Nuclear Engineering 101: Midterm 2 Study Guide

Disclaimer: This is not an official study guide. Stuff ~~might~~ **is** wrong. Use the lecture notes and book!

Note: Everything in this guide is from the text (Krane) or lecture, or office hours and should be cited as completely as possible.

1 Radioactive Decay Review

- The mean lifetime is defined as:

$$\tau = \frac{1}{\lambda}$$

  This is different than the $t_{1/2}$:

$$t_{1/2} = \frac{\text{Log}(2)}{\lambda}$$

- Activity with a production rate R:

$$A(t) = \lambda N(t) = R(1 - e^{\lambda t})$$

2 Nuclear Electromagnetic Moments

Ref: [1, pp. 71-75],[2, Lec 10]
A distribution of electric charge and/or current will create an electric potential. This can be described by the "multipole moment".

Note the values of L are the *order of the moment*, not angular momentum.

2.1 Electric multipoles

- Monopole moment: $L = 0$, this is just the spherical nucleus, the electric potential looks like $V = \frac{Q}{R}$.
- Dipole moment: $L = 1$. The electric dipole is just linear translation of the nucleus. $V = 0$, see below.
- Quadrupole moment: $L = 2$, the electric potential is from the nucleus squishing in one direction and then the other. The amount of quadrupole moment can be calculated:

$$eQ = e \int \psi^*(3z^2 - r^2)\psi \; d^3\vec{r}$$

  If the nucleus is spherical, $\langle z \rangle^2 = \langle x \rangle^2 = \langle y \rangle^2$, so $eQ = 0$. The higher the value of eQ the more deformed the nucleus is.

All electric moments have parity determined by $(-1)^L$. If you want to do some kind of electromagnetic operation on a state (ψ) with some operator O, then you have to evaluate $\int \psi^* O \psi \; dv$. It doesn't matter what the parity of ψ is because they will always multiply (even times even or odd times odd are both always even). If O has negative parity, the whole thing is an odd function and the integral goes to 0. Therefore, to keep things based in reality, all moments with odd angular momentum must not be a thing because they have negative parity ($L = 1, 3, 5$ etc). This is why the dipole doesn't happen.

2.2 Magnetic multipoles

- Monopole moment: $L = 0$ doesn't happen, as far as we know.
- Dipole moment: $L = 1$, nucleons and nuclei have dipole magnetic moments due to their angular momentum (they are charges moving in a loop, therefore magnetic field) and spin:

$$\mu = \frac{e\hbar}{2m} = (g_l l + g_s s)\mu_n$$

  Here l **is** the angular momentum, g_l is a constant (1 for protons, 0 for neutrons), s is the spin, g_s is a constant (positive for protons and negative for neutrons) and μ_n is the "nuclear magneton" (just a constant).[1, pp. 73]

- Quadrupole moment: $L = 3$ doesn't happen.

Just like before, all the moments with odd parity have to not exist. For magnetic moments, the parity is determined by $(-1)^{L+1}$, so all the *even* order moments can't exist.

3 Shell Model

3.1 Evidence

- Ionization energies "jump" at "magic numbers". Something about these make the nucleus more tightly bound, this wouldn't be seen in a liquid drop [2, Lec. 12].
- α-decay: see a big jump in the α-decay of Radon after $N = 128$. This is because the daughter has $N = 126$ (magic number) [2, Lec. 12].

3.2 Shells

- You can use the 3D square well to get a pretty good approximation of what we see, or a parabolic ($1/r^2$) one to get better. But the simple harmonic oscillator gives the best approximation but it's still wrong. [2, Lec. 12].
- Spin-orbit interaction: due to the interaction of the spin and the angular momentum ($\vec{l} \bullet \vec{s}$), you can get two different values of j, $l \pm \frac{1}{2}$. Now two different nucleons will see a *different* potential, so it splits all the states in two. [2, Lec. 13-16], [1, pp. 123-125].
- The size of the split gets bigger with increasing l. The potential is negative so the $J = l + \frac{1}{2}$ states will occur at *lower* energies. High spin states that enter a different lower shell are called *the intruder*. [2, Lec. 13-16]
- Evidence: Magnetic dipole $\mu = \mu_N(g_l l_z + g_s s_z)/\hbar$ should be different for the different spins in the same l level: observations show this is true. [2, Lec. 13-16].
- Energy levels with a given J have degeneracy $(2J + 1)$, which is how many nucleons there can be in that level.
- Works well for **spherical** nuclei and ones that are **close to magic numbers**: 2, 8, 20, 28, 50, 82, 126, 184.
- The ratio of the 4^+ and 2^+ states is usually close to 1:

