# Simulating unknown qubit-unitary inversion with zero-noise extrapolation


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## Abstract


In this work, we demonstrated the feasibility of using zero-noise extrapolation (ZNE), an error mitigation technique, in the unknown qubit-unitary inversion protocol based on work by *Yoshida S, et al. [Physical Review Letters 131.12 (2023)]*. Using noise models and the Mitiq toolkit, we apply ZNE and analyze their deviation from the theoretical results.


## Introduction

During the current Noisy Intermediate-Scale Quantum (NISQ) era, it is crucial to gather and analyze data on how various types of noise impacts the results of our computations, either in QPU and noise models. Such analysis can help us to explore different methods to reduce the effects of noise. So, in addition to the noise modeling, we also need mitigation techniques capable of decreasing the negative effects of noise in the computing.

In this work, a simulation of a qubit-unitary inversion protocol was done using a noise model, taking the calibration data from the real QPU *ibm_brisbane*. Then, an error mitigation technique was applied.

## Unified Noise Model

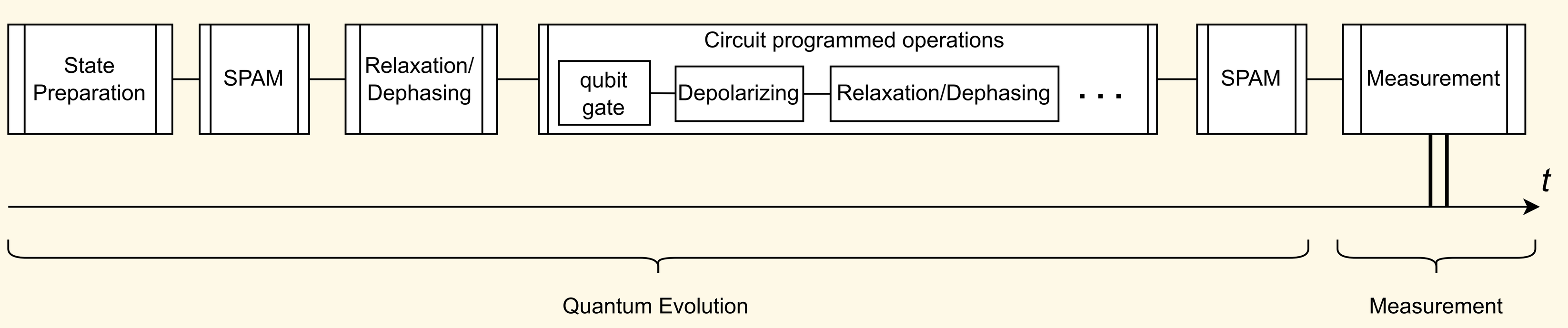


Figure 1. The application of the Unified Noise Model (UNM) in a circuit (*image inspired by [1]*)

## Zero-noise Extrapolation

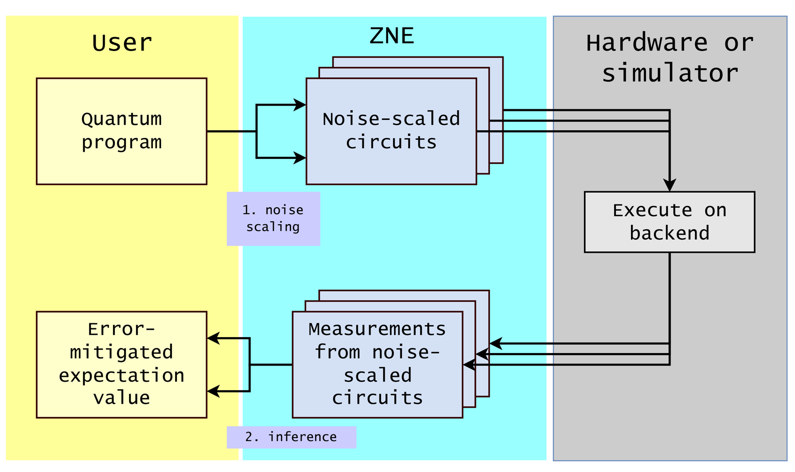


Figure 2. The zero-noise extrapolation [2] workflow (*image inspired by Mitiq [3]*)

