4.9 Intro

In this question, we will perform sensor fusion for two sensor with known variances. This can be achieved with the theory of the last section of this chapter (4.6.4: Sensor fusion with unkown precision) where it starts with sensors of known precision and then goes to the case of unkown precision.

Solution

In the last section of the chapter the book derive an expression for the posterior $p(\mu|D)$. This expression is given by:

$$
\begin{aligned}
&p(\mu|D) = \mathcal{N}(\mu|m_n, \lambda_N^{-1}) \\
&m_n = \frac{\lambda_1 n_1 \bar{y}^{(1)} + \lambda_2 n_2 \bar{y}^{(2)}}{n_1\lambda_1 + n_2\lambda_2} \\
&\lambda_n = \lambda_0 + n_1\lambda_1 + n_2\lambda_2
\end{aligned} \tag{1}
$$

where $\bar{y}^{(i)} = \frac{1}{n_i}\sum_{k=1}^{n_i} y^{(i)}$ is the sample average. Now, we just need to put the expressions for the mean and the variance/precision in terms of the known variables: $y^{(1)}$, $y^{(2)}$, n_1, n_2, v_1 and v_2. Also, note that since the prior is a non-informative Gaussin, we have $\lambda_0 = 0$

$$
\begin{aligned}
&m_n = \frac{\lambda_1 n_1 \bar{y}^{(1)} + \lambda_2 n_2 \bar{y}^{(2)}}{n_1\lambda_1 + n_2\lambda_2} = \\
&\frac{\frac{1}{v_1} n_1 \bar{y}^{(1)} + \frac{1}{v_2} n_2 \bar{y}^{(2)}}{\frac{n_1}{v_1} + \frac{n_2}{v_2}} = \\
&\frac{v_2 n_1 \bar{y}^{(1)} + v_1 n_2 \bar{y}^{(2)}}{v_2 n_1 + v_1 n_2}
\end{aligned} \tag{2}
$$

$$
\begin{aligned}
&\lambda_n = n_1\lambda_1 + n_2\lambda_2 \implies \frac{1}{v_n} = \frac{n_1}{v_1} + \frac{n_2}{v_2} \implies \\
&v_n = \frac{v_1 v_2}{n_1 v_2 + n_2 v_1}
\end{aligned} \tag{3}
$$

Conclusion

In this exercise we performed sensor fusion using two sensors with known variances. The results are what we expected from theory. The posterior mean is a weighted average of the prior (the prior mean is zero) and the sample average of the two sensors. Moreover, since the posterior precision is the sum of the prior precison with the weighted precisions of the sensors and the variance is the inverse of the precision, we arrive at an equation that resembles the equivalent resistor in a circuit with two resistors in parallel. This expression, says that the posterior variance decreases with the number of measurements taken n_1, n_2 and that the it is lower than the original variances v_1 and v_2