$$R_{42} = \frac{E(4^+)}{E(2^+)} \approx 1$$

3.3 Independent Particle Model

- All shells are completely full or empty except for a single particle in the lowest energy of an otherwise empty shell. The nucleus J^π depends only on that last nucleon. [2, Lec. 13-16].
- Expect spherical nucleus, so $eQ \approx 0$, slightly negative for a free proton, very close to 0 for a free neutron.
- Nuclear magnetic moment (μ) is positive and increasing with l for an odd proton, close to 0 and constant with l for neutron.
- The total angular momentum of the nuclei J is equal to the J value of the shell with the single particle in it. The parity is equal to the parity of the shell with the single particle in it, $(-1)^l$. For example, if there is a single particle in the $1p_{3/2}$ state, then $J^\pi = (\frac{3}{2})^-$ because $j = \frac{3}{2}$ and $l = 1$. [2, Lec. 13-16].
- The value J^π is referred to as the spin-parity **or** the total angular momentum of the nucleus. Both of those things mean the same thing *when you're talking about the nucleus.* For a nucleon, spin is the intrinsic angular momentum; the nucleus doesn't have any **intrinsic** angular momentum, only angular momentum due to it's components. We call it spin anyway when referring to the nucleus just to be confusing.
- The independent nucleon always gives the total angular momentum of the shell to the nucleus when in the **ground state.**
- When you are in a **low-lying excited state**, you can line up the nucleons in more interesting way. If you have three nucleons in the $f_{7/2}$ shell and we're in an excited state, we use the notation $(f_{7/2}))^3$. You can add up their angular momentum any way you want, remember each can have angular momentum from $-j$ to $+j$. So these can have $m = \pm\frac{1}{2}, \pm\frac{3}{2}, \pm\frac{5}{2}, \pm\frac{7}{2}$. They can't have the **same** value of m, but they can all add to get, for example $\frac{7}{2} + \frac{5}{2} + \frac{3}{2} = \frac{15}{2}$. So you can get different spin configurations without actually having enough energy to jump the nucleon up to another energy level. [1, pp. 149-151]
- In excited states, the single nucleons can also jump around to other states, changing the J^π of the nucleus. [2, Lec. 13-16]

4 Collective Motion

- Collective nuclear motion comes from the interplay between nucleons making the nuclear potential, and nucleons moving in the nuclear potential. [2, Lec. 13-16]
- Collective motion lowers the energy for the first excited state (2^+ for vibration and rotation). Instead of having to yank a nucleon from a low, tightly bound state, to a high state, you just put enough energy into the system to get all the nucleons vibrating or rotating together. So nuclei where collective motion is seen have much, much lower first excited states.
- Rotations and vibrations and not mutually exclusive. A rotational band can "ride" on top of a vibration, or vice versa.

4.1 Vibration

- A vibration can move over the surface of the nucleus. The order of this vibration is given by lambda (λ), this determines the order of the spherical harmonic that describes the collective vibration.
  - The $\lambda = 1$ vibration is just dipole movement, which is just linear movement of the nucleus. This is not a collective motion state.
  - The $\lambda = 2$ vibration is the quadrupole vibration. The energy is quantized in the *phonon* which is just a discrete unit of vibrational energy. For $\lambda = 2$, each carries exactly $2\hbar\omega$ units of vibrational energy.

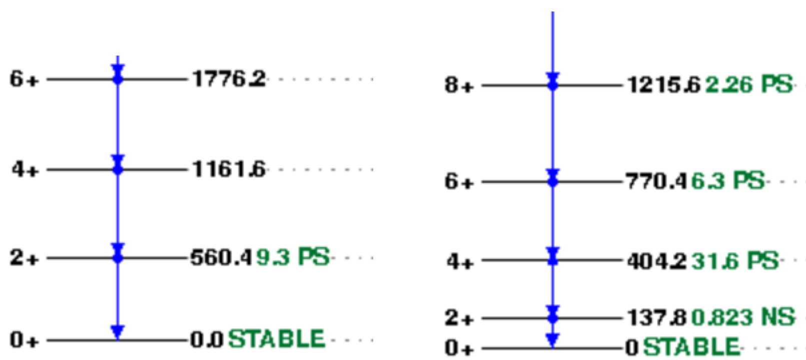
– The $\lambda = 3$ vibration is octopole and I'm pretty sure Lee said this hasn't been observed.

