Le equazioni fanno parte del testo, con anche punteggiature. Se ne cita il numero solo quando ti devi riferire in un luogo diverso. Figure e tabelle NON fanno parte del testo (float) e ci si deve sempre riferire con numero, almeno una volta. Da standardizzare figure 1.1 table 1.1 equation (1.1)

21/8/2013

Chapter 1

~~Physics Motivation for~~ Neutron Skin Thickness Measurement

1.1 The Mainz Radius Experiment

The Mainz Radius Experiment (MREX), at the Mainz nuclear physics institute, is an experimental campaign with the aim of investigating the nature of atomic nuclei, by measuring the neutron skin thickness of ^{208}Pb. The characteristics of atomic nuclei are mainly determined by the strong interaction, whose existence was firstly speculated by Yukawa in 1935. The strong interaction is responsible of a broad range of phenomena: from nature of the nuclei, the compositions of baryons and meson to the exotic structure of Neutron stars. Because of this, ~~the~~ nuclear physics provides many answers to fundamental questions that are ~~important~~ also in other fields of physics~~, too~~. In particular the neutron stars, that are among the most studied astrophysical objects, are ideal to study theories of dense nuclear matter. It can be surprising to think that, despite a difference of so many order of magnitude, neutron rich nuclei and neutron stars have the same basic ~~physics, enshrined~~ description, represented by the the Equation Of State (EOS) of nuclear matter. The Equation Of State represents the fundamental relation between the state variables such as temperature, energy, pressure, and the neutron-proton asymmetry. Specifically, the final goal of the MREX experiment is to determine an important parameter of the EOS, the slope of the symmetry energy at saturation density L, which controls the change in energy due to presence of an asymmetry between neutron and proton densities. This parameters plays an important role for the determination of the radius of the neutron stars and it is also responsible for a peculiar characteristic shown by heavy nuclei: the neutron skin thickness. The neutron skin thickness δr_{np} is a phenomena that affect heavy nuclei, which consists in the accumulation of the excess of neutrons near the surface, producing a neutral layer of neutrons. It is defined ~~in equation 1.1~~ as:

uniforma le formattazione

$$\delta r_{np} = \sqrt{< r_n^2 >} - \sqrt{< r_p^2 >} , \tag{1.1}$$

where $< r^2 >_n$ and $< r^2 >_p$ are the rms radii of proton and neutron distributions. The neutron skin thickness is ~~strongly~~ sensitive to L, so an accurate determination of δr_{np} provides significant constrains on the value of L, which in turn is used as an input to many theoretical models of the structure of the neutrons stars. The determination of δr_{np} presents considerable difficulties. While r_p is ~~known~~ measured with high accuracy, with ~~thanks to the~~ electrons elastic scattering experiments ~~which~~ involving electromagnetic force, the determination of r_n has traditionally relied on hadronic experiments, such as proton-nucleus scattering, π^0 photo-production, α and π nucleus scattering. These processes suffer from large and often uncontrolled theoretical uncertainties that compromise the extraction of the neutron density. The most promising method, that is the least model dependent, is ~~the~~ parity-violating electron scattering, ~~In this reaction~~ where longitudinal polarized electrons are elastically scattered off an unpolarized target. This method consists in the measurement of ~~the~~ asymmetry between right and left handed electrons:

Cross section

$$A_{pv} = \frac{\sigma_R - \sigma_L}{\sigma_R + \sigma_L} \tag{1.2}$$

This process is dominated by the exchange of a virtual photon, which is sensitive to the charge form factor, and of a Z_0 boson, that is sensitive to the weak form factor. ~~Because of the fact that~~ Since the weak charge of the neutron is $Q_w = 0.99$ and the weak charge of the proton is 0.04, the weak form factor contains the information on the neutron density, necessary to measure δr_{np}. In this context, the MREX experiment is an experimental campaign with the aim of measuring the neutron skin thickness via the parity violating scattering with the new MESA electron accelerator, at the Nuclear physics institute of Mainz.

1.2 Nuclear Equation of State (EOS) and Neutron Skin Thickness

During the 30s of the last century, a considerable part of the scientific community was focused in the study of the structure of atomic nuclei. The discovery that every atoms has a positive charged nucleus dates back to 1908, with the famous Rutherford experiment, where alpha particles scatter ~~from~~ on a thin gold foil. In the following years, especially ~~with~~ after the birth of quantum mechanics in the second half of the 1920s, significant progress ~~were~~ was made in the knowledge of atomic nuclei and their properties. In 1935, a significant contribution was given by Carl Friedrich von Weizsäcker and Hans Bethe, ~~that~~ who proposed the semi-empirical mass formula, to approximate the mass of an atomic nucleus [7]. Although some refinements have been made over the years, the general structure of the formula is the same today. The model proposed by Weizsäcker is the application of the liquid-drop model for nuclear matter, where the Nucleus is described as a drop of protons and neutrons, ~~that are~~ assumed to be incompressible and ~~are~~ held together by a nuclear potential. The semi-empirical mass formula states that the mass of a nucleus (with Z protons and N neutrons) is given by

$$m = Zm_p + Nm_n - \frac{E_B(N,Z)}{c^2} \tag{1.3}$$

where E_B is ~~An important terms is~~ the binding energy ~~E_B, that~~ contains 5 parameters:

$$E_B = a_V A - a_s A^{\frac{2}{3}} - a_c \frac{Z^2}{A^{\frac{1}{3}}} - a_{asym} \frac{(N-Z)^2}{A} + \delta(N,Z) \tag{1.4}$$

