Exercícios de Recuperação - MAB

18/07/22

1) a) $y = 5x + 2$

$\hookrightarrow 5x + 2 = 0$

$5x = -2$

$x = -2/5$

b) $f(x) = \frac{x}{2} + 4$

$\hookrightarrow \frac{x}{2} + 4 = 0$

$\frac{x}{2} = -4 \rightarrow x = -8$

c) $f(x) = 3x^2 - 7x + 2$

$\hookrightarrow 3x^2 - 7x + 2 = 0$

$\hookrightarrow \Delta = (-7)^2 - 4(3)(2)$

$\Delta = 49 - 24 = 25$

$x = \frac{-(-7) \pm \sqrt{25}}{2 \cdot 3}$

$\rightarrow x_1 = \frac{-7+5}{6} = -1/3$

$\rightarrow x_2 = \frac{-7-5}{6} = -2$

d) $f(x) = x^2 - 5x + 6$

$\hookrightarrow x^2 - 5x + 6 = 0$

$\hookrightarrow \Delta = (-5)^2 - 4(6)(1)$

$\Delta = 25 - 24 = 1$

$x = \frac{-(-5) \pm \sqrt{1}}{2 \cdot 1}$

$\rightarrow x_1 = \frac{5+1}{2} = 3$

$\rightarrow x_2 = \frac{5-1}{2} = 2$

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2) $f(x) = x^2 - 1 \quad g(x) = \frac{3x}{2} + 2$

a) $f(g(x))$

$$f(g(x)) = \left(\frac{3x}{2} + 2\right)^2 - 1$$

$$= \frac{9x^2}{4} + 6x + 4 - 1 = \frac{9x^2}{4} + 6x + 3$$

b) $g(f(x))$

$$g(f(x)) = \frac{3}{2}(x^2 - 1) + 2$$

$$= \frac{3x^2 - 3}{2} + \frac{2}{1/2} = \frac{3x^2 - 3 + 4}{2} = \frac{3x^2 + 1}{2}$$

3) a) $\frac{3x+2}{2x} \Rightarrow x = \frac{3y+2}{2y}$

$\triangleright\ 2xy = 3y + 2$

$2xy - 3y = 2$

$y(2x - 3) = 2$

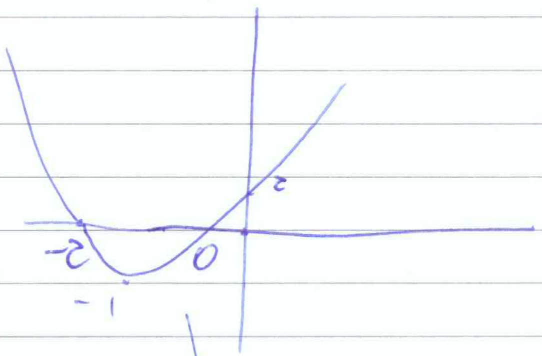
$y = \frac{2}{2x - 3}$

b) $x = 0 \ / \ y = 2$

$\hookrightarrow x^2 + 2x = 0$

$x(x+2) = 0$

$x = 0 \ / \ x = -2$



Função tem dois valores no domínio c/ uma imagem — não é bijetora $\triangleright$ Não admite inversa!

