Exercício 3.9a

quinta-feira, 18 de março de 2021 09:21

a) $\int_{-9}^{-5} \frac{dx}{x+1}$

$U = x + 1$

SE $x = -9$

$U = -9 + 1 = -8$

SE $x = -5$

$U = -5 + 1 = -4$

$$\int_{-9}^{-5} \frac{1}{x+1}\, dx$$

INTEGRAL EM FUNÇÃO DE U.

$U = x + 1$

$U' = 1$

$du = U'dx$

$du = dx$

$$\int_{-8}^{-4} \frac{1}{U}\, du = \ln|U| \Big[_{-8}^{-4}$$

OU

$$= \ln|x+1| \Big[_{-9}^{-5}$$

$$\ln|-5+1| - \ln|-9+1|$$

$$= -(-\ln|-4| + \ln|-8|)$$

$$= -(\ln 8 - \ln 4)$$

$$= -\ln \frac{8}{2} = -\ln 2$$

Exercício 3.9b

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b) $\int_0^{\frac{\pi}{4}} \frac{\sin(2x)}{3+5(\cos(2x))}\, dx$

$U = 3 + 5(\cos(2x))$

$U' = 5(-\sin(2x) \cdot 2)$

$U' = -10\sin(2x)$

$du = U' dx$

$du = -10\sin(2x)\, dx$

$\frac{du}{-10\sin(2x)} = dx$

$U = 3 + 5(\cos(2x))$

SE $x = 0$

$U = 3 + 5(\cancel{\cos}^{1}(2(0)))$

$U = 3 + 5 = 8$

SE $x = \frac{\pi}{4}$

$U = 3 + 5 \cdot \cos\left(\frac{\cancel{2}\pi}{\cancel{4}_2}\right)$

$= 3 + 5\cos\left(\frac{\pi}{2}\right)$

$= 3 - 5(0) = 3$

INT. EM função de U.

$\int_8^3 \frac{\cancel{\sin(2x)}}{U} \frac{du}{-10\cancel{\sin(2x)}}$

$-\frac{1}{10}\int_8^3 \frac{1}{U}\, du = \ln|U| \Big|_8^3$

OU

$= -\frac{1}{10}\ln|3 + 5\cos(2x)| \Big|_0^{\frac{\pi}{4}}$

$-\frac{1}{10}\ln\left|3 + 5\cos\left(2\frac{\pi}{4}\right)\right| - \left(-\frac{1}{10}\right)\ln|3 + 5\cos(2(0))|$

$= -\frac{1}{10}\ln\left|3 + 5\cos\left(\frac{\pi}{2}\right)\right| + 10\ln|3 + 5(1)|$

$= -\frac{1}{10}\ln|3 + 5(0)| + \frac{1}{10}\ln|8|$

$= \frac{1}{10}\ln|8| - \frac{1}{10}\ln|3|$

$= \frac{1}{10}\ln|8| - \ln|3|$

$= \frac{1}{10}\ln\frac{8}{3}$ //

Exercício 3.9c

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c) $\int_0^1 cosh^3 x . \sinh x \ dx$

