9.

Intro

In this question we have to exercise our capacity of getting some information about conditional independence and see if we can infer something more based on it.

Solution

a)

We have two information about conditional independence. The first one is:

$$X \perp W|Z,Y$$

Based on this, we can state that:

$$p(x,w|z,y) = p(x|z,y)p(w|z,y) \tag{1}$$

The second one is:

$$X \perp Y|Z$$

Based on this, we can state that:

$$p(x,y|z) = p(x|z)p(y|z) \tag{2}$$

Now, we have to see if $X \perp Y,W|Z$ is true.

$$\begin{aligned} p(x,y,w|z) = p(x|z)p(y|x,z)p(w|x,y,z) = \\ p(x|z)p(y|z)p(w|y,z) = p(x|z)p(y,w|z) \end{aligned} \tag{3}$$

In (3), the first passage is the chain rule of probability. The second make use of the conditional independences given to us (Y do not depend on X given Z and W do not depend on X given Z and Y). The third passage is putting the joint distribution back together. Therefore, the proposition is **true**.

b)

Again, we have two information about conditional independence. The first one is:

$$X \perp Y|Z$$

Based on this, we can state that:

$$p(x,y|z) = p(x|z)p(y|z) \tag{4}$$

The second one is:

$$X \perp Y|W$$

Based on this, we can state that:

$$p(x,y|w) = p(x|w)p(y|w) \tag{5}$$

Now, we have to see if $X \perp Y|Z, W$ is true.

$$p(x, y|z, w) = p(x|z, w)p(y|z, w) \tag{6}$$

In (6), we used the fact that X and Y are conditionally independent given Z or W. So, the proposition is **true**.

Conclusion

There is not a major conclusion to be mentioned. As it was said it the begin, I think the main purpose of this exercise is just to exercise the brain in inference involving conditional independence.