Exercícios 9.1a - 9.1d (Escalonamento)

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Exercício 9.1: Resolva os seguintes sistemas por dois métodos diferentes. Um deles dever ser pelo produto da matriz inversa.

a)

$$3x + y = 9$$
$$2x + 3y = 13$$

b)

$$x + 2y - z = 2$$
$$x - y + z = 3$$
$$x + y + z = 6$$

c)

$$3x + z = -5$$
$$x - y + z = -2$$
$$2y - z = -3$$

d)

$$5x + 3y - 11z = 13$$
$$4x - 5y + 4z = 18$$
$$9x - 2y + 7z = -25$$

a)

$$3x + y = 9$$
$$2x + 3y = 13$$

$$\left|\begin{array}{cc|c} 3 & 1 & 9 \\ 2 & 3 & 13 \end{array}\right| \quad L_2 = L_2 - \frac{2}{3} L_1$$

$3 - \frac{2}{3}(1) = \frac{9}{3} - \frac{2}{3} = \frac{7}{3}$

$13 - \frac{2}{3}(9) = \frac{39 - 18}{3} = \frac{21}{3}$

$$\left|\begin{array}{cc|c} 3 & 1 & 9 \\ 0 & 7/3 & 21/3 \end{array}\right|$$

$\frac{7}{3}y = \frac{21}{3}$

$y = \frac{21}{3} \cdot \frac{3}{7}$

$= \frac{21 \cdot 2}{21}$

$\boxed{y = 3}$

$3x + y = 9$

$3x + 3 = 9$

$3x = 9 - 3$

$3x = 6$

$x = \frac{6}{3}$

$\boxed{x = 2}$

$y = 3$ | $x = 2$

b)

$$x + 2y - z = 2$$
$$x - y + z = 3$$
$$x + y + z = 6$$

$$\left|\begin{array}{ccc|c} 1 & 2 & -1 & 2 \\ 1 & -1 & 1 & 3 \\ 1 & 1 & 1 & 6 \end{array}\right| \quad \begin{array}{l} L_2 = L_2 - L_1 \\ L_3 = L_3 - L_1 \end{array}$$

$$\left|\begin{array}{ccc|c} 1 & 2 & -1 & 2 \\ 0 & -3 & 2 & 1 \\ 0 & -1 & 2 & 4 \end{array}\right| \quad L_3 = L_3 - \frac{1}{3} L_2$$

$$\left|\begin{array}{ccc|c} 1 & 2 & -1 & 2 \\ 0 & -3 & 2 & 1 \\ 0 & 0 & 4/3 & 11/3 \end{array}\right| \quad \begin{array}{l} 2 - \frac{1}{3}(2) = 2 - \frac{2}{3} = \frac{4}{3} \\ 4 - \frac{1}{3}(1) = 4 - \frac{1}{3} = \frac{11}{3} \end{array}$$

$\frac{4}{3} z = \frac{11}{3}$

$4z = 11$

$z = \frac{11}{4}$

$-3y + 2z = 1$

$-3y + 2\left(\frac{11}{4}\right) = 1$

$-3y + \frac{11}{2} = 1$

$-3y = 1 - \frac{11}{2}$

$-3y = \frac{-9}{2}$

$y = \frac{-9}{(-3)2}$

$y = \frac{3}{2}$

$x + 2y - z = 2$

$x + 2\left(\frac{3}{2}\right) - \frac{11}{4} = 2$

$x + 3 - \frac{11}{4} = 2$

$x + \frac{1}{4} = 2$

$x = 2 - \frac{1}{4}$

$x = \frac{7}{4}$

$x = 7/4$
$y = 3/2$
$z = 11/4$

c)

$$3x + z = -5$$
$$x - y + z = -2$$
$$2y - z = -3$$

$$\left|\begin{array}{ccc|c} 3 & 0 & 1 & -5 \\ \textcircled{1} & -1 & 1 & -2 \\ 0 & 2 & -1 & -3 \end{array}\right| \quad L_2 = L_2 - \frac{1}{3}L_1$$

$$\left|\begin{array}{ccc|c} 3 & 0 & 1 & -5 \\ 0 & -1 & 2/3 & -1/3 \\ 0 & \textcircled{2} & -1 & -3 \end{array}\right| \quad L_3 = L_3 + 2L_2$$