The first two terms a_V, a_s are the volume energy and the surface energy, and conceptually analogous to the volume and surface parameter of the liquid drop model. The volume term represent the energy due to the interaction of each nucleon with the other nearby nucleons. This term is proportional to A $= N+Z$, that is the number of nucleons, which is proportional to the volume, ~~hence the name~~. The second term represents the surface energy, and it is a correction to the volume energy. For the volume energy parameter, it is assumed that each nucleon interacts with a constant number of nearby nucleons. This is not true, because the strong interaction is a short distance interaction, and furthermore the external protons and neutrons have less neighbors to interact with. This correction terms is then proportional to $A^{\frac{2}{3}}$, that is the also proportional to the surface area. The third term with the coefficient a_c denote the binding energy correction due to the ~~electromagnetic~~ Coulomb repulsion between protons. The fourth term ~~is a_{asym}~~, the asymmetry term, ~~and it~~ is proportional to the asymmetry between neutrons and protons. The theoretical justification for this terms is due to the Pauli exclusion principle. Neutrons and protons are distinct type of particles, and occupy different quantum states. Because neutrons/protons are fermions, they can~~'t~~ not occupy a state with the same quantum numbers, therefore higher energy states are progressively filled. If there is an asymmetry between neutrons and protons, for example the number of neutrons is greater than the number of protons, some neutrons will be in higher energy states with respect to the protons. The imbalance between the nucleons causes the energy to be higher with respect to the situation with the equal number of p and n. The last term is the Pairing term, and describes the effect of spin coupling, which ~~leads~~ leads to the formation of proton or neutron pairs. It has a positive/negative values or zero depending on the parity of ~~for even or odd~~ N,Z. We ~~want to focus on~~ stress the fact that the liquid-drop model has the underlying assumption that the nucleons are incompressible. Because of this it is well defined the concept of saturation density, the fact that, at first order, it is almost constant and independent of mass number A. In the context of neutron stars, it is more useful to take the thermodynamic limit in which the number of nucleons and volume are taken to infinity. The binding energy per nucleons can be written as: ??

qui non ci sono ρ_0, α ?

1) Perché spariscon gli altri termini?

$$\epsilon(\rho_0, \alpha) = -\frac{E_B}{A} = -a_V + a_{asym} \overbrace{\boxed{\frac{\rho_n - \rho_p}{\rho_n + \rho_p}}}^{\alpha} \tag{1.5}$$

2) Cosa sono ρ_n, ρ_p ?

devi definirli

In reality, this simple equation is only an approximation, because the nuclear matter does~~n't~~ it behave like an ideal liquid drop, and it is not incompressible. To describe the response of the nuclear matter to density variation, as well as temperature, etc. ~~We~~ we need the equation of state (EOS) of the system, ~~that~~ the fundamental

$\rho_n = \frac{N}{V}$?

Smn! Snm! Cosa mai avete!

relation that binds all these quantities together. For neutron stars, the EOS depends on ρ, the conserved baryon density, and neutron-proton asymmetry α, in the ~~ideal~~ limit of $T = 0$.

$$\epsilon(\rho, \alpha) = \epsilon_{snm}(\rho, \alpha = 0) + \alpha^2 S(\rho) + O(\alpha^4) \tag{1.6}$$

però è strano che nel modello a goccia hai I

The energy density can be expanded in a power series of $\alpha = \dfrac{\rho_n - \rho_p}{\rho_n + \rho_p}$. No odd power of α appears in the expansion, because the strong force does not depend on the isospin; or in other words, neglecting electromagnetic interaction and weak interaction, the equation of state depends only on the relative asymmetry between neutrons and protons, it doesn't matter if such an asymmetry is biased towards protons or neutrons. The term~~s~~ $S(\rho)$ is the symmetry energy, and ~~it~~ represents the cost of converting symmetr~~y~~ic nuclear matter ($\alpha = 0$) to pure neutrons matter, as in the case of neutron star. Now we can proceed considering the saturation density. A further expansion around the saturation density ρ_0 is done, following [20]:

ρ_0 è la sat. density?

$$S(\rho) = J + L \cdot \frac{\rho - \rho_0}{3\rho_0} + \frac{1}{2} K_{sym} \cdot \left(\frac{\rho - \rho_0}{3\rho_0}\right)^2$$
$$\epsilon_{smn}(\rho) = \epsilon_0 + \frac{1}{2} K_0 \left(\frac{\rho - \rho_0}{3\rho_0}\right)^2 \tag{1.7}$$

Several new terms appear in this expression:

- ϵ_0 is the energy per nucleon for symmetric matter at saturation density.
- J is the symmetry energy at saturation density.
- L is the slope of the symmetry energy.
- K_0 is the incompressibility coefficient for symmetr~~y~~ic matter.
- K_{sym} is the incompressibility coefficient for the symmetry energy.

