Nuclear Engineering 101: Final Study Guide

Disclaimer: This is not an official study guide. Stuff ~~might~~ **is** wrong. Use the lecture notes and book!

Note: Everything in this guide is from the text (Krane) or lecture, or office hours and should be cited as completely as possible.

Contents

1 Reactions

1.1 General Information

- The reaction:

$$a + X \rightarrow Y + b$$

Can be written in reaction notation as:

$$X(a, b)Y$$

[1, pp. 378-379]

- A microscopic cross section (σ) represents the "relative probability for the reaction to occur." It can be used in the following equation:

$$R = (\rho d)_{target} \times I_{beam} \times \sigma_{reaction}$$

Where R is the reaction rate (in reactions/sec); (ρd) is the density (in g/cm^3) times the width of the target (in cm), also known as the areal density (put it in atoms/cm^2); I_{beam} is the incident particle flux (in atoms/sec); and $\sigma_{reaction}$ is the microscopic cross section of the reaction occurring. This is only valid when very little of the beam reacts (small σ) and everything moves in straight lines. It can also be expressed as:

$$R = N\phi\sigma$$

Where N is the number of target atoms, ϕ is the flux in (atoms/sec/cm^2) and σ is the same. [2, Lec. 25]

- Microscopic cross sections are generally given in units of barns. 1 barn = 10^{-24} cm^2.
- The cross section is not always constant over angle (it rarely is). So the *differential cross section* is used:
$$\frac{d\sigma}{d\Omega}$$
What is confusing, is this is just a number, in units of barns/steradian. It's representing the fact that some small number of particles ($d\sigma$) will strike our small detector ($d\Omega$). It is dependent on the angle of scatter (θ) and the polraization of the radiation (ϕ). Generally we assume there is no affect due to polarization (things are randomly polarized).
We can find the size of our detector $d\Omega$ in steradians, which is related to the area of our detector (dA) and the distance from the target (r) by:
$$d\Omega = \frac{dA}{r^2}$$
Then, if we know the differential cross section at the angle of our detector, we can multiply to get the reaction cross section for our detector:
$$\sigma_{det} = d\Omega \frac{d\sigma}{d\Omega}$$
This represents something **very specific**. This is the probability that incoming particles striking the target will then be detected by our detector. Based on the size of our detector ($d\Omega$) and our a priori knowledge of the number of particles that will be seen in a small area ($\frac{d\sigma}{d\Omega}$). The value of that differential cross section will probably vary with angle, so you have to know the differential cross section for the angle where your detector is to even use this. More rigorously, you'd integrate over the area of the detector and $\frac{d\sigma}{d\Omega}$ may vary over the integral:
$$\sigma_{det} = \int_{detector} \frac{d\sigma}{d\Omega} d\Omega$$
Or, you can get the total σ by integrating over the whole angle space.
- Conserved quantities in reactions:
  - Total energy
  - Linear momentum
  - Angular momentum
  - Parity $(-1)^l$ (except in weak interactions)
- **Kinematics** For a reaction, $X(a,b)Y$:
$$Q = (m_X + m_a - m_Y - m_b)c^2$$
$$Q = T_Y + T_b - T_X - T_a$$
- Exothermic Q>0 :
$$m_X + m_a > m_Y + m_b$$
$$T_Y + T_b > T_X + T_a$$
- Endothermic Q<0 :
$$m_X + m_a < m_Y + m_b$$
$$T_Y + T_b < T_X + T_a$$
- Reaction reaches excited states of Y:
$$Q_{ex} = (m_X + m_a - m_{Y*} - m_b)c^2 = Q_0 - E_{ex}$$
- Compound nucleus:
$$Q = -T_a = (m_X + m_a - m_{C*})c^2 - E_{ex}$$

1.2 Photo-nuclear Interactions

- A photon interacts with the nucleus directly. For this to happen, we need to have an energy level at the energy of the incoming photon. [2, Lec 25]
- There are three types of photo-nuclear interactions:
  - Spontaneous emission: if the nucleus is in an energy level, it can release a photon to de-excite. This is an intrinsic property of the level.
  - Resonant absorption: if the incoming photon is at the exact same energy of an energy level, it can be absorbed and the nucleus excited to that state. The energy of the state is the resonance energy.
  - Stimulated emission: One photon goes in, two photons come out. This occurs if the incoming photon is at the resonant value. This is the principle by which lasers work (on the atomic scale), but it has not been seen for nuclei.