4) a) $2^x = \frac{1}{16}$

$2^x = \frac{1}{2^4}$

$2^x = 2^{-4} \Rightarrow x = -4$

b) $2^{(x^2-x-16)} = 16$

$2^{(x^2-x-16)} = 2^4$

$\Rightarrow x^2 - x - 16 = 4$

$x^2 - x - 20 = 0$

$\Rightarrow \Delta = (-1)^2 - 4(1)(-20)$

$\Delta = 81$

$x = \frac{-(-1) \pm \sqrt{81}}{2 \cdot 1}$

$\Rightarrow x_1 = \frac{1+9}{2} = 5$

$\Rightarrow x_2 = \frac{1-9}{2} = -4$

5) a) $27^{(x+2)} > 9^{(x+5)}$

$3^{3(x+2)} > 3^{2(x+5)}$

$\Rightarrow 3^{3x+6} > 3^{2x+10}$

$\Rightarrow 3x + 6 > 2x + 10$

$3x - 2x > 10 - 6$

$x > 4$

b) $0{,}5^{(4x+3)} \geq 0{,}25^{(x+5)}$

$\Rightarrow \frac{1}{2}^{(4x+3)} \geq \frac{1}{2}^{2(x+5)}$

$\frac{1}{2}^{(4x+3)} \geq \frac{1}{2}^{(2x+10)}$

$\Rightarrow 4x + 3 \leq 2x + 10$

$\Rightarrow 4x - 2x \leq 10 - 3$

$2x \leq 7$

$x \leq 7/2$

spiral

5) c) $(0{,}008)^x > \sqrt[3]{25}$

$\left(\frac{8}{1000}\right)^x > \sqrt[3]{5^2}$

$\left(\frac{1}{125}\right)^x > 5^{2/3}$

$\left(\frac{1}{5^3}\right)^x > 5^{2/3}$

$\hookrightarrow 5^{-3x} > 5^{2/3}$

$\rightarrow -3x > 2/3$

$3x < -2/3$

$x < -2/9$

d) $2^{(x-1)} + 2^x + 2^{(x+1)} - 2^{(x+2)} + 2^{(x+3)} > 240$

$\hookrightarrow 2^x(2^{-1} + 1 + 2^1 - 2^2 + 2^3) > 240$

$2^x\left(\frac{1}{2} + 1 + 2 - 4 + 8\right) > 3 \cdot 2^4 \cdot 5$

$2^x\left(\frac{1}{2} + 7\right) > 2^4 \cdot 3 \cdot 5$

$2^x\left(\frac{\cancel{15}}{2}\right) > 2^4 \cdot \cancel{3 \cdot 5}$

$2^{x-1} > 2^4$

$\hookrightarrow x - 1 > 4$

$x > 5$

6) a) $\log_4(3x+2) = \log_4(2x+5)$

$\log_4(3x+2) = \log_4(2x+5)$

$\hookrightarrow 3x + 2 = 2x + 5$

$3x - 2x = 5 - 2$

$x = 3$

Condição de Existência:

$3x + 2 > 0$

$x > -2/3$

$2x + 5 > 0$

$x > -5/2$

6) b) $\log_{1/3}(3x^2-4x-17) = \log_{1/3}(2x^2-5x+3)$ 18/07/22

Condição de Existência:

$\cancel{\Delta = (-4)^2 - 4(3)(-17)}$, $3x^2-4x-17 > 0$

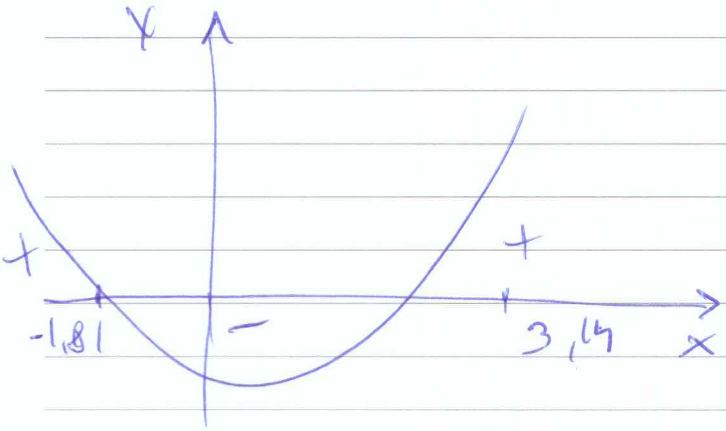
$\Delta = (-4)^2 - 4(3)(-17)$

$\Delta = 16 + 204 = 220$

$x = \dfrac{-(-4) \pm \sqrt{220}}{2(3)}$

$x_1 = \dfrac{4+14,83}{6} \cong 3,14$

$x_2 = \dfrac{4-14,83}{6} \cong -1,81$



Obs.: Acho pouco provável que ele deva cobrar valores quebrados em uma prova que não permite calculadora =D