[2, Lec 13-16]

- Generally occurs for large nuclei with $150 < A < 190$ and $A > 230$..
- Magnetic moment $\mu = 2Z/A$
- A vibration collective structure on top of the 0^+ ground state will be a 2^+ level, followed by a 4^+ level, followed by 6^+, etc. They will be *evenly spaced* in energy, because each level represents the addition of one phonon and each phonon has the same energy. The parity doesn't change because $\lambda = 2$ and parity goes like $(-1)^\lambda$. [2, Lec 13-16]
- Therefore, he ratio of the 4^+ and 2^+ states should almost always be:

$$R_{42} = \frac{E(4^+)}{E(2^+)} \approx 2$$

- For vibration, we expect the quadropole moment (the measure of nuclear deformation) to be $eQ(2^+) = 0$. This is because the nucleus is just a sphere with a surface wave moving over it, so it all averages out to 0. [2, Lec 13-16]



(a) (b)

Figure 1: (a) Shows vibrational structure in Te-120. Notice that the 2^+, 4^+, and 6^+ states are all evenly spaced. (b) Shows a rotational structure in Dy-156. The levels are not evenly spaced, but get larger with I (Both images are from www.nndc.bnl.gov).

4.2 Rotation

- Nuclei with a lot of valence protons and neutrons may statically deform. This is seen with promiscuity:

$$P = \frac{N_p N_n}{N_p + N_n}$$

Where N_p is the number of valence protons, and N_n is the number of valence neutrons. Valence nucleons are the extra nucleons after the last large shell. Around P>4, you see deformation. [2, Lec. 13-16]

- Generally occurs in the ranges $150 < A < 190$ and $A > 230$.
- Expect $eQ \neq 0$ due to deformation of nucleus.
- Magnetic moment $\mu = 2Z/A$
- The deformation parameter β tells you how much it is deformed. [2, Lec. 13-16]

 - $\beta > 0$: Prolate, rotating around the major axis (egg).
 - $\beta < 0$: Oblate, rotating around the minor axis (pumpkin)

- The energy of the states is proportional to the J of the state, and the moment of inertia ($\Im$).

$$E = \frac{\hbar^2}{2\Im} J(J+1)$$

Instead of the angular momentum squared, like in classic mechanics, it is $J(J+1)$ because this is quantum mechanics and that's how it works. Get over it. The energy for rotation levels is therefore tied to the angular momentum J of the level. The values in Table 1 look confusing but bear with me.

Level	**E**	E_γ	ΔE_γ
0^+	0		
2^+	$6(\hbar^2/2\Im)$	$6(\hbar^2/2\Im)$	$8(\hbar^2/2\Im)$
4^+	$20(\hbar^2/2\Im)$	$14(\hbar^2/2\Im)$	$8(\hbar^2/2\Im)$
6^+	$42(\hbar^2/2\Im)$	$22(\hbar^2/2\Im)$	$8(\hbar^2/2\Im)$
8^+	$72(\hbar^2/2\Im)$	$30(\hbar^2/2\Im)$	

Table 1: Rotational energy levels.

The table shows energy levels (E) based on the equation above for a bunch of levels in the nucleus. The E values go up faster than linearly, (6, 20, 42, etc). The next column shows the γ-ray energy that is emitted to go from a given state to the *next lower state*. Those also go up, but with a constant slope. This is shown in the final column, where the ΔE_γ is shown. This means that you'll see an equally spaced series of γ-rays emitted by a rotating nucleus. You can see this in practice if you calculate out the energy differences in the Dy-156 rotational levels in Figure 1. You'll find that every level difference is getting larger by about 100 keV every time. [2, Lec 13-16]

- The difference between two energy levels is given by (from the homework):

$$\Delta E = E_{J^\pi} - E_{(J-1)^\pi} = \frac{\hbar^2 J}{\Im}$$

- All the energy for the rotational band levels are tied up in the rotation. The nucleus does not actually possess any of the energy, and it doesn't affect its state. Much like the Earth's rotation doesn't affect you. This means that the wave functions between rotation states are almost identical, and so there is an enhanced probability of γ-decay between them. Also, the energy from rotation will never cause particle emission, because the rotation hasn't "heated up" the nucleus. It can just "slow down" via γ emission.[2, Lec. 13-16]

- The ratio of the 4^+ and 2^+ states should almost always be close to:

$$R_{42} = \frac{E(4^+)}{E(2^+)} \approx 3.3$$

- Deformation also leads to breaking of degenerate J substates. Just stick with me:
 - Every state has an associated J value (for example, a $p_{3/2}$ state has $J = 3/2$).
 - $(2J+1)$ nucleons can live there because they can have their own m_j values that range from $-j \dots j$, by integer values. The energy level has a *degeneracy* of $(2J+1)$.

