Nuclear Engineering 101: Final Study Guide

Disclaimer: This is not an official study guide. Stuff ~~might~~ **is** wrong. Use the lecture notes and book!

Note: Everything in this guide is from the text (Krane) or lecture, or office hours and should be cited as completely as possible.
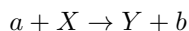
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1.1   General Information

- The reaction:
$$a + X \rightarrow Y + b$$

  Can be written in reaction notation as:
$$X(a, b)Y$$
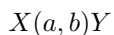
  [1, pp. 378-379]

- A microscopic cross section (σ) represents the "relative probability for the reaction to occur." It can be used in the following equation:

$$R = (\rho d)_{target} \times I_{beam} \times \sigma_{reaction}$$

  Where R is the reaction rate (in reactions/sec); (ρd) is the density (in g/cm^3) times the width of the target (in cm), also known as the areal density (put it in atoms/cm^2); I_{beam} is the incident particle flux (in atoms/sec); and $\sigma_{reaction}$ is the microscopic cross section of the reaction occurring. This is only valid when very little of the beam reacts (small σ) and everything moves in straight lines. It can also be expressed as:
$$R = N\phi\sigma$$

  Where N is the number of target atoms, ϕ is the flux in (atoms/sec/cm^2) and σ is the same. [2, Lec. 25]

- Microscopic cross sections are generally given in units of barns. 1 barn = 10^{-24} cm^2.

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- The cross section is not always constant over angle (it rarely is). So the *differential cross section* is used:

$$\frac{d\sigma}{d\Omega}$$

What is confusing, is this is just a number, in units of barns/steradian. It's representing the fact that some small number of particles ($d\sigma$) will strike our small detector ($d\Omega$). It is dependent on the angle of scatter (θ) and the polraization of the radiation (ϕ). Generally we assume there is no affect due to polarization (things are randomly polarized).

We can find the size of our detector $d\Omega$ in steradians, which is related to the area of our detector (dA) and the distance from the target (r) by:

$$d\Omega = \frac{dA}{r^2}$$

Then, if we know the differential cross section at the angle of our detector, we can multiply to get the reaction cross section for our detector:

$$\sigma_{det} = d\Omega \frac{d\sigma}{d\Omega}$$

This represents something **very specific**. This is the probability that incoming particles striking the target will then be detected by our detector. Based on the size of our detector ($d\Omega$) and our a priori knowledge of the number of particles that will be seen in a small area ($\frac{d\sigma}{d\Omega}$). The value of that differential cross section will probably vary with angle, so you have to know the differential cross section for the angle where your detector is to even use this. More rigorously, you'd integrate over the area of the detector and $\frac{d\sigma}{d\Omega}$ may vary over the integral:

$$\sigma_{det} = \int_{detector} \frac{d\sigma}{d\Omega} d\Omega$$

Or, you can get the total σ by integrating over the whole angle space.

- **Rutherford Differential Scattering:** elastic Coulomb scattering. An incoming particle scatters off the potential of the target. [2, Lec 24]

- **Coulomb Excitation:** inelastic Coulomb scattering. An incoming particle scatters off the potential of a target and leaves some energy behind. This "Coulex" reaction can excite nuclei up rotational bands. [2, Lec. 24]

- Reaction Q-value is to create the final products at rest.

    - Center of Mass Frame: products are at rest, $Q = Q$.
    - Lab Frame: products are *not* at rest. Threshold energy for reaction is:

$$E_{\text{threshold}} = Q\left(\frac{m_a + m_x}{m_x}\right)$$

    for a(X,Y)b.

- Conserved quantities in reactions:

    - Total energy
    - Linear momentum
    - Angular momentum
    - Parity $(-1)^l$ (except in weak interactions)

- **Kinematics** For a reaction, $X(a,b)Y$:

$$Q = (m_X + m_a - m_Y - m_b)c^2$$
$$Q = T_Y + T_b - T_X - T_a$$

- Exothermic Q>0 :

$$m_X + m_a > m_Y + m_b$$
$$T_Y + T_b > T_X + T_a$$

- Endothermic Q<0 :

$$m_X + m_a < m_Y + m_b$$
$$T_Y + T_b < T_X + T_a$$

- Reaction reaches excited states of Y:

$$Q_{ex} = (m_X + m_a - m_{Y*} - m_b)c^2 = Q_0 - E_{ex}$$

- Compound nucleus:

$$Q = -T_a = (m_X + m_a - m_{C*})c^2 - E_{ex}$$

1.2 Photo-nuclear Interactions

- A photon interacts with the nucleus directly. For this to happen, we need to have an energy level at the energy of the incoming photon. [2, Lec 25]

- There are three types of photo-nuclear interactions:

  - Spontaneous emission: if the nucleus is in an energy level, it can release a photon to de-excite. This is an intrinsic property of the level.
  - Resonant absorption: if the incoming photon is at the exact same energy of an energy level, it can be absorbed and the nucleus excited to that state. The energy of the state is the resonance energy.
  - Stimulated emission: One photon goes in, two photons come out. This occurs if the incoming photon is at the resonant value. This is the principle by which lasers work (on the atomic scale), but it has not been seen for nuclei.

