Hopping in Legged Systems—Modeling and Simulation for the Two-Dimensional One-Legged Case

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Abstract—**In this paper a two-dimensional one-legged hopping machine is modeled and simulated in order to better understand legged systems that hop and run. The analysis is focused on balance, dynamic stability, and resonant oscillation for the planar case. A springy leg with nonzero mass, a simple body, and an actuated hinge-type hip are incorporated in the model. Control of the model is decomposed into a vertical hopping part, a horizontal velocity part, and a body attitude part. Estimates of total system energy are used in regulating hopping height in order to initiate hopping, to maintain level hopping, to change from one hopping height to another, and to terminate hopping. Balance and control of forward velocity are explored with three algorithms. First the role of foot placement in balance through a linear algorithm that stabilizes the system and generates low velocity translations from point to point is studied. Second, control of forward velocity is controlled by considering constraints that arise in constant velocity forward travel, and the *CG print* is introduced. In the improved algorithm the foot is placed forward with respect to the center of the *CG print* during flight, and the leg is swept backward during stance. Third, control of body attitude is improved by using the hip actuator to correct pitch errors during stance. The feasibility of decomposing control of running into a height control part, a forward velocity control part, and an attitude control part are verified by simulations.**

I. Introduction

SUBSTANTIAL study has been devoted to understanding legged systems that crawl and walk, but little attention has been given to systems that run and hop [1]–[7]. During crawling and walking, support is provided by at least one leg at all times, but in running and hopping, support is provided only intermittently with intervening periods of ballistic flight. One consequence of intermittent support is the vertical bouncing motion that characterizes running. A second consequence of intermittent support is the intermittent opportunity for the system to change its angular momentum to maintain balance and control attitude. Because angular momentum is conserved when there are no external forces on the system, torques can be applied to the body to change angular momentum only when the system is in contact with the ground. A further consequence of intermittent support is that leg dynamics play an important role in determining a system's behavior—both the pattern and efficiency of a running system's motion are influenced by leg dynamics.

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In this paper we model a hopping system with just one leg, and simulate its behavior as controlled by algorithms that manipulate hopping height and running speed, while maintaining balance and attitude. The purpose is to understand the principles of balance and dynamic stability as they apply to legged systems while ignoring the problem of coupling many legs. Since each leg in multilegged running systems does roughly the same thing as every other leg, the hopping of a one-legged model can be viewed as a fundamental activity through which running and certain types of walking can be better understood.

Hopping is actually a special case of running, in which all legs provide support at the same time. For a system with one leg, running and hopping are the same. The part of the stepping cycle in which the leg is unloaded, called *transfer*, is also the part of the cycle in which no legs give support, called *flight*. During flight motion of the center of gravity of the system is ballistic. The period when the leg provides support is called *stance*, during which behavior of the system is like that of an inverted pendulum.

The model developed here incorporates a springy leg, in imitation of legs found in nature. While rigid massless leg models have sufficed to study walking [5], [8], leg models that include mass and springs are important for understanding running and hopping. The stiffness of the leg influences the vertical oscillatory behavior of the hopping system and governs the details of landing on the ground and of taking off. The resonant interaction between body mass and springy legs in the vertical direction has a profound impact on the behavior of a running system. This has been shown by McMahon and Green [9] in the human, and by Dawson and Taylor [10] and Alexander and Vernon [11] in the hopping kangaroo.

Previous work on balance began with Cannon's control of inverted pendulums that rode on a small powered truck [12]. His experiments included balance of a single pendulum, two pendulums one atop the other, two pendulums side by side, and a long limber pendulum. Hemami and his coworkers [1], [8], [13]–[15], Vukobratovic and his coworkers [5], [16], [17], and others [18]–[22] have studied the dynamic characteristics of a variety of multilink legged models that walk. These models range from a fully static

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walking biped described by Juricic [16] to the dynamically stabilized five-link model of Hemami and Farnsworth [8]. Each of these models relies on continuous contact with the support surface. Additional references to work on walking can be found in the bibliography of [23].

Balance in hopping has also been studied. In 1967 Seifert explored the concept of a hopping vehicle for lunar exploration [24]. The primary means of balance employed in his design were a moment exchange gyro that could stabilize the system by reorienting it in flight, and a foot with locking ankle. Matsuoka [25] analyzed hopping in humans with a one-legged model. He derived a time-optimal state feedback controller that stabilized his system, assuming that the leg could be massless, and that the stance period could be of very short duration. In fact, legs comprise a substantial fraction of a human's mass, and the duration of stance during running for each leg of a biped is typically about 40 percent of the total duration of a stride [25]. Therefore the model used in the present paper includes nonzero leg mass, and a ratio of leg stiffness to body mass that makes it operate in a regime where support time is about 40 percent of stride time.

In this paper control of hopping is presented as two parts: a vertical control part that uses energy measures to regulate hopping height, and a horizontal control part that maintains balance and generates forward travel. Although there are interactions between these activities, their dynamics are not strongly coupled. Unlike Seifert's design, the model used here does not have a moment exchange gyro and the foot is of zero dimension. Therefore the system changes angular momentum only during stance, when hip torques generate reaction forces on the ground, and no torques develop between the foot and the ground.

The section that follows develops the one-legged hopping model and characterizes its behavior. Section III presents an analysis of hopping and the control algorithm used to regulate hopping height. Section IV describes three algorithms that provide balance while hopping in place and running.

II. The Model

The basic components of legged systems are a set of legs and a body to which the legs are attached. For humans and other animals the body has many actuated degrees of freedom whose actions enhance performance and versatility. For instance, stretching of the back efficiently increases stride length for the running quadrupeds. The body also carries the payload and sensors. The most important characteristic of the body is that it forms an elevated mass that must be balanced atop the legs, and that it forms a structure from which torques can be applied to the legs.

Legs typically do two things during locomotion: they change length and they change orientation with respect to the body. This is true for organisms that crawl, walk, run, and hop, and for organisms with two legs, four legs, six legs, and many legs. A leg changes length to propel the body upward and forward, to cushion landings, and to reduce its own moment of inertia and increase its clearance when swung forward. The lengthening and shortening of a leg during these activities is not merely a kinematic action, but a dynamic action governed by the resonant interaction of leg compliance, body mass, and gravity [26]. Energy is stored in springy muscle and tendon when the leg is shortened, and energy is retrieved when the leg is lengthened. Legs swing back and forth to propel, to permit feet to be precisely positioned with respect to the system's center of gravity, and to change angular momentum.

The model used in this paper, shown in Fig. 1, has a single springy leg that articulates with respect to a body about a simple hinge-type hip. The body is represented by a rigid mass, to which the leg is connected. The leg has mass M_1, moment of inertia I_1, and the body has mass M_2, moment of inertia I_2. The center of mass of the leg is located a distance r_1 from the lower tip of the leg, which is the *foot*. The center of mass of the body is located a distance r_2 above the hip.

A control torque τ is generated between the body and the leg at the hip. A simple linear servo is closed around this actuator to position the leg or body. It is of the form

$$\tau(t) = K_P(\theta_1 - \theta_{1,d}) + K_v(\dot{\theta}_1) \tag{1}$$

where

$\theta_{1,d}$ desired leg angle
K_P, K_v feedback gains.

The same feedback rule is used during stance and during flight, but with different values for K_P and K_v, as listed in Appendix II.

The overall length of the leg is influenced by a spring, a position actuator in series with the spring, and a mechanical stop. The leg spring is modeled as though one end is rigidly connected to the foot, with the other end fastened to one side of the position actuator. The mechanical stop, modeled as a very stiff spring with damping, also acts between the foot and one side of the position actuator. The spring acts to lengthen the leg and the mechanical stop acts to shorten it. The spring and mechanical stop are arranged so that only the spring generates forces when $(w - \chi) < k_0$, and only the mechanical stop generates forces when $(w - \chi) > k_0$, see Fig. 1. The spring constant and damping coefficient of the mechanical stop, K_{L2} and B_{L2}, are chosen so that vibrations that occur between the body and leg when the stop is hit decay within a few cycles.

The position actuator, length χ, is arranged in series with the leg spring, acting between the leg spring and the hip. Changes in actuator length typically do work on the leg spring to increase or decrease its stored energy. The leg spring can also absorb energy from the system when it shortens under load of the body, and return energy to the system when it lengthens, accelerating the body upward.