$U = \cosh(x)$

se $x = 0$

$U = \cosh(0) = 1$

se $x = 1$

$U = \cosh(1) = 1{,}54$

$U = \cosh(x)$

$U' = \sinh(x)$

$du = u' dx$

$du = \sinh(x) dx$

$\frac{du}{\sinh(x)} = dx$

$$\int_1^{1{,}54} U^3 . \cancel{\sinh(x)} \frac{du}{\cancel{\sinh(x)}}$$

$$\int_1^{1{,}54} U^3 \, dU = \frac{U^4}{4} \Big[_1^{1{,}54}$$

DU

$$= \frac{\cosh^4(x)}{4} \Big[_0^1$$

$$\frac{\cosh^4(1) - \cosh^4(0)}{4}$$

$$= \frac{1}{4} (\cosh^4(1) - 1)$$

3.9d) $\int_0^{\frac{1}{2}} \sqrt[4]{\sin(\pi x)}\, cos^3(\pi x)\ dx$

$u = \pi x$
$u' = \pi$
$du = \pi dx$
$\frac{du}{\pi} = dx$

SE $x = 0$
$u = \pi(0) = 0$

SE $x = \frac{1}{2}$
$u = \pi\left(\frac{1}{2}\right) = \frac{\pi}{2}$

$$\int_0^{\frac{\pi}{2}} \sqrt[4]{\sin(u)}\, cos^3(u) \frac{du}{\pi}$$

$$\frac{1}{\pi}\int_0^{\frac{\pi}{2}} \sqrt[4]{\sin(u)}\, cos^2(u) \cos u\ dx = \frac{1}{\pi}\int_0^{\frac{\pi}{2}} \sqrt[4]{\sin(u)}\left(1 - sin^2(u)\right)\cos u\ du$$

$$= \frac{1}{\pi}\int_0^{\frac{\pi}{2}} \sqrt[4]{\sin(u)}\,(\cos u - sin^2(u))\cos u)\ du$$

$$= \frac{1}{\pi}\int_0^{\pi} \sqrt[4]{\sin(u)}\,(\cos u - sin^2(u))\cos u)\ du$$

$$= \frac{1}{\pi}\int_0^{\frac{\pi}{2}} \sqrt[4]{\sin(u)}\cos u - \sqrt[4]{\sin(u)}\, sin^2(u)\cos u\ du$$

$$= \frac{1}{\pi}\int_0^{\frac{\pi}{2}} \sqrt[4]{\sin(u)}\cos u\ du - \frac{1}{\pi}\int_0^{\frac{\pi}{2}} \sqrt[4]{\sin(u)}\, sin^2(u)\cos u\ du$$

$$\frac{1}{\pi}\int_0^{\frac{\pi}{2}} \sqrt[4]{\sin(u)}\cos u\ du$$

$z = \sin u$
$z' = \cos u$
$dz = \cos u\, du$
$\frac{dz}{\cos u} = du$

$z = \sin(u)$
SE $u = 0$
$z = \sin(0) = 0$

SE $u = \frac{\pi}{2}$
$z = \text{SEN}\left(\frac{\pi}{2}\right) = 1$

$$\frac{1}{\pi}\int_0^{1} \sqrt[4]{z}\cos u \frac{dz}{\cos u}$$

$$\frac{1}{\pi}\int_0^{1} \sqrt[4]{z}\, dz$$

$$\frac{1}{\pi}\int_0^{1} z^{\frac{1}{4}}\, dz$$

$$\frac{1}{\pi}\frac{z^{\frac{5}{4}}}{\frac{5}{4}} = \frac{1}{\pi}\frac{4}{5}z^{\frac{5}{4}}$$

$$= \frac{4}{5\pi}z^{\frac{5}{4}}\begin{cases}1\\0\end{cases} = \frac{4}{5\pi}sin^{\frac{5}{4}}(u)\begin{cases}\pi/2\\0\end{cases}$$

$$-\frac{1}{\pi}\int_0^{\frac{\pi}{2}} \sqrt[4]{\sin(u)}\, sin^2(u)\cos u \;\, du$$

$$z = \sin u$$
$$z' = \cos u$$
$$dz = \cos u\, du$$
$$\frac{dz}{\cos u} = du$$

Z = SIN(U)

SE U = 0

Z = SIN(0) = 0

SE U = $\frac{\pi}{2}$

Z = SIN($\frac{\pi}{2}$) = 1

$$-\frac{1}{\pi}\int_0^1 \sqrt[4]{z} z^2 \cos u \frac{dz}{\cos u}$$
$$= -\frac{1}{\pi}\int_0^1 \sqrt[4]{z} z^2 \, dz$$
$$= -\frac{1}{\pi}\int_0^1 z^{\frac{1}{4}} z^2 \, dz$$
$$= -\frac{1}{\pi}\int_0^1 z^{\frac{9}{4}} \, dz$$
$$= -\frac{1}{\pi}\frac{z^{\frac{13}{4}}}{\frac{13}{4}} = -\frac{1}{\pi}\frac{4}{13} z^{\frac{13}{4}}$$
$$= -\frac{4}{13\pi} z^{\frac{13}{4}} \begin{cases} 1 \\ 0 \end{cases}$$
$$= -\frac{4}{13\pi} (\sin u)^{\frac{13}{4}} \begin{cases} \pi/2 \\ 0 \end{cases}$$

