5.

Intro

For those who watched 21 (the movie about blackjack), you already know that this question was proposed by the infamous professor Micky Rosa to see if the protagonist Ben Campbell was fit to count cards in the casinos of Las Vegas. For those that didn't watch, go see it on Netflix, it is pretty good. Jokes aside, search in Wikipedia for this problem. It is astonishing how much history and polemic exist behind it.I will give two different explanations to try to convince the reader of the solution.

Solutions

First solution

Events

d1 - prize is on door number 1

d2 - prize is on door number 2

d3 - prize is on door number 3

Each port has a prior of $\frac{1}{3}$ of being the right one. Assume you select door number 2, without losing generality. Now assume that the gameshow host open door number 1 and there is nothing there. Now, we have to calculate the posteriors based on this new event. The trick here is to note one thing. It does not matter what information the gameshow host gives to you, because you made your choice before that information. Thus the posterior of your initially chosen door remains the same. Therefore, the posteriors are:

$$P(d1 = true|d1 = false) = 0$$

$$P(d2 = true|d1 = false) = \frac{1}{3}$$

$$P(d3 = true|d1 = false) = \frac{2}{3}$$

So, is better to change the port number, because you get the double of your initial chance.

Second solution

The sample space of the prize location can be represented as

$$\Omega\ = \{PNN, NPN, NNP\}.$$

Where, P represents the prize location, and N represents nothing.

Assume you can play the game 999 times. It is reasonable to assume that each configuration will happen about 333 of the time. Now assume you always pick port number 1 and you never change the port. Thus, you will get the prize in $\frac{333}{999} = \frac{1}{3}$ of the time (when PNN happen). On the other hand, if you always choose to switch the doors, you would get the prize in $\frac{666}{999} = \frac{2}{3}$ of the time (when NPN or NNP happen). Thus, is always better to change the port.

Conclusion

If the answer still may seem odd to you right know, I think it is good to try thinking in larger number to get a better grasp of the problem. Assume we are given the same problem, but rather than having only 3 doors, we have 10^{100} (Just to be clear, the host will open all the ports, except the port you choose and one more port. The port with the prize cannot be open). This number is so large, that you could spent your entire lifespan playing this game repeatedly and never actually getting the prize, not even once. So, when you choose one port, it is faded to be wrong. Think on selecting a port as saying: 'the prize is not here'. Now, if you choose door A knowing that is the **wrong one**, and the host open all ports with exception of port A and port B, it makes sense to change your choice to port B.