propto \text{Gamma}(d_i|\alpha_d,\theta_i)\,\text{Gamma}(\theta_i|\alpha_d+\alpha_\theta,d_{i-1}+\beta_\theta)$$

$$= \frac{\theta_i^{\alpha_d}}{\Gamma(\alpha_d)} d_i^{\alpha_d-1} e^{-\theta_i d_i} \frac{(d_{i-1}+\beta_\theta)^{\alpha_d+\alpha_\theta}}{\Gamma(\alpha_d+\alpha_\theta)} \theta_i^{\alpha_d+\alpha_\theta-1} e^{-\theta_i(d_{i-1}+\beta_\theta)}$$

$$= \frac{d_i^{\alpha_d-1}(d_{i-1}+\beta_\theta)^{\alpha_d+\alpha_\theta} d_i^{\alpha_d-1}}{\Gamma(\alpha_d)\,\Gamma(\alpha_d+\alpha_\theta)} \theta_i^{\overbrace{2\alpha_d+\alpha_\theta-1}} e^{-\theta_i(d_{i-1}+d_i+\beta_\theta)} \propto \theta_i^{2\alpha_d+\alpha_\theta-1} e^{-\theta_i(d_i+d_{i-1}+\beta_\theta)}$$

$$\boxed{\therefore\ \theta_i \sim \text{Gamma}(2\alpha_d+\alpha_\theta, d_i+d_{i-1}+\beta_\theta)}$$

$$\pi(\gamma_i|\alpha_\gamma,\beta_\gamma,d_i,c_i) \propto \text{Weibull}(c_i|d_i,\gamma_i)\,\pi(\gamma_t|c_{i-1},d_{i-1},\alpha_\gamma,\beta_\gamma)$$

$$= \frac{d_i}{\gamma_i^{d_i}} c_i^{d_i-1} e^{-\left(\frac{c_i}{\gamma_i}\right)^{d_i}} \left(\frac{1}{\gamma_i}\right)^{d_{i-1}+\alpha_\gamma+1} e^{-\frac{\beta_\gamma}{\gamma_i}} e^{-\left(\frac{c_{i-1}}{\gamma_i}\right)^{d_{i-1}}}$$

$$= d_i c_i^{d_i-1} \left(\frac{1}{\gamma_i}\right)^{d_i+d_{i-1}+\alpha_\gamma+1} e^{-\frac{\beta_\gamma}{\gamma_i}} e^{-\left(\frac{c_i}{\gamma_i}\right)^{d_i}} e^{-\left(\frac{c_{i-1}}{\gamma_i}\right)^{d_{i-1}}}$$

$$\propto \left(\frac{1}{\gamma_i}\right)^{d_i+d_{i-1}+\alpha_\gamma+1} e^{-\left(\frac{\beta_\gamma}{\gamma_i}+\left(\frac{c_i}{\gamma_i}\right)^{d_i}+\left(\frac{c_{i-1}}{\gamma_i}\right)^{d_{i-1}}\right)}$$

Kernel de $\pi(\gamma_i|\cdots)$ ∧ $\pi(\theta_i|\cdots)$ para $t=1,\dots,N(t)$.

En la dist. de costos la variable de normalización, no existe analíticamente.

$$\pi(\beta_\theta|\dots) \propto \prod_{i=1}^{N(t)} \text{Gamma}(\theta_i|\alpha_d+\alpha_\theta,d_{i-1}+\beta_\theta)\,\text{Gamma}(\beta_\theta|\alpha_0,\beta_0)$$

$$= \frac{(d_{i-1}+\beta_\theta)^{\alpha_d+\alpha_\theta}}{\Gamma(\alpha_d+\alpha_\theta)} \theta_i^{\alpha_d+\alpha_\theta-1} e^{-\theta_i(d_{i-1}+\beta_\theta)} \frac{\beta_0^{\alpha_0}}{\Gamma(\alpha_0)} \beta_\theta^{\alpha_0-1} e^{-\beta_0\beta_\theta}$$