$$= \frac{1}{\pi}\int_0^{\frac{\pi}{2}} \sqrt[4]{\sin(u)} \cos u \;\, du - \frac{1}{\pi}\int_0^{\frac{\pi}{2}} \sqrt[4]{\sin(u)}\, sin^2(u)\cos u \;\, du$$

$$= \frac{4}{5\pi}(\sin u)^{\frac{5}{4}} - \frac{4}{13\pi}(\sin u)^{\frac{13}{4}} \begin{cases} \pi/2 \\ 0 \end{cases}$$

$$= \frac{4(\sin u)^{\frac{5}{4}}}{5\pi} - \frac{4(\sin u)^{\frac{13}{4}}}{13\pi}$$

$$= \frac{4.13\pi(\sin u)^{\frac{5}{4}} - 4.5\pi(\sin u)^{\frac{13}{4}}}{65\pi^2}$$

$$= \frac{4\pi\left(13(\sin u)^{\frac{5}{4}} - 5(\sin u)^{\frac{13}{4}}\right)}{65\pi^2}$$

$$= \frac{4\left(13(\sin u)^{\frac{5}{4}} - 5(\sin u)^{\frac{5}{4}}(\sin u)^2\right)}{65\pi}$$

$$= \frac{4\left(sin^{\frac{5}{4}}(u)(13 - 5sin^2(u))\right)}{65\pi}$$

$$= \frac{4\left(sin^{\frac{5}{4}}(u)\big(13-5(1-cos^2(u))\big)\right)}{65\pi}$$

$$= \frac{4\left(sin^{\frac{5}{4}}(u)(13-5-5cos^2(u))\right)}{65\pi}$$

$$= \frac{4\left(sin^{\frac{5}{4}}(u)(8-5cos^2(u))\right)}{65\pi}\Big\{\begin{matrix}\pi/2\\0\end{matrix}$$

Como $u = \pi x$, temos que

$$= \frac{4sin^{\frac{5}{4}}(\pi x)\big(8+5cos(\pi x)\big)}{65\pi}\Big\{\begin{matrix}1/2\\0\end{matrix}$$

$$= \frac{4\sqrt[4]{sin^5\left(\pi\frac{1}{2}\right)}\left(8+5cos\left(\pi\frac{1}{2}\right)\right)}{65\pi} - \frac{4\sqrt[4]{sin^5(\pi.0)}\big(8+5cos(\pi.0)\big)}{65\pi}$$

$$= \frac{4\sqrt[4]{1}(8)}{65\pi} - \frac{4\sqrt[4]{0}(26)}{65\pi} = \frac{32}{65\pi} - 0 = 0{,}1567$$

3.9e) $\int_{\frac{\pi}{6}}^{\frac{\pi}{9}} cotg\ (3x)\ dx$

$u = 3x$
$u' = 3$
$du = 3dx$
$\frac{du}{3} = dx$

$U = 3x$

$Se\ x = \frac{\pi}{6}$ | $Se\ x = \frac{\pi}{9}$

$U = 3\frac{\pi}{6} = \frac{\pi}{2}$ | $U = 3\frac{\pi}{9} = \frac{\pi}{3}$

$$\int_{\frac{\pi}{2}}^{\frac{\pi}{3}} cotg\ (u)\ \frac{du}{3} = \frac{1}{3}\int_{\frac{\pi}{2}}^{\frac{\pi}{3}} cotg\ (u)\ du$$

