# FAETERJ-Rio
# Cálculo I
# Professor DSc. Wagner Zanco

## Solução dos Exercícios 2.1 – 1.7

2.1) Use a definição básica de $f'(x)$ como um limite para calcular as derivadas das seguintes funções:

a) $f(x) = 2x - 5$
b) $f(x) = \frac{1}{3}x^2 - 7x + 4$
c) $f(x) = 2x^3 + 3x - 1$
d) $f(x) = x^4$
e) $f(x) = \frac{1}{2-x}$
f) $f(x) = \frac{x}{x+2}$
g) $f(x) = \frac{1}{x^2+1}$

a) $f(x) = 2x - 5$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$f(x+h) = 2(x+h) - 5$$
$$= 2x + 2h - 5$$

$$f(x+h) - f(x) = 2x + 2h - 5 - (2x - 5)$$
$$= 2x + 2h - 5 - 2x + 5$$
$$= 2h$$

$$\frac{f(x+h) - f(x)}{h} = \frac{2h}{h} = 2$$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} 2$$

$$f'(x) = 2$$

b) $f(x) = \frac{1}{3}x^2 - 7x + 4$

$$f'(x) = \frac{2}{3}x - 7$$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$f(x+h) = \frac{1}{3}(x+h)^2 - 7(x+h) + 4$$

$$= \frac{1}{3}(x^2 + 2xh + h^2) - 7x - 7h + 4$$

$$= \frac{1}{3}x^2 + \frac{1}{3}2xh + \frac{1}{3}h^2 - 7x - 7h + 4$$

$$f(x+h) - f(x) = \frac{1}{3}x^2 + \frac{1}{3}2xh + \frac{1}{3}h^2 - 7x - 7h + 4 - \left(\frac{1}{3}x^2 - 7x + 4\right)$$

$$= \frac{1}{3}x^2 + \frac{1}{3}2xh + \frac{1}{3}h^2 - 7x - 7h + 4 - \frac{1}{3}x^2 + 7x - 4$$

$$= \frac{1}{3}2xh + \frac{1}{3}h^2 - 7h$$

$$\frac{f(x+h) - f(x)}{h} = \frac{h\left(\frac{1}{3}2x + \frac{1}{3}h - 7\right)}{h}$$

$$= \frac{1}{3}2x + \frac{1}{3}h - 7$$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{1}{3}2x + \frac{1}{3}h - 7$$

$$= \frac{2}{3}x - 7$$

c) $f(x) = 2x^3 + 3x - 1$

$$f(x+h) = 2(x+h)^3 + 3(x+h) - 1$$

$$2(x+h)^2(x+h) + 3x + 3h - 1$$

$$= 2(x^2 + 2xh + h^2)(x + h) + 3x + 3h - 1$$

$$= 2(x^3 + 2x^2h + xh^2 + x^2h + 2xh^2 + h^3) + 3x + 3h - 1$$

$$f(x + h) = 2x^3 + 4x^2h + 2xh^2 + 2x^2h + 4xh^2 + 2h^3 + 3x + 3h - 1$$

$$f(x + h) - f(x) = 2x^3 + 4x^2h + 2xh^2 + 2x^2h + 4xh^2 + 2h^3 + 3x + 3h - 1 - (2x^3 + 3x - 1)$$

$$= 2x^3 + 4x^2h + 2xh^2 + 2x^2h + 4xh^2 + 2h^3 + 3x + 3h - 1 - 2x^3 - 3x + 1$$

$$= 4x^2h + 2xh^2 + 2x^2h + 4xh^2 + 2h^3 + 3h$$

$$f(x + h) - f(x) = 6x^2h + 6xh^2 + 2h^3 + 3h$$

$$\frac{f(x + h) - f(x)}{h} = \frac{h(6x^2 + 6xh + 2h^2 + 3)}{h}$$

$$= 6x^2 + 6xh + 2h^2 + 3$$

$$f'(x) = \lim_{h \to 0} \frac{f(x + h) - f(x)}{h} = \lim_{h \to 0} \; 6x^2 + 6xh + 2h^2 + 3$$

