$$\propto \prod_{i=1}^{I} \prod_{j=2}^{N(i)} \frac{(d_{ij-1}+\beta_\theta)^{\alpha_d+\alpha_\theta}}{\Gamma(\alpha_d+\alpha_\theta)} \theta_{ij}^{\alpha_d} \alpha_d^{\alpha_0-1} e^{-\alpha_d\beta_0}$$

$$\propto \left(\frac{\alpha_d^{\alpha_0-1} e^{-\alpha_d\beta_0}}{\Gamma(\alpha_d+\alpha_\theta)} \right)^{\sum_{i=1}^{I} N(i)} \prod_{i=1}^{I} \prod_{j=2}^{N(i)} (d_{ij-1}+\beta_\theta)^{\alpha_d+\alpha_\theta} \theta_{ij}^{\alpha_d}$$

$$\Rightarrow \pi(\alpha_\theta \mid (\theta_{ij})_{i=1\, j=1}^{I\, N(i)}) \propto \prod_{i=1}^{I} \prod_{j=2}^{N(i)} \pi(\alpha_\theta \mid (\theta_{ij})_{i=1\, j=1}^{I\, N(i)})$$

$$\times \prod_{i=1}^{I} \prod_{j=2}^{N(i)} \frac{(d_{ij-1}+\beta_\theta)^{\alpha_d+\alpha_\theta}}{\Gamma(\alpha_d+\alpha_\theta)} \theta_{ij}^{\alpha_\theta} \alpha_\theta^{\alpha_0-1} e^{-\alpha_\theta\beta_0}$$

$$\propto \left(\frac{\alpha_\theta^{\alpha_0-1} e^{-\alpha_\theta\beta_0}}{\Gamma(\alpha_d+\alpha_\theta)} \right)^{\sum_{i=1}^{I} N(i)} \prod_{i=1}^{I} \prod_{j=2}^{N(i)} (d_{ij-1}+\beta_\theta)^{\alpha_d+\alpha_\theta} \theta_{ij}^{\alpha_\theta}$$

$$\pi(\beta_\theta \mid (\theta_{ij})_{i=1\, j=1}^{I\, N(i)}) \propto \prod_{i=1}^{I} \prod_{j=2}^{N(i)} \pi(\beta_\theta \mid (\theta_{ij})_{i=1\, j=1}^{I\, N(i)})$$

$$\prod_{i=1}^{I} \prod_{j=2}^{N(i)} (d_{ij-1}+\beta_\theta)^{\alpha_d+\alpha_\theta} \beta_\theta^{\alpha_0-1} e^{-\beta_\theta(\theta_{ij}+\beta_0)}$$

$$\propto (\beta_\theta^{\alpha_0-1})^{\sum_{i=1}^{I} N(i)} \cdot e^{-\beta_\theta \left(\sum_{i=1}^{I} \sum_{j=1}^{N(i)} (\theta_{ij}+\beta_0)\right)} \cdot \prod_{i=1}^{I} \prod_{j=2}^{N(i)} (d_{ij-1}+\beta_\theta)^{\alpha_d+\alpha_\theta}$$

$$\pi(\alpha_\gamma \mid (\gamma_{ij})_{i=1\, j=1}^{I\, N(i)}) \propto \prod_{i=1}^{I} \prod_{j=2}^{N(i)} \pi(\alpha_\gamma \mid (\gamma_{ij})_{i=1\, j=1}^{I\, N(i)})$$

$$\propto \prod_{i=1}^{I} \prod_{j=2}^{N(i)} \left(\frac{1}{\gamma_{ij}}\right)^{\alpha_\gamma} \alpha_\gamma^{\alpha_0-1} e^{-\alpha_\gamma\beta_0}$$

$$\propto \left(\alpha_\gamma^{\alpha_0-1} e^{-\alpha_\gamma\beta_0} \right)^{\sum_{i=1}^{I} N(i)} \prod_{i=1}^{I} \prod_{j=1}^{N(i)} \left(\frac{1}{\gamma_{ij}}\right)^{\alpha_\gamma}$$

$$\pi(\beta_\gamma \mid (\gamma_{ij})_{i=1\, j=1}^{I\, N(i)}) \propto \prod_{i=1}^{I} \prod_{j=1}^{N(i)} \pi(\beta_\gamma \mid (\gamma_{ij})_{i=1\, j=1}^{I\, N(i)})$$

$$\propto \prod_{i=1}^{I} \prod_{j=1}^{N(i)} \beta_\gamma^{\alpha_0-1} e^{-\beta_\gamma\left(\frac{1}{\gamma_{ij}}+\beta_0\right)}$$

$$\propto (\beta_\gamma^{\alpha_0-1})^{\sum_{i=1}^{I} N(i)} e^{-\beta_\gamma\left(\sum_{i=1}^{I} \sum_{j=1}^{N(i)} \frac{1}{\gamma_{ij}}+\beta_0\right)}$$

Log

① $\ln\left(\Pi(\alpha_d \mid (\theta_{ij})_{i=1\ j=1}^{I\ N(t_i)})\right) = \ln\left(\left(\frac{\alpha_d^{\alpha_0-1} e^{-\alpha_d\beta_0}}{\Gamma(\alpha_d+\alpha_\theta)}\right)^{\sum_{i=1}^{I} N(t_i)} \prod_{i=1}^{I}\prod_{j=2}^{N(t_i)} (d_{ij-1}+\beta_\theta)^{\alpha_d+\alpha_\theta}\ \theta_{ij}^{\alpha_d}\right)$

