

Matriz Inversa - Exercício 7.1a

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$$A = \begin{bmatrix} 3 & -7 & 2 \\ -3 & 5 & 1 \\ 6 & -4 & 0 \end{bmatrix}$$

$$A \cdot A^{-1} = I$$

$$\begin{bmatrix} 3 & -7 & 2 \\ -3 & 5 & 1 \\ 6 & -4 & 0 \end{bmatrix} \cdot \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 3a - 7d + 2g & 3b + (-7e) + 2h & 3c + (-7f) + 2i \\ -3a + 5d + 1g & -3b + 5e + 1h & -3c + 5f + 1i \\ 6a + (-4d) + 0(g) & 6b + (-4e) + 0(h) & 6c + (-4f) + 0(i) \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{array}{l|l|l} 3a - 7d + 2g = 1 & 3b - 7e + 2h = 0 & 3c - 7f + 2i = 0 \\ -3a + 5d + g = 0 & -3b + 5e + h = 1 & -3c + 5f + i = 0 \\ 6a - 4d + 0 = 0 & 6b - 4e + 0 = 0 & 6c - 4f + 0 = 1 \end{array}$$

$$\begin{array}{l|l|l|l} 6a = 4d & g = 3a - 5d & a = \frac{4d}{6} & i = 3c - 5f \\ 6b = 4e & e = \frac{6b}{4} & f = \frac{6c - 1}{4} & \end{array}$$

$$\begin{array}{l|l|l} 3a - 7d + 2g = 1 & 6a = 4d & 3a - 7d + 2g = 1 \\ 3a - 7d + 2(3a - 5d) = 1 & 6a = 4(-\frac{1}{11}) & 3(-\frac{2}{33}) - 7(-\frac{1}{11}) + 2g = 1 \\ 3(\frac{4d}{6}) - 7d + 2[3(\frac{4d}{6}) - 5d] = 1 & 6a = -\frac{4}{11} & -\frac{6}{33} + \frac{7}{11} + 2g = 1 \\ \underline{2d - 7d + 4d - 10d = 1} & a = -\frac{4}{6 \cdot 11} = -\frac{4}{66} & \underline{-\frac{6 + 21}{33} + 2g = 1} \\ \underline{-5d - 6d = 1} & \boxed{a = -\frac{2}{33}} & \underline{\frac{15}{33} + 2g = 1} \\ \underline{-11d = 1} & & \underline{2g = 1 - \frac{5}{11}} \end{array}$$

$$-11d = 1$$

$$d = -\frac{1}{11}$$

$$\begin{cases} 2g = 1 - \frac{5}{11} \\ 2g = \frac{6}{11} \\ g = \frac{6}{11} \cdot \frac{1}{2} = \frac{3}{11} \end{cases}$$

$$g = \frac{3}{11}$$

$$3b - 7e + 2h = 0$$

$$3b - 7\left(\frac{6b}{4}\right) + 2h = 0$$

$$\begin{cases} -3b + 5e + h = 1 \\ h = 1 + 3b - 5e \\ h = 1 + 3b - 5\left(\frac{6b}{4}\right) \end{cases}$$

$$3b - \frac{42b}{4} + 2\left(1 + 3b - \frac{30b}{4}\right) = 0$$

$$3b - \frac{42b}{4} + 2 + 6b - \frac{30b}{2} = 0$$

$$\frac{12b - 42b + 8 + 24b - 60b}{4} = 0$$

$$-66b + 8 = 0$$

$$-66b + 8 = 0$$

$$8 = 66b$$

$$b = \frac{8}{66}$$

$$b = \frac{4}{33}$$

$$h = 1 + 3b - \frac{30b}{4}$$

$$= 1 + 3\left(\frac{4}{33}\right) - \frac{30}{4}\left(\frac{4}{33}\right)$$

$$= 1 + \frac{4}{11} - \frac{30}{33}$$

$$= 1 + \frac{4}{11} - \frac{10}{11}$$

$$= \frac{11 + 4 - 10}{11}$$

$$= \frac{5}{11}$$

$$h = \frac{5}{11}$$

$$-3b + 5e + h = 1$$

$$-3\left(\frac{4}{33}\right) + 5e + \frac{5}{11} = 1$$

$$-\frac{4}{11} + 5e + \frac{5}{11} = 1$$

$$5e = 1 + \frac{4}{11} - \frac{5}{11}$$

$$5e = \frac{11 + 4 - 5}{11}$$

$$e = \frac{10}{11} \cdot \frac{1}{5}$$

$$e = \frac{2}{11}$$

$$2c - 7f + 2i = 0$$

$$3C - 7F + 2i = 0$$

$$\begin{cases} -3C + 5F + i = 0 \\ i = 3C - 5F \end{cases}$$

$$3C - 7F + 2(3C - 5F) = 0$$

$$F = \frac{6C - 1}{4}$$

$$3C - 7\left(\frac{6C - 1}{4}\right) + 2\left[3C - 5\left(\frac{6C - 1}{4}\right)\right] = 0$$

$$3C - \frac{7(6C - 1)}{4} + 6C - \frac{10(6C - 1)}{4} = 0$$

$$\frac{12C - 7(6C - 1) + 24C - 10(6C - 1)}{4} = 0$$

$$12C - 7(6C - 1) + 24C - 10(6C - 1) = 4(0)$$

$$\underbrace{12C - 42C + 7}_{-30C} + \underbrace{24C - 60C + 10}_{-36C} = 0$$

$$-66C + 17 = 0$$

$$17 = 66C$$

$$C = \frac{17}{66}$$

$$F = \frac{6\left(\frac{17}{66}\right) - 1}{4}$$

$$= \frac{\frac{17}{11} - 1}{4}$$

$$= \frac{\frac{6}{11}}{4} = \frac{6}{11} \times \frac{1}{4} = \frac{3}{22}$$

$$F = \frac{3}{22}$$

$$i = 3C - 5F$$

$$= 3\left(\frac{17}{66}\right) - 5\left(\frac{3}{22}\right)$$

$$= \frac{17}{22} - \frac{15}{22}$$

$$= \frac{17 - 15}{22} = \frac{2}{22}$$

$$i = \frac{1}{11}$$

$$A = \begin{bmatrix} 3 & -7 & 2 \\ -3 & 5 & 1 \\ 6 & -4 & 0 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} -\frac{2}{33} & \frac{4}{33} & \frac{17}{66} \\ -\frac{1}{11} & \frac{2}{11} & \frac{3}{22} \\ \frac{3}{11} & \frac{5}{11} & \frac{1}{11} \end{bmatrix}$$

$$A \cdot A^{-1} = I$$

$$\begin{bmatrix} 3 & -7 & 2 \\ 3 & 5 & 1 \\ 6 & -4 & 0 \end{bmatrix} \cdot \begin{bmatrix} -\frac{2}{33} & \frac{4}{33} & \frac{17}{66} \\ -\frac{1}{11} & \frac{2}{11} & \frac{3}{22} \\ \frac{3}{11} & \frac{5}{11} & \frac{1}{11} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$