  [2, Lec 25]
- The cross section for this to occur (σ_0) is a function of the nucleus' angular momentum, and internal conversion (IC) factors (α). As the probability of IC rises, photon capture becomes more rare, it's hard to make a nucleus capture photons when it wants to eject electrons. [2, Lec 25]
- The width of the emitted state:
$$\Gamma = \frac{\hbar}{\tau}$$
The longer the mean lifetime (τ), the more well defined the energy level's value is. [2, Lec 25]

1.2.1 Resonance Absorption

- The resonance energy can be affected by any recoil that will result from the capture. This is because the nucleus needs both enough energy to be in its new excited state, **and** enough energy to recoil; so the incoming photon needs to have a little bit more than the expected resonance energy. As shown in Figure 1, the resonance energy has been shifted up by the recoil energy E_R from the expected value ΔE. [2, Lec 25]

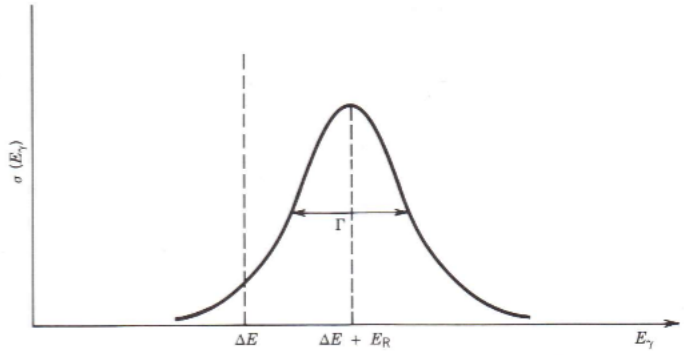

Figure 1: Krane figure 10.23. [1].

- This has an exactly opposite effect on the emission spectrum. The absorption spectrum was shifted *up* because the incoming photon needed extra energy to recoil the nucleus. The emission spectrum is shifted *down* because the recoil takes some of the energy of the emitting photon. [2, Lec. 25]
- **Doppler broadening:** thermal motion makes the nuclei move back and forth, so the incoming photons energy looks doppler shifted either higher or lower. Therefore, a photon with energy just off

the resonance may actually be absorbed because the relative motion can shift its energy to the resonance value. A wider energy range can now be absorbed by the resonance, so the peak gets wider or *broadens*. [1, pp. 363]

- Doppler broadening can can cause overlap between the emission and absorption peaks. [2, Lec. 25]
- **Mossbauer Effect:** If a nuclei is in a crystal lattice, its recoil will be inhibited by the fact that *its stuck in a lattice.* You're not just causing one nuclei to move, but all the ones around it, this makes the recoil energy very low, and therefore minimizes the shift in resonance energy by recoil. This allows you to nail down the *actual* resonance energy. [2, Lec 25]

1.2.2 Giant Dipole Resonance

High energy photons can "ionize" the entire nucleus by creating dipole motion between all the protons and all the neutrons. This means that all the protons moving together and all the neutrons are moving together, and these two groups are in a resonance with each other. This only occurs when the incoming photon energy is very high, >12 MeV. [2, Lec 25]

1.3 Coulomb Scattering

1.3.1 Elastic (Rutherford)

- A particle approaches the nucleus at a distance b (the impact parameter) and scatters off the coulomb potential. The particle follows a hyperbolic path. [1, pp. 396]

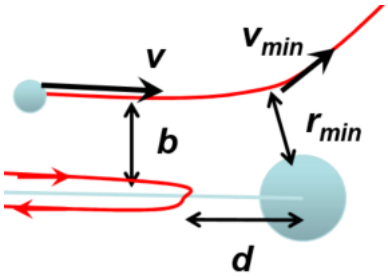

Figure 2: Geometry for elastic coulomb scatter. [2, Lec 24]

- Based on the geometry in Figure 2:

  Incoming particle (at long distances):

$$V = 0 \text{ (far away, the incoming particle has negligible potential energy)}$$
$$T = \frac{1}{2}mv^2$$
$$\ell = mvb$$

  Target particle:

$$V = \frac{Z_1 Z_2 e^2}{4\pi\epsilon_0}\frac{1}{d}$$
$$T = 0$$
$$\ell = 0$$

Incoming particle (at the minimum distance r_{min}):

$$V = \frac{Z_1 Z_2 e^2}{4\pi\epsilon_0}\frac{1}{r_{min}}$$
$$T = \frac{1}{2}mv_{min}^2$$
$$\ell = mv_{min}r_{min}$$

No energy or angular momentum is transferred to the target nucleus (hence elastic scattering). [2, Lec 24]

1.3.2 Inelastic (Coulex)

- **Coulomb Excitation:** inelastic Coulomb scattering. An incoming particle scatters off the potential of a target and leaves some energy behind. This "Coulex" reaction can excite nuclei up rotational bands. [2, Lec. 24]
- Reaction Q-value is to create the final products at rest.
  - Center of Mass Frame: products are at rest, $Q = Q$.
  - Lab Frame: products are *not* at rest. Threshold energy for reaction is:

$$E_{\text{threshold}} = Q\left(\frac{m_a + m_x}{m_x}\right)$$

for a(X,Y)b.