While the position actuator is represented as an ideal source, the finite response time of a physical actuator is taken into consideration by requiring that χ increase and decrease with a quadratic trajectory:

$$\chi(t) = \chi_0 + kt^2 \tag{2}$$

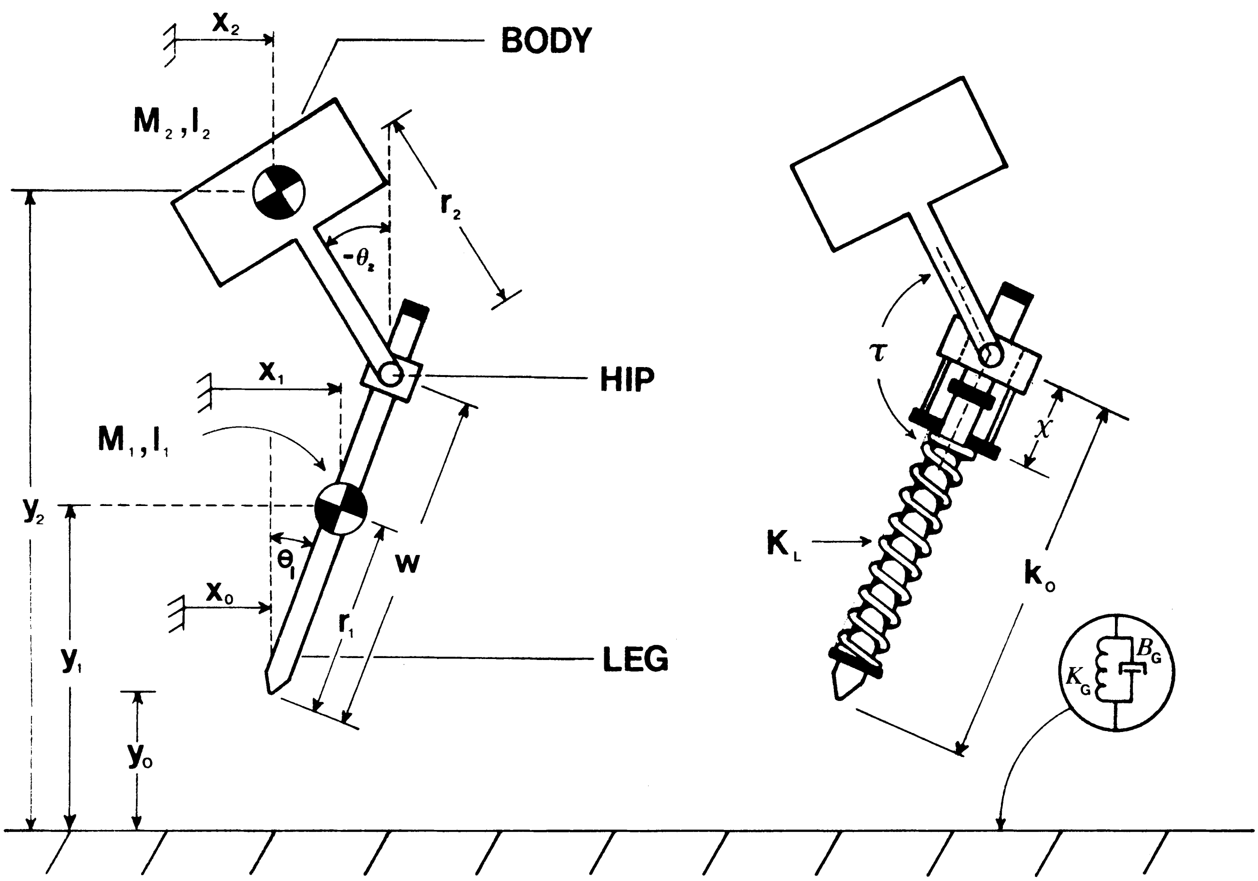

Fig. 1. Planar one-legged model used for analysis and simulation. The body and leg, each having mass and moment of inertia, are connected by a hinge joint at which torque is generated. The leg consists of a spring in series with a position actuator. The support surface is springy itself in two dimensions. The model is restricted to motion in the plane. See Appendix I for equations of motion and Appendix II for model parameters used in simulations. Leg angle θ_1, leg actuator length χ, and hip actuator torque τ are shown positive. Body angle θ_2 is shown negative.

where

χ_0 initial length of position actuator
k timing constant.

Stroke of the position actuator is limited to $\chi_{\min} < \chi < \chi_{\max}$, with $\chi_{\min} > 0$. The importance of this arrangement of actuator, spring, and mechanical stop is that during support, rhythmic activation of the actuator can excite resonant oscillations in the spring–mass system formed by body and leg. As these oscillations build in amplitude, the system will leave the ground and hop.

The support surface is modeled as a two-dimensional spring K_G, and damper B_G. One dimension of the spring acts vertically, the other horizontally, with no interaction between the two. The spring and damper influence the hopper only when the foot is in contact with the ground, $y_0 < 0$. During flight the coefficients of spring and damper are zero. Each time the foot touches the ground, the rest position of the horizontal ground spring is reset to the point at which the foot first touches. The damping coefficient is chosen to make vibrations between the foot and ground negligible, while the coefficient of friction between the foot and ground is assumed to be so large that slipping never occurs. This springy model of the ground represents both the compliances that might be present in a support surface, and those found in a well-designed foot.

The leg actuator acts between the body and leg spring. Lengthening the actuator when the leg is providing support does positive work on the system by compressing the leg spring and accelerating the body mass upward. Shortening the actuator during support does negative work on the system. Energy is injected into the system over a number of hopping cycles by lengthening the position actuator during support and shortening it during flight. By changing the phase of these actions it is possible to remove energy from the system.

The leg mass, M_1, represents that portion of the leg below the spring, the rest being included in M_2. We also assumed that the stiffness of the ground is much greater than the stiffness of the leg, $K_G \gg K_L$. The following analysis applies to repetative hopping in which periods of support alternate with periods of flight. When the leg provides support, the model is a spring–mass oscillator with natural frequency:

$$\omega_n = \sqrt{\frac{K_L}{M_2}}. \tag{3}$$

During repetative hopping, each stance interval has duration:

$$T_{st} = \frac{\pi}{\omega_n} = \pi\sqrt{\frac{M_2}{K_L}}. \tag{4}$$

During flight the model is a gravity–mass oscillator. A full hopping cycle has period

$$T = \pi\sqrt{\frac{M_2}{K_L}} + \sqrt{\frac{8H}{g}} \tag{5}$$

where

K_L stiffness of leg spring
H hopping height measured at the foot
g acceleration of gravity.

For the duration of support to equal the duration of flight, hopping height must be

$$H = \frac{\pi^2 M_2 g}{8K_L}. \tag{6}$$

Equations of motion for the model with respect to a ground-centered coordinate system were derived using d'Alembert's principle. The resulting system of nonlinear coupled differential equations is given in Appendix I. These equations describe the system's behavior for both stance and flight, with the characteristic equation of the ground spring responsible for introducing the multiphase nature of the model. Computer simulations used standard digital numerical integration techniques to determine system behavior as a function of time [27]. The numerical constants used for simulation are given in Appendix II.

The simple nature of this model captures the important aspects of dynamic locomotion while keeping complications to a minimum: since there is only one leg, active balance is studied directly while avoiding the problems of coupling between legs and of multiple support phases. At the same time, vertical oscillations are easily studied by using a leg model that includes mass, spring, and position actuator. Three-dimensional dynamics are avoided by considering the planar case.

III. Vertical Control

The task of synthesizing a system that will control behavior of the one-legged model was broken into two sets of algorithms. One set of algorithms controls forward running velocity, allowing the system to translate from place to place while maintaining the body in an upright posture. This is called horizontal control, about which more is said in the next section. This section discusses a separate control algorithm that controls vertical motion. It generates stable resonant oscillations that cause the locomotion system to hop off the ground, and it controls the height of each hop.

An important function of this hopping motion is to establish a regular cycle of activity within which horizontal and attitude control can take place. A wheel changes its point of support continuously and gradually while bearing weight Unlike a wheel, a leg changes its point of support all at once and must be unloaded to do so. Therefore, in order for a legged system to balance and to make forward progress there must be periods of support when the leg bears weight making the foot immobile, and there must be other periods when the leg is unloaded and the foot free to move. Such an alternation between a loaded phase and an unloaded phase is observed in the legs of all legged systems. For the present system this alternation is the hopping cycle. There are four well-defined events in the hopping cycle.

- *Lift-Off*: The moment at which the foot loses contact with the ground.
- *Top*: The moment in flight when the body has peak altitude and vertical motion changes from upward to downward.
- *Touchdown*: The moment the foot makes contact with the ground.
- *Bottom*: The moment in stance when the body has minimum altitude and vertical motion of the body changes from downward to upward.

