6.

Intro

In this exercise we must determine which set of numbers are sufficient to determine the conditional probability vector $\vec{P}(H|e1, e2)$. As one can expect, the set of number will different when we assume that the events E1 and E2 are conditionally independent given H.

Before starting, I would like to give a shout out to github user andrewgilbert12, for warning me that the previous version of item (b) was incomplete and with the wrong answer marked. As we're going to see, the conditional independency allow us to make one additional simplification regarding the joint distribution, which is crucial to find the right answer.

Solution

a)

Using Bayes' rule:

$$P(H|e1, e2) = \frac{P(e1, e2|H)P(H)}{P(e1, e2)} \tag{1}$$

Therefore, we need $P(e1, e2)$, $P(H)$ and $P(e1, e2|H)$ to calculate the probability vector. This correspond to choice (ii).

b)

Knowing that $E1 \perp E2|H$ we can perform one more step on (1):

$$\begin{aligned} P(H|e1, e2) = \frac{P(e1, e2|H)P(H)}{P(e1, e2)} = \\ \frac{P(e1|H)P(e2|H)P(H)}{P(e1, e2)} \end{aligned} \tag{2}$$

Originally, I said that this was the final answer. If that were the case, then $P(e1, e2)$, $P(H)$, $P(e1|H)$ and $P(e2|H)$ would be the sufficient set of numbers to calculate the probability vector. This correspond to choice (i).

Nevertheless, there is one more step that we need to perform. For this, let's marginalize the joint distribution in terms of H

$$P(e1, e2) = \sum_h P(e1, e2, h) = \sum_h P(h)P(e1, e2|h) = \sum_h P(h)P(e1|h)P(e2|h) \tag{3}$$

In the last step, we made use of the conditional independency property between $E1$ and $E2$. So, we just demonstrated that the joint distribution $P(e1, e2)$ can be decomposed in terms of $P(H)$, $P(e1|H)$ and $P(e2|H)$. Thus, the **right answer** of this item is choice (iii).

Conclusion

As we can see, in order to calculate the posterior there is a set of **sufficient** numbers that should be previously known. The set of numbers required can change, if we make more assumptions about the random variables. In this particular exercise, we saw in (b) that assuming $E1 \perp E2|H$ changed the set of numbers required for Bayes' calculation.