FAETERJ-Rio
Cálculo I
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Solução dos Exercícios 1.18a – 1.18h

1.18a) $\lim_{x\to 0} \frac{\sin 8x}{x}$ $\lim_{x\to 0} \frac{\sin x}{x} = 1$

$$\lim_{x\to 0} \frac{\sin 8x}{x} = \lim_{x\to 0} 8\frac{\sin 8x}{8x} = 8\lim_{x\to 0} \frac{\sin 8x}{8x} = 8.(1) = 8$$

1.18b) $\lim_{x\to 0} \frac{\sin(102x)}{\sin(51x)}$

$$\lim_{x\to 0} \frac{\sin(102x)}{\sin(51x)} = \lim_{x\to 0} \frac{102x\frac{\sin(102x)}{102x}}{51x\frac{\sin(51x)}{51x}} = \lim_{x\to 0} \frac{102\frac{\sin(102x)}{102x}}{51\frac{\sin(51x)}{51x}}$$

$$= \frac{\lim_{x\to 0} 102\frac{\sin(102x)}{102x}}{\lim_{x\to 0} 51\frac{\sin(51x)}{51x}} = \frac{102\lim_{x\to 0}\frac{\sin(102x)}{102x}}{51\lim_{x\to 0}\frac{\sin(51x)}{51x}} = \frac{102.(1)}{51.(1)} = \frac{102}{51} = 2$$

1.18c) $\lim_{x\to 0} \tan x \, cossec \, x$

$$\lim_{x\to 0} \tan x \, cossec \, x = \lim_{x\to 0} \frac{\sin x}{\cos x}\frac{1}{\sin x} = \lim_{x\to 0} \frac{1}{\cos x} = \frac{1}{1} = 1$$

1.18d) $\lim_{x\to 0} \frac{1 - cos^2 x}{xsin\, x}$

Sabendo que $sin^2 x = 1 - cos^2 x$, temos que

$$\lim_{x\to 0} \frac{1 - cos^2 x}{xsin\, x} = \lim_{x\to 0} \frac{sin^2 x}{xsin\, x} = \lim_{x\to 0} \frac{\sin x \sin x}{xsin\, x}$$

$$= \lim_{x\to 0} \frac{\sin x}{x} = 1$$

1.18e) $\lim_{x\to 0} \frac{\frac{\tan x}{\cos x}}{x}$

$$\lim_{x\to 0} \frac{\frac{\tan x}{\cos x}}{x} = \lim_{x\to 0} \frac{\frac{\frac{\sin x}{\cos x}}{\cos x}}{x} = \lim_{x\to 0} \frac{\sin x}{\cos x} \frac{x}{\cos x}$$

$$= \lim_{x\to 0} \frac{x \sin x}{\cos x \cos x} = \frac{0.0}{1.1} = 0$$

1.18f) $\lim_{x\to 1} \frac{\sin(x^2+x-2)}{x^2-1}$

$$\lim_{x\to 1} \frac{\sin(x^2+x-2)}{x^2-1} = \lim_{x\to 1} \frac{\sin\big((x-1)(x+2)\big)}{(x-1)(x+1)}$$

$$= \lim_{x\to 1} \frac{\sin\big((x-1)(x+2)\big)}{(x-1)} \frac{1}{(x+1)}$$

$$= \lim_{x\to 1} \frac{\sin\big((x-1)(x+2)\big)}{(x-1)} \lim_{x\to 1} \frac{1}{(x+1)}$$

$$\lim_{x\to 1} \frac{1}{(x+1)} = \frac{1}{2}$$

$$\lim_{x\to 1} \frac{\sin\big((x-1)(x+2)\big)}{(x-1)}$$

$$u = x - 1$$

$$x = u + 1$$

$$u = 1 - 1 = 0$$

Assim,

$$\lim_{u\to 0} \frac{\sin\big((u)(u+1+2)\big)}{(u)} = \lim_{u\to 0} \frac{\sin\big(u(u+3)\big)}{u} = \lim_{u\to 0} (u+3) \frac{\sin\big(u(u+3)\big)}{u(u+3)}$$

