Nuclear Engineering 101: Midterm 2 Study Guide

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Nuclear Electromagnetic Moments

Ref: [1, pp. 71-75],[2, Lec 10]
A distribution of electric charge and/or current will create an electric potential. This can be described by the "multipole moment".

Note the values of L are the *order of the moment*, not angular momentum.

Electric multipoles

- Monopole moment: $L = 0$, this is just the spherical nucleus, the electric potential looks like $V = \frac{Q}{R}$.
- Dipole moment: $L = 1$. The electric dipole is just linear translation of the nucleus. $V = 0$, see below.
- Quadrupole moment: $L = 2$, the electric potential is from the nucleus squishing in one direction and then the other. The amount of quadrupole moment can be calculated:

$$eQ = e\int \psi^*(3z^2 - r^2)\psi \; d^3\vec{r}$$

  If the nucleus is spherical, $\langle z\rangle^2 = \langle x\rangle^2 = \langle y\rangle^2$, so $eQ = 0$. The higher the value of eQ the more deformed the nucleus is.

All electric moments have parity determined by $(-1)^L$. If you want to do some kind of electromagnetic operation on a state (ψ) with some operator O, then you have to evaluate $\int \psi^* O\psi \; dv$. It doesn't matter what the parity of ψ is because they will always multiply (even times even or odd times odd are both always even). If O has negative parity, the whole thing is an odd function and the integral goes to 0. Therefore, to keep things based in reality, all moments with odd angular momentum must not be a thing because they have negative parity ($L = 1, 3, 5$ etc). This is why the dipole doesn't happen.

Magnetic multipoles

- Monopole moment: $L = 0$ doesn't happen, as far as we know.
- Dipole moment: $L = 1$, nucleons and nuclei have dipole magnetic moments due to their angular momentum (they are charges moving in a loop, therefore magnetic field).

$$\mu = \frac{e\hbar}{2m}l = g_l l \mu_n$$

  Here l **is** the angular momentum, g_l is a constant (1 for protons, 0 for neutrons) and μ_n is the "nuclear magneton" (just a constant).

- Quadrupole moment: $L = 3$ doesn't happen.

Just like before, all the moments with odd parity have to not exist. For magnetic moments, the parity is determined by $(-1)^{L+1}$, so all the *even* order moments can't exist.

Shell Model

Evidence

- Ionization energies "jump" at "magic numbers" like 2, 10, 18, 36, 54, 86. Something about these make the nucleus more tightly bound, this wouldn't be seen in a liquid drop [2, Lec. 12].
- α-decay: see a big jump in the α-decay of Radon after $N = 128$. This is because the daughter has $N = 126$ (magic number) [2, Lec. 12].

Shells

- You can use the 3D square well to get a pretty good approximation of what we see, or a parabolic ($1/r^2$) one to get better. But the simple harmonic oscillator gives the best approximation but it's still wrong. [2, Lec. 12].
- Spin-orbit interaction: due to the interaction of the spin and the angular momentum ($\vec{l} \bullet \vec{s}$), you can get two different values of j, $l \pm \frac{1}{2}$. Now two different nucleons will see a *different* potential, so it splits all the states in two. [2, Lec. 13-16], [1, pp. 123-125].
- The size of the split gets bigger with increasing l. The potential is negative so the $J = l + \frac{1}{2}$ states will occur at *lower* energies. High spin states that enter a different lower shell are called *the intruder*. [2, Lec. 13-16]
- Evidence: Magnetic dipole $\mu = \mu_N(g_l l_z + g_s s_z)/\hbar$ should be different for the different spins in the same l level. Observations show this is true. [2, Lec. 13-16].
- Works well for **spherical** nuclei and ones that are **close to magic numbers**.

Independent Particle Model

- All shells are completely full or empty except for a single particle in the lowest energy of an otherwise empty shell. The nucleus J^π depends only on that last nucleon. [2, Lec. 13-16].
- The total angular momentum of the nuclei J is equal to the J value of the shell with the single particle in it. The parity is equal to the parity of the shell with the single particle in it, $(-1)^l$. For example, if there is a single particle in the $1p_{3/2}$ state, then $J^\pi = (\frac{3}{2})^-$ because $j = \frac{3}{2}$ and $l = 1$. [2, Lec. 13-16].
- The value J^π is referred to as the spin-parity **or** the total angular momentum of the nucleus. Both of those things mean the same thing *when you're talking about the nucleus.* For a nucleon, spin is the intrinsic angular momentum; the nucleus doesn't have any **intrinsic** angular momentum, only angular momentum due to it's components. We call it spin anyway when referring to the nucleus just to be confusing.
- The independent nucleon always gives the total angular momentum of the shell to the nucleus when in the **ground state.**
- When you are in an **excited state**, you can line up the nucleons in more interesting way. If you have three nucleons in the $f_{7/2}$ shell and we're in an excited state, we use the notation $(f_{7/2}))^3$. You can add up their angular momentum any way you want, remember each can have angular momentum from $-j$ to $+j$. So these can have $m = \pm\frac{1}{2}, \pm\frac{3}{2}, \pm\frac{5}{2}, \pm\frac{7}{2}$. They can't have the **same** value of m, but they can all add to get, for example $\frac{7}{2} + \frac{5}{2} + \frac{3}{2} = \frac{15}{2}$. [1, pp. 149-151]
- In excited states, the single nucleons can also jump around to other states, changing the J^π of the nucleus. [2, Lec. 13-16]

References

[1] Kenneth S. Krane. *Introductory Nuclear Physics*. John Wiley & Sons, Inc., 3rd edition, 1988.

[2] Lee Bernstein. Nuclear engineering class lectures. Fall 2015.