In equation 1.7 appears a new quantity: L, the slope of the symmetry energy. This is a key component of the EOS, whose value~~s~~ is an important parameter to determine the radius of neutron star. L quantifies the difference between the symmetry energy at saturation (as in the nuclear core) and the symmetry energy at lower densities, as in the nuclear surface. L is also related to the pressure P at saturation density, as seen in equation 1.10. Giving the EOS in term of ρ and α, the pressure can be written as:

$$P = \rho^2 \frac{\partial \epsilon(\rho, \alpha)}{\partial \rho} \tag{1.8}$$

Equation 1.8 can be simply derived from the first principle of thermodynamics. We know write ϵ_{snm} making explicit all the dependencies:

$$\epsilon(\rho, \alpha) = (\epsilon_0 + \alpha^2 J) + \alpha^2 L x + \frac{1}{2}(K_0 + \alpha^2 K_{sym}) x^2 \tag{1.9}$$

$\frac{\partial}{\partial \rho} = \frac{1}{3\rho_0}\frac{\partial}{\partial x}$

we substitute $x = \dfrac{\rho - \rho_0}{3\rho_0}$. Considering pure neutron matter $\alpha = 1$, the pressure at saturation density P_0 can be easily computed withe the formula (1.8). The result is:

$$P_0 \simeq \frac{1}{3} \rho_0 L \tag{1.10}$$

From this expression we learn that the slope of the symmetry energy is essential to determine the pressure for densities near saturation. Such conditions are encountered in nuclei and in the core of neutron stars. The contribution of the symmetric term $\epsilon_{snm}(\rho)$ vanishes, and at first order the pressure depends only on L. Because of this, it becomes more clear the link between L and the neutron skin thickness. Let's consider the case of the ^{208}Pb, with an excess of 44 neutrons. Placing the excess of neutrons in the surface of the nucleus is discouraged by the surface term a_S, which tends to minimize the area. However, if the excess of neutrons is placed in the core of the nucleus, ~~this is~~ it increases the symmetry energy $S(\rho)$. In the end the neutron skin is the result of the

Non mi sembra che hai definito δr_{np}

competitions between the surface tension and the slope of the symmetry energy. Measurements of the neutron skin have been performed by the PREX collaboration at Thomas Jefferson National Accelerator Facility in Virginia [3]. However the precision attained was insufficient to distinguish between the various competing models which describe the relation between δr_{np} and L. In fact, theoretical models, while predicting different values of L, show that there is a strong correlation between these two quantities, as show in figure (1.1).

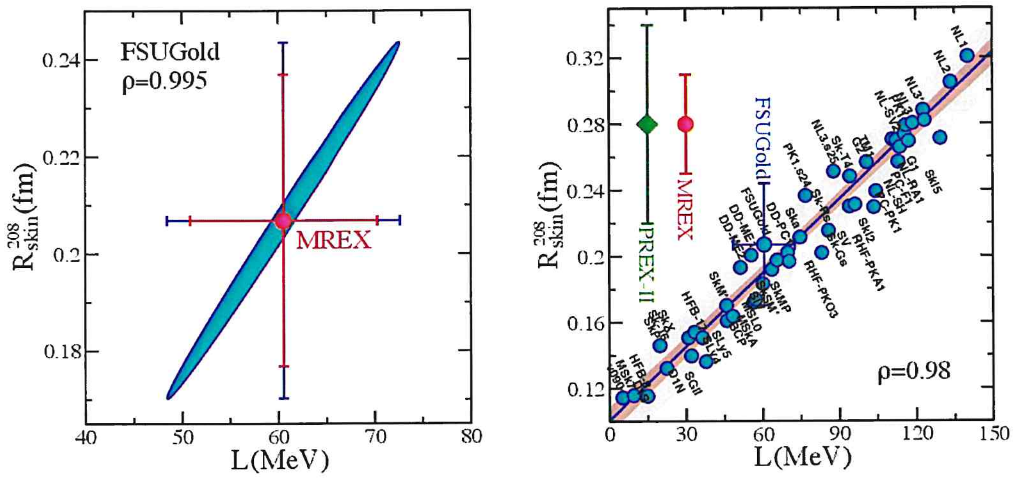


Figure 1.1: *On the right* Neutron skin thickness of ^{208}Pb as a function of the slope of the symmetry energy L. The error bars represent ± 0.06 fm and ± 0.03 fm for the future experiments of PREX-II and MREX. Notice the different scale for x and y axis, a small uncertainty for the neutron skin measurement correspond to ~~an~~ higher uncertainty for the values of L. *On the left* Covariance ellipse displaying the correlation between L and the neutron skin thickness, for FSUGold model. The covariance ρ is equal to 0.995.

for various theoretical models

A strong linear correlation is evident, then measuring the neutron skin gives access to L.

1.3 Parity-violating Scattering Experiment

In the last years, the parity violating electron scattering seems to be a promising method for the determination of the neutron-skin thickness of ^{208}Pb. The choice of lead is due to the significant neutron excess and stability of lead nuclei (^{208}Pb is a double magic nucleus). The advantage of this method is that it is free from the many uncertainties associated to strong interaction. The main disadvantage is the need ~~necessity~~ to accumulate large statistics, because the reaction are mediated by the weak interaction, associated with smaller scattering amplitude compared to electromagnetic and strong interactions. The parity violating scattering is highly sensitive to the neutron density because, as mentioned above, since the weak charge of the neutron is higher compared to the weak charge of the proton. In this reaction, longitudinally polarized electrons are elastically scattered off a lead target. The important quantity to determine is the parity violating asymmetry A_{PV}, the difference in cross section between the scattering of right and left handed electrons.

Apv oppure A_{PV} DECIDI!

$$A_{PV} = \frac{\sigma_R - \sigma_L}{\sigma_R + \sigma_L} \tag{1.11}$$

The theoretical calculation of A_{PV} includes ~~concern~~ the interference between the exchange of virtual γ and Z^0. In the Born approximation A_{PV} is directly proportional to the weak form factor, and ~~it~~ is given by ~~the formula 1.12:~~

$$A_{PV} \simeq \frac{G_F Q^2}{4\pi\alpha} \cdot \frac{Q_W F_W(Q^2)}{Z F_{ch}(Q^2)} \tag{1.12}$$

wWhere G_F is the Fermi constant, Q^2 is the transferred momentum, Z and Q_W are the electric and weak charge of the nucleus, The charge form factor of the lead nucleus is known with high accuracy (precision of 0.02 %), so in this limit the only quantity that is unknown is $F_W(Q^2)$. In the long wavelength approximation, the weak form factor at single value of momentum transfer is given by:

, F_{ch} and F_W are the charged and weak form factors of the nucleus.