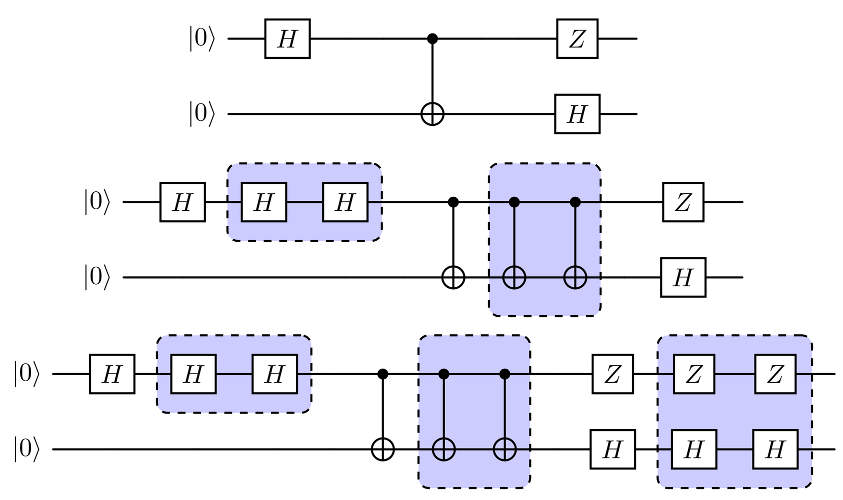

Figure 3. Circuits with different scale factors and scaled from the left.

We can divide the ZNE technique into two steps. The first step, the noise scaling, where we increase the noise strength $\lambda$ of the circuit. In this work, we focus in the unitary folding method. It works by adding to the circuits different unitary gates $G$ and its inverse $G^\dagger$, so as to not affect the computing while increasing $\lambda$. The second step, extrapolation, involves the statistical method to get the zero-noise limit. There are different extrapolation methods that achieves different results, for example, Richardson, polynomial and exponential.

## Unknown Qubit-unitary Inversion

**How the protocol works?**

- It uses four calls of a qubit-unitary operation $U_{in} \in SU(2)$ in sequence of fixed quantum operations $V^{(1)}$ and $V^{(2)}$.
- For an input quantum state $|\phi_{in}\rangle \otimes |\psi^-\rangle \otimes |0\rangle^{\otimes 4}$ results in $|\psi U_{in}\rangle \otimes U_{in}^{-1}|\phi_{in}\rangle \otimes |0\rangle^4$, where $|\psi U_{in}\rangle = (U_{in} \otimes I)|\psi^-\rangle = (U_{in} \otimes I)\frac{1}{\sqrt{2}}(|01\rangle - |10\rangle)$.
- From the output quantum state, $|\psi_{U_{in}}\rangle$ can be used for a new run of the protocol with the same $U_{in}$. This permit only using three $U_{in}$ operations, instead of four.

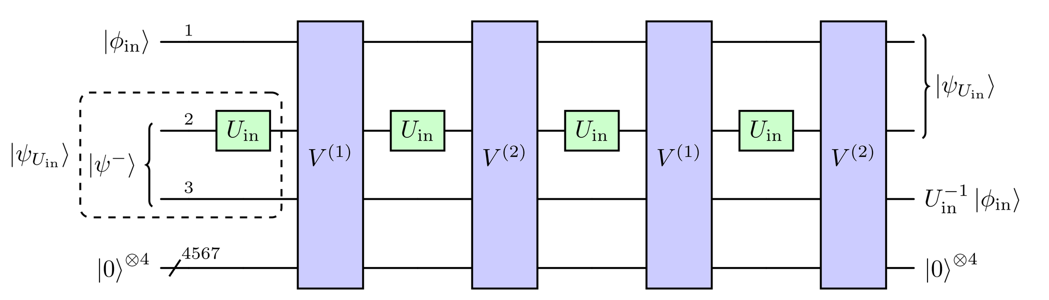

Figure 4. Unknown qubit-unitary inversion protocol, by *Yoshida S, et al. [Physical Review Letters 131.12 (2023)]* [4].