> **Note:** Per Andrew, there won't be anything on the Nielsen model on the exam. This is the following few bullets about the energy levels splitting in a deformed nucleus.

– When the nucleus becomes deformed, this isn't true anymore. Now that there is a preferred direction (the axis of rotation), nucleons with different values of m_j will split out.
– The level splits into all of it's different components. The $+j$ and $-j$ components have the same energy because the nucleus is symmetric.
– The energy levels now depend on how long the nucleon spends close or far from the nucleus, depending on it's angular momentum. See Figure 2.

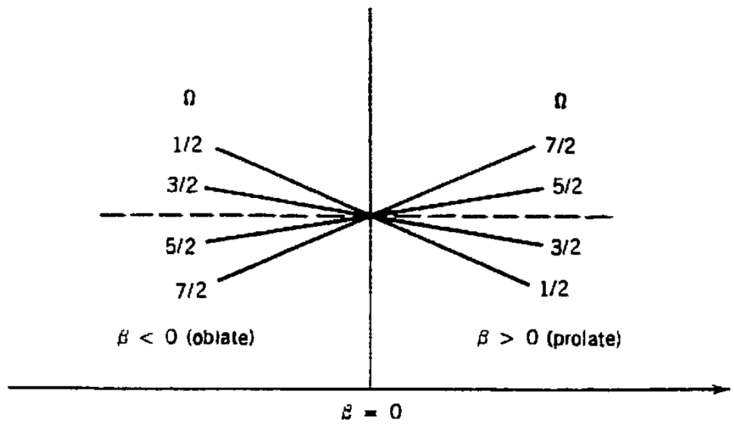


Figure 2: Krane Figure 5.27, splitting of the levels of the $f_{7/2}$ orbit in a deformed nucleus.[1, pp. 153]

[1, pp.151-153]

5 α-decay

- Preferred by heavy nuclei to remove 5-8 MeV through decrease of mass. This is a coulomb repulsion effect and is preferred over other particles because it is very efficient (B/A).
- The α-decay half-life can be estimated using a barrier tunneling model. You can calculate the frequency of collision with the potential well and use the transmission probability to calculate a half-life. This isn't the real model and only represents an electromagnetic baseline to give us insight into what is happening. It isn't *nuclear*. [2, Lec. 18]
- Any angular momentum carried away by the α-particle is **purely orbital**. The spins all couple pairwise to 0. The angular momentum change from α-decay can range from:

$$|I_i - I_f| \leq l_\alpha \leq I_i + I_f$$

With parity change:

$$\Delta\pi = (-1)^{l_\alpha}$$

The simplest decay is any $0 \to X$ state, because the only value for l_α is X. If that doesn't give the right parity change, the decay is *forbidden*. In this case it really means *forbidden*, unlike β-decay, which plays fast and loose with the term. [1, pp. 257-258].

- α-decay can populate many different states. Each one has a different Q-value (given by the Q value for decay to the ground state, minus the excitation energy).

$$Q = Q_0 - E_{ex}$$

Where Q_0 is the Q value to the ground state and E_{ex} is the excitation energy of the excited daughter nucleus. [1, pp. 257-258].

- The Q-value also gives you the total energy of the two decay fragments:

$$Q = T_{X'} + T_\alpha$$

If the original nucleus X was at rest, you can use conservation of momentum ($p_\alpha = P_{X'}$) to get:

$$T_\alpha = \frac{Q}{(1 + m_\alpha/m_{X'})}$$

$$= Q(1 - 4/A) \quad \text{with } A \gg 4$$

The α particle usually has 98% of the Q value. The typical Q value is about 5 MeV. [1, pp. 248]

- The intensity depends on how similar the initial and final wave functions are, and the angular momentum (l_α). The higher the l_α, the less likely the decay. This is because the coulomb barrier is higher with larger angular momentum due to the centrifugal term. [1, pp. 257-258].

- The hinderence factor (HF) tells us about the similarity between the parent and daughter states.:
  - $1 < H < 4$ Favored: α-particle is from two low-lying pairs of nucleons.
  - $4 < H < 10$: favorable overlap between initial and final state.
  - $10 < H < 100$: spin projections are parallel. Unfavorable overlap between initial and final states.
  - $100 < H < 1000$: spin projections are parallel, change in parity.
  - $H > 1000$: spin-flip, change in parity.