Soggetto verbo oggettto -> e poi il resto,

$$F_W(Q^2) = \frac{1}{Q_W}\int \rho_W(r)\frac{sin(Qr)}{Qr}d^3r = (1 - \frac{Q^2}{6}R_W^2 + \frac{Q^4}{120}R_W^4 + ...) \tag{1.13}$$

The form factor is normalized in such a way that $F_W(Q^2 = 0) = 1$. The weak charge radius correspond to $R_W^2 = -6\frac{\partial F_W}{\partial Q^2}\Big|_{Q^2=0}$. Now it is clear that parity-violating experiment are a promising method to extract information about neutron density. The difficulty ~~effort~~ is represented by the small values of A_{pv} asymmetry. Typical values are on the order of $1\,ppm$ or less, for lead target. This requires high statistic to reduce the uncertainty of the measurement and high control over the systematic effects. In 2012 the PREX collaboration measured for the first time through parity-violating scattering ~~experiment~~ the neutron skin, ~~the values obtained is reported in 1.14:~~ obtaining

radius or skin ?

$$\delta r_{np} = 0.33^{+0.16}_{-0.18}\text{fm} \tag{1.14}$$

The error associated to this first measurement is not enough small to provide significant constraints on the values of L. ~~Because of this,~~ The MREX experiment has the objective of measuring the neutron radius of lead with a precision of 0.5% (±0.03 fm). This high precision is needed to decrease the uncertainty associated to L. For example, the left plot in 1.1, shows the correlation between the neutron skin thickness of ^{208}Pb and the slope of the symmetry energy as predicted by FSUGold model ([11]). With a precision of ±0.03 fm, L is determined with ±12.1 MeV. A new 2019 ~~recent~~ measurement, ~~in 2019, by~~ of the PREX collaboration [5], ~~measure the neutron skin thickness with a precision of~~ obtain $\delta r_{np} = 0.283 \pm 0.071$fm. With new measurement that will be performed in MESA accelerator by MREX, the precision will be improved further by a factor 2.

1.3.1 Neutron Star Radius

We mentioned that the slope of the symmetry energy L is strongly correlated to the neutron skin thickness of ^{208}Pb and also to R_{ns}. We can go deeper in the discussion stating that the maximum neutron-star mass and radius are uniquely constrained by the E0S ([19]). The maximum mass depends on the energy density dependence of the pressure, that must be high enough to oppose the gravitational collapse into a black hole. Moreover, stellar radii are strongly dominated by the pressure of degenerate nuclear matter near the saturation density. The connection between the radius of compact object and pressure is enclosed in the Volkov-Oppenheimer equation; resolving the equation for a compact symmetric object gives the relation between radius and pressure P_c at the center of the star (formal proof in [18]).

$$R^2 = \frac{3}{8\pi\rho}\frac{1-(\rho+P_c)^2}{(\rho+3P_c)^2} \tag{1.15}$$

But the pressure P_c is, in large part, determined by the symmetry energy of the equation of state, so there should be a strong correlation between L and the neutron star radius R_{ns}. In the end, different theoretical models [22] confirms the connection between L and R_{ns}, for example we show the covariance ellipses predicted by FSUGold model between the slope of the symmetry energy L and the stellar radii ? in figure 1.2

From these consideration, astronomical observations of Mass and radii of neutron stars represents for important constrains on the EOS, and are valuable for understanding the behaviour and physics of the atomic nuclei. Astronomical observations of the neutron star radius rely traditionally on photometric measurements, assuming that thermal emission of light from the surface follow a blackbody spectrum at uniform temperature. These measurement are affected by systematic uncertainties that are typically of a couple of kilometers. However, the situation is rapidly changing with the advent ~~beginning~~ of ~~the~~ gravitational wave detection. The first observation of the binary neutron star merger by the LIGO-Virgo collaboration opened a new path to measure the neutron-stars radius [2]. In fact, the gravitational wave generated by the merging of two neutron stars depends on a property called tidal deformability; this parameters describes the tendency of a neutron star to deform in response of the gravitational field of its companion star. This parameters Λ, is highly sensitive to the ratio of the stellar radius to the Schwarzschild radius:

$$\Lambda = \frac{2}{3}k_2(\frac{c^2 R_{NS}}{GM})^5 \tag{1.16}$$

In this expression, M and R_{NS} are the neutron star mass and radius, and k_2 is the second tidal Love number [8], which is computed from the quadrupole component of the gravitational field induced by the companion.

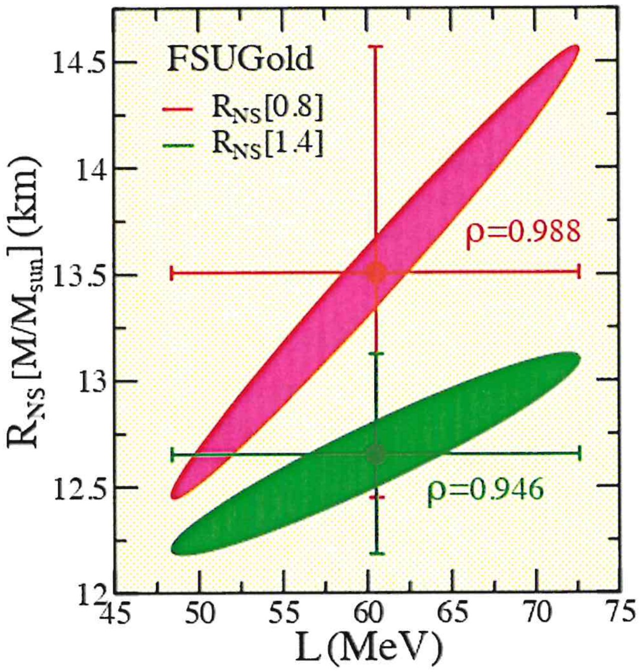


Figure 1.2: Covariance ellipses between slope of the symmetry energy and stellar radii, for 0.8 and 1.4 solar masses, predicted by the relativistic density model FSUGold.

