Simulating unknown qubit-unitary inversion with zero-noise extrapolation

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***Abstract*—In this work, we demonstrated the feasibility of using Zero-Noise Extrapolation (ZNE), an error mitigation technique, in the unknown qubit-unitary inversion protocol based on work by *Yoshida S, et al. [Physical Review Letters 131.12 (2023)]*. Using noise models and the Mitiq toolkit, we apply ZNE and analyze their deviation from the theoretical results.**

I. Introduction

During the current Noisy Intermediate-Scale Quantum (NISQ) era, it is crucial to gather and analyze data on how various types of noise impacts the results of our computations, either in QPU and noise models. Such analysis can help us to explore different methods to reduce the effects of noise. So, in addition to the noise modelling, we also need mitigation techniques capable of decreasing the negative effects of noise in the computing.

In this work, a simulation of a qubit-unitary inversion protocol was done using a noise model, taking the calibration data from a real QPU. Then, an error mitigation tecnique was applied.

II. Unified Noise Model

The Unified Noise Model (UNM) [1] is constructed using three different quantum noise channels. The depolarizing channel essentially simulates a bit-flip and phase-flip operations due to error in the hardware. The State Preparation and Measurement (SPAM), that involves a Pauli X error after the initial state preparation and before the measurement. Finally, the thermal Decoherence and Dephasing, referring to the physical qubits and their interaction with the environment. It involves two types of noise (dephasing and thermal decoherence) occurring over time in form of excitation/de-excitation of the quantum state. In figure 1 is shown how the UNM is applied to a quantum circuit.

III. Zero-noise extrapolation

Zero-noise extrapolation (ZNE) is a noise mitigation technique [2], [3]. It works by intentionally scaling the noise of a quantum circuit to then extrapolate the zero-noise limit of an observable of interest. We can divide the ZNE technique into two steps. The first step, the noise scaling, where we increase the noise strength λ of the circuit. In this work, we focus in the unitary folding method. It works by adding to the circuits different unitary gates G and its inverse $G^\dagger$, so as to not affect the computing while increasing λ. The second step, extrapolation, involves the statistical method to get the zero-noise limit. There are different extrapolation methods that achieves different results, for example, Richardson, polynomial and exponential.

IV. Unknown qubit-unitary Inversion

The circuit in which we have done our different experiments make use of the implementation for an exact and deterministic protocol of qubit-unitary inversion [5]. This protocol uses four calls of a qubit-unitary U_{in} in sequence of some fixed quantum operations V^1 and V^2 to get from an input $|\phi_{in}\rangle$ the output $U_{in}^{-1}|\phi_{in}\rangle$. Specifically, applying the protocol on an input quantum state $|\phi_{in}\rangle \otimes |\psi^-\rangle \otimes |0\rangle^{\otimes 4}$ results in $|\psi U_{in}\rangle \otimes U_{in}^{-1}|\phi_{in}\rangle \otimes |0\rangle^{\otimes 4}$, where $|\psi U_{in}\rangle = (U_{in} \otimes I)|\psi^-\rangle = (U_{in} \otimes I)\frac{1}{\sqrt{2}}(|01\rangle - |10\rangle)$. In figure 2, it can be seen the quantum circuit that implements the protocol.

V. Experiments and results

The experiments were done using *ibm_brisbane* QPU and its calibration data for the UNM. The circuit to be run will be the unknown qubit-unitary inversion followed by the necessary operations to obtain the identity. The idea is that, given n circuit shots, the ideal counts should be n for the observable '000'. To achieve the mentioned ideal counts, we shall apply $U_{in}^\dagger$ on the first qubit and U_{in} on the third qubit. Finally, we have to de-prepare the states $|\phi_{in}\rangle$ and $|\psi^-\rangle$ so as to get quantum state $|0\rangle^{\otimes 3}$.

The basis gates are different when running in simulators than when running in QPUs. The $ibm\_brisbane$ QPU has ECR as two-qubits basis gate, so the tranpilations of circuits are in the basis gates $[ID, RZ, SX, X, ECR]$. While for simulations, we will replace ECR for CX because of lack of support to the new gate ECR in the framework Mitiq. Due to the fact that the gate ECR when transpiled into $[ID, RZ, SX, X, CX]$ makes use of a CX gate, switching between ECR and CX does not make a significant difference in the noise simulation.