$= \sum_{i=1}^{I} N(t_i) * \left[\ln\left(\alpha_d^{\alpha_0-1} \cdot e^{-\alpha_d\beta_0}\right) - \ln\left(\Gamma(\alpha_d+\alpha_\theta)\right)\right] + \ln\left(\prod_{i=1}^{I}\prod_{j=2}^{N(t_i)} (d_{ij-1}+\beta_\theta)^{\alpha_d+\alpha_\theta}\ \theta_{ij}^{\alpha_d}\right)$

$= \sum_{i=1}^{I} N(t_i) * \left[(\alpha_0-1)\ln(\alpha_d) - \alpha_d\beta_0 - \ln\left(\Gamma(\alpha_d+\alpha_\theta)\right)\right] + \sum_{i=1}^{I}\sum_{j=2}^{N(t_i)}\left[(\alpha_d+\alpha_\theta)\ln(d_{ij-1}+\beta_\theta) + \alpha_d \cdot \ln(\theta_{ij})\right]$

② $\ln\left(\Pi(\alpha_\theta \mid (\theta_{ij})_{i=1\ j=1}^{I\ N(t_i)})\right) = \ln\left(\left(\frac{\alpha_\theta^{\alpha_0-1} e^{-\alpha_\theta\beta_0}}{\Gamma(\alpha_d+\alpha_\theta)}\right)^{\sum_{i=1}^{I} N(t_i)} \prod_{i=1}^{I}\prod_{j=2}^{N(t_i)} (d_{ij-1}+\beta_\theta)^{\alpha_d+\alpha_\theta}\ \theta_{ij}^{\alpha_\theta}\right)$

$= \sum_{i=1}^{I} N(t_i) * \left[(\alpha_0-1)\ln(\alpha_\theta) - \alpha_\theta\beta_0 - \ln\Gamma(\alpha_d+\alpha_\theta)\right] + \sum_{i=1}^{I}\sum_{j=2}^{N(t_i)}\left[(\alpha_d+\alpha_\theta)\ln(d_{ij-1}+\beta_\theta) + \alpha_\theta \cdot \ln(\theta_{ij})\right]$

③ $\ln\left(\Pi(\beta_\theta \mid (\theta_{ij})_{i=1\ j=1}^{I\ N(t_i)})\right) = \ln\left((\beta_\theta^{\alpha_0-1})^{\sum_{i=1}^{I} N(t_i)}\ e^{-\beta_\theta\left(\sum_{i=1}^{I}\sum_{j=1}^{N(t_i)}(\theta_{ij}+\beta_0)\right)} \prod_{i=1}^{I}\prod_{j=2}^{N(t_i)} (d_{ij-1}+\beta_\theta)^{\alpha_d+\alpha_\theta}\right)$

$= \left[\sum_{i=1}^{I} N(t_i) * (\alpha_0-1)\cdot\ln(\beta_\theta)\right] - \beta_\theta\left(\sum_{i=1}^{I}\sum_{j=1}^{N(t_i)}(\theta_{ij}+\beta_0)\right) + \sum_{i=1}^{I}\sum_{j=2}^{N(t_i)}\left[(\alpha_d+\alpha_\theta)\ln(d_{ij-1}+\beta_\theta)\right]$

④ $\ln\left(\Pi(\alpha_\gamma \mid (\gamma_{ij})_{i=1\ j=1}^{I\ N(t_i)})\right) = \ln\left((\alpha_\gamma^{\alpha_0-1}\ e^{-\alpha_\gamma\beta_0})^{\sum_{i=1}^{I} N(t_i)} \prod_{i=1}^{I}\prod_{j=1}^{N(t_i)} \left(\frac{1}{\gamma_{ij}}\right)^{\alpha_\gamma}\right)$

$= \sum_{i=1}^{I} N(t_i) * \left[\ln\left(\alpha_\gamma^{\alpha_0-1} e^{-\alpha_\gamma\beta_0}\right)\right] + \ln\left(\prod_{i=1}^{I}\prod_{j=1}^{N(t_i)}\left(\frac{1}{\gamma_{ij}}\right)^{\alpha_\gamma}\right)$

$= \sum_{i=1}^{I} N(t_i) * \left[(\alpha_0-1)\ln(\alpha_\gamma) - \alpha_\gamma\beta_0\right] + \sum_{i=1}^{I}\sum_{j=1}^{N(t_i)}\left(\alpha_\gamma * - \ln(\gamma_{ij})\right)$

⑤ $\ln\left(\pi(\beta_\gamma \mid (\gamma_{ij})_{i=1\ j=1}^{I\ N(t_i)})\right) = \ln\left((\beta_\gamma^{\alpha_0-1})^{\sum_{i=1}^{I} N(t_i)}\ e^{-\beta_\gamma\left(\sum_{i=1}^{I}\sum_{j=1}^{N(t_i)}\left(\frac{1}{\gamma_{ij}}+\beta_0\right)\right)}\right)$

$= \sum_{i=1}^{I} N(t_i) \cdot (\alpha_0-1)\cdot\ln(\beta_\gamma) - \beta_\gamma\left(\sum_{i=1}^{I}\sum_{j=1}^{N(t_i)}\left(\frac{1}{\gamma_{ij}}+\beta_0\right)\right)$