From the first detection, an upper limit of $R_{NS}^{1.4} < 13.76\,\mathrm{km}$ was placed on the radius of a neutron star with a 1.4 solar masses. Because of the strong correlation between R_{NS}, L and δr_{np}, this is a indirect constrain on the neutron skin thickness of ^{208}Pb. An upper limit of $\delta r_{np} < 0.25\,\mathrm{fm}$ was obtained. This limits is ~~not~~ in ~~consistent~~ slight tension with the larger values measured by PREX collaboration~~,~~ this can suggest ing that the symmetry energy, for slightly higher density as in neutron stars, decreases~~,~~ with respect to the typical density found in atomic nuclei. This increment and decrement may be a n ~~proof~~ indication of the presence of phase transition in the interior of neutron stars.

<0.25 e 0.283±0.071 sono consistenti. Al massimo puoi dire "slight tension"

1.4 Transverse Asymmetry

The parity-violating scattering has numerous advantages for extracting the neutron-skin thickness of nuclei. However, the asymmetry to measure is rather small. To measure such asymmetries, it is ~~strictly~~ necessary to reduce ~~at most~~ as much as possible the systematic effects, that can alter the result of the measurement. One of the ~~principal~~ main source s of background for the measurement of A_{PV} is a different process that concerns transverse polarized electrons. The different polarization of the electrons produce an asymmetry, called beam normal single spin asymmetry (BNSSA), or transverse asymmetry A_n. ~~Because~~ Since such asymmetries are typically one ~~order~~ of magnitude higher than the parity-violating ones, a small normal component of the beam polarization during parity-violating experiments can produce a systematic effect that changes the final result. The subject of this thesis is the measurement of transverse asymmetry A_n for carbon target, performed at MAMI, the Mainz microton accelerator. The choice of carbon target is due to the fact that the transverse asymmetry for ^{12}C is well known and already measured at MAMI; the expected asymmetry is roughly $20\,ppm$, thus it is particularly suited for ~~a~~ the commissioning of the new experimental setup. Such asymmetries are challenging because they require calculation of box diagrams with intermediate excited states [15], ~~the theory~~ which will be threated in chapter 2. After the determination on A_n for ^{12}C, the next phase of the MREX experiment will be the determination of the transverse asymmetry for ^{208}Pb. As already mentioned, this is mandatory to constrain the systematic effects ~~of~~ in PV experiment s. However, it is also interesting because ~~for~~ in the last measurement performed by PREX [4] the transverse asymmetry for ^{208}Pb target is compatible with zero, with a complete disagreement with the theoretical predictions. ~~Because~~ Since the theoretical prediction for hydrogen, helium, carbon and zirconium are in agreement with theory, a second,

With respect to.

is be compared with

independent measurement of the BNSSA for lead ~~are~~ interesting, to ~~confirm~~ the measurement performed by PREX, or ~~to reject that.~~

?

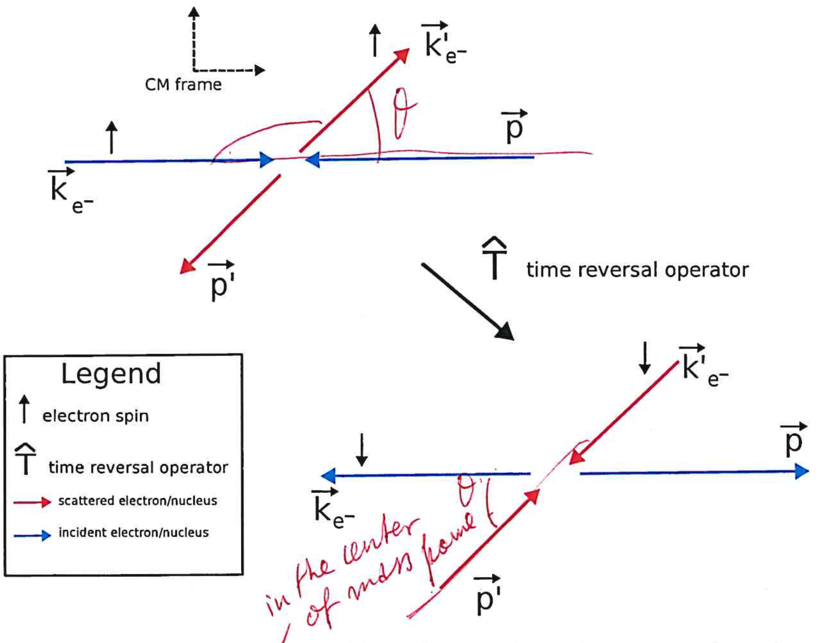
Chapter 2

Transverse Asymmetry

This chapter is ~~focused on describing~~ introduces the theory behind the transverse asymmetry. The transverse asymmetry arises from the interference between two scattering amplitudes (the exchange of one and two virtual photons, ~~respectively~~) and it is deeply connected with the Time-reversal operator. These two scattering amplitudes given by electromagnetic interaction between the incident electron and the nucleus, are explained in this chapter, together with the limits of the theory. The chapter ~~ends by~~ also presenting the problem of the anomalous observation made by PREX of zero A_n for lead target. In the end we discuss the general formula of A_n and we study how the accuracy of a measurement vary ies increasing the statistics.