$2x^2-5x+3 > 0$

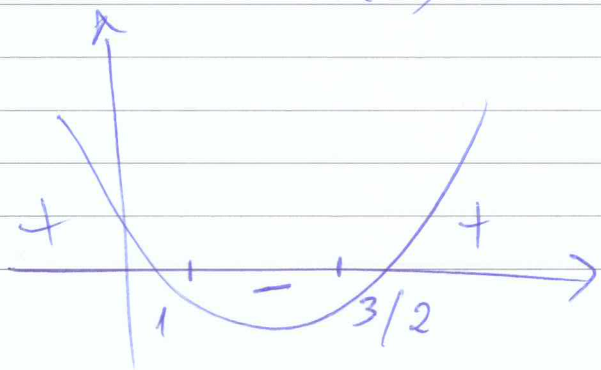
$\Delta = (-5)^2 - 4(2)(3)$

$\Delta = 25-24 = 1$

$x = \dfrac{-(-5) \pm \sqrt{1}}{2(2)}$

$x_1 = \dfrac{5+1}{4} = \dfrac{6}{4} = 3/2$

$x_2 = \dfrac{5-1}{4} = 1$



6) b) Resolvendo o logaritmo: 18/07/22

$3x^2 - 4x - 17 = 2x^2 - 5x + 3$

$3x^2 - 2x^2 - 4x + 5x - 17 - 3 = 0$

$x^2 + x - 20 = 0$

$\Delta = (1)^2 - 4(1)(-20)$

$\Delta = 1 + 80 = 81$

$x = \dfrac{-1 \pm \sqrt{81}}{2 \cdot 1}$

$x_1 = \dfrac{-1+9}{2} = 4$ ✓

$x_2 = \dfrac{-1-9}{2} = -5$ ✓

As soluções atendem às condições de existência!

c) $\log_{\frac{1}{3}}(2x^2 - 9x + 4) = 2$

Condição de existência:

$2x^2 - 9x + 4 > 0$

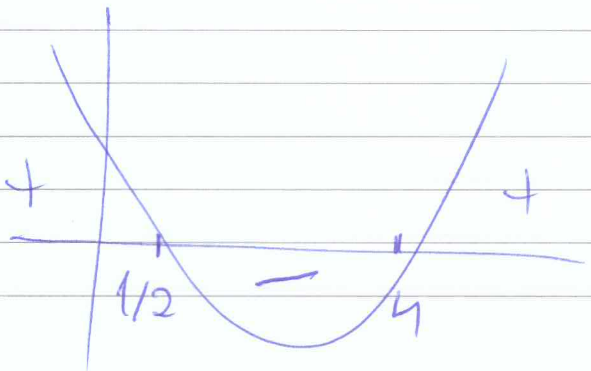
$\Delta = (-9)^2 - 4(2)(4)$

$\Delta = 81 - 32 = 49$

$x = \dfrac{-(-9) \pm \sqrt{49}}{2(2)}$

$x_1 = \dfrac{9+7}{4} = 4$

$x_2 = \dfrac{9-7}{4} = 1/2$



6) c) Resolvendo o logaritmo: 18/04/22

$\log_{\frac{1}{3}}(2x^2-9x+4)=2$

$2x^2-9x+4=\left(\frac{1}{3}\right)^2$

$2x^2-9x+4=1/9$

$2x^2-9x+\underset{1/9}{4}-\frac{1}{9}=0$

$2x^2-9x+\frac{35}{9}=0$

$\Delta=(-9)^2-4(2)\left(\frac{35}{9}\right)$

$\Delta=\underset{1/9}{81}-\frac{280}{9}=\frac{729-280}{9}=\frac{449}{9}$

$x=\frac{-(-9)\pm\sqrt{\frac{449}{9}}}{2(2)}\cong\frac{9\pm\frac{21,18}{3}}{4}\cong\frac{9\pm 7,06}{4}$