$$= \lim_{u\to 0} (u+3) \lim_{u\to 0} \frac{\sin\big(u(u+3)\big)}{u(u+3)} = 3. \lim_{u\to 0} \frac{\sin\big(u(u+3)\big)}{u(u+3)}$$

$$t = u(u+3)$$

$$t = 0(0+3) = 0$$

Assim,

$$\lim_{t\to 0}\frac{\sin(t)}{t}=1$$

Deste forma,

$$\lim_{x\to 1}\frac{\sin\big((x-1)(x+2)\big)}{(x-1)}=\lim_{u\to 0}\frac{\sin\big(u(u+3)\big)}{u(u+3)}$$

$$=\lim_{u\to 0}(u+3)\lim_{u\to 0}\frac{\sin\big(u(u+3)\big)}{u(u+3)}=3.1=3$$

Assim,

$$\lim_{x\to 1}\frac{\sin(x^2+x-2)}{x^2-1}=\lim_{x\to 1}\frac{\sin\big((x-1)(x+2)\big)}{(x-1)}\lim_{x\to 1}\frac{1}{(x+1)}=3\frac{1}{2}=\frac{3}{2}$$

1.18g) $\lim_{x\to 0}\frac{1-\sqrt[3]{\cos x}}{1-\cos x}$

$$\lim_{x\to 0}\frac{1-\sqrt[3]{\cos x}}{1-\cos x}=\lim_{x\to 0}\frac{1-\sqrt[3]{\cos x}}{1-\cos x}\frac{1+\sqrt[3]{\cos x}+cos^{\frac{2}{3}}x}{1+\sqrt[3]{\cos x}+cos^{\frac{2}{3}}x}$$

$$=\lim_{x\to 0}\frac{1+\sqrt[3]{\cos x}+cos^{\frac{2}{3}}x-\sqrt[3]{\cos x}-\sqrt[3]{\cos x}\sqrt[3]{\cos x}-\sqrt[3]{\cos x}\,cos^{\frac{2}{3}}x}{(1-\cos x)\left(1+\sqrt[3]{\cos x}+cos^{\frac{2}{3}}x\right)}$$

$$=\lim_{x\to 0}\frac{1+cos^{\frac{2}{3}}x-cos^{\frac{2}{3}}x-cos^{\frac{1}{3}}x cos^{\frac{2}{3}}x}{(1-\cos x)\left(1+\sqrt[3]{\cos x}+cos^{\frac{2}{3}}x\right)}$$

$$=\lim_{x\to 0}\frac{1-\cos x}{(1-\cos x).\left(1+\sqrt[3]{\cos x}+cos^{\frac{2}{3}}x\right)}$$

$$=\lim_{x\to 0}\frac{1-\cos x}{(1-\cos x)}\frac{1}{\left(1+\sqrt[3]{\cos x}+cos^{\frac{2}{3}}x\right)}$$

$$= \lim_{x\to 0} \frac{1}{\left(1 + \sqrt[3]{\cos x} + cos^{\frac{2}{3}} x\right)} = \frac{1}{1+1+1} = \frac{1}{3}$$

1.18h) $\lim_{x\to 0} \frac{sin^2 x}{x^3 - x^2}$

$$\lim_{x\to 0} \frac{sin^2 x}{x^3 - x^2} = \lim_{x\to 0} \frac{\sin x \sin x}{x^2(x-1)} = \lim_{x\to 0} \frac{\sin x}{x} \frac{\sin x}{x} \frac{1}{x-1}$$

$$= \lim_{x\to 0} \frac{\sin x}{x} \lim_{x\to 0} \frac{\sin x}{x} \lim_{x\to 0} \frac{1}{x-1} = \frac{1}{0-1} = \frac{1}{-1} = -1$$