

11.**Intro**

This is a classical math question about the 1D Gaussian. The author does most of the heavy lifting for us and also give us 2 hints. Thus, it should be no problem to derive the result.

Solution

Hint 1: Separate the integral into a product of two terms:

$$Z^2 = \int_0^{2\pi} \int_0^{\infty} r \exp\left(\frac{-r^2}{2\sigma^2}\right) dr d\theta = \int_0^{2\pi} d\theta \int_0^{\infty} r \exp\left(\frac{-r^2}{2\sigma^2}\right) dr = 2\pi \int_0^{\infty} r \exp\left(\frac{-r^2}{2\sigma^2}\right) dr \quad (1)$$

Hint 2: The author give us the result of the integral on r :

$$Z^2 = 2\pi \int_0^{\infty} r \exp\left(\frac{-r^2}{2\sigma^2}\right) dr = 2\pi\sigma^2 \int_0^{\infty} \frac{r}{\sigma^2} \exp\left(\frac{-r^2}{2\sigma^2}\right) dr = 2\pi\sigma^2 \left[-\exp\left(\frac{-\infty^2}{2\sigma^2}\right) + \exp\left(\frac{0}{2\sigma^2}\right)\right] = 2\pi\sigma^2 \quad (2)$$

Therefore, the normalization constant is:

$$Z = \sqrt{2\pi\sigma^2} \quad (3)$$

Conclusion As mentionet in the intro, this is a classical exercise about the 1D Gaussian. It usually appears in probability and calculus books. The most important passage is done by the author, where he uses the power of multivariable calculus to express a unidimensional problem in a simpler way. It is also interesting to mention that the normalization constant is a function of the standrad deviation σ , but not of the mean μ . This makes a lot of sense, since a change in σ changes the function shape and a change in μ only translates the curve in the horizontal axes.