$$\frac{1}{3} ln|\sin u| \begin{cases} \pi/3 \\ \pi/2 \end{cases}$$

ou

$$= \frac{1}{3} ln|\sin 3x| \Big|_{\pi/6}^{\pi/9}$$

$$= \frac{1}{3} ln|\sin 3x| \Big|_{\pi/6}^{\pi/9}$$

$$= \frac{1}{3} ln \left|\sin\left(3\frac{\pi}{9}\right)\right| - \frac{1}{3} ln \left|\sin 3\frac{\pi}{6}\right| = \frac{1}{3} ln \left|\sin\left(\frac{\pi}{3}\right)\right| - \frac{1}{3} ln \left|\sin\left(\frac{\pi}{2}\right)\right|$$

$$= \frac{1}{3}\left(ln\left|\frac{\sqrt{3}}{2}\right| - ln|1|\right)$$

$$= \frac{1}{3}\left(ln\left|\frac{\sqrt{3}}{2}\right|\right)$$

3.9f) $\int_0^1 x^2 - 2cos\,(x)\ dx$

$$= \int_0^1 x^2\,dx - \int_0^1 2cos\,(x)\ dx$$

$$\int_0^1 x^2\,dx = \frac{x^3}{3}\ \Big|_0^1$$

$$-\int_0^1 2cos\,(x)\ dx$$

$$= -2\int_0^1 cos\,(x)\ dx = -2\sin x\ \Big|_0^1$$

$$\int_0^1 x^2 - 2cos\,(x)\ dx = \frac{x^3}{3} - 2\sin x\ \Big|_0^1$$

$$= \frac{1^3}{3} - 2\sin(1) - \left(\frac{0^3}{3} - 2\sin(0)\right)$$

$$= \frac{1}{3} - 2\sin(1)$$

3.9g) $\int_{-1}^{3} 3x^2 - 2x + 1 \; dx$

$$\int_{-1}^{3} 3x^2 \; dx - \int_{-1}^{3} 2x \; dx + \int_{-1}^{3} 1 \; dx$$

$$3\int_{-1}^{3} x^2 \; dx - 2\int_{-1}^{3} x \; dx + \int_{-1}^{3} 1 \; dx$$

$$= 3\frac{x^3}{3} - 2\frac{x^2}{2} + x \; \Big|_{-1}^{3}$$

$$= 3\frac{3^3}{3} - 2\frac{3^2}{2} + 3 - \left(3\frac{-1^3}{3} - 2\frac{-1^2}{2} - 1\right)$$

$$= 27 - 9 + 3 - (-1) + 1 + 1$$

$$= 27 - 9 + 3 + 1 + 1 + 1 = 24$$

3.9h) $\int_0^{\frac{\pi}{4}} cos\; x \;\; dx$

$=\sin x \Big|\begin{cases} \pi/4 \\ 0 \end{cases}$

$=\sin\left(\frac{\pi}{4}\right) - \sin(0) \; = \frac{\sqrt{2}}{2} - 0 \; = \frac{\sqrt{2}}{2}$

3.9i) $\int_1^{16} \sqrt{x^3} \;\; dx$

$$= \int_1^{16} x^{\frac{3}{2}} \; dx \; = \frac{x^{\frac{5}{2}}}{\frac{5}{2}} = \frac{2}{5}x^{\frac{5}{2}} = \frac{2}{5}\sqrt{x^5} \Big|\begin{cases} 16 \\ 1 \end{cases}$$

$$= \frac{2}{5}\sqrt{16^5} - \frac{2}{5}\sqrt{1^5} = \frac{2}{5}16^2\sqrt{16} - \frac{2}{5}(1)$$

$$= \frac{2}{5}256.4 - \frac{2}{5} = \frac{2048 - 2}{5} = \frac{2046}{5}$$

3.9j) $\int_4^{5} \frac{2}{\sqrt{x}} - x \;\; dx$

$$\int_4^{5} \frac{2}{\sqrt{x}} - x \;\; dx = \int_4^{5} \frac{2}{\sqrt{x}} \; dx - \int_4^{5} x \;\; dx$$