Any spin flip $\rightarrow$ over 1000. Any parity change $\rightarrow$ over 100. Non-favorable overlap $\rightarrow$ over 10. See Table. 2. [2, Lec. 18]

H	**Nuclear State Overlap**	**Parity Change**	**Spins**
<10	Favorable	None	Parallel
10–100	Unfavorable	None	Parallel
100–1000	Unfavorable	Yes	Parallel
>1000	Unfavorable	Yes	Spin-flip

Table 2: Summary of α-decay hinderance factors.

- α-decay spectrum has sharp peaks, each represents a decay to different excited states. Higher E_α, lower excitation energy. The excited states then usually release γ-ray photons. [1, pp. 262-264]

- In an even-even nucleus, the alpha decay to the ground state is usually *very strong*. The $0^+ \rightarrow 0^+$ decay is not inhibited by differences between the wave functions (they are very similar. By the same logic, the decay of odd-A nuclei may be *very weak* because the two states may be significantly different. [1, pp.264]

6 β-decay

- Preferred by nuclei that want to move down the mass parabola to find the optimal charge ratio. This relates directly to the Semi-Empirical-Mass-Formula. β-decay can do a number of things to increase the Binding Energy of the nucleus: increase symmetry to reduce ($a_{sym}(A - 2Z)^2/A$), decrease the repulsion due to protons, and increase the amount of pairing.

- Energy release distribution is continuous because there are two particles emitted so they can share any proportion of the energy. [1, 2, pp. 273, Lec 19]
- Three kinds:

$$\beta^- : n \rightarrow p + e^- + \bar{\nu}_e$$
$$\beta^+ : p \rightarrow n + e^+ + \nu_e$$
$$EC : p + e^- \rightarrow n + \nu_e$$

Each has a different energy release:

$$Q_{\beta^-} = [m(^AX) - m(^AX')]c^2$$
$$Q_{\beta^+} = [m(^AX) - m(^AX') - 2m_e]c^2$$
$$Q_\epsilon = [m(^AX) - m(^AX')]c^2 - B_n$$

For β^-, the electron masses all cancel when using atomic masses because $-Zm_e + (Z+1)m_e - m_e = 0$, that is the electrons accounted for in the atomic masses for the original atom, final atom and leftover electron all cancel. They don't cancel for β^+ because $-Zm_e + (Z-1)m_e - m_e = -2m_e$. This means that β^+ decay has a threshold energy, because the difference in masses must be larger than $2m_e$ for the Q to be greater than 0. Electron capture has a threshold energy equal to the binding energy of the electron B_n. [1, pp. 274-276]

- The binding energy can be estimated using:

$$B_n = 13.6eV\frac{Z^2}{n^2}$$

where n is the principle number for the electron.

6.1 Fermi Theory

- There are three things that go into determining the spectrum of beta decay energy values:

$$N(p) \propto \text{Statistical factor} * \text{Coulomb Interaction (electrodynamic)} * \text{Nuclear Matrix Element}$$
$$N(p) \propto p^2(Q_\beta - T_e)^2 * F(Z_d, T_e) * |M_{fi}|^2 * S(p, q)$$

These terms are:

  - Statistical factor: related to the number of final states accessible. Decays like to happen when there are a lot of final states that are open.
  - Fermi function (F): the influence of the atomic coulomb field. This is the **electrodynamic** part.
  - Matrix element ($M_{fi}^2 * S(p, q)$): This is the **Nuclear** part. It is kind of like the hinderance factors for α-decay and represents the effect of the initial and final states. The S function only comes into play in **forbidden** decays, when the electron and neutrino have angular momentum ($l \neq 0$, s and q are terms for the electron and neutrino angular momenta.

[1, pp.281-282].