  [2, Lec 25]

- The cross section for this to occur (σ_0) is a function of the nucleus' angular momentum, and internal conversion (IC) factors (α). As the probability of IC rises, photon capture becomes more rare, it's hard to make a nucleus capture photons when it wants to eject electrons. [2, Lec 25]

- The width of the emitted state:

$$\Gamma = \frac{\hbar}{\tau}$$

The longer the mean lifetime (τ), the more well defined the energy level's value is. [2, Lec 25]

1.2.1 Resonance Absorption

- The resonance energy can be affected by any recoil that will result from the capture. This is because the nucleus needs both enough energy to be in its new excited state, **and** enough energy to recoil; so the incoming photon needs to have a little bit more than the expected resonance energy. As shown in Figure 1, the resonance energy has been shifted up by the recoil energy E_R from the expected value ΔE. [2, Lec 25]

- This has an exactly opposite effect on the emission spectrum. The absorption spectrum was shifted *up* because the incoming photon needed extra energy to recoil the nucleus. The emission spectrum is shifted *down* because the recoil takes some of the energy of the emitting photon. [2, Lec. 25]
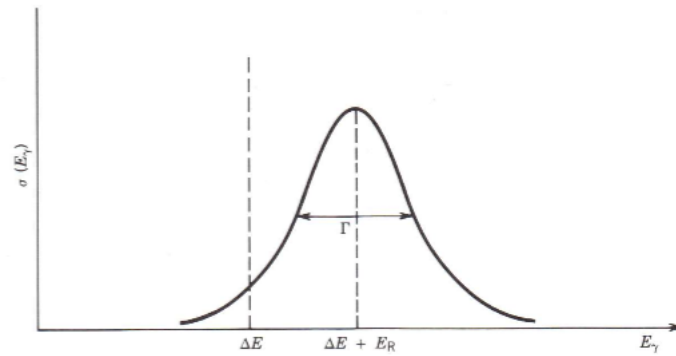
Figure 1: Krane figure 10.23. [1].

- **Doppler broadening:** thermal motion makes the nuclei move back and forth, so the incoming photons energy looks doppler shifted either higher or lower. Therefore, a photon with energy just off the resonance may actually be absorbed because the relative motion can shift its energy to the resonance value. A wider energy range can now be absorbed by the resonance, so the peak gets wider or *broadens.* [1, pp. 363]

- Doppler broadening can can cause overlap between the emission and absorption peaks. [2, Lec. 25]

- **Mossbauer Effect:** If a nuclei is in a crystal lattice, its recoil will be inhibited by the fact that *its stuck in a lattice.* You're not just causing one nuclei to move, but all the ones around it, this makes the recoil energy very low, and therefore minimizes the shift in resonance energy by recoil. This allows you to nail down the *actual* resonance energy. [2, Lec 25]

1.2.2 Giant Dipole Resonance

High energy photons can "ionize" the entire nucleus by creating dipole motion between all the protons and all the neutrons. This means that all the protons moving together and all the neutrons are moving together, and these two groups are in a resonance with each other. This only occurs when the incoming photon energy is very high, >12 MeV. [2, Lec 25]

1.3 Direct Reactions

- Occur on time frames of $10^{-21} - 10^{-22}$s. [2, Lec 25]

- These are collisions off the nucleons on the surface. They occur fast enough and with high enough energy that the incoming particle actually "sees" the individual protons and neutrons instead of the nucleus as a whole. This happens because the wavelength of the incoming particle is inversely proportional to momentum, and therefore energy:

$$\lambda = \frac{h}{p}$$

So higher energy particles can interact with individual nucleons. [2, Lec 25]

- Different incoming angles affect the cross sections for reactions. This is because the incoming particle is a wave, and the nucleus is a big wave made up of little waves, so you can have constructive and destructive interference as the two interact. They can add up to make a reaction or less likely based on angle. [2, Lec 25]

1.3.1 Kinematics

For an incoming particle a, outgoing particle b and product, as shown in Figure 2 The momentum is conserved:
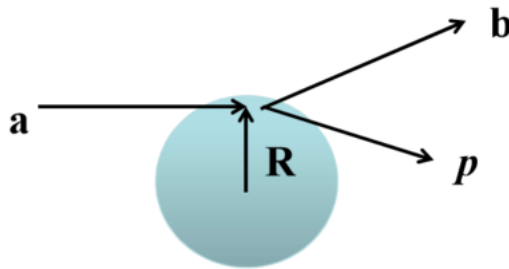
Figure 2: Direction reaction kinematics diagram.

$$\vec{p}_{product} = \vec{p}_a - \vec{p}_b$$

Based on the radius R that the incoming particle comes in, there is some amount of angular momentum l transferred to the product (spinning up the nucleus):

$$l = Rp$$

Using conservation of energy, you can also relate the final product momentum (p) to the incoming and outgoing particles:

$$p^2 = p_a^2 + p_b^2 - 2p_a 2p_b \cos\theta$$

If you solve for l you can figure out what J^π the product will be left in. In summary:

1. Figure out the momentum of the product using the momenta of the products and the angle of collision (that p^2 formula).

2. Solve for the angular momentum transfer l using R ($l = Rp$).

3. Add $l \pm \frac{1}{2}$ to the original J^π of the target to get the final spin.

4. Parity change will go $\Delta\pi = (-1)^l$ where that l is the angular momentum transfer from step 2.

1.4 Compound Reactions

2 Neutron Physics

3 Fission

4 Fusion

References

[1] Kenneth S. Krane. Introductory Nuclear Physics. John Wiley & Sons, Inc., 3rd edition, 1988.

[2] Lee Bernstein. Nuclear engineering class lectures. Fall 2015.

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