$$\propto (d_{i-1}+\beta_\theta)^{\alpha_d+\alpha_\theta} \beta_\theta^{\alpha_0-1} e^{-\theta_i d_{i-1}} e^{-\beta_\theta\theta_i} e^{+\beta_0\beta_\theta} = \underline{(d_{i-1}+\beta_\theta)^{\alpha_d+\alpha_\theta} \beta_\theta^{\alpha_0-1} e^{-\beta_\theta(\theta_i+\beta_0)}}$$

$$\pi(\alpha_d|\ldots) \propto \prod_{i=1}^{N(t)} \text{Gamma}(\theta_i|\alpha_d+\alpha_\theta, d_{i-1}+\beta_\theta)\, \text{Gamma}(\alpha_d|\alpha_0,\beta_0)$$

$$= \frac{(d_{i-1}+\beta_\theta)^{\alpha_d+\alpha_\theta}}{\Gamma(\alpha_d+\alpha_\theta)} \theta_i^{\alpha_d+\alpha_\theta-1} e^{-\theta_i(d_{i-1}+\beta_\theta)} \frac{\beta_0^{\alpha_0}}{\Gamma(\alpha_0)} \alpha_d^{\alpha_0-1} e^{-\alpha_d\beta_0}$$

$$\propto \frac{(d_{i-1}+\beta_\theta)^{\alpha_d+\alpha_\theta}}{\Gamma(\alpha_d+\alpha_\theta)} \theta_i^{\alpha_d} \alpha_d^{\alpha_0-1} e^{-\alpha_d\beta_0}$$

$$\pi(\alpha_\theta|\ldots) \propto \prod_{i=1}^{N(t)} \text{Gamma}(\theta_i|\alpha_d+\alpha_\theta, d_{i-1}+\beta_\theta)\, \text{Gamma}(\alpha_\theta|\alpha_0,\beta_0)$$

$$= \frac{(d_{i-1}+\beta_\theta)^{\alpha_d+\alpha_\theta}}{\Gamma(\alpha_d+\alpha_\theta)} \theta_i^{\alpha_d+\alpha_\theta-1} e^{-\theta_i(d_{i-1}+\beta_\theta)} \frac{\beta_0^{\alpha_0}}{\Gamma(\alpha_0)} \alpha_\theta^{\alpha_0-1} e^{-\alpha_\theta\beta_0}$$

$$\propto \frac{(d_{i-1}+\beta_\theta)^{\alpha_d+\alpha_\theta}}{\Gamma(\alpha_d+\alpha_\theta)} \theta_i^{\alpha_\theta} \alpha_\theta^{\alpha_0-1} e^{-\alpha_\theta\beta_0}$$

$$\pi(\alpha_\gamma|\ldots) \propto \prod_{i=1}^{N(t)} \left(\frac{1}{\gamma_i}\right)^{d+\alpha_\gamma+1} e^{-\frac{\beta_\gamma}{\gamma_i}} e^{-\left(\frac{c}{\gamma_i}\right)^d} \text{Gamma}(\alpha_\gamma|\alpha_0,\beta_0).$$

$$= \left(\frac{1}{\gamma_i}\right)^{d+\alpha_\gamma+1} e^{-\frac{\beta_\gamma}{\gamma_i}} e^{-\left(\frac{c_{i-1}}{\gamma_i}\right)^{d_i}} \frac{\beta_0^{\alpha_0}}{\Gamma(\alpha_0)} \alpha_\gamma^{\alpha_0-1} e^{-\alpha_\gamma\beta_0}$$

$$\propto \left(\frac{1}{\gamma_i}\right)^{\alpha_\gamma} \alpha_\gamma^{\alpha_0-1} e^{-\alpha_\gamma\beta_0}$$