$$= 6x^2 + 3$$

d) $f(x) = \; x^4$

$$f(x + h) = (x + h)^4 = (x + h)^2(x + h)^2$$

$$= (x^2 + 2xh + h^2)(x^2 + 2xh + h^2)$$

$$= x^4 + 2x^3h + x^2h^2 + 2x^3h + 4x^2h^2 + 2xh^3 + x^2h^2 + 2xh^3 + h^4$$

$$= x^4 + 4x^3h + 6x^2h^2 + 4xh^3 + h^4$$

$$f(x + h) - f(x) = x^4 + 4x^3h + 6x^2h^2 + 4xh^3 + h^4 - (x^4)$$

$$= 4x^3h + 6x^2h^2 + 4xh^3 + h^4$$

$$\frac{f(x + h) - f(x)}{h} = \frac{h(4x^3 + 6x^2h + 4xh^2 + h^3)}{h}$$

$$= 4x^3 + 6x^2h + 4xh^2 + h^3$$

$$f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h} = \lim_{h\to 0}\ 4x^3 + 6x^2h + 4xh^2 + h^3$$
$$f'(x) = 4x^3$$

e) $f(x) = \frac{1}{2-x}$

$$f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

$$f(x+h) = \frac{1}{2-(x+h)}$$

$$f(x+h)-f(x) = \frac{1}{2-x-h} - \frac{1}{2-x}$$

$$\frac{a}{b} - \frac{c}{d} = \frac{ad-bc}{bd}$$

$$= \frac{2-x-(2-x-h)}{(2-x-h)(2-x)}$$

$$= \frac{2-x-2+x+h}{(2-x-h)(2-x)}$$

$$f(x+h)-f(x) = \frac{h}{4-2x-2h-2x+x^2+xh}$$

$$\frac{f(x+h)-f(x)}{h} = \frac{h}{4-2x-2h-2x+x^2+xh}.\frac{1}{h}$$

$$= \frac{1}{4-2x-2h-2x+x^2+xh}$$

$$f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h} = \lim_{h\to 0}\left(\frac{1}{4-2x-2h-2x+x^2+xh}\right)$$

$$= \frac{\lim_{h\to 0} 1}{\lim_{h\to 0} 4-2x-2h-2x+x^2+xh}$$

$$= \frac{1}{4-2x-2x+x^2}$$

$$= \frac{1}{4 - 4x + x^2} = \frac{1}{(x-2)^2}$$

f) $f(x) = \frac{x}{x+2}$

$$f(x+h) = \frac{x+h}{x+h+2}$$

$$f(x+h) - f(x) = \frac{x+h}{(x+h+2)} - \frac{x}{x+2}$$

$$= \frac{(x+2)(x+h) - (x+h+2)x}{(x+h+2)(x+2)}$$

$$= \frac{x^2 + 2x + xh + 2h - (x^2 + xh + 2x)}{(x+h+2)(x+2)}$$

$$= \frac{x^2 + 2x + xh + 2h - x^2 - xh - 2x}{(x+h+2)(x+2)}$$

$$f(x+h) - f(x) = \frac{2h}{(x+h+2)(x+2)}$$

$$\frac{f(x+h) - f(x)}{h} = \frac{2h}{(x+h+2)(x+2)}\frac{1}{h}$$

$$= \frac{2}{(x+h+2)(x+2)}$$

$$f'(x) = \lim_{h\to 0}\frac{f(x+h) - f(x)}{h} = \lim_{h\to 0}\left(\frac{2}{(x+h+2)(x+2)}\right)$$