- You can plot:

$$(Q - T_e) \propto \sqrt{\frac{N(p)}{p^2F(Z', p)}}$$

Which *should* give you a straight line (Fermi-Kurie plot) that intersects the x-axis at the Q value. For forbidden decays, the line is **not** straight, you have to put that $S(p, q)$ factor in on the bottom to make it straight. This is the *shape factor*, which then gives you a straight line. [1, pp. 282]

- Total decay rate: you can integrate some messy function to get a messy function for total decay rate:

$$\lambda = \text{Matrix Element Term} * \int F\text{-function and statistical factor}$$

The full equation is in Krane, page 282. What is important is that the term outside the $\int$ is the **nuclear** part, and the integral is the **electrodynamic** part. Those are the two things (and some constants) that determine the decay rate. You can just crunch the integral and look up values, which is f, or the *Fermi Integral*. Then, if you do $\frac{f}{\lambda}$ or just $ft_{1/2}$, then the integrals cancel and all you have is the Matrix Element part, the **nuclear** part. This lets us compare $ft_{1/2}$ values, which are *only dependent on nuclear properties*. It might be useful to think about the ft value as just a *corrected* half-life. Corrected to remove electrodynamic stuff. [1, 2, pp. 282-283, Lecs. 19-21].

- ft values have a huge range (10^3 to 10^{20}) so we use $\log ft$ instead.

6.2 Allowed Decays

- Electron and neutrino are created at the origin ($r = 0$), so their angular momentum is $l = 0$. There are two modes based on spin:
  - **Fermi Decay**: the spins of the neutrino and electron are anti-parallel, so total $S = 0$.
  - **Gamow-Teller Decay:** the spins are parallel, so the total $S = 1$.

  The difference in the nuclear spin parity (I^π) before and after the decay will come from the l and S that the electron and neutrino carry away. So, if you have Fermi decay ($l = 0$ and $S = 0$), there can't be any change in the nuclear spin $\Delta I = |I_i - I_t| = 0$. If you have Gamow-Teller decay ($l = 0$ and $S = 1$), you have $\Delta I = 0$ or 1.

> **TRICKY PART:** You can have $\Delta I = 0$ in Gamow-Teller decay because you can couple a vector of length 1 onto a vector and end up with the same vector magnitude. This **does not work** if you started with $I_i = 0$ because you can't add 1 to 0 and get 0. This means that a beta decay from $I_i = 0$ to $I_f = 0$ *must* be Fermi decay.

Therefore, all allowed decays have:

$$\Delta I = 0, 1 \quad \Delta\pi = \text{no}$$

[1, 2, pp. 289, Lec. 19-21]

- **Superallowed Decays:** all super-allowed decays are allowed decays, but not all allowed decays are super-allowed. All that super-allowed means is that the ft value is 3-4. So they are just super-likely.

6.3 Forbidden Decays

- Any beta-decay that has $l \neq 0$ is **forbidden**. This doesn't mean that it doesn't happen, it just means it's not very likely. This is because angular momentum $\vec{l} = \vec{r} \bullet \vec{p}$. The radius where the β-decay is on the order of the nuclear radius (≈ 6 fm) which is *super* small. The small radius means a small angular momentum, so it's very unlikely that β-decay will occur with $l \neq 0$. But it does, rarely. [2, Lec. 19-21].

- The n^{th} forbidden decay will have $l = n$, and both of the spin-arrangements (Fermi for $S = 0$ and Gamow-Teller for $S = 1$) described above. Therefore, you can have:

$$\Delta I = 0, 1, 2 \ldots (l + 1) \quad \Delta\pi = (-1)^l$$

The ΔI goes up to $l + 1$ because Gamow-Teller can have the spin $S = 1$ lined up with the angular momentum. [1, 2, pp.291,Lec.19-21]

- β-decay will occur via the "lowest" decay that it can. That is, it will use an allowed decay or the lowest nth forbidden decay that can accomplish the transition. You have to look at *both* parts of the initial and final states, I **AND** $\Delta\pi$ to figure out which it is. Also, β-decay tries to populate states the result in the largest energy release (it wants to get to the ground state). [2, Lec 19-21]
- Based on the description above, as you get higher in the forbidden decays, 1st forbidden to 2nd forbidden etc, the decays become less likely. See Table 3. [2, Lec. 19-21]

Type of β-decay	**Log(ft)**
Superallowed	3.5
Allowed	4-7.5
1st forbidden	6-9
2nd forbidden	10-13
3rd forbidden	14-20
4th forbidden	≈ 23

Table 3: Log(ft) values for types of β-decay

6.4 Electron Capture

- Almost always, the electron is captured from the inner-most S orbital, and the neutrino emitted is mono-energetic. Auger electrons may result from the cascade of electrons as they move down to fill the vacancy left by the lower shells. [2, Lec. 19-21]

6.5 Helicity

- Helicity: helicity is just a property that we define based on a particles spin and momentum:

$$h = \frac{\vec{s} \bullet \vec{p}}{|\vec{s} \bullet \vec{p}|}$$

  Neutrinos have specific helicity based on if they are a neutrino or anti-neutrino:

  - **ALL** neutrinos (ν) have helicity $h = -1$
  - **ALL** anti-neutrinos ($\bar{\nu}$) have helicity $h = +1$
  - All electrons **from β-decay** have helicity $h = -\frac{v}{c}$.
  - All positrons **from β-decay** have helicity $h = +\frac{v}{c}$.

