**5.5**

**Intro**

In this exercise, we will solve the classic Newsvendor problem, which appears very often in econometrics courses. In simple words, the problem asks us to determine which is the optimal quantity of some product to buy in order to have the maximum profit. I think this is the most basic formulation of the problem. It does not consider inventory, which simplifies the solution and it also make sense when our product is something like a newspaper. Also, all the quantities and the pdf $f$ are supposed to be stationary. Tough this assumption is not correct or useful on the long run, it might give good predictions considering small windows of time.

**Solution**

Since we want to estimate our expected profit, it is necessary to consider all the possible scenarios. In this case there are two. Either we buy a quantity $Q$ that it is not enough to supply the demand $D$ and loose the chance of getting a bigger profit or we buy a quantity bigger than the actual demand, and wast some money on unsold merchandise. This combination of situations can be modeled by the following expression:

$$E_\pi(Q) = \int_Q^\infty (P-C)Qf(D)dD + \int_0^Q (P-C)Df(D)dD - \int_0^Q C(Q-D)f(D)dD \tag{1}$$

Equation 1 express the expected profit as a function of $Q$. In order to find its global optimum we have to search the extremities and the points where the derivative is equal to 0. Since $Q \in [0, +\infty)$, its only extrimity is $Q = 0$. Since if we don't purchase anything we will not have any profit, this cannot be the optimum profit. So, all that's is left is to differentiate Equation 1. However, before doing that, we will follow the author suggestion and simplify the expression.

$$\begin{aligned}
&E_\pi(Q) = (P-C)Q\int_Q^\infty f(D)dD + (P-C)\int_0^Q Df(D)dD - \\
&CQ\int_0^Q f(D)dD + C\int_0^Q Df(D)dD = \\
&(P-C)Q(1-F(Q)) + P\int_0^Q Df(D)dD - CQF(Q) = \\
&(P-C)Q - PQF(Q) + P\int_0^Q Df(D)dD
\end{aligned} \tag{2}$$

Now, all that is left to do is to differentiate (2) with respect to $Q$ and equal it to zero.

$$\begin{aligned}
&\frac{d}{dQ}E_\pi(Q) = (P-C) - PF(Q) - PQf(Q) + PQf(Q) \\
&(P-C) - PF(Q^*) = 0 \iff F(Q^*) = \frac{P-C}{P}
\end{aligned} \tag{3}$$

**Conclusion**

In this exercise, we solved the newsvendor problem. We started by modeling the expected profit as a function of the quantity $Q$. Then, we simplified the expression and found its global optimum by differentiation. The final expression $F(Q^*) = \frac{P-C}{P} = 1 - \frac{C}{P}$ tell us that the parametric model $F$ responsible for predicting $D$ is crucial in determining the optimal purchase quantity.

As a final remark, we should be careful with one aspect of our model. Looking at $F(Q^*) = 1 - \frac{C}{P}$, it appears that as we increase the price with respect to the cost, the optimal amount $Q^*$ increases. Therefore, we could put the price really high and our model would suggest that $Q^*$ would also be really high, making our profit as big as we like! The problem with this line of thought is twofold. First, the pdf $f(D)$ should not have a right tail, because there is a limit on how big the demant can be. Therefore, the quantity $Q^*$ cannot increase as we like. Second, in our model the demand $D$ is not a function of the selling price $P$, which does not correspond to reality. We can get away with this simplification if we set the price fix and estimate a distribution $F(D)$ based on this price. But we must be very careful to not overextend this simple model. To those more interested in this kind of problem, I think books in Econometrics, inventory and revenue management can have a more comprehensive treatment on the subject.