$$2\int_4^5 x^{-\frac{1}{2}}\ dx = 2\frac{x^{\frac{1}{2}}}{\frac{1}{2}} = 2\frac{2}{1}x^{\frac{1}{2}} = 4\sqrt{x} + C$$

$$-\int_4^5 x\ dx = -\frac{x^2}{2} + C$$

Assim,

$$\int_4^5 \frac{2}{\sqrt{x}} - x\ dx = 4\sqrt{x} - \frac{x^2}{2}\begin{cases}5\\4\end{cases}$$

$$4\sqrt{5} - \frac{5^2}{2} - \left(4\sqrt{4} - \frac{4^2}{2}\right) = 4\sqrt{5} - \frac{25}{2} - 4\sqrt{4} + \frac{16}{2}$$

$$=\frac{16-25}{2} + 4\left(\sqrt{5} - \sqrt{4}\right) = -\frac{9}{2} + 4\left(\sqrt{5} - 2\right) = -\frac{9}{2} + 4\sqrt{5} - 8$$

$$\frac{-9-16}{2} + 4\sqrt{5} = -\frac{25}{2} + 4\sqrt{5} = 12{,}5 + 4(2{,}236)$$

$$= -12{,}5 + 8{,}9442 = -3{,}5558$$

3.9k) $\int_0^1 \sqrt{x^2 - 6x + 9}\ dx$

$$= \int_0^1 \sqrt{(x-3)^2}\ dx = \int_0^1 x - 3\ dx = \int_0^1 x\ dx - \int_0^1 3\ dx$$

$$= \frac{x^2}{2} - 3x\ \Big|_0^1$$

$$= \frac{1^2}{2} - 3.1 - \left(\frac{0^2}{2} - 3.0\right) = \frac{1}{2} - 3 = -\frac{5}{2}$$

3.9l) $\int_0^2 1 - \frac{1}{2}x\ dx$

$$= \int_0^2 1\ dx - \int_0^2 \frac{1}{2}x\ dx = \int_0^2 1\ dx - \frac{1}{2}\int_0^2 x\ dx$$

$$= x - \frac{1}{2}\frac{x^2}{2} = x - \frac{x^2}{4}\ \Big|_0^1$$

$$2 - \frac{2^2}{4} - \left(0 - \frac{0^2}{4}\right) = 2 - 1 - 0 = 1$$

3.9m) $\int_{-1}^2 |2x + 3|\ dx$

$$|2x+3| = \begin{cases} (2x+3)\ \ se\ 2x+3 \geq 0 \\ -(2x+3)\ \ se\ 2x+3 < 0 \end{cases}$$

$2x + 3 \geq 0$	$2x + 3 < 0$
$2x \geq -3$	$2x < -3$
$x \geq -\frac{3}{2}$	$x < -\frac{3}{2}$

Como o intervalo definido na integral resultará sempre em valores positivos, temos que $|2x + 3| = 2x + 3$ no intervalo [-1, 2]. Assim,

$$\int_{-1}^2 |2x+3|\ dx = \int_{-1}^2 2x + 3\ dx$$

$$\int_{-1}^2 2x + 3\ dx = \int_{-1}^2 2x\ dx + \int_{-1}^2 3\ dx$$

$$= 2\frac{x^2}{2} + 3x \Big|_{-1}^{2}$$

$$= 2\frac{2^2}{2} + 3.2 - \left(2\frac{-1^2}{2} + 3(-1)\right) = 4 + 6 - (1 - 3) = 10 + 2 = 12$$

Apresentamos, a seguir, outra forma de resolver a integral, utilizando o método de substituição simples.