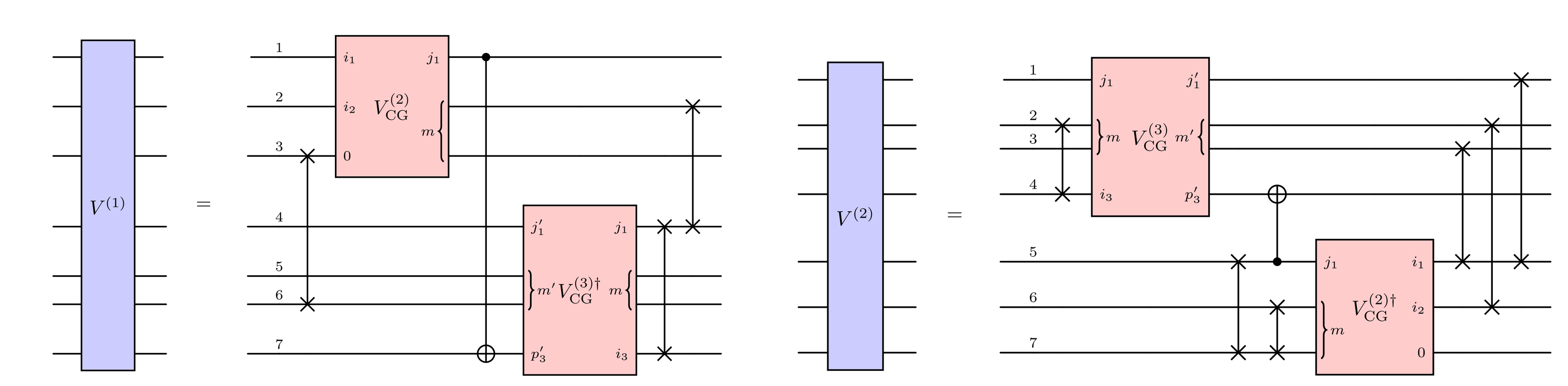

Figure 5. Unitary operators $V^{(1)}$ and $V^{(2)}$ in the qubit-unitary inversion circuit shown above. They make use of the Clebsch-Gordan transform operations $V_{CG}^{(2)}$ and $V_{CG}^{(3)}$

## Noise model experiments

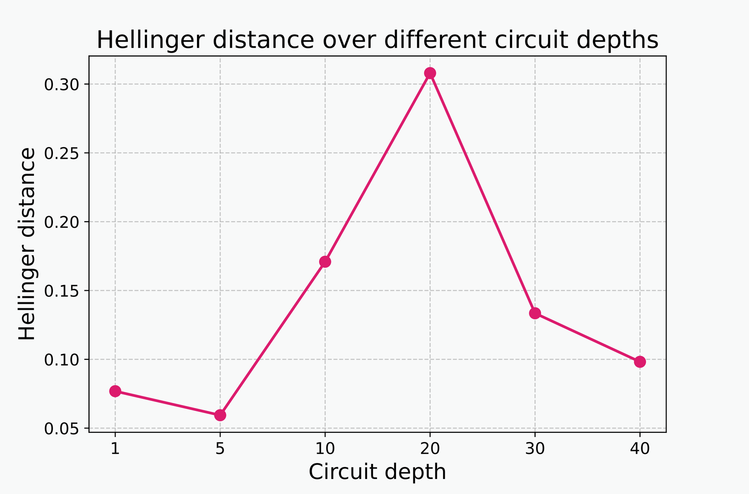


Figure 6. Hellinger distances between *ibm_brisbane* counts and a UNM counts for random circuits and different depth.

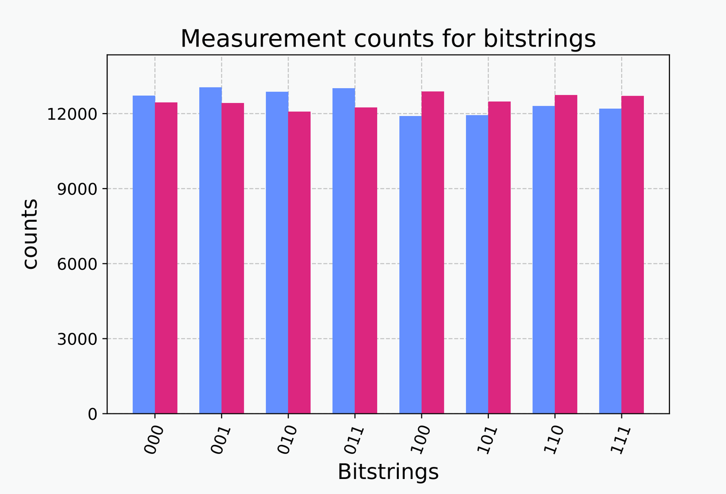


Figure 7. Counts given the unknown unitary protocol run on *ibm_brisbane* and a UNM. Blue: QPU. Red: UNM.

## Error mitigation experiments

The ZNE strategies tested are

- LinearOne: Linear Factory with scale factors 1 and 6.
- RichOne: Richardson Factory with scale factors 1, 2 and 6.
- RichTwo: Richardson Factory with scale factors 1, 1.22, 1.44, 1.66 and 2.
- RichThree: Richardson Factory with scale factors 1, 1.26, 1.52, 3 and 6.

