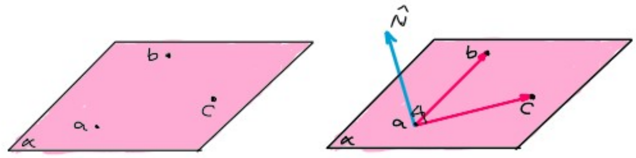
Exercícios 11.20 - 11.24

quinta-feira, 9 de novembro de 2023 16:42

Exercício 11.20: Encontre a equação do plano α, determinado pelos pontos $a = (1,0,1), b = (2,2,4)$ e $c = (2,1,0)$.



$\vec{ab} = b - a = (2,2,4) - (1,0,1) = (1,2,3)$

$\vec{ac} = c - a = (2,1,0) - (1,0,1) = (1,1,-1)$

FAZENDO O PRODUTO VETORIAL $\vec{ab} \times \vec{ac}$, TEREMOS COMO O RESULTADO UM VETOR ORTOGONAL AO PLANO ONDE SE ENCONTRAM OS VETORES $\vec{ab} \times \vec{ac}$.

$$D = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ 1 & 2 & 3 \\ 1 & 1 & -1 \end{vmatrix}$$

$$\vec{ab} \times \vec{ac} = \det(D) = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} & \hat{x} & \hat{y} \\ 1 & 2 & 3 & 1 & 2 \\ 1 & 1 & -1 & 1 & 1 \end{vmatrix} \begin{aligned} &= -2\hat{x} + 3\hat{y} + \hat{z} - (3\hat{x} - \hat{y} + 2\hat{z}) \\ &= -2\hat{x} + 3\hat{y} + \hat{z} - 3\hat{x} + \hat{y} - 2\hat{z} \\ &= -5\hat{x} + 4\hat{y} - \hat{z} \end{aligned}$$

$\underbrace{2\hat{z} \quad 3\hat{x} \quad -\hat{y}}_{3\hat{x} - \hat{y} + 2\hat{z}} \qquad \underbrace{-2\hat{x} \quad 3\hat{y} \quad \hat{z}}_{-2\hat{x} + 3\hat{y} + \hat{z}}$

$$\vec{ab} \times \vec{ac} = -5\hat{x} + 4\hat{y} - \hat{z}$$

EQUAÇÃO DO PLANO $\Rightarrow Ax + By + Cz + D = 0$,

EM QUE (A, B, C) SÃO COORDENADAS DO VETOR NORMAL, ASSIM,

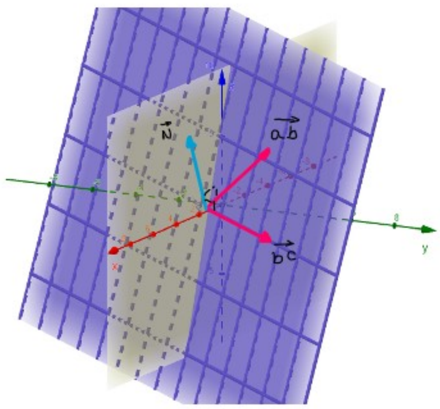
$E_{Plano} = -5x + 4y - z + D = 0$

SUBSTITUINDO (x, y, z) POR UM PONTO CONHECIDO NO PLANO, TEMOS QUE

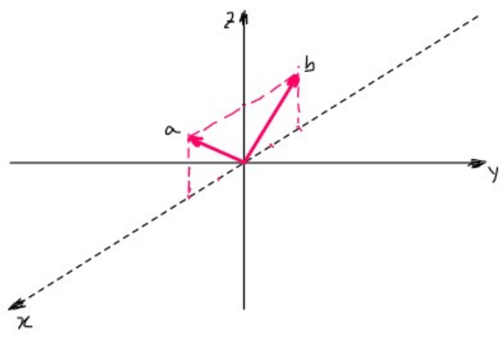
$$\begin{aligned} E_{Plano} &= -5(2) + 4(1) - 1(0) + D = 0 \\ & -10 + 4 - 0 + D = 0 \\ & -6 + D = 0 \\ & D = 6 \end{aligned}$$

PLANO α

$$\boxed{E_{Plano} = -5x + 4y - z + 6 = 0}$$



Exercício 11.21: Dado os vetores $a = (2,0,2)$ e $b = (-2,0,2)$, encontre a equação do plano xz.