1.4 Direct Reactions

- Occur on time frames of $10^{-21} - 10^{-22}$s. [2, Lec 25]
- These are collisions off the nucleons on the surface. They occur fast enough and with high enough energy that the incoming particle actually "sees" the individual protons and neutrons instead of the nucleus as a whole. This happens because the wavelength of the incoming particle is inversely proportional to momentum, and therefore energy:

$$\lambda = \frac{h}{p}$$

So higher energy particles can interact with individual nucleons. [2, Lec 25]
- Different incoming angles affect the cross sections for reactions. This is because the incoming particle is a wave, and the nucleus is a big wave made up of little waves, so you can have constructive and destructive interference as the two interact. They can add up to make a reaction or less likely based on angle. [2, Lec 25]

1.4.1 Kinematics

For an incoming particle a, outgoing particle b and product, as shown in Figure 3 The momentum is conserved:

$$\vec{p}_{product} = \vec{p}_a - \vec{p}_b$$

Based on the radius R that the incoming particle comes in, there is some amount of angular momentum l transferred to the product (spinning up the nucleus):

$$l = Rp$$

Using conservation of energy, you can also relate the final product momentum (p) to the incoming and outgoing particles:

$$p^2 = p_a^2 + p_b^2 - 2p_a 2p_b \cos\theta$$

If you solve for l you can figure out what J^π the product will be left in. In summary:

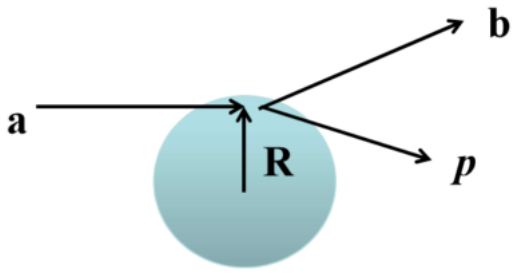

Figure 3: Direction reaction kinematics diagram.

1. Figure out the momentum of the product using the momenta of the products and the angle of collision (that p^2 formula).
2. Solve for the angular momentum transfer l using R ($l = Rp$).
3. Add $l \pm \frac{1}{2}$ to the original J^π of the target to get the final spin.
4. Parity change will go $\Delta\pi = (-1)^l$ where that l is the angular momentum transfer from step 2.

[2, Lec 25]

1.5 Compound Reactions

- Reactions with a definite intermediate state:

  $$a + X \rightarrow C^* \rightarrow Y + b$$

  Where C^* represents the compound nucleus. [1, pp. 416]

- Works best for particles with low incident energy (10-20 MeV), to reduce the chance the particle can escape with its energy and identity. [1, pp. 416]

- Occur on time frames of $10^{-16} - 10^{-18}$ seconds. The time-scale for decay is large compared to the time-scale for formation. This means that nuclei don't "remember" how they were formed; the formation process has no affect on what eventually happens to the nuclei (decay, etc). Only total energy and angular momentum information is retained. [2, Lec 26]

2 Neutron Physics

- Free neutrons are unstable, they will β-decay into protons with a half-life of 10.6 minutes. [1, pp. 444]
- Neutron energies are listed in Table 1.

	Energy
Thermal	$\approx$ 0.025 eV
Epithermal	$\approx$ 1 keV
Fast	100 keV - 10 MeV

Table 1: Neutron Energies [1, pp.445]

2.1 Attenuation

- As neutrons move through a material, they are absorbed and scattered, which will reduce the overall intensity of the beam. We consider scattered neutrons to be gone because they generally scatter away from the beam, so we don't see them anymore. Also, we usually define intensity with a given energy, so scattering lowers their energy and they leave our intensity.
- The loss of intensity (I) of neutrons of a *given energy* in a distance dx of material:

  $$dI = -I\sigma_t n dx$$

  Where σ_t is the total cross section of the material (absorption plus scattering) and n is the number of atoms per unit volume of the material. To make this useful, solve:

  $$I = I_0 e^{-\sigma_t n x}$$

  Where x is the distance the neutrons are traveling through the material. Remember, this isn't a decrease in the total number of neutrons, just the ones in our beam with the amount of energy we shot them into the material with. The scattering will make lower-energy neutrons, so they aren't actually gone for reals. [1, pp. 448]