$$\pi(\beta_\gamma|\ldots) \propto \prod_{i=1}^{N(t)} \left(\frac{1}{\gamma_i}\right)^{d_i+\alpha_\gamma+1} e^{-\frac{\beta_\gamma}{\gamma_i}} e^{-\left(\frac{c_{i-1}}{\gamma_i}\right)^{d_i}} \text{Gamma}(\beta_\gamma|\alpha_0,\beta_0)$$

$$= \left(\frac{1}{\gamma_i}\right)^{d_i+\alpha_\gamma+1} e^{-\frac{\beta_\gamma}{\gamma_i}} e^{-\left(\frac{c_{i-1}}{\gamma_i}\right)^{d_i}} \frac{\beta_0^{\alpha_0}}{\Gamma(\alpha_0)} \beta_\gamma^{\alpha_0-1} e^{-\beta_\gamma\beta_0}$$

$$\propto \beta_\gamma^{\alpha_0-1} e^{-\beta_\gamma\left(\frac{1}{\gamma_i}+\beta_0\right)} \qquad \sim \beta_\gamma \sim \text{Gamma}\left(\alpha_0, \frac{1}{\gamma_i}+\beta_0\right).$$

Resumen de funciones

— i —

- Duraciones $(\theta_i | d_{i-1})$

$$\theta | d \sim Gamma(\alpha_d + \alpha_\theta, d + \beta_\theta) \quad \left(\theta^{\alpha_d + \alpha_\theta - 1} \quad e^{-\theta(d + \beta_\theta)} \right)$$

- Costos $(\gamma_i | c_{i-1}, d_i)$

$$\left(\frac{1}{\gamma}\right)^{d + \alpha_\gamma + 1} e^{-\left(\left(\frac{\beta_\gamma}{\gamma}\right) + \left(\frac{c}{\gamma}\right)^d\right)}$$

— ii Verosimilitud —

- $\pi(\theta_i | \alpha_d, \alpha_\theta, \beta_\theta)$

$$\theta_i \sim Gamma(2\alpha_d + \alpha_\theta, d_i + d_{i-1} + \beta_\theta) \quad \left(\theta_i^{2\alpha_d + \alpha_\theta - 1} \quad e^{-\theta_i(d_i + d_{i-1} + \beta_\theta)} \right)$$

- $\pi(\gamma_i | \alpha_\gamma, \beta_\gamma, c_i)$

$$\left(\frac{1}{\gamma_i}\right)^{d_i + d_{i-1} + \alpha_\gamma + 1} \quad e^{-\left(\frac{\beta_\gamma}{\gamma_i} + \left(\frac{c_i}{\gamma_i}\right)^{d_i} + \left(\frac{c_{i-1}}{\gamma_i}\right)^{d_{i-1}}\right)}$$

- $\pi(\alpha_d | \dots)$

$$\frac{(d_{i-1} + \beta_\theta)^{\alpha_d + \alpha_\theta}}{\Gamma(\alpha_d + \alpha_\theta)} \quad \theta_i^{\alpha_d} \quad \alpha_d^{\alpha_0 - 1} \quad e^{-\alpha_d \beta_0}$$

- $\pi(\alpha_\theta | \dots)$

$$\frac{(d_{i-1} + \beta_\theta)^{\alpha_d + \alpha_\theta}}{\Gamma(\alpha_d + \alpha_\theta)} \quad \theta_i^{\alpha_\theta} \quad \alpha_\theta^{\alpha_0 - 1} \quad e^{-\alpha_\theta \beta_0}$$

- $\pi(\beta_\theta | \dots)$

$$(d_{i-1} + \beta_\theta)^{\alpha_d + \alpha_\theta} \quad \beta_\theta^{\alpha_0 - 1} \quad e^{-\beta_\theta(\theta_i + \beta_0)}$$