3.9m) $\int_{-1}^{2} |2x + 3| \; dx$

$u = 2x + 3$ $u' = 2$ $\frac{du}{2} = dx$	Os novos limites inferior e superior são $u = 2(-1) + 3 = 1$ $u = 2(2) + 3 = 7$

Assim,

$$\int_{1}^{7} |u| \; \frac{du}{2}$$

Uma vez que $1 < u < 7$, *u* será sempre positivo e $|u| = u$. Desta forma

$$\frac{1}{2}\int_{1}^{7} u \; du = \frac{1}{2}\frac{u^2}{2} = \frac{u^2}{4} \Big|_{1}^{7}$$

$$\frac{7^2}{4} - \frac{1^2}{4} = \frac{49 - 1}{4} = 12$$

3.9n) $\int_{-3}^{4} |x + 2| \; dx$

$$|x + 2| = \begin{cases} (x + 2) \; \; se \; x + 2 \geq 0 \\ -(x + 2) \; \; se \; x + 2 < 0 \end{cases}$$

$x + 2 \geq 0$ $x \geq -2$	$x + 2 < 0$ $x < -2$

Como no intervalo definido [-3, 4] na integral de $|x + 2|$ a função retornará valores negativos para $x < -2$, temos que dividir a integral em duas partes. Assim,

$$\int_{-3}^{4} |x + 2| \ dx = \int_{-3}^{-2} -(x + 2) \ dx + \int_{-2}^{4} x + 2 \ dx$$

$$= \int_{-3}^{-2} -x - 2 \ dx + \int_{-2}^{4} x + 2 \ dx$$

$$= -\frac{-2^2}{2} - 2(-2) - \left(-\frac{-3^2}{2} - 2(-3)\right) + \frac{4^2}{2} + 2.4 - \left(\frac{-2^2}{2} + 2(-2)\right)$$

$$= -2 + 4 - \left(-\frac{9}{2} + 6\right) + 8 + 8 - (2 - 4)$$

$$= 2 - \left(\frac{3}{2}\right) + 16 + 2 == \frac{4 - 3 + 32 + 4}{2} = \frac{37}{2}$$

39a) $\displaystyle\int_4^9 \frac{\sqrt{x}}{1-x}\,dx$

$u=\sqrt{x}$

$u'=\frac{1}{2\sqrt{x}}$

$du=\frac{1}{2\sqrt{x}}dx$

$2\sqrt{x}\,du=dx$

$$\int \frac{u}{1-u^2}\,2u\,du = 2\int \frac{u^2}{1-u^2}\,du = 2\int -\frac{u^2}{u^2-1}\,du$$

$$= -2\int \frac{u^2}{u^2-1}\,du = -2\int \frac{u^2 \underbrace{-1+1}}{u^2-1}\,du$$

$$= -2\int \frac{u^2-1}{u^2-1} + \frac{1}{u^2-1}\,du = -2\int 1 + \frac{1}{u^2-1}\,du$$

$$= -2\left(\int 1\,du + \int \frac{1}{u^2-1}\,du\right)$$

$$\frac{1}{u^2-1} = \frac{1}{(u-1)(u+1)} = \frac{A}{u-1} + \frac{b}{u+1}$$

$$1 = A(u+1) + b(u-1)$$

$x=1$

$1 = A(1+1) + b(1-1)$

$1 = 2A$

$A=\frac{1}{2}$

$x=-1$

$1 = A(-1+1) + b(-1-1)$

$1 = -2b$

$b = -\frac{1}{2}$

$$\frac{\frac{1}{2}}{u-1} + \frac{\frac{-1}{2}}{u+1} = \frac{1}{2(u-1)} - \frac{1}{2(u+1)}$$

$$-2\left(\int 1\,du + \int \frac{1}{u^2-1}\,du\right) = -2\left(\int 1\,du + \int \frac{1}{2(u-1)}\,du - \int \frac{1}{2(u+1)}\,du\right)$$

$$= -2\left(\int 1\,du + \frac{1}{2}\int \frac{1}{u-1}\,du - \frac{1}{2}\int \frac{1}{u+1}\,du\right)$$

$$= -2\left(u + \frac{1}{2}\ln|u-1| - \frac{1}{2}\ln|u+1|\right)$$

$$= -2u - \ln|u-1| + \ln|u+1|$$

$$= \ln\left|\frac{u+1}{u-1}\right| - 2u$$

$$\boxed{= \ln\left|\frac{\sqrt{x}+1}{\sqrt{x}-1}\right| - 2\sqrt{x} + C}$$

$$\int_4^9 \frac{\sqrt{x}}{1-x}\,dx = \ln\left|\frac{\sqrt{x}+1}{\sqrt{x}-1}\right| - 2\sqrt{x}\,\Bigg[_4^9$$