2.2 Collisions

- The main process by which neutrons slow down (lose energy, also called the process of moderation) is collisions.
- In an elastic collision with a nucleus of atomic mass A, a neutron with incoming energy E will have final energy:

  $$\frac{E'}{E} = \frac{A^2 + 1 + 2A\cos\theta}{(A+1)^2}$$

  [1, pp.448]
- The maximum energy loss is from a head-on collision ($\theta = 180°$), where the above equation reduces to:

  $$\left(\frac{E'}{E}\right)_{min} = \left(\frac{A-1}{A+1}\right)^2$$

  This is the "min" value because it's the minimum final energy (E') that the neutron can have after the collision. As A gets larger, the energy transferred goes down, the neutron just kind of grazes off. The equation is maximized for $A = 1$ (hydrogen) which is why water is so good at moderating. [1, pp. 448]
- We usually use the average energy lost after each collision (actually, the log of that amount), represented by squiggle (ξ) (This is actually the greek letter *xi* but no one knows how to pronounce that. If you do, shut up it's squiggle now).

  $$\xi = \left[\log \frac{E}{E'}\right]_{av}$$

  This can be related to the final energy after the neutron after n collisions:

  $$\log E'_n = \log E - n\xi$$

  This is good for problems where you want to figure out how many collisions are required to thermalize a neutron. There is a complex equation for the value of ξ assuming isotropic scattering:

  $$\xi = 1 + \frac{(A-1)^2}{2A} \log \frac{A-1}{A+1}$$

  For hydrogen ($A = 1$), $\xi = 1$. [1, pp.449-450]

2.3 Capture

- Most of the time, neutron capture on most massive nuclei form compound nuclei. [2, Lec 26]
- Neutron capture immediately makes a nucleus with energy:

  $$E = S_n + E_n$$

  Where S_n is the neutron separation energy and E_n is the energy of the incoming neutron. [2, Lec 27]
- Neutron capture rates are higher with higher Q values. The Q value in this case is just equal to the neutron separation energy, S_n, so the higher S_n, the higher the chance of capturing a neutron. This is why capture rates near the "valley of stability" are high, because S_n is large. [2, Lec 27]
- Why does this happen? Nuclei like to go to states with lots of available transitions. The larger the Q-value, the more options the nuclei has for the decay that follows. Therefore, the higher the Q value (S_n), the higher the chance of neutron capture. [2, Lec 27]
- You can figure out the energy and spin parity of the nuclei immediately following neutron capture. The final spin of the nuclei is:

  $$I' = I + \ell + s$$

  and the parity change:

  $$\Delta\pi = (-1)^{\ell}$$

  If the neutron is thermal, the captured neutron will probably have no angular momentum ("s-wave capture"). So $I' = I \pm \frac{1}{2}$; unless $I = 0$, in which it is always $I' = \frac{1}{2}$. Parity doesn't change because $\ell = 0$. [1, pp.463]
- After capture, the excited capture state will γ-decay down into all the accessible excited states of the compound nucleus. Many of them are accessible because S_n is usually pretty high. Accessible means any states that the excited state can go to via $E1$ radiation, described below. [1, pp. 463]
- After capturing a neutron, the primary transition that follows is dominated by Electric Dipole, $E1$ ($\Delta I = 0$ or 1 and $\Delta\pi = -1$). The transition will populate all of these possible states. Magnetic dipole radiation and higher multipole radiation are usually present, but are usually far less intense than $E1$. [2, Lec 27]

  Example: If neutron capture results in a compound nucleus in an energy state (at S_n) with $J^{\pi} = 2^+$, the following primary transition will populate states: $1^-, 2^-, 3^-$.
- In summary:
  - A neutron with energy E_n is captured and makes a compound nucleus with energy $E = E_n + S_n$.
  - The compound nucleus has a spin parity related to it's old spin-party (I) and the spin and angular momentum of the neutron.
  - This excited compound nucleus will γ-decay via $E1$ (electric dipole) radiation down to many other states. (Primary γ-rays)
  - Unless it went down to the ground state, more decays will occur. (Secondary γ-rays)

3 Fission

4 Fusion

References

[1] Kenneth S. Krane. Introductory Nuclear Physics. John Wiley & Sons, Inc., 3rd edition, 1988.

[2] Lee Bernstein. Nuclear engineering class lectures. Fall 2015.