These events are each detected from behavior of the state variables, and are used to determine four distinct states. The regular cyclic progression among these states suggests use of a finite state sequencer to organize control of the system.

Vertical control must initiate hopping, control hopping height, change between different hopping heights, and terminate hopping. These tasks can be accomplished by regulating the energy in the oscillating spring–mass system formed by the springy leg and the mass of the body. Hopping is initiated by exciting the spring/mass oscillator with the position actuator until hopping velocity is reached —when the energy in the system is sufficiently large to overcome gravity, the foot leaves the ground and hopping begins. At this point the system becomes a spring/mass, gravity/mass oscillator.

The height of a hop can be controlled by measuring and manipulating the system's energy. For the simplified case in which motion is primarily vertical, angles and angular rates of the leg and body, θ_1, $\dot{\theta}_1$, θ_2, and $\dot{\theta}_2$ are negligible. The total vertical energy is

$$\begin{aligned} E_{\text{STANCE}} &= \mathrm{PE}_g(M_1) + \mathrm{PE}_g(M_2) + \mathrm{KE}(M_1) + \mathrm{KE}(M_2) \\ &\quad + \mathrm{PE}_e(M_1) + \mathrm{PE}_e(M_2) \\ &= M_1 g y_1 + M_2 g y_2 + \frac{1}{2}M_1\dot{y}_1^2 + \frac{1}{2}M_2\dot{y}_2^2 \\ &\quad + \frac{1}{2}K_L(k_0 - w + \chi)^2 + \frac{1}{2}K_G y_0^2 \end{aligned} \tag{7}$$

where

PE_g gravitational potential energy
PE_e elastic potential energy
KE kinetic energy
g acceleration of gravity
k_0 rest length of the leg spring
K_G stiffness of the ground, $K_G = 0$ for $y_0 > 0$

Additional variables are defined in Fig. 1.

The expressions for potential energy were chosen so that they are zero when the hopper is standing vertically with the leg spring extended to its rest length and with the foot just touching the ground. When the leg spring is at rest length it just touches the mechanical stop. As (7) shows, energy may be stored in the leg spring, in the ground spring, and in the motion of the body and leg masses.

Energy is lost to the ground damping throughout stance and to air resistance throughout the hopping cycle, but such losses are generally small [28] and are disregarded. Significant energy losses occur at two events in the hopping cycle, TOUCHDOWN and LIFT-OFF. At TOUCHDOWN the leg is very suddenly brought to rest by dissipating its kinetic energy in ground damping:

$$E_{\text{TD-LOSS}} = \text{KE}(M_1) = \frac{1}{2} M_1 \dot{y}_{1,\text{TD}-}^2 \tag{8}$$

where

$\dot{y}_{1,\text{TD}-}^2$ is the vertical velocity just before TOUCHDOWN.

At LIFT-OFF damping in the stiff region of the leg spring dissipates a fraction of the system's kinetic energy. This fraction can be calculated by equating the system's linear momentum just before and after LIFT-OFF. Since the leg is stationary during stance its vertical velocity is zero. When the leg extends fully during stance the hopper leaves the ground, accelerating the leg from rest to $\dot{y}_{1,\text{LO}+}$. After LIFT-OFF the leg and body move at the same rate. Equating linear momentum before and after LIFT-OFF:

$$\dot{y}_{2,\text{LO}+} = \frac{M_2}{(M_1 + M_2)} \dot{y}_{2,\text{LO}-}. \tag{9}$$

Substituting (9) into (7), we can find the kinetic energies before and after LIFT-OFF. The loss associated with accelerating the leg upward is

$$E_{\text{LO-LOSS}} = \frac{M_1}{M_1 + M_2} \text{KE}_{\text{LO}-} = \frac{M_1 M_2}{2(M_1 + M_2)} \dot{y}_{2,\text{LO}-}^2 \tag{10}$$

where

$KE_{\text{LO}-}$	total kinetic energy just before LIFT-OFF
$\dot{y}_{2,\text{LO}-}^2$	vertical velocity of the body just before LIFT-OFF
Subscript LO−	just before LIFT-OFF.

The fraction $M_2/(M_1 + M_2)$ represents a fundamental efficiency of the leg. It is maximized when the ratio of leg mass to body mass is minimized. This can be done by minimizing the unsprung mass of the leg. To compensate for losses the vertical controller operates the position actuator to increase the vertical energy. When χ changes from χ_i to $\chi_i + \Delta\chi$ with $w < k_0$, then there is an energy change

$$\Delta E_\chi = K_L \left[\frac{1}{2} \Delta\chi^2 + \Delta\chi(\chi_i - w + k_0) \right]. \tag{11}$$

Energy is removed when $\Delta\chi$ is negative. For a given $\Delta\chi$ the magnitude of ΔE depends on the length of the leg and the position actuator. More work is done when the spring is compressed than when it is relaxed. Lengthening the actuator at BOTTOM and shortening during flight causes the total hopping energy to increase. Shortening the actuator at BOTTOM and lengthening during flight causes the total hopping energy to decrease, eventually to zero.

The task of the vertical control algorithm is to manipulate the altitude to which the system will bounce. If the leg spring assumes its rest length during flight, all energy takes the form of gravitational potential when $\dot{y}_2 = 0$ at the top of each hop. It is possible, therefore, to predict the height of the next hop at any time during stance. All energy is in the form of kinetic energy at LIFT-OFF.

$$E_{\text{LO}-} = \text{KE}_{\text{LO}-} = E_{\text{STANCE}}. \tag{12}$$

The fractional loss of kinetic energy at LIFT-OFF is known from (10). Neglecting ground damping and air resistance losses, the total energy during flight is obtained in terms of variables available during stance by combining (10) and (7):

$$E_{\text{FLIGHT}} = \frac{M_2}{M_1 + M_2} \Big[M_1 g y_1 + M_2 g y_2 + \frac{1}{2}\left(M_1 \dot{y}_1^2\right) + \frac{1}{2}\left(M_2 \dot{y}_2^2\right) + \frac{1}{2}\left(K_L (k_0 - w + \chi)^2\right) + \frac{1}{2}\left(K_G y_0^2\right) \Big] \tag{13}$$

For the system to hop to height H the total vertical energy must be

$$E_H = M_1 g(H + r_1) + M_2 g(H + k_0 + r_2). \tag{14}$$

During stance the energy change needed to produce a hop of height H, ΔE_H, can be supplied or removed by the vertical actuator. From (11) the linear actuator must extend by

$$\Delta\chi = -(\chi - w + k_0) + \sqrt{(\chi - w + k_0)^2 + \frac{2\Delta E_H}{K_L}}. \tag{15}$$

A parameter that is important to the mechanical design of a legged system is how much the leg spring must compress and the leg shorten during the stance portion of the hopping cycle. The maximum compression of the leg spring during stance is a function of body and leg mass, leg stiffness, and hopping height. It can be found from (14) and (15):

$$\Delta w = \frac{M_2 g}{K_L} + \sqrt{\frac{M_2^2 g^2}{K_L^2} + \frac{2(M_1 + M_2)^2 g H}{M_2 K_L}}. \tag{16}$$

Simulations were used to evaluate application of this vertical control algorithm to the model. Each time BOTTOM occurred, indicated by $\dot{y}_0$ changing sign from negative to positive, (7) and (13) were used to predict hopping height. The length of the leg actuator, χ, was then increased or decreased accordingly. Fig. 2 plots the vertical position of the foot and body, and the actuator length during a period of increasing hopping height, and during stable hopping. Starting at rest, the system executed a positive work cycle on each hop until the vertical energy increased to the specified value. This level was then maintained. Since the stroke of the position actuator was limited to $\chi_{\max}$, the

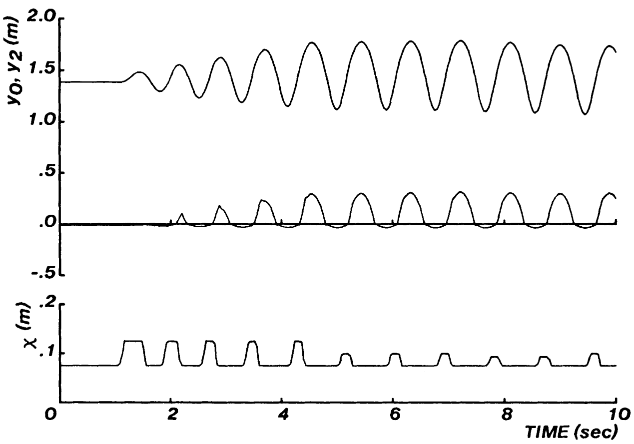

Fig. 2. Vertical hopping. Starting from rest, vertical energy was increased until desired hopping height was attained. Hopping height was regulated through the position actuator that acts in series with the leg spring. Note different vertical scales. Top curve—elevation of hip. Middle curve—elevation of foot. Bottom curve—length of leg actuator.