$$= \ln\left|\frac{\sqrt{9}+1}{\sqrt{9}-1}\right| - 2\sqrt{9} - \left(\ln\left|\frac{\sqrt{4}+1}{\sqrt{4}-1}\right| - 2\sqrt{4}\right)$$

$$= \ln\left|\frac{3+1}{3-1}\right| - 2.(3) - \ln\left|\frac{2+1}{2-1}\right| + 2.(2)$$

$$= \ln 2 - 6 - \ln 3 + 4$$

$$= \ln\left|\frac{2}{3}\right| - 2 = \boxed{-2{,}405}$$

3.9p) $\int_{-3}^{-1} \frac{x^4}{\sqrt{3-x}}\, dx$

$u = 3 - x$ $u' = -1$ $-du = dx$ $x = 3 - u$	Os novos limites superior e inferior são $u = 3 - (-1) = 4$ $u = 3 - (-3) = 6$	$\int_6^4 \frac{(3-u)^4}{\sqrt{u}} - du$ $= -\int_6^4 \frac{(3-u)^4}{\sqrt{u}} du$

$$(3-u)^4 = (3-u)^2(3-u)^2 = (9 - 6u + u^2)(9 - 6u + u^2)$$
$$= 81 - 54u + 9u^2 - 54u + 36u^2 - 6u^3 + 9u^2 - 6u^3 + u^4$$

$$= -\int_6^4 \frac{(3-u)^4}{\sqrt{u}} du$$

$$= -\int_6^4 \frac{81 - 54u + 9u^2 - 54u + 36u^2 - 6u^3 + 9u^2 - 6u^3 + u^4}{\sqrt{u}} du$$

$$= -\int_6^4 \frac{81 - 108u + 54u^2 - 12u^3 + u^4}{\sqrt{u}} du$$

$$= -\int_6^4 (81 - 108u + 54u^2 - 12u^3 + u^4)u^{-\frac{1}{2}} du$$

$$= -\int_6^4 81u^{-\frac{1}{2}} - 108uu^{-\frac{1}{2}} + 54u^2u^{-\frac{1}{2}} - 12u^3u^{-\frac{1}{2}} + u^4u^{-\frac{1}{2}} du$$

$$= -\int_6^4 81u^{-\frac{1}{2}} - 108u^{\frac{1}{2}} + 54u^{\frac{3}{2}} - 12u^{\frac{5}{2}} + u^{\frac{7}{2}} du$$

$$= -\left(\int_6^4 81u^{-\frac{1}{2}} du - \int_6^4 108u^{\frac{1}{2}} du + \int_6^4 54u^{\frac{3}{2}} du - \int_6^4 12u^{\frac{5}{2}} du + \int_6^4 u^{\frac{7}{2}} du\right)$$

$$= -81\int_6^4 u^{-\frac{1}{2}} du + 108\int_6^4 u^{\frac{1}{2}} du - 54\int_6^4 u^{\frac{3}{2}} du + 12\int_6^4 u^{\frac{5}{2}} du - \int_6^4 u^{\frac{7}{2}} du$$

$$= -81\frac{u^{\frac{1}{2}}}{\frac{1}{2}} + 108\frac{u^{\frac{3}{2}}}{\frac{3}{2}} - 54\frac{u^{\frac{5}{2}}}{\frac{5}{2}} + 12\frac{u^{\frac{7}{2}}}{\frac{7}{2}} - \frac{u^{\frac{9}{2}}}{\frac{9}{2}}$$

$$= -81\frac{2}{1}u^{\frac{1}{2}} + 108\frac{2}{3}u^{\frac{3}{2}} - 54\frac{2}{5}u^{\frac{5}{2}} + 12\frac{2}{7}u^{\frac{7}{2}} - \frac{2}{9}u^{\frac{9}{2}}$$