$$= \frac{\lim_{h\to 0} 2}{\lim_{h\to 0}(x+h+2)(x+2)}$$

$$= \frac{2}{\lim_{h\to 0}(x+h+2)\lim_{h\to 0}(x+2)}$$

$$= \frac{2}{(x+2)(x+2)} = \frac{2}{(x+2)^2}$$

g) $f(x) = \frac{1}{x^2+1}$

$$f(x+h) = \frac{1}{(x+h)^2+1}$$

$$= \frac{1}{x^2+2xh+h^2+1}$$

$$f(x+h) - f(x) = \frac{1}{x^2+2xh+h^2+1} - \frac{1}{x^2+1}$$

$$= \frac{x^2+1-(x^2+2xh+h^2+1)}{(x^2+2xh+h^2+1)(x^2+1)}$$

$$= \frac{x^2+1-x^2-2xh-h^2-1}{(x^2+2xh+h^2+1)(x^2+1)}$$

$$= \frac{-2xh-h^2}{(x^2+2xh+h^2+1)(x^2+1)}$$

$$f(x+h) - f(x) = \frac{h(-2x-h)}{(x^2+2xh+h^2+1)(x^2+1)}$$

$$\frac{f(x+h)-f(x)}{h} = \frac{h(-2x-h)}{(x^2+2xh+h^2+1)(x^2+1)}\frac{1}{h}$$

$$= \frac{-2x-h}{(x^2+2xh+h^2+1)(x^2+1)}$$

$$f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h} = \lim_{h\to 0}\left(\frac{-2x-h}{(x^2+2xh+h^2+1)(x^2+1)}\right)$$

$$= \frac{\lim_{h\to 0} -2x-h}{\lim_{h\to 0}(x^2+2xh+h^2+1)(x^2+1)}$$

$$= \frac{\lim_{h\to 0} -2x-h}{\lim_{h\to 0}(x^2+2xh+h^2+1)\lim_{h\to 0}(x^2+1)}$$

$$\frac{-2x}{(x^2+1)(x^2+1)} = -\frac{2x}{(x^2+1)^2}$$

2.2) Use a regra 5 para calcular as derivadas dos seguintes polinômios

a) $f(x) = 3x^3 - 4x^2 + 5x - 2$
b) $f(x) = -8x^5 + \sqrt{3}x^3 + 2\pi x^2 - 12$
c) $f(x) = 3x^{13} - 5x^{10} + 10x^2$

$$Dx(x^n) = nx^{n-1}$$

a)

$$f'(x) = 3.3x^2 - 2.4x^1 + 5$$

b)

$$f'(x) = -40x^4 + 3\sqrt{3}x^2 + 4\pi x$$

c)

$$f'(x) = 13.3x^{12} - 10.5x^9 + 2.10x$$

$$= 39x^{12} - 50x^9 + 20x$$

2.3) Determine a derivada de:

a) $Dx(3x^7 - \frac{1}{5}x^5)$
b) Encontre $\frac{d(3x^2-5x+1)}{dx}$
c) Se $y = \frac{1}{2}x^4 + 5x$, obtenha $\frac{dy}{dx}$
d) Calcule $\frac{d(3t^7-12t^2)}{dt}$
e) Se $U = \sqrt{2}x^5 - 3x^3$, obtenha $DxU$

a) $f(x) = 3x^7 - \frac{1}{5}x^5$

$$f'(x) = 7.3x^6 - 5.\frac{1}{5}x^4 = 21x^6 - x^4$$

b) $f(x) = 3x^2 - 5x + 1$

$$f'(x) = 6x - 5$$

c) $y = \frac{1}{2}x^4 + 5x$

$$y' = 2x^3 + 5$$

d) $f(t) = 3t^7 - 12t^2$

$$f'(t) = 21t^6 - 24t$$

e) $U = \sqrt{2}x^5 - 3x^3$

$$U' = 5\sqrt{2}x^4 - 9x^2$$

2.4) Encontre as equações reduzidas das retas tangentes aos gráficos das seguintes funções, nos pontos especificados.

