**5.2**

**Intro**

In this exercise we will solve a multi-output linear regression exercise. One of the major aspects of this particular exercise is that the outputs are **independent**. As a result, we essentitly have in our hands two separate linear regression problems with the same input. Furthermore, if we take a closer look on the output values of $y_1$ and $y_2$, we can see that they are the same, with exception of the order of appereance. Let's jump to the exercise and see how this will affect the result.

**Solution**

Expanding Equation 7.92, our model becomes:

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} -\hat{w}_1{}^t- \\ -\hat{w}_2{}^t- \end{bmatrix} \begin{bmatrix} \phi_1(x) \\ \phi_2(x) \end{bmatrix} \tag{1}$$

For the full dataset, the model becomes:

$$\begin{bmatrix} y_1^{(1)} & y_1^{(2)} & \dots & y_1^{(n)} \\ y_2^{(1)} & y_2^{(2)} & \dots & y_2^{(n)} \end{bmatrix} = \begin{bmatrix} -\hat{w}_1{}^t- \\ -\hat{w}_2{}^t- \end{bmatrix} \begin{bmatrix} \phi_1^{(1)} & \phi_1^{(2)} & \dots & \phi_1^{(n)} \\ \phi_2^{(1)} & \phi_2^{(2)} & \dots & \phi_2^{(n)} \end{bmatrix} \tag{2}$$

From Equation 2 we conclude that our multi-output problem is nothing more than the two following single output linear regression:

$$\begin{aligned} y_1 &= \phi(X)\hat{w}_1 \\ y_2 &= \phi(X)\hat{w}_2 \end{aligned} \tag{3}$$

Solving the two problems using ordinary least squares we find the following weight vectors:

$$\begin{aligned} \hat{w}_1 &= [-\frac{4}{3}\ \frac{4}{3}]^t \\ \hat{w}_2 &= [-\frac{4}{3}\ \frac{4}{3}]^t \end{aligned} \tag{4}$$

As we can see in Equation 4, the weight vectors are equal for both outputs

**Conclusion**

In this exercise, we performed multi-output linear regression. The problem was straightfoward because the independenc between the outputs made possible to decompose our original problem into two single output linear regression sharing the same input. We solved the problem using ordinary least squares and discovered that the weight matrix are equal for both outputs. This is no coincidence because taking a closer look on the outputs, we see that $y_1$ and $y_2$ have the same pairs $(x, y)$ of input/output. The only difference is that they do not appear in the same order.

The big conclusion of this exercise is that we should not fear the multidimensional output case, because is easy to extend the single output analysis to tackle it. A more challenging probelm would be one involving multi-output linear regression with dependency between the output variables.