$$= -162u^{\frac{1}{2}} + \frac{216}{3}u^{\frac{3}{2}} - \frac{108}{5}u^{\frac{5}{2}} + \frac{24}{7}u^{\frac{7}{2}} - \frac{2}{9}u^{\frac{9}{2}}$$

$$= -162\sqrt{u} + \tfrac{216}{3}\sqrt{u^3} - \tfrac{108}{5}\sqrt{u^5} + \tfrac{24}{7}\sqrt{u^7} - \tfrac{2}{9}\sqrt{u^9}\ \Big|_6^4$$

$$= -162\sqrt{4} + \frac{216}{3}\sqrt{4^3} - \frac{108}{5}\sqrt{4^5} + \frac{24}{7}\sqrt{4^7} - \frac{2}{9}\sqrt{4^9}$$
$$- \left(-162\sqrt{6} + \frac{216}{3}\sqrt{6^3} - \frac{108}{5}\sqrt{6^5} + \frac{24}{7}\sqrt{6^7} - \frac{2}{9}\sqrt{6^9}\right)$$

$$= -162\sqrt{4} + \frac{216}{3}4\sqrt{4} - \frac{108}{5}4^2\sqrt{4} + \frac{24}{7}4^3\sqrt{4} - \frac{2}{9}4^4\sqrt{4}$$
$$- \left(-162\sqrt{6} + \frac{216}{3}6\sqrt{6} - \frac{108}{5}6^2\sqrt{6} + \frac{24}{7}6^3\sqrt{6} - \frac{2}{9}6^4\sqrt{6}\right)$$

$$= -162\sqrt{4} + \frac{216}{3}4\sqrt{4} - \frac{108}{5}16\sqrt{4} + \frac{24}{7}64\sqrt{4} - \frac{2}{9}256\sqrt{4}$$
$$- \left(-162\sqrt{6} + \frac{216}{3}6\sqrt{6} - \frac{108}{5}36\sqrt{6} + \frac{24}{7}216\sqrt{6} - \frac{2}{9}1296\sqrt{6}\right)$$

$$= -162.2 + \frac{216}{3}4.2 - \frac{108}{5}16.2 + \frac{24}{7}64.2 - \frac{2}{9}256.2$$
$$- \left(-162\sqrt{6} + \frac{216}{3}6\sqrt{6} - \frac{108}{5}36\sqrt{6} + \frac{24}{7}216\sqrt{6} - \frac{2}{9}1296\sqrt{6}\right)$$

$$= -324 + \frac{216}{3}8 - \frac{108}{5}32 + \frac{24}{7}128 - \frac{2}{9}512$$
$$- \sqrt{6}\left(-162 + \frac{216}{3}6 - \frac{108}{5}36 + \frac{24}{7}216 - \frac{2}{9}1296\right)$$

$$= -324 + 8\left(+\frac{216}{3} - \frac{108}{5}4 + \frac{24}{7}16 - \frac{2}{9}64\right)$$
$$- \sqrt{6}\left[-162 + 6\left(\frac{216}{3} - \frac{108}{5}6 + \frac{24}{7}36 - \frac{2}{9}216\right)\right]$$

$$= -324 + 8\left(+\frac{216}{3} - \frac{432}{5} + \frac{384}{7} - \frac{128}{9}\right) - \sqrt{6}\left[-162 + 6\left(\frac{216}{3} - \frac{648}{5} + \frac{864}{7} - \frac{432}{9}\right)\right]$$

$$= -324 + 8(+72 - 86{,}4 + 54{,}85 - 14{,}22)$$

$$-\sqrt{6}[-162 + 6(72 - 129{,}6 + 123{,}42 - 48)]$$

$$= -324 + 8(26{,}23) - \sqrt{6}[-162 + 6(17{,}82)]$$

$$= -324 + 209{,}84 - \sqrt{6}(-55{,}08)$$

$$= -324 + 209{,}84 + 134{,}91 = 20{,}75$$