# Lecture 1

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Let $X$ be a random variable → data generating process
Let $x$ be a realization (data)

$X \in Supp[X] = \{ X : p(X) > 0 \}$
"Support"

**Discrete random variables:** $|supp(X)| \leq |\mathbb{N}|$

size(# of elements) — natural #'s

**Probability Mass function (PMF):** $p(x) = P(X=x)$

Two definitions $\quad p: R \longrightarrow [0,1]$
$\quad p: Supp(x) \longrightarrow (0,1]$

**Cumulative Distribution Function (CDF)**

$$F(x) = P(X \leq x) = \sum_{\{y : y \in Supp(x) \text{ and } y \leq x\}} p(y)$$

$F : \mathbb{R} \rightarrow [0,1)$

**Continuous R.V.'s**

$|Supp(x)| = |\mathbb{R}|$ → uncountable infinite
CDF is defined the same as for discrete r.v.'s

$$f(x) = \frac{d}{dx}[F(x)], \quad Supp(x) = \{ X : f(x) > 0 \}$$

Probability density function (PDF)

By fundamental Thm. of Calculus

$$P(\underbrace{X \in [a,b)}_{a \le b}) = F(b) - F(a) = \int_a^b f(t)\,dt$$

<u>Examples of Discrete r.v.'s</u>

(I) $X \sim Bern(p) = \underbrace{p^x (1-p)^{1-x}}_{p(x)}$ , $Supp(x) = \{0, 1\}$

(II) $X \sim Binom(n,p) = \underbrace{\binom{n}{x} p^x (1-p)^{n-x}}_{p(x)}$ , $Supp(x) = \{0, 1, \dots, n)$

"<u>Parameters</u>" → numerical characteristics that define a distribution

For Bernoulli, $p$ is the "probability of success" (If 1 is the definition of success)

$p \in (0,1)$

$X \sim Bern(0) = Deg(0) = \{ 0$ w.p. 1

Degenerate ⇒ r.v. that does NOT make things happen

$X \sim Bern(1) = Deg(1) = \{ 1$ w.p. 1

$p = 0$ or $p = 1$ are degenerate cases. As a convention, exclude degenerate from the parameter space.

<u>Parameter Space</u> The set of all parameter values which are non degenerate

Parameter space for Binomial $p \in (0,1)$
$n \in \{1, 2, \dots\} = \mathbb{N}$

## Continuous r.v. Examples

$X \sim Exp(\lambda) = \lambda e^{-\lambda x}$ $\quad$ $Supp(X) = (0, \infty)$

$\lambda \in (0, \infty)$

$X \sim N(\mu, \sigma^2) = \frac{1}{\sqrt{2\pi\sigma^2}} \cdot e^{-\frac{1}{2\sigma^2}(X-\mu)^2}$ $\quad$ $Supp(X) = \mathbb{R}$

$\mu \in \mathbb{R}$

$\sigma^2 \in (0, \infty)$

$Var(X) = E[(X-\mu)^2]$

Let $\theta$ be the unknown parameter and
$\vec{\theta}$ be multiple unknown parameters and
$\Theta$ "Capital Theta" be the parameter space

$X \sim Bern(\theta) = \theta^x (1-\theta)^{1-x}$

$X \sim N(\theta_1, \theta_2)$

## Parametric Model

$\tilde{F} = \{ p(x; \theta) : \theta \in \Theta$

$\tilde{F}$: Space of Parametric models

$p(x; \theta)$: PMF/PDF

$\Theta$: capital Theta

If $X_1, X_2, \dots, X_n$ are random variables and $P(X_1, X_2, \dots, X_n; \vec{\theta})$ is the joint mass function / joint density function

If $X_1, \dots, X_n \overset{iid}{\sim}$

$$\Rightarrow P(X_1, X_2, \dots, X_n; \vec{\theta}) = P_1(X_1, \vec{\theta}) P_2(X_2; \vec{\theta}) \cdot \dots \cdot P_n(X_n; \vec{\theta})$$

$$= \prod_{i=1}^{n} P(X; \vec{\theta}) \quad \text{If } X_1, \dots, X_n \overset{iid}{\sim} \quad = \prod_{i=1}^{n} P(X_i; \vec{\theta})$$

In the real world, observe "data" $x = <0,0,1,0,1,0>$

Step I  Pick $\tilde{F}$. Pick a model. (Beyond the scope of this course)

Step II  Inference (i.e. figure out the role of $\vec{\theta}$)

Three goals of Inference

I) Pair Estimation. Provide best single guess of $\theta$

II) Confidence Set. Provide a range of possible $\theta$ values

III) Theory Testing. Test plausibility that $\theta$ is some value or belongs to some set