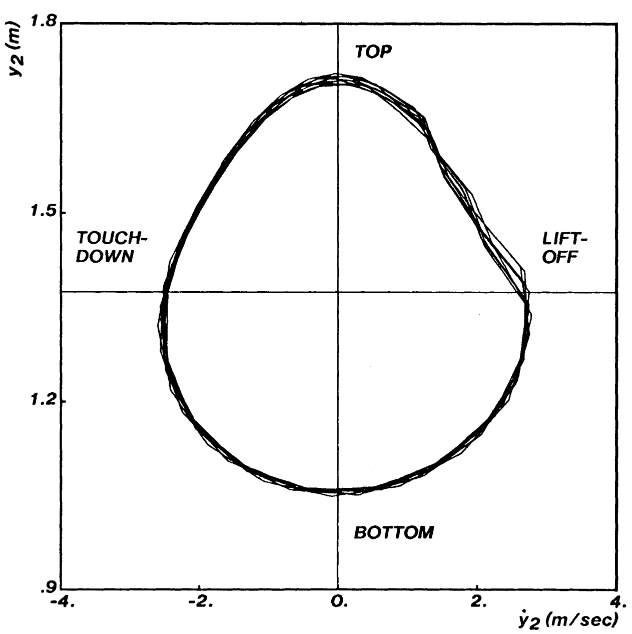

Fig. 3. Phase plot for vertical hopping. Four synchronization events are indicated where curve crosses axes. Data are from stable part of Fig. 2. The rough part of the curves between LIFT-OFF and TOP indicate the damped vibration that occurred when the mechanical stop was hit. Note that position is plotted on the ordinate, velocity is on the abscissa, and the action advances in a counterclockwise direction.

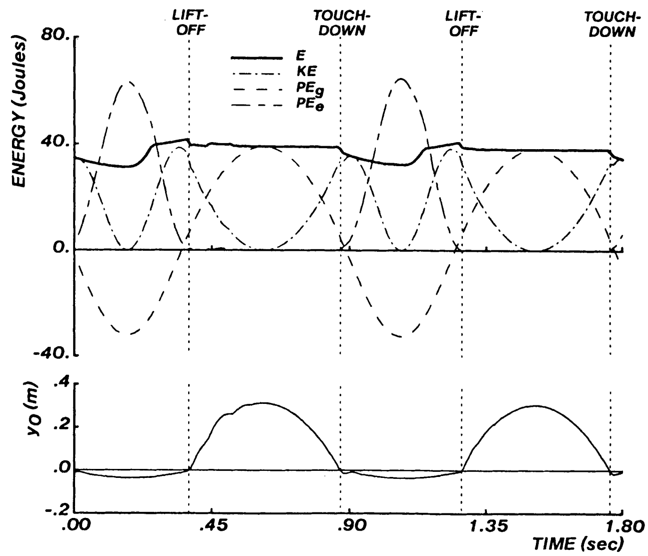

Fig. 4. Vertical energy for two hopping cycles at constant hopping height. Total energy, kinetic energy, gravitational potential, and elastic potential are shown. Primary losses of energy occur at TOUCHDOWN and LIFT-OFF, indicated by the vertical dotted lines. Data are from Fig. 2, $t = 7.5$ to 9.3 s.

energy that could be injected on a single cycle was limited. A number of cycles therefore was required to achieve the desired hopping height.

The last 2 seconds of data from Fig. 2 are replotted in the phase plane in Fig. 3. The body velocity is plotted on the abscissa and body altitude is plotted on the ordinate. The parabolic trajectory during flight was caused by constant gravitational acceleration, and the harmonic trajectory during stance was due to the spring. The four events that synchronized actions of the controller to behavior of the hopping system, LIFT-OFF, TOP, TOUCHDOWN, and BOTTOM are indicated in the figure.

Fig. 4 is a plot of vertical energy during two cycles of fixed height hopping. A lossless system would produce a perfectly flat total energy line. The primary losses occur when the foot strikes the ground, and when it leaves the ground, as indicated by (8) and (10). Energy increases during the latter part of stance, when the actuator lengthens. These data are similar in qualitative detail, to those obtained for the kangaroo by Alexander and Vernon [11].

Fig. 5 shows an 80 s simulation sequence of vertical hopping in which desired height was adjusted a number of times. It was mentioned earlier that the leg actuator could be used to remove energy from the system, to reduce hopping height. The figure shows that a descent employing active damping ($t = 45$) was more rapid than one relying on passive system losses ($t = 65$). In general, the algorithm obtained good control of hopping height.

The time at which the leg is shortened during a steady state hop cycle can be manipulated to optimize hopping according to a variety of criteria.

- When the leg is shortened at LIFT-OFF, ground clearance of the foot during flight is optimized. This is important when terrain is uneven or when large horizontal swinging motions of the leg occur during flight, as when the model translates at high speed. If the leg is not short during swing it may become difficult to avoid *stubbing the toe*. Shortening at LIFT-OFF also minimizes the leg's moment of inertia during flight, so the leg can be swung forward faster and with less angular effect on the body.
- When the leg is shortened at TOP, the time between vertical actuations is maximized. This strategy could permit use of an actuator of lower bandwidth.
- When the leg is shortened at TOUCHDOWN, then the ground impact forces on the foot are minimized. This strategy is normally used by humans when they are asked to hop in place on a flat floor.

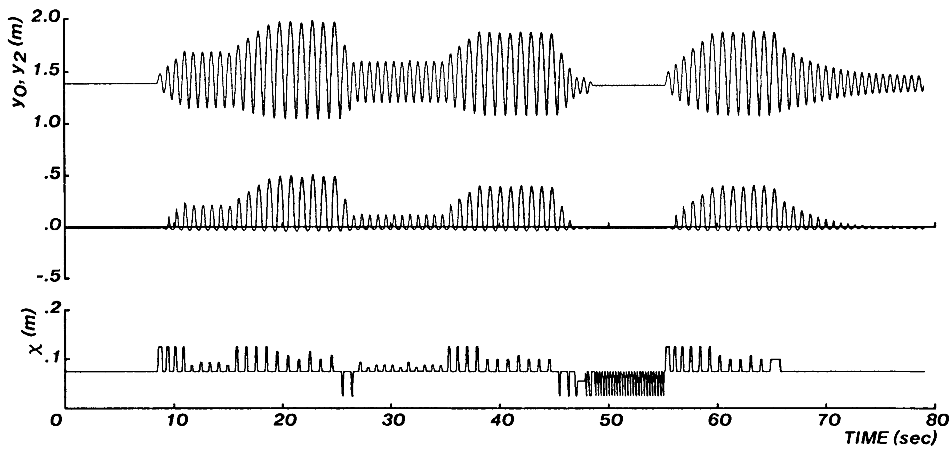

Fig. 5. Vertical hopping sequence. At times $t = 9, 15, 25, 35, 45, 55, 65$ desired hopping height $H = 1.7, 2.0, 1.6, 1.9, 1.4, 1.9, 1.4$. The descent beginning at $t = 45$ was actively damped, while the descent at $t = 65$ was passive. Top curve—elevation of hip. Middle curve—elevation of foot. Bottom curve—length of leg actuator.

It is also possible to shorten the leg at LIFT-OFF, lengthen it again just before the next TOUCHDOWN, and let it shorten during the landing. This strategy, apparently used by humans when running, maximizes ground clearance and simultaneously minimizes impact forces on the foot. However this is accomplished at the expense of additional actuator bandwidth. Although more energy is required for these extra lengthening and shortening motions, the leg can be swung forward more efficiently with a lower moment of inertia.

IV. Horizontal Control

The preceding section discusses the vertical hopping behavior of the model, and a method for controlling hopping height. We now turn to the question of balance, and control of travel from place to place. These two phenomena are intimately related. The tasks of a balance algorithm are to ensure that there are no unwanted horizontal motions, that horizontal motions of adequate velocity are generated when necessary, and that the locomotion system does not tip over.

An important characteristic of dynamically stabilized legged systems is that they are always tipping, but their control system ensures that the tipping motions are controlled and orderly. Two mechanisms can be used to control balance and horizontal travel: foot placement and hip motion. Placement of the foot with respect to the center of gravity of a locomotion system has a powerful influence on the tipping and horizontal motion of the system. Gravity generates a moment about the foot proportional to the horizontal displacement of the foot from the center of gravity. Foot placement can be adjusted during the flight part of each hop to influence attitude and translation during the next stance interval. The pattern of hip motion between the body and leg during stance influences the system's total angular momentum. Such motions change the momentum of the system only during stance when there is adequate friction to hold the foot firmly in place on the ground.

