Lec 1

note: #: number

Let X be a r.v. Let x be a realization (data)

r.v. → data gathering process

$x \in \text{Supp}[X]$

Supp → support

Discrete random variable X

$|\text{Supp}[X]| \leq |\mathbb{N}|$ countable infinity

$|\text{Supp}[X]|$: numbers of elements; $\mathbb{N}$: natural #

probability mass function (PMF)

$p(x) := P(X = x)$

Definition: $p: \mathbb{R} \to [0,1]$

$p: \text{supp}[X] \to [0,1]$

Cummulative Distribution function (cdf)

$F(x) := P(X \leq x) = \sum p(y)$

$\{y : y \in \text{Supp}(X) \text{ and } y \leq x\}$

$F: \mathbb{R} \to [0,1]$

Continuous r.v's $|\text{supp}(X)| = |\mathbb{R}|$ uncountable infinity

$f(x) := \frac{d}{dx}(F(x))$ pdf

if $a \leq b$ $P(X \in [a,b]) = F(b) - F(a) = \int_a^b f(x)\,dx$

Example of Discrete

① $X \sim Bern(p)$ "Bernoulli" $= \underbrace{p^x (1-p)^{1-x}}_{p(x)}$ pmf only 2 #

$Supp(X) = \{0, 1\}$

$$p(x) = \begin{cases} p & x=1 \\ 1-p & x=0 \end{cases}$$

② $X \sim Binom(n, p) := \underbrace{\binom{n}{x} p^x (1-p)^{n-x}}_{pmf}$ $Supp(X) = \{0, 1, \dots n\}$ n+1 #

"parameter" a thinking knob

For 1, p is the probability of sucess (if 1 is ~~the def~~ of success)

$p \in 0, 1$ $X \sim Bern(0) = Deg(0)$ "Degenerate" ↪ not random

$X \sim Bern(1) = Deg(1) := \{$always 1 with prob 1$\}$

$p=0$ or $p=1$ are degenerate cases, we exclude them from parameter space

parameter space: the set of all parameter values which are not degenerate.

Parameter space for Binomial $p \in (0,1)$

$n \in \{1, 2, \dots\} = \mathbb{N}$

Continuous r.v's

$X \sim Exp(\lambda) := \lambda e^{-\lambda x}$ $f(x)$ exponential

$Supp(x) \in [0, \infty)$ $\lambda \in (0, \infty)$

$X \sim N(\mu, \sigma^2) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$ normal r'v's $Supp(x) = \mathbb{R}$

$\mu \in \mathbb{R}$ $\sigma^2 \in (0, +\infty)$ $\sigma^2 = Var(x) := E[(x-\mu)^2]$

Let θ be the unknown parameter

and $\vec{\theta}$, , , , , parameters. and H be parameter space

vector

$x \sim Bern(\theta) := \theta^x (1-\theta)^{1-x}$

$x \sim N(\theta_1, \theta_2)$ parameter model

$\tilde{F} : \{P(x;\theta) : \theta \in \Theta\}$

pmf

space of parameter model

If $X_1, X_2, \dots, X_n$ are r.v's

$p(x_1, \dots, x_n; \vec{\theta})$ is joint mass function (Jmf)
joint density function (Jdf)

If $X_1, \dots, X_n$ indep then

$$p(x_1, \dots, x_n; \vec{\theta}) = p(x_1, \vec{\theta}) \cdot p(x_2, \vec{\theta}) \cdots p(x_n, \vec{\theta}) = \prod_{i=1}^{n} p_i(x_i, \vec{\theta})$$

if iid

$$p(x_1, \dots, x_n; \vec{\theta}) = \prod_{i=1}^{n} p(x_i, \vec{\theta})$$

In the real world, you observe "data" $X = \langle 0, 0, 1, 0, 1, 0 \rangle$

Step I: pick $\tilde{F}$ pick a model, beyond the scope of this course.

Step II: Inference (figure out the value of $\vec{\theta}$)

The of inference

(I) point estimation provide a single guessing of θ

(II) Confident set provide a range of likely θ values

(III) Theory testing test possibility that θ is some value or belongs to some set.