The rates calculated are

- **Mitigation success rate**, the probability of extrapolating the observable $000$ measurement counts nearer to the total number of shots $n = 10000$.
- **Relative Error Rate (RER) over all mitigations**, an ordinary relative error rate where the formula's measured value are the $000$ extrapolated counts and the real value is $n = 10000$.
- **RER only over succeed mitigations**.
- **RER only over failed mitigations**.

| Strategy | Rate | Mean | Median | Min. | Max. | $q_1$ | $q_3$ |
|---|---|---|---|---|---|---|---|
| *No mitigation* | RER | - | 0.8704 | 0.8636 | 0.8753 | 0.8682 | 0.8725 |
| *LinearOne* | Success | 0.76 | - | - | - | - | - |
| | RER (succeed) | - | 0.8695 | 0.8612 | 0.875 | 0.8670 | 0.8718 |
| | RER (failed) | - | 0.875 | 0.8699 | 0.8786 | 0.8735 | 0.8763 |
| | RER (all) | - | 0.8705 | 0.8612 | 0.8824 | 0.8681 | 0.8738 |
| *RichOne* | Success | 0.69 | - | - | - | - | - |
| | RER (succeed) | - | 0.8647 | 0.8431 | 0.8762 | 0.8565 | 0.8694 |
| | RER (failed) | - | 0.8767 | 0.8703 | 0.8917 | 0.8749 | 0.8823 |
| | RER (all) | - | 0.8693 | 0.8431 | 0.8951 | 0.8605 | 0.8749 |
| *RichTwo* | Success | 0.15 | - | - | - | - | - |
| | RER (succeed) | - | 0.65 | 0.08 | 0.86 | 0.41 | 0.76 |
| | RER (failed) | - | 1.57 | 0.03 | 3.93 | 1 | 2.38 |
| | RER (all) | - | 1.24 | 0.03 | 3.93 | 0.79 | 2.08 |
| *RichThree* | Success | 0.6 | - | - | - | - | - |
| | RER (succeed) | - | 0.69 | 0.40 | 0.86 | 0.62 | 0.77 |
| | RER (failed) | - | 1 | 0.87 | 1.25 | 0.91 | 1.06 |
| | RER (all) | - | 0.81 | 0.33 | 1.33 | 0.66 | 0.96 |

Table 1. Statistics for the 100 mitigations sample experiment over ZNE strategies. Not all rates have all the statistics calculated, if that is the case, then the table cell will be completed with a $-$.