Actually, there is a third mechanism that can influence the balance and horizontal travel of a legged system. Since a leg is not always vertical, it is possible to influence the system's horizontal behavior by modulating the forces generated along the leg's axis. Effective use of this mechanism, however, requires intimate coordination between vertical and horizontal control. In order to keep the vertical control mechanisms separate from the horizontal control mechanisms, and thereby obtain simplicity in the control, this mechanism is not used. Rather, we permit the pattern of axial leg forces to be dictated solely by the requirements of regulating resonant vertical hopping.

The remainder of this section explores three algorithms for controlling balance and travel. The first method relies solely on foot placement as a means for effecting horizontal control, with no hip motion during stance. The second method uses an improved foot placement algorithm that is developed by considering the kinematic constraints imposed by constant velocity locomotion. The improved foot placement is combined with hip motion that sweeps the leg backwards during stance. The third method uses the same foot placement as method two, but gets better control of body attitude by servoing the body angle during stance.

Method 1: Foot Placement

Each time a hopping system touches the ground the foot can be positioned horizontally to influence the translation and tipping of the system. When hopping upright in place, $(\theta_2 = 0, \dot{\theta}_2 = 0, \dot{x}_2 = 0)$, movement of the foot to one side during flight causes the body to tip and translate toward the other side during stance. Similar rules hold when the system is not upright and stationary. Therefore, if the hip angle is kept fixed during stance, i.e. $(\theta_2 - \theta_1)_{\mathrm{LO}} = (\theta_2 - \theta_1)_{\mathrm{TD}}$, then the model behaves qualitatively like a one link inverted pendulum. It is not precisely an inverted pendulum because the leg shortens during stance.

A simple algorithm to balance the model uses linear feedback to place the foot. Two factors determine where the foot should be placed: the projection of center of

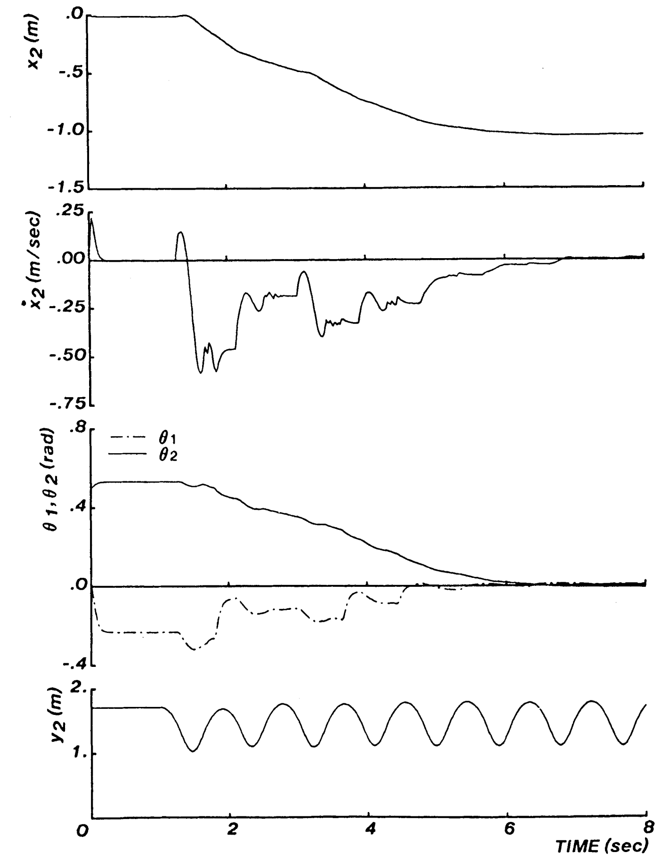

Fig. 6. Test of foot placement controller. At $t = 0$ initial error in body attitude was 0.5 rad. At $t = 1$ gravity is turned on and hopping begins. State errors approached zero in about 7 s. $K_1 = 0.05$, $K_2 = 0.3$, $K_3 = 0.1$.

gravity and an error function of state variables. First, the projection of the center of gravity x_{CG} is calculated. Then a linear function of errors in state x_{ERR} is added. Model kinematics are then used to calculate a leg angle that places the foot. Since the leg has mass, movement of the leg changes the projection of the center of gravity—simultaneous equations are solved. The following analysis is done in a coordinate system that translates with the hip.

Find horizontal position of center of gravity

$$x_{CG} = \frac{(r_1 - w)M_1 \sin(\theta_1) + r_2 M_2 \sin(\theta_2)}{M_1 + M_2}. \quad (17)$$

Calculate linear combination of state errors to provide corrective feedback.

$$x_{ERR} = K_1(\dot{x}_2 - \dot{x}_{2,d}) + K_2(\theta_2 - \theta_{2,d}) + K_3(\dot{\theta}_2) \quad (18)$$

where

$\dot{x}_{2,d}, \theta_{2,d}$	desired values for $\dot{x}_2$ and θ_2
K_1, K_2, K_3	feedback gains.

Place foot at TOUCHDOWN.

$$x_{TD} = x_{CG} + x_{ERR} \quad (19)$$

Take kinematics of model into account.

$$w \sin(\theta_1) = -x. \quad (20)$$

Substitute (17) into (19) and the result into (20), and solve for foot placement with respect to hip.

$$x_{TD} = w \frac{r_2 M_2 \sin(\theta_2) + (M_1 + M_2) x_{ERR}}{r_1 M_1 + w M_2}. \quad (21)$$

Apply leg kinematics again to obtain corresponding leg angle.

$$\theta_1 = -\mathrm{Arcsin}\left[\frac{r_2 M_2 \sin(\theta_2) + (M_1 + M_2) x_{ERR}}{r_1 M_1 + w M_2}\right] \quad (22)$$

The servo given in (1) is used to move the leg to this angle.

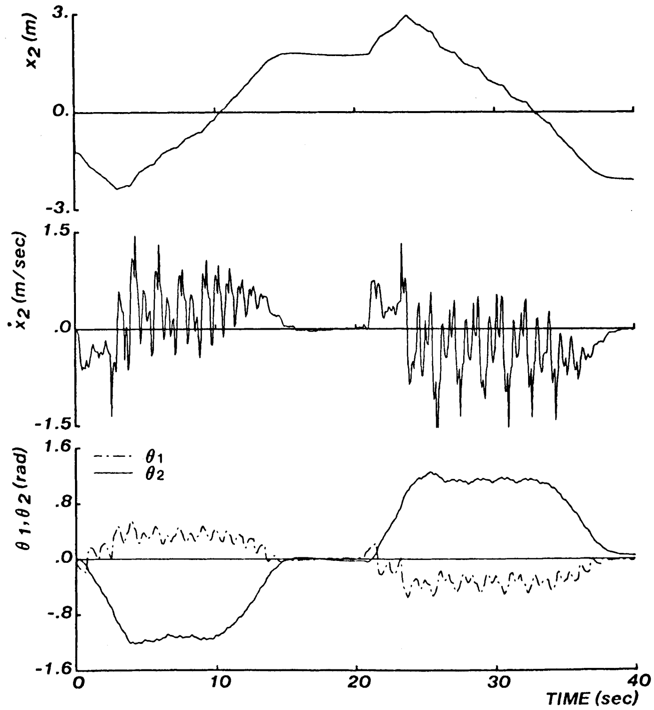

Fig. 7. Response of model to step in $x_{2,d}$ using foot placement algorithm. Horizontal position errors generated rate setpoints as described in text. $K_1 = 0.18$, $K_2 = 0.3$, $K_3 = 0.4$.

Figs. 6 and 7 show results from simulation using the foot placement algorithm. The data of Fig. 6 show correction of a large error in body attitude that was introduced through the initial conditions. The 0.5 rad initial error in body attitude θ_2 was corrected in about six hops. In this simulation horizontal position was not controlled, so the transient error in horizontal velocity causes the system to come to rest some distance from the origin. Fig. 7 shows the response of the same foot placement algorithm to a pair of step changes in desired horizontal position. Control of x_2 is accomplished by implementing a position controller of the form

$$\dot{x}_{2,d} = \min\{K(x_2 - x_{2,d}), \dot{x}_{2d,\max}\}. \quad (23)$$

Desired body angle $\theta_{2,d}$ is also manipulated during translations. Balance using this method was stable with a variety of initial conditions and body attitudes.

