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flipping the coin — Let X be a random variable.
data generating process.

after got the result — Let x be a realization (data)

$x \in \text{Supp}[X]$ ("support") $:= \{x : p(x) > 0\}$

Discrete random variable X

$|\text{Supp}[X]| \leq |\mathbb{N}|$ ← countable infinity

size [# of elements] · natural #'s

Probability Mass Function (PMF)

$P(x) := P(X = x)$

Two definitions

I $P : \mathbb{R} \rightarrow [0, 1]$

II $P : \text{Supp}[X] \rightarrow (0, 1]$

Cumulative Distribution Function (CDF)

$F(x) := \underbrace{P(X \leq x)}_{\text{monotonically increasing}} = \sum P(y) \, [y : y \in \text{Supp}[X] \ \& \ y \leq x]$

$F : \mathbb{R} \rightarrow [0, 1]$

Continuous random variable's

$|\text{Supp}[X]| = |\mathbb{R}|$ ← uncountable infinity

Cumulative distribution function (CDF) is defined the same as for discrete random variable's.

$f(x) := \frac{d}{dx}[F(x)]$, $Supp(x) := \{x : f(x) > 0\}$

Probability Density Function (PDF)

By fundamental theorem of calculus
$a \leq b$,
$$P(X \in [a,b]) = F(b) - F(a) = \int_a^b f(x)\,dx$$

Examples of discrete random variable's

distributed as

I $X \sim Bern(p) := \underbrace{p^x (1-p)^{1-x}}_{p(x)}$, $Supp(x) = \{0, 1\}$

"Bernoulli" (proof for Bern)

II $X \sim Binom(n,p) := \underbrace{\binom{n}{x} p^x (1-p)^{n-x}}_{p(x)}$, $supp(x) = \{0, 1, \ldots, n\}$

"Binomial" (proof for Binom)

"Parameters"

A tuning knob
For Bernoulli, p is the "prob. of success" (if 1 is the definition of success)
$p \in (0,1)$

$X \sim Bern(0) = Deg(0) = \{0$ wp 1

"degenerous" are deterministic (not the random)

$X \sim Bern(1) = Deg(1) = \{1$ wp 1

$p=0$ or $p=1$ are degenerous cases.
As a $\boxed{\text{caution}}$ (convention), we exclude then from the parameter space

Parameter space

The set of all parameters values which are non degenerate.

parameter space for binomial
$p \in (0,1)$
$n \in \{1, 2, \ldots, n\} = \mathbb{N}$ (natural numbers)

Continuous random variable Examples. (cont. doesn't have proof)

$X \sim Exp(\lambda) := \underbrace{\lambda e^{-\lambda x}}_{f(x) \to PDF}$
exponent
$Supp(x) = (0, \infty)$
$(\lambda \in (0, \infty))$

Example
$X \sim N(\mu, \sigma^2) := \frac{1}{\sqrt{2\pi\sigma^2}} \cdot e^{-\frac{1}{2\sigma^2}(x-\mu)^2}$ (PDF)
normal random variable.
$Supp(x) = \mathbb{R}$ $\quad \sigma^2 = Var(x) := E[(x-\mu)^2]$
$\mu \in \mathbb{R}$
$\sigma^2 \in (0, \infty)$

Let θ be the unknown parameter.
And $\vec{\theta}$ be the unknown parameters.
And Θ be the parameter space.
capital theta.

$X \sim Bern(\theta) := \theta^x (1-\theta)^{1-x}$
$X \sim N \underbrace{(\theta_1, \theta_2)}_{\vec{\theta}}$

Parametric model

$\mathcal{F} := \{ p(x;\theta) : \theta \in \Theta \}$

space of parametric models — PMF/PDF — PMF "needs" the parameter value

If $x_1, x_2, \dots, x_n$ are random variables
$p(x_1, x_2, \dots, x_n ; \vec{\theta})$ = is the joint mass function / joint density function.

If $x_1, \dots, x_n \overset{ind}{\sim}$ (independent)

$$p(x_1, x_2, \dots x_n ; \vec{\theta}) \overset{ind}{=} p_1(x_1 ; \vec{\theta})\, p_2(x_2 ; \vec{\theta}), \dots, p_n(x_n ; \vec{\theta})$$

$$= \prod_{i=1}^{n} p_i(x_i ; \vec{\theta}) = \prod_{i=1}^{n} p(x_i ; \vec{\theta})$$

If $x_1, \dots x_n \overset{iid}{\sim}$ independent & identically distributed.

In the real world, you observe "data"
$x = \langle 0, 0, 1, 0, 1, 0 \rangle$

Step I) Pick $\mathcal{F}$; Pick a model
Beyond the scope of the course

Step II) Inference (i.e figure out the value of $\vec{\theta}$)

Three goals of Inference (good guess $x = \langle 0,0,1,0,1,0 \rangle$ $2/6 = 1/3$)

① Point estimates: provide best single guess of θ (denoted $\hat{\theta}$)

② Confident set: Provide range of likely θ values.

③ Theory testing: Test plausibilities that θ is some value or develop to some set.
how realistic