2.1 Description of the Process

The Beam Normal Single Spin Asymmetry, which we will refer to ~~for brevity~~ as Transverse Asymmetry, originates from the interference of two scattering process. The theory of the electron scattering against a spin 0 target is extensively treated in [15]. To understand why the interference of this two scattering amplitude gives rise to an asymmetry, we first have to look at the kinematics of the experiment, shown in figure 2.1;



in the center of mass frame

Figure 2.1: Scheme of the scattering process. In blue the incident electron and nucleus, in red the outgoing electron and nucleus. All the quantities are referred to the center of mass frame. The small arrow over the vector represent the electron spin, ~~aligned~~ lying in the normal plane.

Where all the momenta are measured respect to the center of mass frame. In the figure we can ~~confront~~ compare the two situation before and after applying the Time-reversal operator, ~~$\hat{\Theta}$~~ $\hat{T}$. Looking at the picture we can

Since I do something, ... etc

Because of my doing something, ... etc

perché usi $\hat{\Theta}$ invece di $\hat{T}$?
Cmq scegli.

understand that :

- Before applying $\hat{\Theta}$, we have the incident electron with $\vec{k}$ momenta and the nucleus with $\vec{P}$ momenta, after applying $\hat{\Theta}$ we have that the incident/outgoing electron and the incident/outgoing nucleus are exchanged.
- ? The $\hat{\Theta}$ operator acts also on the spin of the electron. Because we are considering process where the spin doesn't flip, the two situations are not equivalent.
- Considering that the process is elastic, the kinematic is the same, taking $\vec{p}$ and $\vec{k}$ as the initial particle momenta, or $\vec{p}'$ and $\vec{k}'$.

NON SI CAPISCE COSA VUOI DIRE

The time-reversal operator ~~seems to~~ connects the two different cases of UP and DOWN polarized electron. Our ~~effort~~ goal is to measure the asymmetry between the two cross sections

~~NON SI E' CAPITO DA QUANTO HAI DETTO.~~

$$A = \frac{\sigma_\uparrow - \sigma_\downarrow}{\sigma_\uparrow + \sigma_\downarrow} \tag{2.1}$$

~~And it's particularly clear that a~~ A non-zero asymmetry depends on how the time-reversal act on the elastic amplitude of the process.
With this idea, let's see in more detail the $\hat{\Theta}$. We know that $\hat{\Theta}$ is an anti-unitary operator that can be always seen as:

$$\hat{\Theta} = U \cdot K$$

Where U is an unitary operator, while K is the complex conjugation operator that generates the complex conjugate of ~~each~~ the coefficients in front ~~of it~~ the state vectors. If we consider a ket describing a system we have that:

$$Kc\,|\alpha\rangle = c^* K\,|\alpha\rangle \tag{2.2}$$

Now, let's consider H as the hamiltonian of our system. We want to apply the $\hat{\Theta}$ operator. We can now use the assumption that the hamiltonian consist of two term, which correspond to ~~the~~ two different scattering process. Because of the electromagnetic interaction conserve CP, so also T is conserved, we know in advance that each piece of the hamiltonian commute with $\hat{\Theta}$. Now let's see what happen for an hamiltonian which has an imaginary part:

$$H = H_R + iH_{Im} \quad ; \quad \hat{\Theta}H\hat{\Theta}^{-1} = \hat{\Theta}H_R\hat{\Theta}^{-1} + \hat{\Theta}iH_{Im}\hat{\Theta}^{-1} \Rightarrow H_R - iH_{Im} \neq H \tag{2.3}$$

what we understand from these simple calculation is that to give rise to an asymmetry, we expect an imaginary part of the scattering amplitude different from zero.
At the α leading order, the two process of the electron-Nucleus scattering that give rise to the asymmetry involve the exchange of one-photon-exchange (OPE) and two-photon-exchange (TPE). The Feynman diagrams that describes the processes are the following:

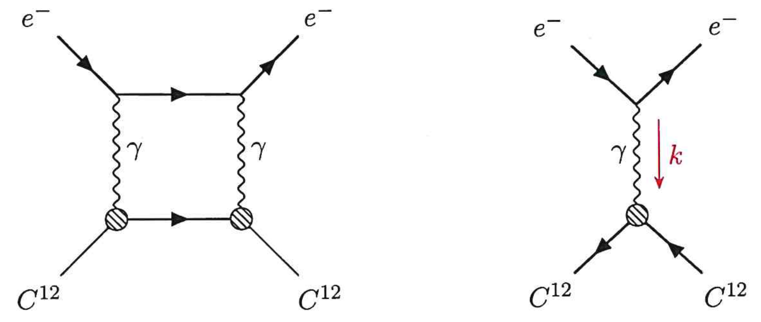


Figure 2.2: TPE and OPE diagrams in electron nucleus scattering.