  Note that the helicity rules for neutrinos are always always true, but the ones for electrons/positrons are only true *if the electron is from β-decay.*

7 γ-decay

- γ-rays are photons generated from the decay between two nuclear states. The radiation comes from the electric and magnetic fields generated by charge and current in the nucleus. Therefore, there can be radiation caused by electric or magnetic fields. [1, pp.330]
- The radiation fields have multipole orders, as before ($L = 1$ is dipole, $L = 2$ is quadrupole, etc). The parity of the radiation fields are:

$$\pi(ML) = (-1)^{L+1}$$
$$\pi(EL) = (-1)^{L}$$

  [1, pp.330]

- Therefore, for a given ML or EL transition, we can calculate the allowable range of angular momentum change between the initial and final states:

$$|I_i - I_f| \leq L \leq I_i + I_f \quad (\text{no } L = 0)$$

$$\Delta\pi = \text{no: even electric, odd magnetic}$$

$$\Delta\pi = \text{yes: odd electric, even magnetic}$$

  There are no $L = 0$ transitions because the monopole moment is just the electric field, which does not vary with time. Therefore, the $E0$ and $M0$ transitions are not possible *by the emission of a single photon* (it is possible with internal conversion). [1, pp. 334]

- The lowest L-value transition is usually the most probable, raising the value of L by one makes the next transition on the order of 10^5 times less probable. Electric transitions are about 100 times more likely than magnetic transitions. [1, pp. 335]

- Weiskoff estimates: using the model in which a single particle releases a single photon, we can calculate decay rates. These may be *way* off because the state that is decaying may be some collective motion of all the nucleons. Therefore, the standing wave that is holding the energy of the state may constructively interfere and release a γ much faster than the estimate would say. Or, it could slow down the decay if it adds destructively. [2, Lec. 22]

7.1 Internal Conversion

- An electromagnetic process that occurs with γ-decay, and "competes" to be mechanism for decay. The nucleus does not emit a photon, but the fields eject an electron from the nucleus. This creates peaks on the β^- energy spectrum (because it's releasing electrons at finite energies). [2, pp. 341]

- If the ΔJ is large, internal conversion becomes much more probable compared to γ-decay, because a γ-decay with such high L is very rare. It is also more probable if the ΔE between the states is close to x-ray energy (10-100 keV). It gets much less probable when the energy gets higher (5 MeV) or high electron shell (K, L, M).

- The total decay probability is due in some part to the regular γ-decay, as well as the internal conversion (multiple holes in the same bucket). We define a ratio of decay probabilities α:

$$\alpha = \frac{\lambda_e}{\lambda_\gamma}$$

  It is more useful in this form:

$$\lambda_t = \lambda_\gamma(1 + \alpha)$$

  The α can be split up into specific values for each shell, α_K, α_L, α_M, etc. In that case you just add them all up to get the total α.

- The value of α varies with Z, n_{atom} and is best defined between $B_e < E_\gamma < 2m_ec^2$. Around B_e the value of α varies wildly, and above $2m_ec^2$, pair production becomes important. [2, Lec. 24]

8 EM interaction with matter

- **Photoelectric Effect:** An incident photon interacts with an electron and ejects it (called a photoelectron) and radiation. This is much more probable with outer electrons, as they have much lower binding energies. Inner shell electrons have a much lower probability of becoming photoelectrons, so they have a much lower *cross section* for this kind of absorption. [2, Lec. 24]

- **Comptom Scattering:** Dominates at lower energies than Photoelectric effect, photon scatters off electron. [2, Lec. 24]

- **Pair Production:** Dominates at high energies $E_\gamma > 1.022$MeV, the photon forms and electron and positron. (1.022 MeV = $2m_e$).

8.1 Isomerism

- Look at the pictures, this is all from Lecture 24 [2, Lec. 24]
- An Isomeric state is a long lived excited state that exists for three reasons.
- Shape: transition is inhibited by a change in shape (deformed to round), sometimes seen as a long lived $0^+ \to 0^+$ transition.
- K-trap: Once deformed (and only when deformed) if the momentum vector k is projected along the axis of rotation, it will stay in this state.
- Spin isomer: if there is max alignment in a high angular momentum state due to spins, the emitted photon would need to have a large multipole moment (large L), which is rare.