$x_1\cong\frac{9+7,06}{4}\cong 4,02\cong 4 \to 4,015$

$x_2=\frac{9-7,06}{4}\cong \cancel{0,49 \cong} 0,5 \rightsquigarrow 0,485$

$x_1=4,015$
$x_2=0,485$ $\gg$ Atendem às condições de existência do log

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d) $x^{\log_x(x+3)} = 7$

$\hookrightarrow x^{\log_x(x+3)} = 7$

$x^{\frac{\log(x+3)}{\log x}} = 7$

$\log\left(x^{\left(\frac{\log(x+3)}{\log x}\right)}\right) = \log 7$

$\frac{\log(x+3)}{\cancel{\log x}} \cdot \cancel{\log x} = \log 7$

$\log(x+3) = \log 7$

$\hookrightarrow x+3 = 7$

$x = 4$ //

Condição de Existência:

$x+3>0$

$\rightarrow x > -3$

$\rightarrow x > 0$

$\rightarrow x \neq 1$

7) a) $\log(x^2-x-2) < \log(2x+4)$

Condições de Existência:

$x^2 - x - 2 > 0$

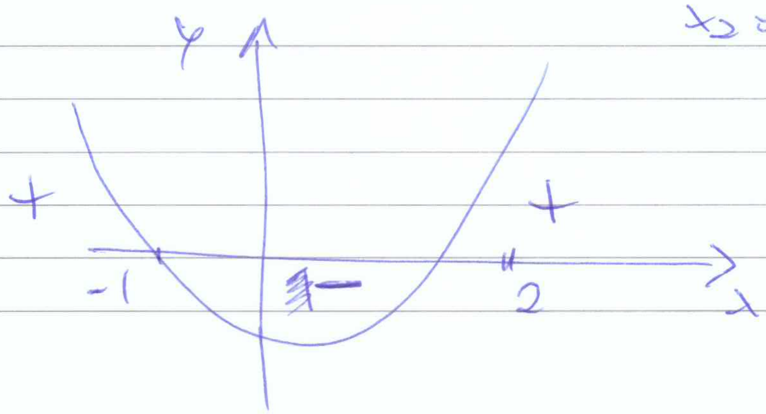
$\hookrightarrow \Delta = (-1)^2 - 4(1)(-2)$

$\Delta = 1+8 = 9$

$x = \frac{-(-1) \pm \sqrt{9}}{2 \cdot 1}$

$\rightarrow x_1 = \frac{1+3}{2} = 2$

$x_2 = \frac{1-3}{2} = -1$



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7) $2x-4>0$

$2x>4$

$x>2$

Resolvendo o logaritmo:

$\log(x^2-x-2) < \log(2x-4)$

$\hookrightarrow x^2-x-2 < 2x-4$

$x^2-3x+2<0$

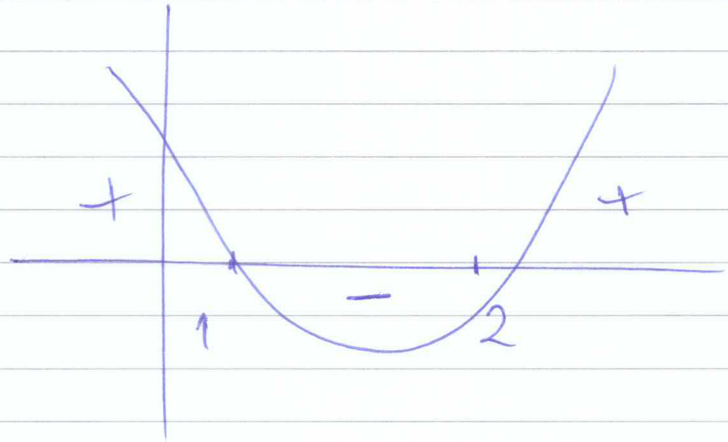
$\Delta=(-3)^2-4(1)(2)$

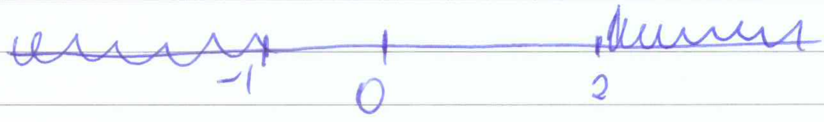
$\Delta = 9-8 = 1$

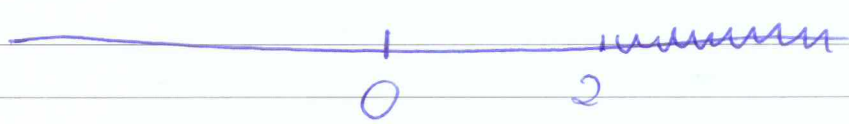
$x = \dfrac{-(-3)\pm\sqrt{1}}{2(1)}$

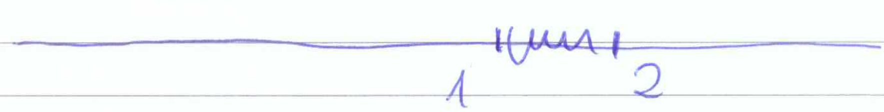
$x_1 = \dfrac{3+1}{2} = 2$

$x_2 = \dfrac{3-1}{2} = 1$



Condição 1: 

Condição 2: 

Condição 3: 

R.: $\nexists\, x \in \mathbb{R}$

7) b) $\log_2(x^2+x-2) \le 2$ 18/07/22

Condição de existência:

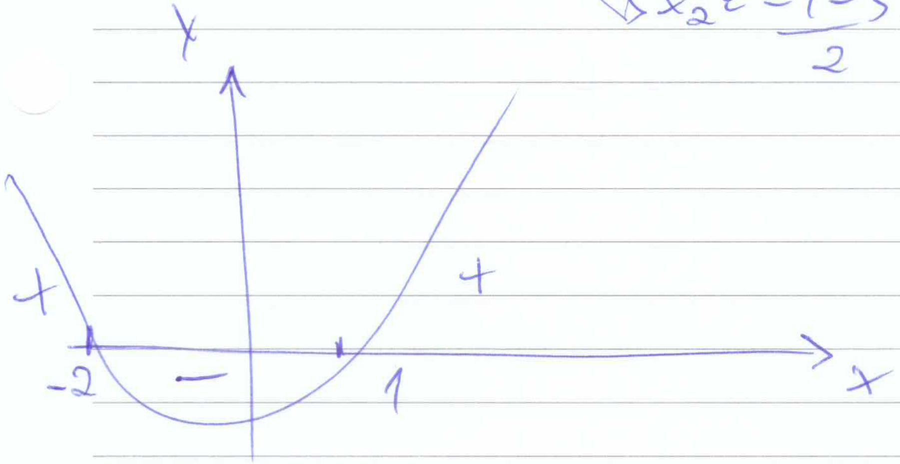
$x^2+x-2>0$

$\Delta=(1)^2-4(1)(-2)$

$\Delta = 1+8=9$

$x = \frac{-1\pm\sqrt{9}}{2\cdot 1}$ $\rightarrow x_1 = \frac{-1+3}{2} = 1$

$\rightarrow x_2 = \frac{-1-3}{2} = -2$



Resolvendo o logaritmo:

$\log_2(x^2+x-2) \le 2$

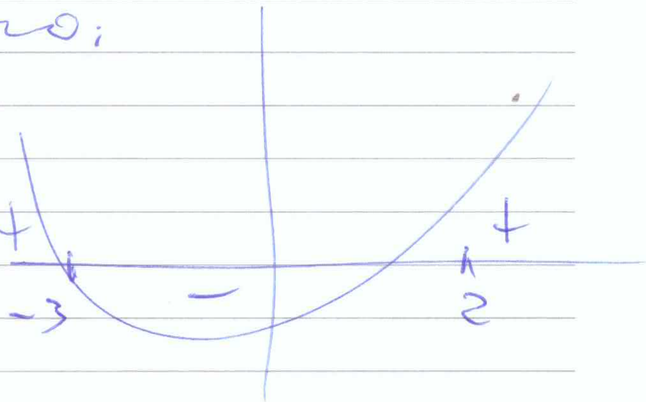
$\rightarrow x^2+x-2 \le 2^2$

$x^2+x-2 \le 4$

$x^2+x-6 \le 0$

$\Delta = 1^2-4(1)(-6)$

$\Delta = 1+24=25$



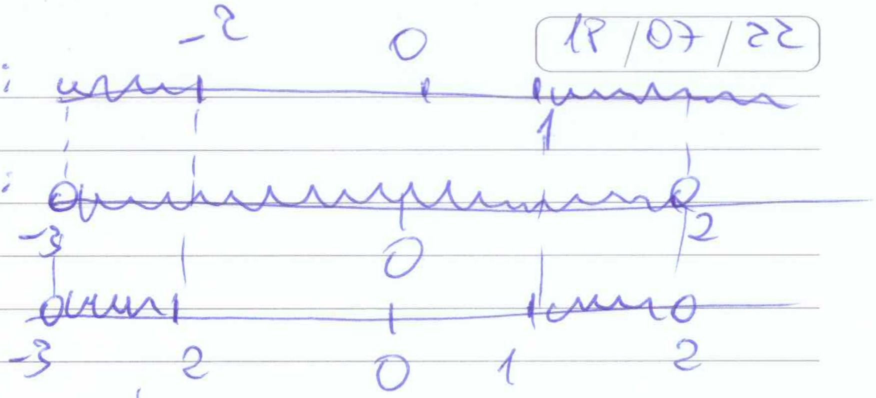
$x = \frac{-1\pm\sqrt{25}}{2(1)}$ $\rightarrow x_1 = \frac{-1+5}{2} = 2$

$\rightarrow x_2 = \frac{-1-5}{2} = -3$

7) b) Condição 1:

Condição 2:

Final:



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↳ $R = \{x \in \mathbb{R} \mid [-3,-2[\cup]1,2]\}$

c) $\log_5 (x-2) + \frac{1}{\log_{(x-3)} 5} > \log_5 2$

$\log_5 (x-2) + \frac{1}{\log_{(x-3)} 5} > \log_5 2$

$\log_5 (x-2) + \frac{1}{\frac{\log_5 5 = 1}{\log_5 (x-3)}} > \log_5 2$

$\log_5 (x-2) + \log_5 (x-3) > \log_5 2$

$\log_5 [(x-2)(x-3)] > \log_5 2$

$\log_5 (x^2 - 2x - 3x + 6) > \log_5 2$

$\log_5 (x^2 - 5x + 6) > \log_5 2$

↳ $x^2 - 5x + 6 > 2$

$x^2 - 5x + 4 > 0$

$\Delta = (-5)^2 - 4(1)(4)$

$\Delta = 25 - 16 = 9$

$x = \frac{-(-5) \pm \sqrt{9}}{2 \cdot 1}$ → $x_1 = \frac{5+3}{2} = 4$ → $x_2 = \frac{5-3}{2} = 1$

Condições de existência:

$x - 2 > 0$

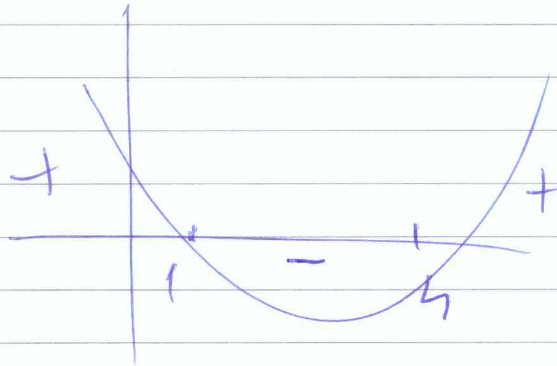
$x > 2$

$x - 3 > 0$

$x > 3$

$x - 3 \neq 1$

$x \neq 4$



R: $\{x > 4)$

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7) d) $|2+\log_2 x| \geq 3$

1ª Possibilidade:

$2+\log_2 x \geq 3$

$\log_2 x \geq 1$

$x \geq 2$

Condição de existência:

$x > 0$

2ª Possibilidade:

$-2-\log_2 x \geq 3$

$-\log_2 x \geq 5$

$\log_2 x \leq -5$

$x \leq \frac{1}{2^5} \rightarrow x \leq \frac{1}{32}$

R: $\{x \in \mathbb{R} \mid x \leq \frac{1}{32}$ ou $x \geq 2\}$

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