a) $f(x) = x^2 - 5x + 2$ , em $x = -1$
b) $f(x) = 4x^3 - 7x^2$, em $x = 3$
c) $f(x) = x^4 + 2x^2 + 3$, em $x = 0$

a) $f(x) = x^2 - 5x + 2$

$$m = f'(x) = 2x - 5$$

Em $x = -1$

$$m = 2(-1) - 5 = -7$$

$$f(-1) = (-1)^2 - 5(-1) + 2$$

$$= 1 + 5 + 2 = 8$$

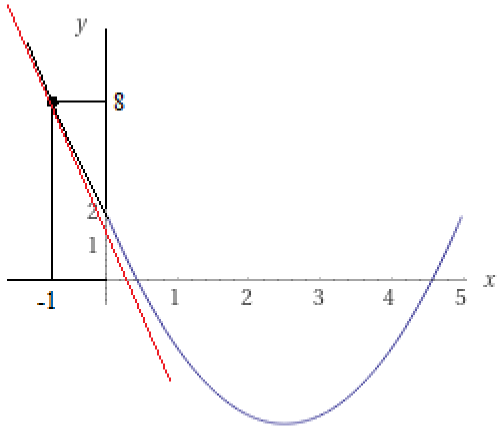
$$y = mx + b$$

$$8 = -7(-1) + b$$

$$8 = 7 + b$$

$$b = 1$$

$$y = -7x + 1$$




b) $f(x) = 4x^3 - 7x^2$

$$m = f'(x) = 12x^2 - 14x$$
$$m = 12(3)^2 - 14(3)$$
$$m = 108 - 42 = 66$$

$$f(3) = 4(3)^3 - 7(3)^2$$

$$= 108 - 63 = 45$$

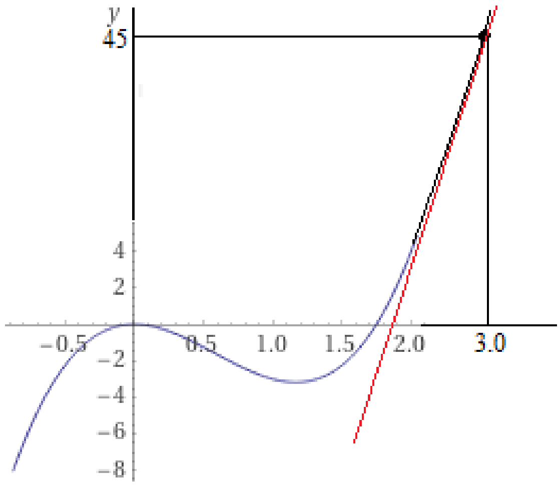
$$y = mx + b$$

$$45 = 66(3) + b$$

$$45 = 198 + b$$

$$b = 45 - 198 = -153$$

$$y = 66x - 153$$




c) $f(x) = x^4 + 2x^2 + 3$

$$m = f'(x) = 4x^3 + 4x$$

$$m = 4(0)^3 + 4(0) = 0$$

$$f(0) = (0)^4 + 2(0)^2 + 3 = 3$$

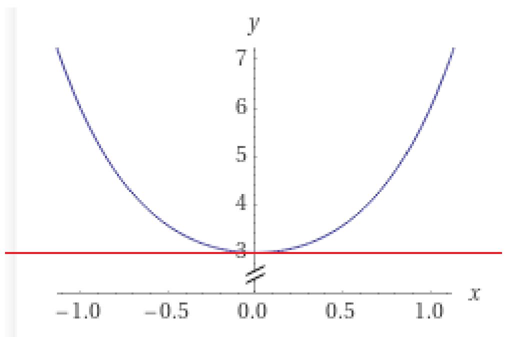
$$y = mx + b$$

$$3 = 0 + b$$

$$b = 3$$

$$y = 0.x + 3$$

$$y = 3$$



2.5) Determine a equação reduzida da reta normal ao gráfico $y = x^3 - x^2$, no ponto onde $x = 1$.

