Exercícios 6.2a - 6.2e

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EXERCÍCIO 6.2: ENCONTRE OS VALORES DE X, Y E Z UTILIZANDO O MÉTODO DE ESCALONAMENTO DE MATRIZES.

a)

$$x + 2y - z = 5$$
$$-x + y + z = -2$$
$$x + 2y - 3z = 7$$

$$\left|\begin{array}{ccc|c} 1 & 2 & -1 & 5 \\ -1 & 1 & 1 & -2 \\ 1 & 2 & -3 & 7 \end{array}\right| \begin{array}{l} \\ L2 = L2 + L1 \\ L3 = L3 - L1 \end{array}$$

$$\left|\begin{array}{ccc|c} 1 & 2 & -1 & 5 \\ 0 & 3 & 0 & 3 \\ 0 & 0 & -2 & 2 \end{array}\right|$$

$$-2z = 2$$
$$z = \frac{2}{-2}$$
$$\boxed{z = -1}$$

$$3y + 0z = 3$$
$$3y = 3$$
$$y = \frac{3}{3}$$
$$\boxed{y = 1}$$

$$x + 2y - z = 5$$
$$x + 2(1) - (-1) = 5$$
$$x + 2 + 1 = 5$$
$$x + 3 = 5$$
$$x = 5 - 3$$
$$\boxed{x = 2}$$

b)

$$\begin{cases} x + y + z = 1 \\ 2x + y - 3z = 4 \\ 3x + 2y - 2z = 5 \end{cases}$$

$$\left|\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 2 & 1 & -3 & 4 \\ 3 & 2 & -2 & 5 \end{array}\right| \begin{array}{l} \\ L2 = L2 - 2L1 \\ L3 = L3 - 3L1 \end{array}$$

$$\left|\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 0 & -1 & -5 & 2 \\ 0 & -1 & -5 & 2 \end{array}\right| \begin{array}{l} \\ \\ L3 = L3 - L2 \end{array}$$

$$\left|\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 0 & -1 & -5 & 2 \\ 0 & 0 & 0 & 0 \end{array}\right|$$

$$0z = 0$$

$$z = \frac{0}{0} = \text{INDETERMINAÇÃO}$$

$$z = \frac{0}{0} = \text{INDETERMINAÇÃO}$$

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c) $$\begin{cases} 2x + y + z = 3 \\ -x + 2y - z = 0 \\ x - 3y + z = -1 \end{cases}$$

$$\left|\begin{array}{ccc|c} 2 & 1 & 1 & 3 \\ -1 & 2 & -1 & 0 \\ 1 & -3 & 1 & -1 \end{array}\right| \quad \begin{array}{l} L_2 = L_2 + \frac{1}{2} L_1 \\ L_3 = L_3 - \frac{1}{2} L_1 \end{array}$$

$$\left|\begin{array}{ccc|c} 2 & 1 & 1 & 3 \\ 0 & 5/2 & -1/2 & 3/2 \\ 0 & -7/2 & 1/2 & -5/2 \end{array}\right| \quad L_3 = L_3 + \frac{7}{5} L_2$$

$$-\frac{7}{2} + \frac{7}{\cancel{5}}\left(\frac{\cancel{5}}{2}\right) = 0$$

$$\frac{1}{2} + \frac{7}{5}\left(-\frac{1}{2}\right) = \frac{1}{2_{/5}} - \frac{7}{10_{/1}} = \frac{5-7}{10} = -\frac{2}{10} = -\frac{1}{5}$$

$$-\frac{5}{2} + \frac{7}{5}\left(\frac{3}{2}\right) = -\frac{5}{2_{/5}} + \frac{21}{10_{/1}} = \frac{-25+21}{10} = -\frac{4}{10} = -\frac{2}{5}$$

$$\left|\begin{array}{ccc|c} 2 & 1 & 1 & 3 \\ 0 & 5/2 & -1/2 & 3/2 \\ 0 & 0 & -1/5 & -2/5 \end{array}\right|$$

$$-\frac{1}{\cancel{5}} z = -\frac{2}{\cancel{5}}$$

$$-z = -2 \quad ^{(-1)}$$

$$\boxed{z = 2}$$

$$\frac{5}{\cancel{2}} y - \frac{1}{\cancel{2}} z = \frac{3}{\cancel{2}}$$

$$5y - z = 3$$

$$5y - 2 = 3$$

$$5y = 3 + 2$$

$$y = \frac{5}{5}$$

$$\boxed{y = 1}$$

$$2x + y + z = 3$$

$$2x + 1 + 2 = 3$$

$$2x + 3 = 3$$

$$2x = 3 - 3$$

$$x = \frac{0}{3}$$

$$\boxed{x = 0}$$

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$$x = 0$$
$$y = 1$$
$$z = 2$$