Non mi torna questa discussione. Sono le ampiezze, non l'hamiltoniana che hanno una parte immaginaria

The important quantity to compute the cross section is the scattering amplitude. The scattering amplitude is given by the two contributions: the exchange of a single virtual photon A_1 and the terms given by the two photon exchange A_2. In general we can write that the total scattering amplitude S:

$$S = \frac{e^2}{Q^2}\overline{u}(k')[m_e A_2 + A_1 \not{P}]\overline{u}(k) \tag{2.4}$$

Where in this expression $\vec{P} = \frac{\vec{p}+\vec{p}'}{2}$. The second term A_1 has a simple expression, given by the form factor of the nucleus:

$\vec{P}'$ non si legge

$$A_1 = 2ZF_N(Q^2)$$

This expression is obtained if we look at ~~the~~ figure 2.2. For the one photon exchange the first vertex connects the incident and scattered electron, whose expression is given by $-ie\gamma^\mu$. The second vertex connect the carbon nucleus with the virtual photon. The carbon is threated like as a spin 0 particle, and the contribution due to the charge density in enshrined in the form factor. The lagrangian term for a vertex of this type is given by the formula:

$$\mathcal{L}_{interaction} = +ieA_\mu(\Phi\partial^\mu\Phi^\dagger - \Phi^\dagger\partial_\mu\Phi) \tag{2.5}$$

For the spin 0 field Φ. This is not the only piece of the lagrangian: there exist another term of interaction, that involves a vertex with four particles which is not of our interest. This interaction term give rise to the Feynmann rule for spin 0 particle, and we have to substitute for this vertex:

$$-ie(p+p')_\mu$$

And we recognize, apart from a factor 2, $\not{P}$ which multiplies A_1. The last term is the ~~feynmann~~ Feynmann propagator for the photon, that give the $\frac{1}{Q^2}$ term. This first part of the scattering amplitude is T-even, and it is purely real, so it is the imaginary part of the two photon exchange which give rise to the asymmetry. The expression that connects the amplitude with the transverse asymmetry is given by:

$$A_n = -\frac{m_e}{\sqrt{s}}tan(\frac{\theta_{CM}}{2})\frac{\mathrm{Im}(A_2)}{ZF_N(Q^2)} \tag{2.6}$$

Looking at this formula, the theoretical effort to compute the transverse asymmetry is given by the imaginary part of A_2. The calculation of this quantity is theoretically challenging, due to the fact that at energies of $\simeq 1\,\mathrm{GeV}$ of incident electrons, contributions from intermediate excited states become important. Because of this, the contribution of A_2 are given by the sum of elastic intermediate state and inelastic terms, which involve hadronic excitations.

Le frasi incomprensibili.

2.1.1 Hadronic Tensor

The imaginary part A_2 is related to the two-photon exchange. To compute this quantity, we have to perform an integration over the internal momenta of the electron k_1 (see figure 2.2). This contribution, following [15], is given by:

$$\mathrm{Im}(A_2) = e^4\frac{1}{(2\pi)^2}\int\frac{l_{\mu\nu}\cdot W^{\mu\nu}}{2E_1 Q_1^2 Q_2^2}d^3\vec{k_1} \tag{2.7}$$

Two new terms appear in this expression. The first term is $l_{\mu\nu}$, named leptonic tensor. This term is given computing the Amplitude for the upper part of the diagram, which involve the incident and scattered electron:

$$l_{\mu\nu} = \overline{u}(k')\gamma_\nu(\not{k}_1 + m_e)\gamma_\mu u(k) \tag{2.8}$$

In this expression is immediate to recognize the feynmann rules for fermion vertex. The term $(k_1 \not{+} m_e)$ comes from the fermion propagator of the internal electron, which is:

$$\frac{i(\not{p}+m)}{p^2+m^2}$$

The other term is $W^{\mu\nu}$, the hadronic tensor. For the elastic contribution this term is simply given by the feynmann rules for vertex with spin 0 particles, with the proper correction of the form factor, so we can write:

$$W_{\mu\nu} = \pi\delta((p+k-k_1)^2 - M^2)(2p+q_1)_\mu(2p'+q_2)_\nu \times Z^2 F_N(Q_1)F_N(Q_2) \tag{2.9}$$

At this point, one can substitute in the integral above, and compute the contribution of the transverse asymmetry due to the elastic term. This first terms scales with the nuclear charge $Z\alpha$, and this is important for electron scattering with heavy nuclei. However, this mechanism is important in the energy range of few MeV, and has a minor impact, although not negligible, for higher energy, such as the energy of interest for this thesis. For the inelastic contributions, the structure of the hadronic tensor is different. Realistic estimate are given only for nearly forward scattering angles. The hadronic tensor is given in terms of the structure functions $W_{1,2}$

$$W^{\mu\nu} = 2\pi W_1(\omega^2, Q_1^2)\Big(-g^{\mu\nu} + \frac{P^\mu q_1^\nu + P^\nu q_2^\mu}{(P\overline{K})} - \frac{q_1 q_2}{(P\overline{K})^2}P^\mu P^\nu\Big) \tag{2.10}$$

Several assumption are made to t~~h~~reat this new term. The structure function, for forward scattering angles, can be approximated by a function containing the Compton form factor of the nucleus, neglecting some dependence on $Q_{1,2}$ that let to simplify the integral in equation 2.6. It is beyond our scope to go into a detailed description, which can be found in the articles ([16], [15], [17]). We emphasize however that for the estimation of the inelastic intermediate state, theoretical calculation are affected by the approximation of forward angles and other assumptions due to lack of data in the dependence of some important variables, such as the Compton form factor for carbon 12, the Compton slope parameter and the use of the approximated Callavan-Gross relation. In summary, the theoretical prediction for the transverse asymmetry are reliable for small scattering angle, that correspond to lower values in transfer momentum Q; the experimental data measure by PREX [4] for ^1H, ^4He, and ^{12}C at Q values of 0.31 GeV, 0.28 GeV and 0.1 GeV, respectively, are in agreement with the theoretical prediction. The measurement performed at MAMI for ^{12}C [10] ~~are~~ is with higher values of transfer momentum ($Q = 0.2$ GeV) and shows a ~~discrete~~ reasonable agreement with the theoretical prediction, considering also the systematic uncertainties associated to the poorly known Compton slope parameter. While the theory presented so far is quite successful in describing the data, it fails completely with ^{208}Pb. PREX report for lead 0 asymmetry. This ~~strictly~~ disagreement, although remove the presence of systematic effects due to the BNSSA for PV experiment, suggest to repeat the measure, besides being a theoretical challenge. Also for this reason, after the measurement with carbon, a new measurement of the transverse asymmetry with lead is scheduled.