A. Scaling depth on random circuits

An analysis over random circuits was done. The goal is to analyze how different are the resultant counts distribution of random circuits run on the QPU against the ones run on a

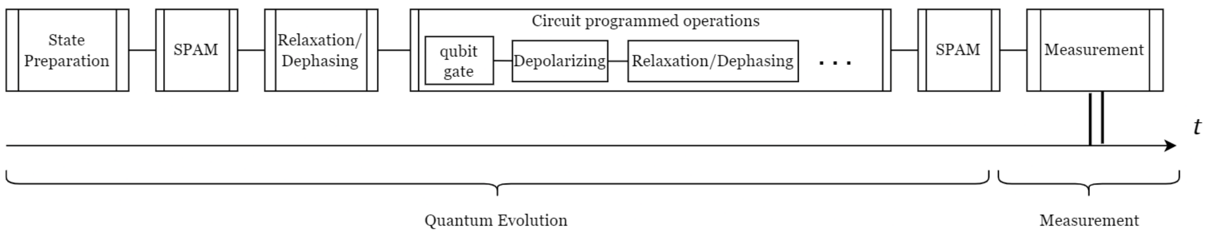

Fig. 1: The application of the Unified Noise Model in a circuit (*image inspired by [1]*)

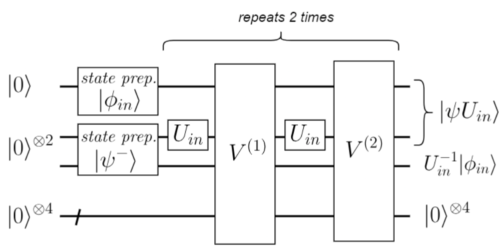

Fig. 2: Unknown qubit-unitary inversion protocol, where given an unknown unitary U_{in} and a quantum state $|\psi_{in}\rangle$, returns $U_{in}^{-1}|\phi_{in}\rangle$.

UNM. For that, we use the *Hellinger distance* metric. This is a 0 to 1 rate with two probabilistic distributions as inputs, where a 0 value means that the two distributions are equal.

The results of the experiment can be seen in figure 3. Despite we still doesn't reach the same Hellinger distance values from [1], this preliminary UNM implementation will be useful for our experiments.

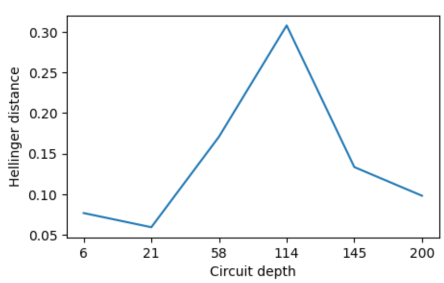

Fig. 3: Hellinger distances for random circuits with different depth.

B. Noise on circuit

From now on, we will keep using for the experiments the circuit mentioned at the introduction of section V. The idea is to compare the counts results of applying the circuit on a QPU against a UNM. This experiment was run with 100000 circuit shots. In figure 4, it can be seen a quasi-uniform distribution in both counts for QPU and UNM. The form of the distribution is because of the long circuit depth. The distribution is consistent with figure 3, where for the longest circuit depths, the hellinger distance start decreasing.

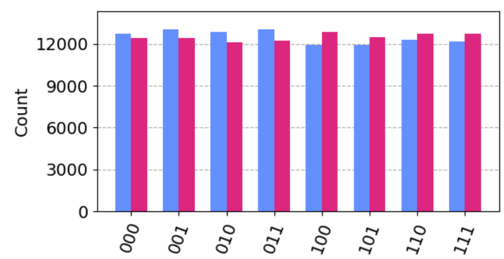

Fig. 4: Counts given the Qubit Unitary Inversion protocol run on *ibm_brisbane* QPU and a UNM with the QPU calibration data. Blue: QPU. Red: UNM.

C. Noise mitigation

We benchmark different ZNE configurations with the UNM being applied to the circuit. For the ZNE implementation, we used the Mitiq framework [4]. The configurations used are:

- *linearOne*: Linear Factory with scale factors 1 and 6.
- *richOne*: Richardson Factory with scale factors 1, 2 and 6.
- *richTwo*: Richardson Factory with scale factors 1, 1.22, 1.44, 1.66 and 2.
- *richThree*: Richardson Factory with scale factors 1, 1.26, 1.52, 3 and 6.

For each configuration, we run a sample of 100 mitigations over the observable '000', with 10000 circuit shots. We consider each mitigation as succeed or failed, where succeed means that the extrapolated count for '000' was closer to $n = 10000$. The ideal counts are n, so a succeed mitigation should extrapolate the counts nearer to that value. Given this distinction, we will calculate some statistics with the following rates:

- Mitigation success rate
- Relative Error Rate (RER) only over succeed mitigations
- RER only over failed mitigations
- RER over all mitigations

The rates statistics are shown on table I.

We also calculate statistics about the expectation values of each configuration. Focusing on *richTwo*, figure 5 (*left side*) shows the respective means and standard deviations. The figure (*right side*) also shows the results over the same *ibm_brisbane* calibration data, but decreasing a 40% the two qubits gates noise rate. The differences over the statistics are easily visible and corresponds with their respective noise strength.