Limit cycle oscillations in horizontal velocity were observed in the data, especially at forward velocities greater than about 0.25 m/s. Maximum rate of travel is limited because motion of the leg is restricted during stance by the

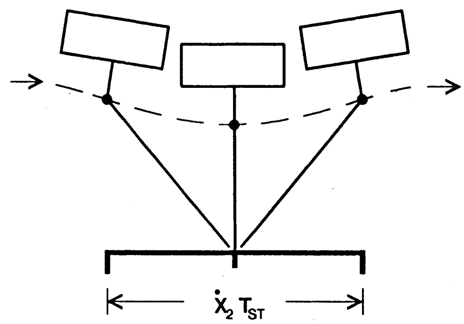

Fig. 8. When the foot is placed in the center of the CG print, there is symmetry in the model's motion. Running from left to right, the left-most drawing shows model's configuration just before TOUCHDOWN, the center drawing shows configuration at BOTTOM, and right most drawing shows configuration just after LIFT-OFF.

fixed hip angle. Another consequence of the fixed hip during stance is that the body remains inclined at a rather steep angle, with the leg extended under the body during travel; see Fig. 7. The leg does not sweep back and forth through large angles as it would in natural running.

Method 2: Leg Sweeping

The leg sweeping algorithm is based on a generalization of the foot placement approach just described. It arises when one thinks about the constraints imposed by constant velocity travel, the legged system's kinematics, the forces generated between the foot and the ground, and the need to balance. We start by finding the nominal motion that will maintain constant forward velocity with no tipping, and then modify this motion to eliminate deviations from the desired state. The resulting algorithm has two parts: one part that places the foot to control balance and tipping, and another part that moves the hip during stance to control forward velocity.

In natural biped running each leg extends forward during flight so that the foot first touches the ground some distance in front of the body. During stance the leg sweeps backward with respect to the body. The foot then leaves the ground some distance behind the body and the other foot extends forward. There is a symmetry in this motion about the point half way through stance, when the leg is directly under the center of gravity. Fig. 8 diagrams the symmetry. This symmetrical motion causes no tipping because the center of gravity spends about an equal time in front of the point of support, and an equal time behind it. The gravitational tipping moments average to zero throughout stance.

In order to achieve symmetry of this sort for the one-legged model, the control system must determine the locus of points over which the center of gravity will travel during the next stance period. We call this locus a *CG print*, in analogy to a footprint. The length of the CG print is just the product of the forward velocity and the duration of stance. In the steady state the control system places the foot in the center of the CG print.

When the attitude of the body deviates from its desired value, the control system moves the foot from the center of

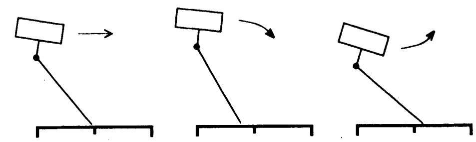

Fig. 9. Three types of behavior when foot is placed in CG print. (a) Foot is placed in center of the CG print. No angular change in model, horizontal velocity is unmodified. (b) Foot is placed toward rear of CG print, causing body to tip forward during stance interval. (c) Foot is placed toward front of CG print, causing body to tip rearward during stance interval. Horizontal lines indicate CG print, locus of projection of center of gravity during stance.

the CG print to correct the error. Placing the foot forward of the center of the CG print will create a net backward tipping moment during stance, while placing it behind the center of the CG print will create forward tipping. Fig. 9 illustrates the three cases. A linear combination of state errors determines how far to move the foot from the center of the CG print. The method uses the same rule for placing the foot with respect to the center of the CG print as the last method used for placing the foot with respect to the projection of the center of gravity.

Motion of the hip during stance generates forces between the ground and the foot that determine how the system's forward velocity will change. If the system is to progress at a constant forward rate with no forward acceleration, then the horizontal component of all forces acting on the system must be zero. Viewing the problem in a coordinate system that moves forward with the hip, the task is to make the foot sweep backward at the same rate as the ground. The leg sweeping algorithm accomplishes this task by calculating a target angle for the hip at each moment during stance. The target angle is based on the rate of forward travel, the instantaneous length of the leg, the time that passed since TOUCHDOWN, and the placement of the foot relative to the hip at TOUCHDOWN. Under nominal conditions this motion will cause the resultant force acting on the foot to be vertical.

When the forward velocity deviates from the desired value, the sweeping motion of the leg no longer results in a match between the speed of the foot and the ground. The result will be an accelerating or retarding force that corrects the velocity error. Faster leg sweeping will increase forward velocity, and slower sweeping will retard it.

If the duration of stance, the horizontal velocity of the body, and the geometry of the vehicle are known, then an appropriate leg angle for TOUCHDOWN and a sweeping function for the leg can be calculated. The duration of stance can be recorded from the previous hop. When the horizontal velocity is $\dot{x}_2$, the horizontal distance traversed during stance, the length of the CG print, is

$$\Delta x_{\text{STANCE}} = \dot{x}_2 T_{\text{ST}}. \tag{24}$$

Combining (24) with (21) a new equation for foot placement is obtained.

$$x_{\text{TD}} = w \frac{r_2 M_2 \sin(\theta_2) + (M_1 + M_2) x_{\text{ERR}}}{r_1 M_1 + w M_2} + \frac{\Delta x_{\text{STANCE}}}{2}. \tag{25}$$

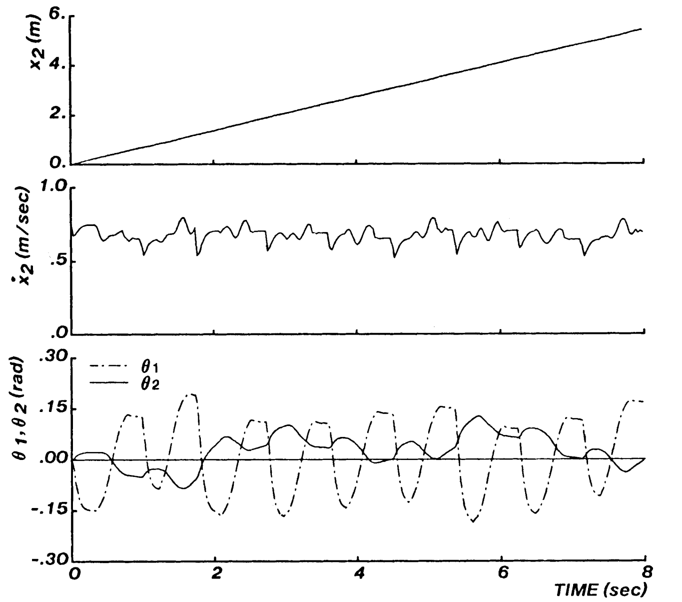

Fig. 10. Running at constant horizontal rate is generated by algorithm that places the foot in the center of the CG print, and then sweeps the leg backward during stance. Rate of travel is 0.75 m/s. ($K_1 = 0.2$, $K_2 = 0.1$, $K_3 = 0.1$).

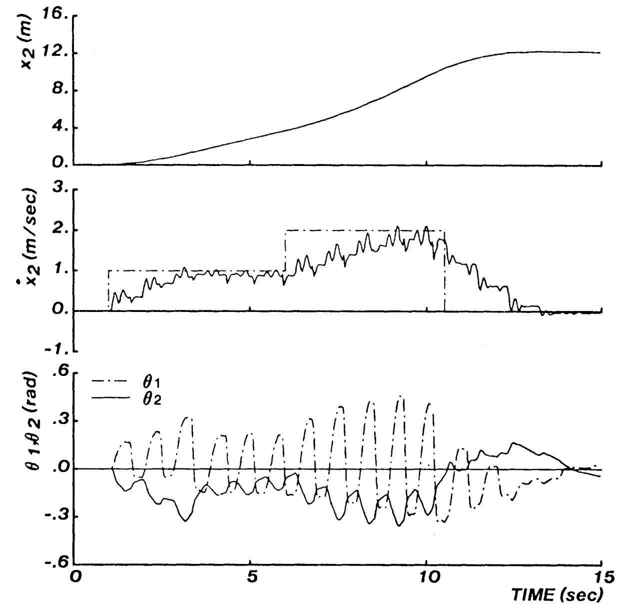

Fig. 11. Leg sweeping algorithm controls running as desired velocity, $\dot{x}_{2,d}$ (shown stippled in second plot), changed in steps ($K_1 = 0.2$, $K_2 = 0.1$, $K_3 = 0.1$, $\Delta\dot{x}_{\max} = 0.45$).