$1 - \frac{1}{3}(1) = 1 - \frac{1}{3} = \frac{2}{3}$

$-2 - \frac{1}{3}(-5) = -2 + \frac{5}{3} = -\frac{1}{3}$

$$\left|\begin{array}{ccc|c} 3 & 0 & 1 & -5 \\ 0 & -1 & 2/3 & -1/3 \\ 0 & 0 & 1/3 & -11/3 \end{array}\right|$$

$-1 + 2\left(\frac{2}{3}\right) = -1 + \frac{4}{3} = \frac{1}{3}$

$-3 + 2\left(-\frac{1}{3}\right) = -3 - \frac{2}{3} = -\frac{11}{3}$

$\frac{1}{3}z = -\frac{11}{3}$

$z = -11$

$-y + \frac{2}{3}z = -\frac{1}{3}$

$-y + \frac{2}{3}(-11) = -\frac{1}{3}$

$-y - \frac{22}{3} = -\frac{1}{3}$

$-y = -\frac{1}{3} + \frac{22}{3}$

$-y = \frac{-1+22}{3}$

$-y = \frac{21}{3}$

$-y = 7 \quad (-1)$

$y = -7$

$3x + 0y + z = -5$

$3x - 11 = -5$

$3x = -5 + 11$

$x = \frac{6}{3}$

$x = 2$

$x = 2$

$y = -7$

$z = -11$

d)

$$5x + 3y - 11z = 13$$

$$4x - 5y + 4z = 18$$

$$9x - 2y + 7z = -25$$

$$\left|\begin{array}{ccc|c} 5 & 3 & -11 & 13 \\ \textcircled{4} & -5 & 4 & 18 \\ \textcircled{9} & -2 & 7 & -25 \end{array}\right| \quad L_2 = L_2 - \frac{4}{5}L_1 \quad L_3 = L_3 - \frac{9}{5}L_1$$

$-5 - \frac{4}{5}(3) = -5 - \frac{12}{5} = \frac{-25-12}{5} = -\frac{37}{5}$

$4 - \frac{4}{5}(-11) = 4 + \frac{44}{5} = \frac{20+44}{5} = \frac{64}{5}$

$18 - \frac{4}{5}(13) = 18 - \frac{52}{5} = \frac{90-52}{5} = \frac{38}{5}$

$-2 - \frac{9}{5}(3) = -2 - \frac{27}{5} = \frac{-10-27}{5} = -\frac{37}{5}$

$7 - \frac{9}{5}(-11) = 7 + \frac{99}{5} = \frac{35+99}{5} = \frac{134}{5}$

$-25 - \frac{9}{5}(13) = -25 - \frac{117}{5} = \frac{-125-117}{5} = -\frac{242}{5}$

$$\left|\begin{array}{ccc|c} 5 & 3 & -11 & 13 \\ 0 & -37/5 & 64/5 & 38/5 \\ 0 & \textcircled{-37/5} & 134/5 & -242/5 \end{array}\right| \quad L_3 = L_3 - L_2$$

$\frac{134}{5} - \frac{64}{5} = \frac{134-64}{5} = \frac{70}{5}$

$$\left|\begin{array}{ccc|c} 5 & 3 & -11 & 13 \end{array}\right|$$

$$\left|\begin{array}{ccc|c} 5 & 3 & -11 & 13 \\ 0 & -37/5 & 64/5 & 38/5 \\ 0 & 0 & 70/5 & 56 \end{array}\right|$$

$$\frac{134}{5} - \frac{64}{5} = \frac{134-64}{5} = \frac{70}{5}$$

$$-\frac{242}{5} - \frac{38}{5} = \frac{-242-38}{5} = \frac{-280}{5} = -56$$

$\frac{70}{5} z = -56$

$70z = -56.5$

$z = -\frac{280}{70}$

$z = -4$

$-\frac{37}{5}y + \frac{64}{5}z = \frac{38}{5}$

$-37y + 64(-4) = 38$

$-37y - 256 = 38$

$-37y = 38 + 256$

$-37y = 294$

$y = \frac{-294}{37}$

$5x + 3y - 11z = 13$

$5x + 3\left(-\frac{294}{37}\right) - 11(-4) = 13$

$5x - \frac{882}{37} + 44 = 13$

$5x = 13 - 44 + \frac{882}{37}$

$5x = -31 + \frac{882}{37}$

$5x = \frac{-1147 + 882}{37}$

$5x = -\frac{265}{37}$

$x = -\frac{265}{(5)\,37}$

$x = -\frac{53}{37}$

$x = -\frac{53}{37}$

$y = -\frac{294}{37}$

$z = -4$