D) $$\begin{cases} 2x + y - z = 5 \\ -x + 2y + z = 2 \\ x + y + 2z = 1 \end{cases}$$

$$\frac{1}{2} - k\left(\frac{5}{2}\right) = 0 \qquad \frac{1}{2} \cdot \frac{2}{5} = k$$

$$\frac{1}{2} = \frac{5}{2} k \qquad k = \frac{1}{5}$$

$$\left|\begin{array}{ccc|c} 2 & 1 & -1 & 5 \\ -1 & 2 & 1 & 2 \\ 1 & 1 & 2 & 1 \end{array}\right| \quad \begin{array}{l} L_2 = L_2 + \frac{1}{2} L_1 \\ L_3 = L_3 - \frac{1}{2} L_1 \end{array}$$

$$\begin{cases} -1 + \frac{1}{2}(2) = -1 + 1 = 0 \\ 2 + \frac{1}{2}(1) = 2 + \frac{1}{2} = \frac{5}{2} \\ 1 + \frac{1}{2}(-1) = 1 - \frac{1}{2} = \frac{1}{2} \\ 2 + \frac{1}{2}(5) = 2 + \frac{5}{2} = \frac{9}{2} \end{cases}$$

$$\begin{cases} 1 - \frac{1}{2}(2) = 1 - 1 = 0 \\ 1 - \frac{1}{2}(1) = 1 - \frac{1}{2} = \frac{1}{2} \\ 2 - \frac{1}{2}(-1) = 2 + \frac{1}{2} = \frac{5}{2} \\ 1 - \frac{1}{2}(5) = 1 - \frac{5}{2} = -\frac{3}{2} \end{cases}$$

$$\left|\begin{array}{ccc|c} 2 & 1 & -1 & 5 \\ 0 & 5/2 & 1/2 & 9/2 \end{array}\right|$$

$$\frac{1}{2} - \frac{1}{\cancel{5}}\left(\frac{\cancel{5}}{2}\right) = \frac{1}{2} - \frac{1}{2} = 0$$

$$\left[\begin{array}{ccc|c} 2 & 1 & -1 & 5 \\ 0 & 5/2 & 1/2 & 9/2 \\ 0 & 1/2 & 5/2 & -3/2 \end{array}\right] \quad L_3 = L_3 - \frac{1}{5}L_2$$

$$\begin{cases} \frac{1}{2} - \frac{1}{\cancel{5}}\left(\frac{\cancel{5}}{2}\right) = \frac{1}{2} - \frac{1}{2} = 0 \\ \frac{5}{2} - \frac{1}{5}\left(\frac{1}{2}\right) = \frac{5}{2_{/5}} - \frac{1}{10_{/1}} = \frac{25-1}{10} = \frac{24}{10} = \frac{12}{5} \\ -\frac{3}{2} - \frac{1}{5}\left(\frac{9}{2}\right) = -\frac{3}{2_{/5}} - \frac{9}{10_{/1}} = \frac{-15-9}{10} = \frac{-24}{10} = \frac{-12}{5} \end{cases}$$

$$\left[\begin{array}{ccc|c} 2 & 1 & -1 & 5 \\ 0 & 5/2 & 1/2 & 9/2 \\ 0 & 0 & 12/5 & 12/5 \end{array}\right]$$

$\frac{12}{5}z = -\frac{12}{5}$

$z = -\frac{12}{5} \cdot \frac{5}{12}$

$\boxed{z = -1}$

$\frac{5}{2}y + \frac{1}{2}z = \frac{9}{2}$

$\frac{5}{2}y + \frac{1}{2}(-1) = \frac{9}{2}$

$\frac{5}{2}y - \frac{1}{2} = \frac{9}{2}$

$\frac{5}{2}y = \frac{9}{2} + \frac{1}{2}$

$\frac{5}{2}y = \frac{10}{2}$

$y = \cancel{5} \cdot \frac{2}{\cancel{5}}$

$\boxed{y = 2}$

$2x + y - z = 5$

$2x + 2 - (-1) = 5$

$2x + 2 + 1 = 5$

$2x + 3 = 5$

$2x = 5 - 3$

$2x = 2$

$x = \frac{2}{2}$

$\boxed{x = 1}$

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$x = 1$

$y = 2$

$z = -1$

E)

$$\begin{aligned} x + 2y + 3z &= 3 \\ 2x + 4y + 6z &= 6 \\ x - y + z &= 4 \end{aligned}$$

$$\left[\begin{array}{ccc|c} 1 & 2 & 3 & 3 \\ 2 & 4 & 6 & 6 \\ 1 & -1 & 1 & 4 \end{array}\right] \quad \begin{array}{l} L_2 = L_2 - 2L_1 \\ L_3 = L_3 - L_1 \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & 2 & 3 & 3 \\ 0 & 0 & 0 & 0 \\ 0 & -3 & -2 & 1 \end{array}\right]$$

$0z = 0$

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