- $\pi(\alpha_\gamma | \dots)$

$$\left(\frac{1}{\gamma_i}\right)^{\alpha_\gamma} \cdot \alpha_\gamma^{\alpha_0 - 1} \quad e^{-\alpha_\theta \beta_0}$$

- $\pi(\beta_\gamma | \dots)$

$$\beta_\gamma \sim Gamma\left(\alpha_0, \frac{1}{\gamma_i} + \beta_0\right) \quad \left(\beta_\gamma^{\alpha_0 - 1} \quad e^{-\beta_\gamma\left(\frac{1}{\gamma_i} + \beta_0\right)} \right)$$

$$\to \ln \pi(\gamma_i|\alpha_\gamma,\beta_\gamma,c_i) = \underline{-(d_i+d_{i-1}+\alpha_\gamma+1)\ln(\gamma_i) - \left(\frac{\beta_\gamma}{\gamma_i} + \left(\frac{c_i}{\gamma_i}\right)^{d_i} + \left(\frac{c_{i-1}}{\gamma_i}\right)^{d_{i-1}}\right)}$$

$$\to \ln \pi(\alpha_d|\dots) = \underline{\alpha_d+\alpha_\theta \ln(d_{i-1}+\beta_\theta) - \ln\Gamma(\alpha_d+\alpha_\theta) + \alpha_d \ln\theta_i + (\alpha_0-1)\ln\alpha_d - \alpha_d\beta_0}$$

$$\to \ln \pi(\alpha_\theta|\dots) = \alpha_d+\alpha_\theta \ln(d_{i-1}+\beta_\theta) - \ln\Gamma(\alpha_d+\alpha_\theta) + \alpha_\theta \ln\theta_i + (\alpha_0-1)\ln\alpha_\theta - \alpha_\theta\beta_0$$

$$\to \ln \pi(\beta_\theta|\dots) = \alpha_d+\alpha_\theta \ln(d_{i-1}+\beta_\theta) + (\alpha_0-1)\ln\beta_\theta - \beta_\theta(\theta_i+\beta_0)$$

$$\to \ln \pi(\alpha_\gamma|\dots) = -\alpha_\gamma \ln(\gamma_i) + (\alpha_0-1)\ln\alpha_\gamma - \alpha_\gamma\beta_0$$

alpha_d_sim → 15.23967

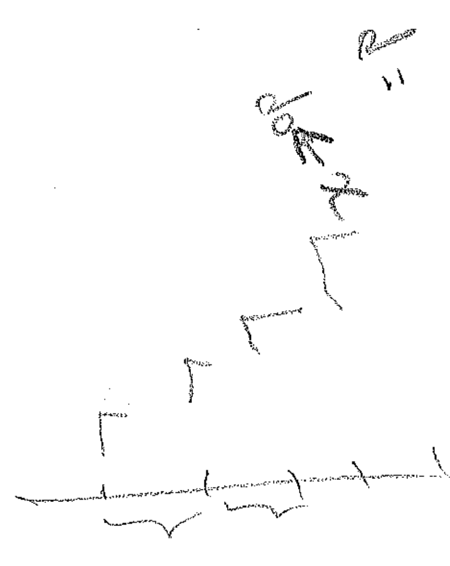
alpha_theta_sim → 24.776281

$$\begin{array}{c} d_i = \end{array} \begin{pmatrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{pmatrix} \qquad \begin{array}{ccccc} \theta_1 & \theta_2 & \theta_3 & \theta_4 & \theta_5 \\ \hline \cdot & \to & \to & \to & \to \end{array}$$

for d in D:

for i in I

sample $\theta \sim Ga(\dots)$



Data

$$\begin{array}{cccc} ind & t & d_i & \theta_i \end{array}$$

$n = \#obs \times \#ind.$

$$\begin{array}{c|ccc} & 3 & d & c \\ n_1 & ind_1 & d_1 & c_1 \\ \vdots & \vdots & \vdots & \vdots \\ N & ind_N & d_N & c_N \end{array}$$