9 Reactions

- The reaction:
$$a + X \to Y + b$$
Can be written in reaction notation as:
$$X(a, b)Y$$
[1, pp. 378-379]
- A microscopic cross section (σ) represents the "relative probability for the reaction to occur." It can be used in the following equation:
$$R = (\rho d)_{target} \times I_{beam} \times \sigma_{reaction}$$
Where R is the reaction rate (in reactions/sec); (ρd) is the density (in g/cm^3) times the width of the target (in cm), also known as the areal density (put it in atoms/cm^2); I_{beam} is the incident particle flux (in atoms/sec); and $\sigma_{reaction}$ is the microscopic cross section of the reaction occurring. This is only valid when very little of the beam reacts (small σ) and everything moves in straight lines. It can also be expressed as:
$$R = N\phi\sigma$$
Where N is the number of target atoms, ϕ is the flux in (atoms/sec/cm^2) and σ is the same. [2, Lec. 25]
- Microscopic cross sections are generally given in units of barns. 1 barn = 10^{-24} cm^2.
- The cross section is not always constant over angle (it rarely is). So the *differential cross section* is used:
$$\frac{d\sigma}{d\Omega}$$
What is confusing, is this is just a number, in units of barns/steradian. It's representing the fact that some small number of particles ($d\sigma$) will strike our small detector ($d\Omega$). It is dependent on the angle of scatter (θ) and the polraization of the radiation (ϕ). Generally we assume there is no affect due to polarization (things are randomly polarized).

  We can find the size of our detector $d\Omega$ in steradians, which is related to the area of our detector (dA) and the distance from the target (r) by:
$$d\Omega = \frac{dA}{r^2}$$
Then, if we know the differential cross section at the angle of our detector, we can multiply to get the reaction cross section for our detector:
$$\sigma_{det} = d\Omega \frac{d\sigma}{d\Omega}$$

This represents something **very specific**. This is the probability that incoming particles striking the target will then be detected by our detector. Based on the size of our detector ($d\Omega$) and our a priori knowledge of the number of particles that will be seen in a small area ($\frac{d\sigma}{d\Omega}$). The value of that differential cross section will probably vary with angle, so you have to know the differential cross section for the angle where your detector is to even use this. More rigorously, you'd integrate over the area of the detector and $\frac{d\sigma}{d\Omega}$ may vary over the integral:

$$\sigma_{det} = \int_{detector} \frac{d\sigma}{d\Omega} d\Omega$$

Or, you can get the total σ by integrating over the whole angle space.

- **Rutherford Differential Scattering:** elastic Coulomb scattering. An incoming particle scatters off the potential of the target. [2, Lec 24]
- **Coulomb Excitation:** inelastic Coulomb scattering. An incoming particle scatters off the potential of a target and leaves some energy behind. This "Coulex" reaction can excite nuclei up rotational bands. [2, Lec. 24]
- Reaction Q-value is to create the final products at rest.
  - Center of Mass Frame: products are at rest, $Q = Q$.
  - Lab Frame: products are *not* at rest. Threshold energy for reaction is:

    $$E_{\text{threshold}} = Q\left(\frac{m_a + m_x}{m_x}\right)$$

    for a(X,Y)b.
- Conserved quantities in reactions:
  - Total energy
  - Linear momentum
  - Angular momentum
  - Parity $(-1)^l$ (except in weak interactions)
- **Kinematics** For a reaction, $X(a,b)Y$:

  $$Q = (m_X + m_a - m_Y - m_b)c^2$$
  $$Q = T_Y + T_b - T_X - T_a$$

- Exothermic Q>0 :

  $$m_X + m_a > m_Y + m_b$$
  $$T_Y + T_b > T_X + T_a$$

- Endothermic Q<0 :

  $$m_X + m_a < m_Y + m_b$$
  $$T_Y + T_b < T_X + T_a$$

- Reaction reaches excited states of Y:

  $$Q_{ex} = (m_X + m_a - m_{Y*} - m_b)c^2 = Q_0 - E_{ex}$$

- Compound nucleus:

  $$Q = -T_a = (m_X + m_a - m_{C*})c^2 - E_{ex}$$

References

[1] Kenneth S. Krane. Introductory Nuclear Physics. John Wiley & Sons, Inc., 3rd edition, 1988.

[2] Lee Bernstein. Nuclear engineering class lectures. Fall 2015.