4.

Intro

This question is a reinforcement of the point made on page 30, where the author use Bayes' rule to give a correct medical diagnosis.

Solution

Events

D - The patient have the disease

T - The test for the disease is positive

We want to know the probability of having the disease, given that the result of the test was positive. In probability terms, we want to know $P(D = true|T = true)$.

By Bayes' rule

$$P(D = true|T = true) = \frac{P(T = true|D = true)P(D = true)}{P(T = true)} \tag{1}$$

Now, we only need to compute the three terms from the right.

$$P(D = true) = \frac{1}{10,000} \tag{2}$$

,because the disease strikes one in 10,000 people.

$$P(T = true|D = true) = 0.99 \tag{3}$$

, because of the accuracy of the test.

$$\begin{aligned} P(T = true) = P(T = true, D = true) + P(T = true, D = false) \\ = P(T = true|D = true)P(D = true) + P(T = true|D = false)P(D = false). \end{aligned} \tag{4}$$

$$P(D = false) = \frac{9,999}{10,000} \tag{5}$$

$$P(T = true|D = false) = 1 - P(T = false|D = false) = 0.01 \tag{6}$$

, because the probability of the test to be negative when the patient do not have the disease is also 0.99.

Substituting (2), (3), (5) and (6) in (4), we have:

$$P(T) = \frac{0.99}{10,000} + \frac{9,999}{10,000} * 0.01 \approx 0.01. \tag{7}$$

Finally, the probability of actually having the disease is:

$$P(D = true|T = true) = \frac{0.99}{10,000 * 0.01} = 0.99\% \tag{8}$$

Conclusion

From the final result, we saw that the actual chance of having the disease is very small, despite the of the test result and its accuracy. This shows the importance that a prior can have in the final result.