Configuration	Rate	Mean	Median	Min.	Max.	q_1	q_3
No mitigation	RER	-	0.8704	0.8636	0.8753	0.8682	0.8725
LinearOne	Success	0.76	-	-	-	-	-
	RER (succeed)	-	0.8695	0.8612	0.875	0.8670	0.8718
	RER (failed)	-	0.875	0.8699	0.8786	0.8735	0.8763
	RER (all)	-	0.8705	0.8612	0.8824	0.8681	0.8738
RichOne	Success	0.69	-	-	-	-	-
	RER (succeed)	-	0.8647	0.8431	0.8762	0.8565	0.8694
	RER (failed)	-	0.8767	0.8703	0.8917	0.8749	0.8823
	RER (all)	-	0.8693	0.8431	0.8951	0.8605	0.8749
RichTwo	Success	0.15	-	-	-	-	-
	RER (succeed)	-	0.65	0.08	0.86	0.41	0.76
	RER (failed)	-	1.57	0.03	3.93	1	2.38
	RER (all)	-	1.24	0.03	3.93	0.79	2.08
RichThree	Success	0.6	-	-	-	-	-
	RER (succeed)	-	0.69	0.40	0.86	0.62	0.77
	RER (failed)	-	1	0.87	1.25	0.91	1.06
	RER (all)	-	0.81	0.33	1.33	0.66	0.96

TABLE I: Statistics for the 100 mitgations sample experiment over ZNE configurations. Not all rates have all the statistics calculated, if that is the case, then the table cell will be completed with a $-$. The success rate is defined as the probability of extrapolating a nearer value to 10000. RER is an ordinary relative error rate where the formula's measured value is the extrapolated count and the real value is 10000. The RER (succeed), specifies that the statistics were calculated only on the succeed mitigations. RER (failed), statistics only on the failed mitigations. RER (all), statistics on all the sample.

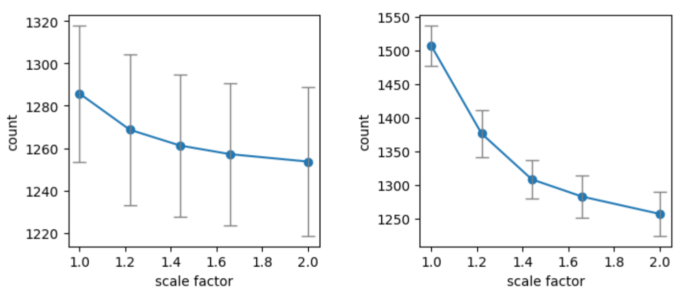

Fig. 5: Statistics of $richTwo$ expectation values. The blue points represents the means and the gray lines represent the standard deviations. (Left) The experiment was run with $ibm\_brisbane$ calibration data. (Right) The same calibration data was used, but with a decrease of 40% over the two qubits gates noise rate.

VI. Discussion

The *richOne* and *linearOne* configurations offers similar results over all relative error statistics. Both shows good/bad results when having succeed/failed mitigations, but $linearOne$ is more probable to succeed, predicting that the counts of the observable should be increased. Unfortunately, it didn't offer a relevant decrease of relative error when succeeding, making it not so useful when used alone. On the other hand, $richTwo$ configuration achieves the lowest relative error rate when succeeding, making it the configuration that is closest to the ideal count. But, given that it has the lowest mitigation success rate, is not a reliable method.

The $richThree$ is the most balanced configuration. It takes the best of $linearOne$ and $richTwo$. That means, an acceptable mitigation success rate and acceptable relative error when succeeding. It would be interesting to combine $linearOne$ and $richTwo$. We could take advantage of the mitigation success rate of $linearOne$ to predict the gradient that the zero-noise limit should have. We could run as many $richTwo$ executions as required to achieve an extrapolation with the same gradient of $linearOne$. When founding a convenient $richTwo$ extrapolation, we will take advantage of the low relative error rates that this configuration achieves when succeeding.

The long standard deviations and the low gradients formed by the means on the $ibm\_brisbane$ experiment are solved with a 40% decrease over the two qubits gates noise rate.

VII. Conclusions

We have demonstrated the feasibility of using ZNE for the unknown qubit-unitary inversion protocol proposed in [5]. Results with noise models are analyzed considering the effects of different parameters in ZNE, so as to identify the best configuration to use. Considering future hardware improvement that can be done on $ibm\_brisbane$, our results shows that the usefulness that ZNE can offer will reach significant impact for the protocol, with a 40% decrease over the two qubits gates noise. Further experiments are necessary to expand over our results and models to explore different applications.

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