For vertical hopping in place, where $\dot{x}_2 = 0$, (25) reduces to (21) of the last section. During stance leg angle must change in a specified way in order to satisfy the symmetry argument for zero net ground force and moment. At time t during stance

$$x(t) = x_{\mathrm{TD}} - \frac{(t - t_{\mathrm{TD}})}{T_{\mathrm{ST}}} \Delta x_{\mathrm{STANCE}}. \tag{26}$$

Once again, taking the kinematics of the model into account, a leg sweeping function can be found

$$\theta_1(t) = -\mathrm{Arcsin}\left(\frac{x(t)}{w}\right). \tag{27}$$

Though the leg is vertical only momentarily during each stride, a sweeping servo employing (26) and (27) resolves leg springiness into the vertical direction. Horizontal foot motion is rigid. Figs. 10, 11, and 12 show the results of simulating this algorithm. Fig. 10 shows a constant velocity translation at 0.75 m/s. These data show that horizontal velocity was well controlled, with only small variations within each cycle. The leg swung forward during flight to place the foot, and swept backward during stance to minimize horizontal ground forces. The leg and the body counteroscillated. Body attitude was kept to within 0.1 rad of vertical, with a distinct 0.3 Hz oscillation superimposed on the stepping oscillation.

To accommodate forward acceleration a compromise between actual and desired velocity was used to calculate the length of the CG print.

$$\Delta x_{\mathrm{STANCE}} = \begin{cases} (\dot{x}_2 - \Delta\dot{x}_{\max})T_{\mathrm{ST}} & \text{for } \dot{x}_{2,d} < (\dot{x}_2 - \Delta\dot{x}_{\max}) \\ (\dot{x}_2 + \Delta\dot{x}_{\max})T_{\mathrm{ST}} & \text{for } \dot{x}_{2,d} > (\dot{x}_2 + \Delta\dot{x}_{\max}) \\ \dot{x}_{2,d}T_{\mathrm{ST}} & \text{otherwise.} \end{cases} \tag{28}$$

$\Delta\dot{x}_{\max}$ limits the magnitude of sudden changes in desired velocity. Equation (28) replaces (24). The controller employing this algorithm stretches the leg forward and lengthens the stride in order to accelerate the model.

Fig. 11 demonstrates regulation of forward velocity as the system accelerates. The model starts hopping in place, then accelerates to 1 m/s, then to 2 m/s, and then slows to a stop. At 2 m/s, the body deviates from vertical by about 0.3 rad. Other acceleration data are shown in cartoon form in Fig. 12 where the pattern of motion can be more easily visualized. The paths of the body and foot are indicated by a string of dots that correspond to equal time intervals. The overall elapsed time is about 10 seconds. The rightmost strides in this cartoon show that clearance of the foot above the ground was reduced as stride and speed increased. The model's tendency to stub its toe as clearance is reduced is the factor that limits maximum speed.

Method 3: Servo Attitude

The leg sweeping algorithm maintained an even forward velocity, but it did not control the attitude of the body with any precision. In Fig. 11 the body deviates from its erect position by about 0.3 rad when running at full speed. Instead of using the hip actuator to sweep the leg according to the running rate, method 3 uses the hip actuator to erect the body during stance. Placement of the foot in the CG print is as before using (18), (24), and (25).

$$\tau(t) = K_P(\theta_2 - \theta_{2,d}) + K_V(\dot{\theta}_2) \tag{29}$$

where $\theta_{2,d}$ is selected on each hop to make the average body angle zero.

This algorithm controls forward velocity through placement of the foot, and the accelerations that result from tipping. Since the hip servo erects the body during stance,

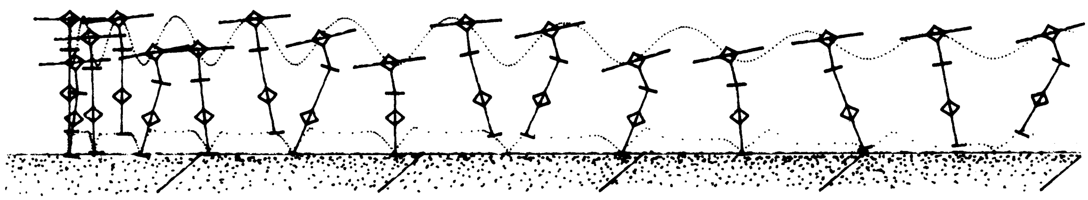

Fig. 12. Cartoon of running controlled by leg sweeping algorithm. Model accelerates from standing start to about 2.2 m/s in 10 s. The dotted lines represent the paths of hip and foot. (20 ms/dot, 0.6 s/stick figure.) Maximum speed is limited by clearance between foot and ground. When the stride becomes too long the model *stubs its toe*.

placement of the foot with respect to the center of the CG print exclusively controls forward velocity, $K_2 = K_3 = 0$. Fig. 13 shows the behavior of this algorithm when executing the same sequence of steps in forward velocity that were used to produce Fig. 11.

Method 3 is simpler to implement than the leg sweeping algorithm. It is not necessary to servo a hip trajectory during stance, as required by (26). A setpoint for desired body angle is specified once during stance, and another setpoint for leg angle is specified during flight. The fore and aft swinging motions of the leg that characterize running are not explicitly programmed, but result from interactions between the servos that alternate satisfying these two setpoints.

Method 3 is also simple because it allows control of running to be decomposed into three separate parts. One part controls hopping height by regulating the amount of thrust delivered on each hop. A second part controls forward velocity by placing the foot with respect to the center of the CG print. The third part controls attitude of the body by servoing the hip joint during stance. No explicit coupling between these parts is required.

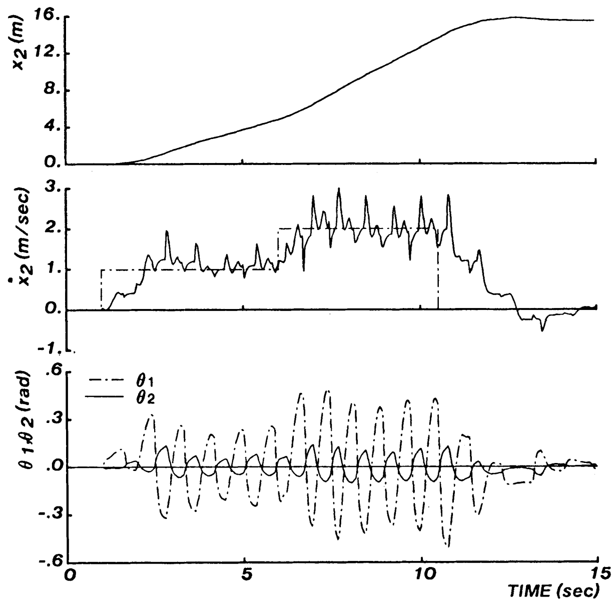

Fig. 13. Method 3 algorithm shown responding to step changes in desired velocity, $\dot{x}_{2,d}$ (shown stippled). The algorithm generates hip torque during stance to control attitude of body. ($K_1 = 0.15$, $K_2 = 0.0$, $K_3 = 0.0$, $\Delta \dot{x}_{\max} = 0.5$.)

V. Conclusion

The long range goal of this work is to develop an understanding of balance and dynamics in legged locomotion that will help to explain behavior observed in biological legged systems, and to lay the groundwork needed to construct useful legged vehicles. The purpose of this paper is to describe results obtained from modeling and simulating a legged system that hops on one leg. Such a model encourages focus on the problem of balance, without attending to the coordination of many legs.

The model consists of a body, an actuated hinge-type hip, and a leg. The leg is massful and springy, and its length can be controlled by a position actuator. We separate control of the model into a vertical hopping part and a horizontal balance part. Vertical control takes advantage of the springy leg to achieve resonant hopping motion. The control system regulated hopping height using a measure of vertical energy to control the thrust delivered on each hop.

Horizontal control ensures that the body is maintained in an upright posture, and that the rate of forward travel is well controlled. We explore in the following three algorithms for horizontal control—method 1, the foot placement algorithm, method 2, the leg sweeping algorithm, and method 3, the attitude control algorithm:

- *Method 1*: The foot placement algorithm places the foot with respect to the projection of the center of gravity, and holds the hip fixed during stance. It maintained balance when hopping in place, but could move forward only slowly.
- *Method 2*: The leg sweeping algorithm uses the CG print to calculate both where to put the foot on each step, and a hip trajectory that will control forward travel. It controlled forward velocity with good precision, but control of body angle was poor.
- *Method 3*: The attitude control algorithm also uses the CG print to place the foot, but hip torque during stance controls body attitude. This simpler algorithm controls forward velocity and attitude with good precision.