$$m = y' = 3x^2 - 2x$$

Em $x = 1$

$$m = 3(1)^2 - 2(1) = 1$$

O coeficiente angular da reta normal ($m_2$)

$$m.m_2 = -1$$

$$m_2 = \frac{-1}{1} = -1$$

Em $x = 1$, a equação reduzida da reta normal a reta tangente é:

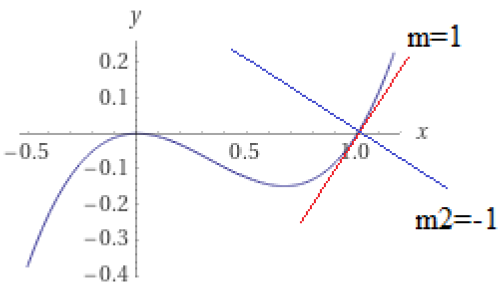
$$y = (1)^3 - (1)^2 = 0$$

$$y = m_2.x + b$$

$$0 = -1(1) + b$$

$$b = 1$$

$$y = -x + 1$$



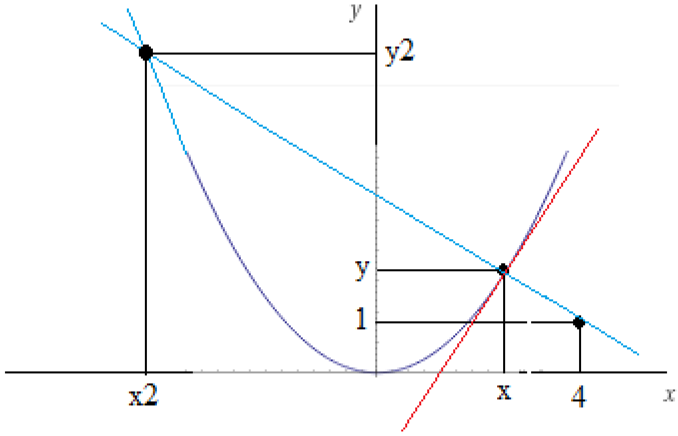
2.6) Obtenha os pontos do gráfico $y = \frac{1}{2}x^2$, nos quais a reta normal passa pelo ponto (4,1).

$$m = y' = x$$

O coeficiente angular da reta normal ($m_2$)

$$m.m_2 = -1$$

$$m_2 = -\frac{1}{x}$$




$$m_2 = \frac{y-1}{x-4}$$

$$-\frac{1}{x} = \frac{y-1}{x-4}$$

$$-\frac{1}{x} = \frac{\left(\frac{1}{2}x^2\right)-1}{x-4}$$

$$-\frac{1}{x}(x-4) = \frac{x^2}{2} - 1$$

$$-1 + \frac{4}{x} = \frac{x^2}{2} - 1$$

$$\frac{4}{x} = \frac{x^2}{2}$$

$$4.2 = x.x^2$$

$$x^3 = 8$$

$$x = \sqrt[3]{8} = 2$$

$$y(2) = \frac{2^2}{2} = 2$$

$$m_2 = -\frac{1}{x}$$

Em *x=2*

$$m_2 = -\frac{1}{2}$$

$$m_2 = \frac{y_2 - 2}{x_2 - 2}$$

$$-\frac{1}{2} = \frac{\frac{{x_2}^2}{2} - 2}{x_2 - 2}$$

$$-\frac{1}{2}(x_2 - 2) = \frac{{x_2}^2}{2} - 2$$

$$-\frac{x_2}{2} + 1 = \frac{{x_2}^2}{2} - 2$$

$$\frac{{x_2}^2}{2} - 2 + \frac{x_2}{2} - 1 = 0$$

$$\frac{{x_2}^2}{2} - 2 + \frac{x_2}{2} - 1 = 0$$

$$\frac{1}{2}{x_2}^2 + \frac{1}{2}x_2 - 3 = 0$$

$$x'_2 = -3 \text{ e } x''_2 = 2$$

Como $x_2$ é negativo, $x_2 = -3$

$$y_2(-3) = \frac{(-3)^2}{2} = \frac{9}{2}$$

$$(x, y) = (2{,}2)$$

$$(x_2, y_2) = \left(-3, \frac{9}{2}\right)$$

2.7) Especifique todas as retas que satisfazem as seguintes equações:

a) Passa pelo ponto (0,2) e é tangente a curva $y = x^4 - 12x + 50$
b) Passa pelo ponto (1,5) e é tangente a curva $y = 3x^3 + x + 4$

a)

$$m = y' = 4x^3 - 12$$

$$m = \frac{y_2 - y}{x_2 - x}$$

$$4{x_2}^3 - 12 = \frac{{x_2}^4 - 12x_2 + 50 - 2}{x_2 - 0}$$

$$x_2\left(4{x_2}^3 - 12\right) = {x_2}^4 - 12x_2 + 48$$

$$4{x_2}^4 - 12x_2 = {x_2}^4 - 12x_2 + 48$$

$$4{x_2}^4 - {x_2}^4 - 48 = 0$$

$$3{x_2}^4 = 48$$

$${x_2}^4 = 16$$

$$x_2 = \sqrt[4]{16} = \pm 2$$

Em $x = 2$

$$y(2) = 2^4 - 12.2 + 50$$

$$y(2) = 16 - 24 + 50 = 42$$

Em $x = -2$

$$y(-2) = -2^4 - 12.(-2) + 50$$

$$y(-2) = 16 + 24 + 50 = 90$$

Os pontos das retas que interceptam o gráfico (-2,90) e (2,42).

Em $x = 2$

$$m = 4(2^3) - 12 = 20$$

$$y = 42$$

$$y = mx + b$$

$$42 = 20(2) + b$$

$$b = 2$$

$$y = 20x + 2$$

Em $x = -2$

$$m = 4(-2^3) - 12 = -44$$

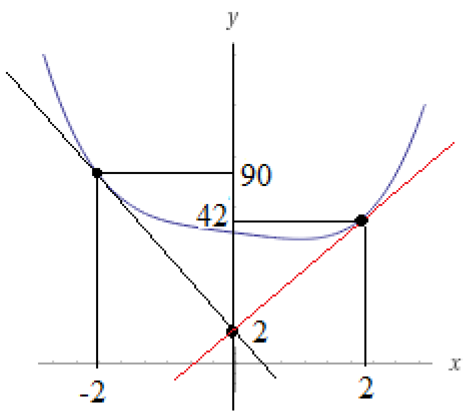
$$y = 90$$

$$y = mx + b$$

$$90 = -44(-2) + b$$

$$b = 2$$

$$y = -44x + 2$$




Em $x = 0$

$$m = y' = 4(0)^3 - 12 = -12$$

$$m = -12$$

**Gabarito**:

2.1a) 2. 2.1b) $\frac{2}{3}x - 7$. 2.1c) $6x^2 + 3$. 2.1d) $4x^3$. 2.1e) $\frac{1}{(2-x)^2}$. 2.1f) $\frac{2}{(2+x)^2}$. 2.1g) $\frac{2x}{(x^2+1)^2}$. 2.1a) $9x^2 - 8x + 5$. 2.2b) $-40x^4 + 3\sqrt[2]{3}x^2 + 4\pi x$.

2.2.c) $39x^{12} - 50x^9 + 20x$. 2.3a) $21x^6 - x^4$. 2.3b) $6x - 5$. 2.3c) $2x^3 + 5$. 2.3d) $21t^6 - 24t$. 2.3e) $5\sqrt{2}x^4 - 9x^2$. 2.4a) $-7x + 1$. 2.4b) $66x - 153$. 2.4c) 3. 2.5) $y = -x + 1$. 2.6) (2, 2) e (-3, 4,5). 2.7a) $y = 20x + 2$ e $y = -44x + 2$. 27b) $y = \frac{85}{4}x - \frac{65}{4}$.