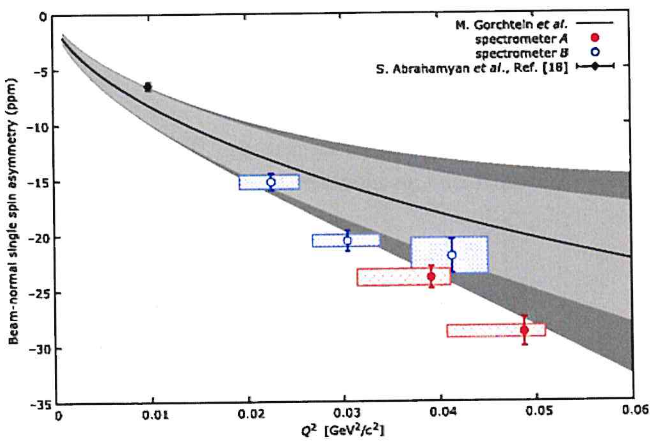


Figure 2.3: Transverse asymmetry measured at MAMI for ^{12}C target [10]. Theoretical calculation for E_{beam} = 570 MeV is shown.

2.1.2 Model Description

$$A_N = C_0 \cdot log(\frac{Q^2}{m_e^2 c^2}) \frac{F_{Compton}(Q^2)}{F_{ch}(Q^2)} \tag{2.11}$$

2.2 State of the Experiment

We have seen so far how the Transverse Asymmetry is related to the interference between two scattering amplitude, and the theoretical model used to describe the process. The goal from an experimental point of view is to measure this quantity. The challenge is to obtain a valid measure of A_n, which is of the order of 20 part per million (ppm), taking into consideration all the possible effects that can interfere. To measure A_n, the straightforward method is to prepare an electron beam, with polarized electron, and send it to a fixed target. The scattered electrons are then collected by a detector placed at a certain angle, ~~and now it's possible to obtain~~ with the transverse asymmetry ~~applying the formula~~: obtained by

$$A_N(Q,p) = \frac{N_\uparrow(Q) - N_\downarrow(Q)}{N_\uparrow(Q) + N_\downarrow(Q)} \cdot (\frac{1}{p}) \tag{2.12}$$

perchè ???

where we have explained the dependence on the transmitted impulse, on the degree of polarization of the beam. In an experiment of this type, several requests are necessary to have an effective data acquisition:

- The accelerator must produce a polarized beam, stable over the time, with an high polarization percentage, in order to amplify the effect.
- The Beam energy needs to be quite stable, and should not depend on the Polarization state of the electrons. A change in the Beam energy associated with the polarization state, can lead to a different count rate for $N_\uparrow$ and $N_\downarrow$, would make a contribution that would be added to that of the physical process
- The beam must be correctly aligned with the target, and stable. ~~Again~~ y if the position of the target changes according to the polarization of the electrons, it will produce another contribution to the total asymmetry.
- The beam current should not depend on the polarization state of the electrons. If the beam source depends on the polarization, we will have a difference in the event rate and then another false asymmetry.
- ~~it's~~ It's necessary to reject possible double elastic scattering events, which may contribute to the total asymmetry.

All this demands can be satisfied with an accelerator that has stabilization devices with great precision and that can sustain high beam intensities. This last request is necessary to accumulate enough statistics to measure the transverse asymmetry with an accuracy about 1 ppm, in view of the future PV experiments. We can quantify how the statistical error varies according to the amount of data available. With the assumption that $N_{\uparrow,\downarrow}$ are gaussian distributed variables, we can compute the expected variance

$$Var[A_n] = \frac{1 - A^2}{N_\uparrow + N_\downarrow} \tag{2.13}$$

For a single measurement of the A_n. For n multiple measurement s ~~n~~, the variance scales as $\frac{1}{n}$. Because A_n is expected to be smaller respect to 1, we can approximate the above formula:

$$V[A_n] = \frac{1}{2N \cdot n} \tag{2.14}$$

The error associated to the reconstructed asymmetry is the square root of the above quantity. If we impose that the error must be $\leq 1ppm$ we can easily obtain that the quantity $n \cdot N$:

$$n \cdot N \leq \frac{1}{2} \cdot 10^{12}$$

??

lose?

We will see later that achievable rates $N_{\uparrow,\downarrow}$ are in the range (20000,60000) counts per event for a carbon target. This number can not be increased at will by acting on the beam current. The first reason is oblivious: obvious the beam source can produce only a certain amount of electrons before loosing, furthermore a beam with great intensity for an extended periods of time can damage the carbon target up to the risk of melting it. Another idea might be to increase the thickness of the target, to take advantage of the larger cross section. However this does not take into account that by doing so the number of double scattering event is increased. To avoid this the scientific community that deals with these nuclear physics measurements respect the convention that the target thickness should be less than the 10% of the radiation length of the material.