Control of running in the one-legged model can be decomposed into height control, a forward velocity control, and attitude control. The data presented in this paper were obtained from computer simulations of the one-legged model. We have completed a set of experiments on a physical one-legged hopping machine, that is similar to the model used here. Results from those physical experiments, reported elsewhere [23], demonstrate the system's ability to hop in place, to run at various rates up to about 1.2 m/s, to withstand horizontal disturbance forces, and to leap over small obstacles.

Acknowledgment

I am indebted to Ivan Sutherland for his early and continued support for this project, and to Nancy Cornelius, Marc Donner, Matt Mason, and Cynthia Hibbard for their incisive reviews. One of the reviewers made an important contribution to (16).

Appendix I
Equations of Motion for Model of Fig. 1

These equations were derived from free body diagrams of the leg and body using d'Alembert's principle.

$$\ddot{y}_1 = \ddot{y}_0 - r_1\left(\ddot{\theta}_1 \sin(\theta_1) + \dot{\theta}_1^2 \cos(\theta_1)\right) \tag{30}$$

$$\ddot{x}_1 = \ddot{x}_0 + r_1\left(\ddot{\theta}_1 \cos(\theta_1) - \dot{\theta}_1^2 \sin(\theta_1)\right) \tag{31}$$

$$\begin{aligned}\ddot{y}_2 = \ddot{y}_0 &+ \ddot{w}\cos(\theta_1) - w\ddot{\theta}_1 \sin(\theta_1) - w\dot{\theta}_1^2 \cos(\theta_1)\\ &- r_2\left(\ddot{\theta}_2 \sin(\theta_2) + \dot{\theta}_2^2 \cos(\theta_2)\right) - 2\dot{w}\dot{\theta}_1 \sin(\theta_1)\end{aligned} \tag{32}$$

$$\begin{aligned}\ddot{x}_2 = \ddot{x}_0 &+ \ddot{w}\sin(\theta_1) + w\ddot{\theta}_1 \cos(\theta_1) - w\dot{\theta}_1^2 \sin(\theta_1)\\ &+ r_2\left(\ddot{\theta}_2 \cos(\theta_2) - \dot{\theta}_2^2 \sin(\theta_2)\right) + 2\dot{w}\dot{\theta}_1 \cos(\theta_1)\end{aligned} \tag{33}$$

$$M_1\ddot{y}_1 = F_y - F_T\cos(\theta_1) + F_N \sin(\theta_1) - M_1 g \tag{34}$$

$$M_1\ddot{x}_1 = F_x - F_T\sin(\theta_1) - F_N \cos(\theta_1) \tag{35}$$

$$\begin{aligned}I_1\ddot{\theta}_1 &= -F_x r_1 \cos(\theta_1) + F_y r_1 \sin(\theta_1)\\ &\quad - F_N(w - r_1) - \tau(t)\end{aligned} \tag{36}$$

$$M_2\ddot{y}_2 = F_T\cos(\theta_1) - F_N \sin(\theta_1) - M_2 g \tag{37}$$

$$M_2\ddot{x}_2 = F_T\sin(\theta_1) + F_N \cos(\theta_1) \tag{38}$$

$$\begin{aligned}I_2\ddot{\theta}_2 &= F_T r_2 \sin(\theta_2 - \theta_1)\\ &\quad - F_N r_2 \cos(\theta_2 - \theta_1) + \tau(t)\end{aligned} \tag{39}$$

where

(x_0, y_0)	coordinates of the foot
(x_1, y_1)	coordinates of the leg's CG
(x_2, y_2)	coordinates of the body's CG
F_x, F_y	horizontal and vertical forces on the foot
F_T, F_N	forces acting at the hip between the leg and body. F_T acts tangent to the leg, and F_N acts perpendicular to the leg.

These equations are expressed in terms of the state variables $\theta_1, \theta_2, x_0, y_0, w$ by eliminating x_1, y_1, x_2, y_2, F_N, and F_T:

$$\begin{aligned}\cos(\theta_1)&(M_2Ww + I_1)\ddot{\theta}_1 + M_2r_2W\cos(\theta_2)\ddot{\theta}_2\\ &+ M_2W\ddot{x}_0 + M_2W\sin(\theta_1)\ddot{w}\\ = {}& WM_2\left(\dot{\theta}_1^2 W\sin(\theta_1) - 2\dot{\theta}_1\dot{w}\cos(\theta_1)\right.\\ &\left.+ r_2\dot{\theta}_2^2\sin(\theta_2) + r_1\dot{\theta}_1^2\sin(\theta_1)\right)\\ &- r_1F_x\cos^2(\theta_1) + \cos(\theta_1)\left(r_1F_y\sin(\theta_1) - \tau(t)\right)\\ &+ F_KW\sin(\theta_1)\end{aligned} \tag{40}$$

$$\begin{aligned}-\sin(\theta_1)&(M_2Ww + I_1)\ddot{\theta}_1 - M_2r_2W\sin(\theta_2)\ddot{\theta}_2\\ &+ M_2W\ddot{y}_0 + M_2W\cos(\theta_1)\ddot{w}\\ = {}& WM_2\left(\dot{\theta}_1^2W\cos(\theta_1) + 2\dot{\theta}_1\dot{w}\sin(\theta_1) + r_2\dot{\theta}_2^2\cos(\theta_2)\right.\\ &\left.+ r_1\dot{\theta}_1^2\cos(\theta_1) - g\right)\\ &+ r_1F_x\cos(\theta_1)\sin(\theta_1) - \sin(\theta_1)\\ &\left(r_1F_y\sin(\theta_1) - \tau(t)\right) + F_KW\cos(\theta_1)\end{aligned} \tag{41}$$

$$\begin{aligned}\cos(\theta_1)&(M_1r_1W - I_1)\ddot{\theta}_1 + M_1W\ddot{x}_0\\ &= W\left(M_1r_1\dot{\theta}_1^2\sin(\theta_1) - F_K\sin(\theta_1) + F_x\right)\\ &\quad - \cos(\theta_1)\left(F_yr_1\sin(\theta_1) - F_xr_1\cos(\theta_1) - \tau(t)\right)\end{aligned} \tag{42}$$

$$\begin{aligned}-\sin(\theta_1)&(M_1r_1W - I_1)\ddot{\theta}_1 + M_1W\ddot{y}_0\\ &= W\left(M_1r_1\dot{\theta}_1^2\cos(\theta_1) - F_K\cos(\theta_1) + F_y - M_1g\right)\\ &\quad - \sin(\theta_1)\left(F_yr_1\sin(\theta_1) - F_xr_1\cos(\theta_1) - \tau(t)\right)\end{aligned} \tag{43}$$

$$\begin{aligned}-\cos(\theta_2 - \theta_1)&I_1r_2\ddot{\theta}_1 + I_2w\ddot{\theta}_2\\ &= W\left(F_Kr_2\sin(\theta_2 - \theta_1) + \tau(t)\right) - r_2\cos(\theta_2 - \theta_1)\\ &\quad \cdot\left(r_1F_y\sin(\theta_1) - r_1F_x\cos(\theta_1) - \tau(t)\right)\end{aligned} \tag{44}$$

where

$$W = w - r_1$$

$$F_K = \begin{cases} K_L(k_0 - w + \chi) & \text{for } (k_0 - w + \chi) > 0\\ K_{L2}(k_0 - w + \chi) - B_{L2}\dot{w} & \text{otherwise}\end{cases} \tag{45}$$

$$F_x = \begin{cases} -K_G(x_0 - x_{TD}) - B_G\dot{x}_0 & \text{for } y_0 < 0\\ 0 & \text{otherwise}\end{cases} \tag{46}$$

$$F_y = \begin{cases} -K_Gy_0 - B_Gy_0 & \text{for } y_0 < 0\\ 0 & \text{otherwise.}\end{cases} \tag{47}$$

Appendix II
Simulation Parameters

$M_1 = 1$ kg $\qquad M_2 = 10$ kg

$I_1 = 1$ kg-m^2 $\qquad I_2 = 10$ kg-m^2

$r_1 = 0.5$ m $\qquad r_2 = 0.4$ m

$k_0 = 1$ m

$K_L = 10^3$ Nt/m

$K_{L2} = 10^5$ Nt/m $\qquad B_{L2} = 125$ Nt-s/m

$K_G = 10^4$ Nt/m $\qquad B_G = 75$ Nt-s/m

$H = 0.4$ m

$$K_P = \begin{cases} 1800 \text{ Nt-m/rad} & \text{for } y_0 \leqslant 0\\ 1200 \text{ Nt-m/rad} & \text{otherwise}\end{cases}$$

$$K_V = \begin{cases} 200 \text{ Nt-m-s/rad} & \text{for } y_0 \leqslant 0\\ 60 \text{ Nt-m-s/rad} & \text{otherwise.}\end{cases}$$

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