Matriz Inversa - exercício 7.1 b - d

terça-feira, 12 de setembro de 2023 14:00

$$B = \begin{bmatrix} 1 & 2 & -1 \\ 0 & -5 & 3 \\ 0 & 0 & \frac{3}{5} \end{bmatrix}$$

$$B \cdot B^{-1} = I$$

$$\begin{vmatrix} 1 & 2 & -1 \\ 0 & -5 & 3 \\ 0 & 0 & 3/5 \end{vmatrix} \begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}$$

$$\begin{vmatrix} a + 2d + (-g) & b + 2e + (-h) & c + 2f + (-i) \\ 0 + (-5d) + 3g & 0 + (-5e) + 3h & 0 + (-5f) + 3i \\ 0 + 0 + \frac{3}{5}g & 0 + 0 + \frac{3}{5}h & 0 + 0 + \frac{3}{5}i \end{vmatrix} = \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}$$

$$\begin{cases} a + 2d - g = 1 \\ 0 - 5d + 3g = 0 \\ 0 + 0 + \frac{3}{5}g = 0 \end{cases} \qquad \begin{cases} b + 2e - h = 0 \\ 0 - 5e + 3h = 1 \\ 0 + 0 + \frac{3}{5}h = 0 \end{cases} \qquad \begin{cases} c + 2f - i = 0 \\ 0 - 5f + 3i = 0 \\ 0 + 0 + \frac{3}{5}i = 1 \end{cases}$$

$$\frac{3}{5}g = 0 \qquad \frac{3}{5}h = 0 \qquad \frac{3}{5}i = 1$$

$$3g = 5d \qquad 3i = 5f \qquad i = c + 2f$$

$$h = b + 2e \qquad \boxed{g = 0} \qquad \boxed{h = 0}$$

$$\boxed{i = \frac{5}{3}} \qquad i = \frac{5}{3}f \qquad \boxed{f = \frac{3}{5}i = 1}$$

$$3g = 5d \qquad 3(0) = 5d \qquad d = \frac{0}{5} \qquad \boxed{d = 0}$$

$$a + 2d - g = 1 \qquad a + 2(0) - 0 = 1 \qquad \boxed{a = 1}$$

$$-5e + 3h = 1 \qquad 3h - 1 = 5e \qquad 3(0) - 1 = 5e \qquad -1 = 5e \qquad \boxed{e = -\frac{1}{5}}$$

$b + 2e - h = 0$

$b + 2(-\frac{1}{5}) - (0) = 0$

$b - \frac{2}{5} = 5$

$\boxed{b = \frac{2}{5}}$

$c + 2f - i = 0$

$c + 2 - \frac{5}{3} = 0$

$c + \frac{1}{3} = 0$

$\boxed{c = -\frac{1}{3}}$

$$\boxed{B^{-1} = \begin{vmatrix} 1 & 2/5 & -1/3 \\ 0 & -1/5 & 1 \\ 0 & 0 & 5/3 \end{vmatrix}}$$

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c)

$$C = \begin{vmatrix} 3 & 1 \\ 2 & -4 \end{vmatrix}$$

$$C \cdot C^{-1} = I$$

$$\begin{vmatrix} 3 & 1 \\ 2 & -4 \end{vmatrix} \cdot \begin{vmatrix} a & b \\ c & d \end{vmatrix} = \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix}$$

$$\begin{vmatrix} 3a + 1c & 3b + 1d \\ 2a + (-4)c & 2b + (-4)d \end{vmatrix} = \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix}$$

$3a + c = 1$

$2a - 4c = 0$

$3b + d = 0$

$2b - 4d = 1$

$2a = 4c$

$a = \frac{4c}{2}$

$3b = -d$

$b = \frac{-d}{3}$

$$3\left(\frac{\cancel{4}^{2}c}{\cancel{2}}\right)+c=1$$

$$6c+c=1$$

$$7c=1$$

$$\boxed{c=\frac{1}{7}}$$

$$a=\frac{\cancel{4}^{2}(1/7)}{\cancel{2}}=\frac{2}{7}$$

$$\boxed{a=\frac{2}{7}}$$

$$2b-4d=1$$

$$2\left(\frac{-d}{3}\right)-4d=1$$

$$-\frac{2}{3}d-4d=1$$

$$d\left(-\frac{2}{3}-4\right)=1$$

$$d\left(\frac{-2-12}{3}\right)=1$$

$$d\left(\frac{-14}{3}\right)=1$$

$$\boxed{d=\frac{-3}{14}}$$

$$b=\frac{-d}{3}$$

$$=-\left(\frac{-3/14}{3}\right)=\frac{\cancel{3}}{14}\times\frac{1}{\cancel{3}}$$

$$\boxed{b=\frac{1}{14}}$$

$$\boxed{C^{-1}=\begin{vmatrix} 2/7 & 1/14 \\ 1/7 & -3/14 \end{vmatrix}}$$

$$D=\begin{vmatrix} 1 & 2 & 3 \\ 9 & -2 & 1 \\ 1 & 2 & 3 \end{vmatrix}$$

$$D.D^{-1}=I$$

$$\begin{vmatrix} 1 & 2 & 3 \\ 9 & -2 & 1 \\ 1 & 2 & 3 \end{vmatrix}\cdot\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix}=\begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}$$

$$|\, a+2d+3g \quad b+2e+3h \quad c+2f+3i \,| \quad |\, 1 \quad 0 \quad 0 \,|$$

$$\begin{vmatrix} a+2d+3g & b+2e+3h & c+2f+3i \\ 9a+(-2)d+g & 9b+(-2)e+h & 9c+2f+i \\ a+2d+3g & b+2e+3h & c+2f+3i \end{vmatrix} = \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}$$

$$\begin{cases} a+2d+3g=1 \\ 9a-2d+g=0 \\ a+2d+3g=0 \end{cases} \quad \begin{cases} b+2e+3h=0 \\ 9b-2e+h=1 \\ b+2e+3h=0 \end{cases} \quad \begin{cases} c+2f+3i=0 \\ 9c+2f+i=0 \\ c+2f+3i=1 \end{cases}$$

PROPRIEDADE DA MATRIZ: SE DUAS LINHAS DA MATRIZ FOREM IGUAIS, SEU DETERMINANTE SERÁ ZERO.

A MATRIZ D NÃO ADMITE INVERSA, UMA VEZ QUE $\det(D) = \nexists$.