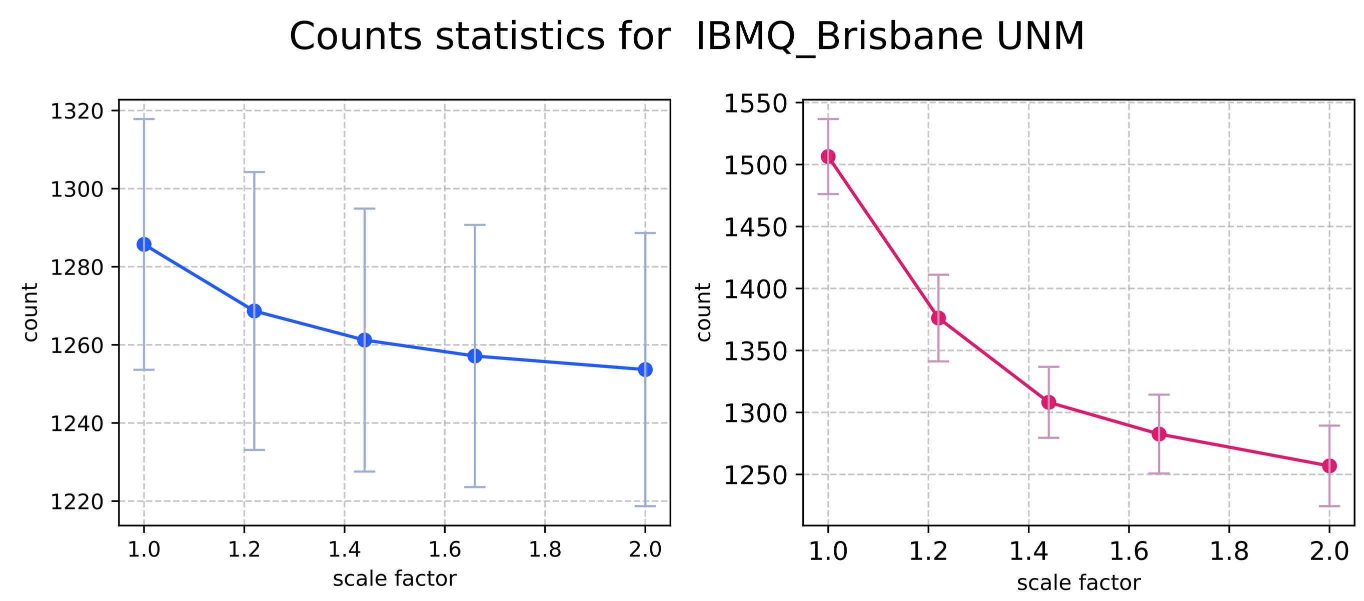


Figure 8. Statistics of RichTwo expectation values. The points represents the means and the whiskers represent the standard deviations. (Left) The experiment was run with *ibm_brisbane* calibration data on the UNM. (Right) The same calibration data was used for the UNM, but with a decrease of $40\%$ over the two-qubit gates noise rate.

## Discussion

**linearOne**

- Similar relative error statistics that *richOne*
- Higher success probability than *richOne*
- Unfortunately, it do not offer a relevant relative error decrease when succeeding

**richTwo**

- Offers the lowest relative error when succeeding
- Unfortunately, it also has the lowest success probability

**richThree**

- A balanced configuration between *linearOne* and *richTwo*
- It has an acceptable success probability and relative error when succeeding

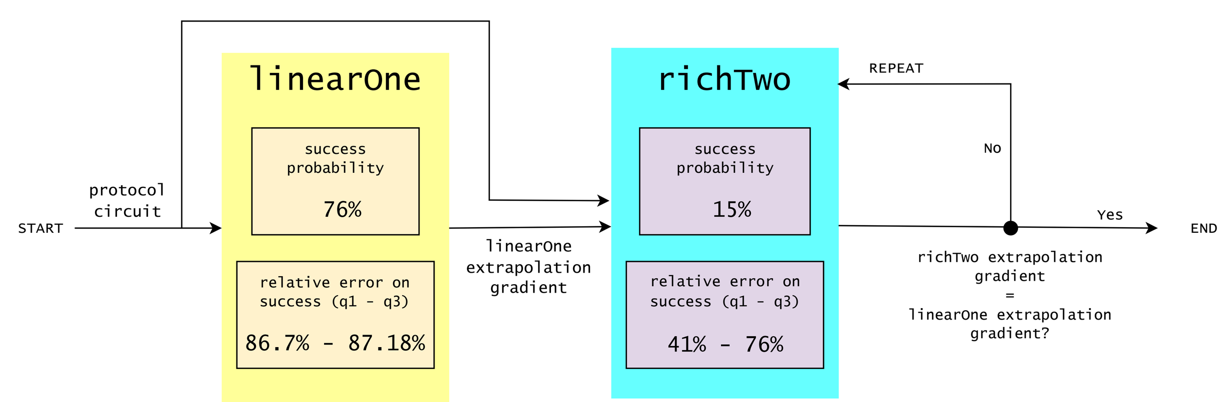


Figure 9. Proposed protocol for combining linearOne and richTwo

## Conclusions

We demonstrated the feasibility of ZNE for the unknown qubit-unitary inversion protocol. Noise models results are analyzed considering the effects of different ZNE parameters, so as to identify the best configuration to use. Considering future hardware improvements that can be done on *ibm_brisbane*, our results shows that the usefulness that ZNE can offer will reach significant impact for the protocol, with a $40\%$ decrease over the two-qubit gates noise.

## References


[1] K. Georgopoulos, C. Emary, and P. Zuliani, "Modeling and simulating the noisy behavior of near-term quantum computers," *Physical Review A*, vol. 104, no. 6, p. 062432, 2021.

[2] D. Bultrini, M. H. Gordon, P. Czarnik, A. Arrasmith, M. Cerezo, P. J. Coles, and L. Cincio, "Unifying and benchmarking state-of-the-art quantum error mitigation techniques," *Quantum*, vol. 7, p. 1034, 2023.

[3] R. LaRose, A. Mari, S. Kaiser, P. J. Karalekas, A. A. Alves, P. Czarnik, M. El Mandouh, M. H. Gordon, Y. Hindy, A. Robertson, *et al.*, "Mitiq: A software package for error mitigation on noisy quantum computers," *Quantum*, vol. 6, p. 774, 2022.

[4] S. Yoshida, A. Soeda, and M. Murao, "Reversing unknown qubit-unitary operation, deterministically and exactly," *Physical Review Letters*, vol. 131, no. 12, p. 120602, 2023.



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