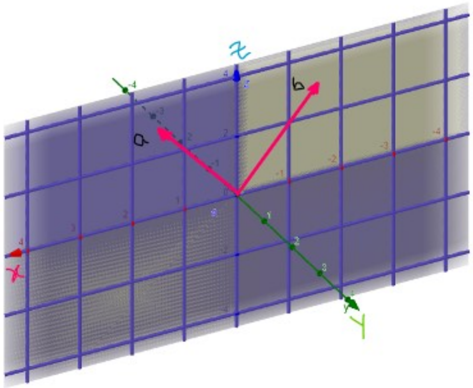
$$\begin{aligned} a \times b &= (2\hat{x}, 0\hat{y}, 2\hat{z}) \times (-2\hat{x}, 0\hat{y}, 2\hat{z}) \\ &= -4\cancel{\hat{x} \times \hat{x}}^{0} + 4\hat{x} \times \hat{z} - 4\hat{z} \times \hat{x} + 4\cancel{\hat{z} \times \hat{z}}^{0} \\ &= -4\hat{y} - 4\hat{y} = -8\hat{y} \end{aligned}$$

$E_{plano} = -8y + D = 0$

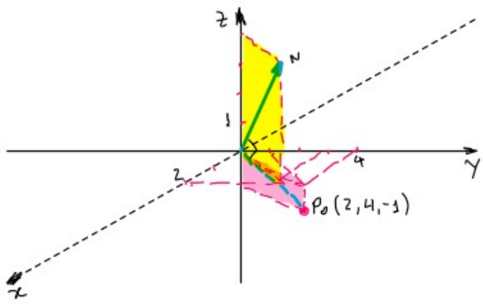
SUBSTITUINDO y POR UM PONTO CONTIDO DO PLANO, TEMOS QUE

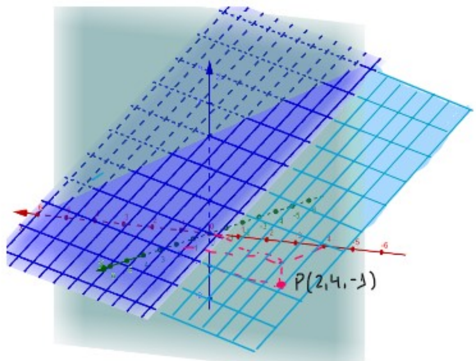
$-8(0) + D = 0$ $\qquad E_{plano} = -8y = 0$

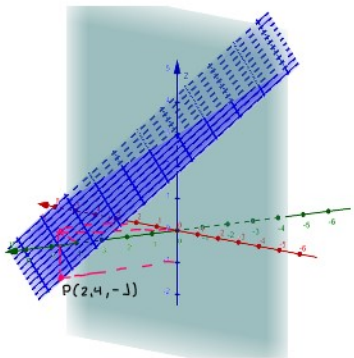
$D = 0$



Exercício 11.22: Determine a equação do plano que passa pelo ponto $a = (2,4,-1)$ e tem como vetor normal $n = (2,3,4)$.







$P_0 = (2,4,-1)$

$N = (2,3,4)$

A EQUAÇÃO DO PLANO PODE ESCRITA DA FORMA

$e_{plano} = A(x-x_0) + B(y-y_0) + C(z-z_0) = 0$

(A, B, C) SÃO AS COORDENADAS DO VETOR NORMAL.

ASSIM,

$e_{plano} = 2(x-2) + 3(y-4) + 4(z-(-1)) = 0$

$= 2x - 4 + 3y - 12 + 4z + 4 = 0$

$\boxed{e_{plano} = 2x + 3y + 4z - 12 = 0}$

Exercício 11.23: Dados os vetores $v = (-2,0,3,)$ e $w = (2,3,-2,)$, o produto vetorial $v \times w$ tem como resultado o vetor $\boldsymbol{n}$. Determine a equação do plano onde se encontram os vetores v e w.

$$D = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ -2 & 0 & 3 \\ 2 & 3 & -2 \end{vmatrix}$$

$$v \times w = \det(D) = \left|\begin{matrix} \hat{x} & \hat{y} & \hat{z} \\ -2 & 0 & 3 \\ 2 & 3 & -2 \end{matrix}\;\begin{matrix} \hat{x} & \hat{y} \\ -2 & 0 \\ 2 & 3 \end{matrix}\right|$$

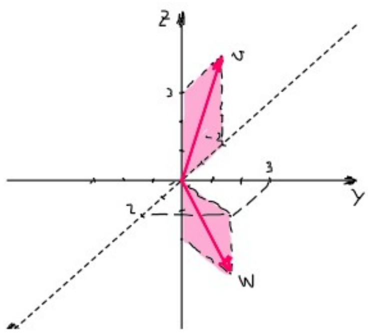
$$\underbrace{0 \quad 9\hat{x} \quad 4\hat{y}}_{9\hat{x}+4\hat{y}} \qquad \underbrace{0 \quad 6\hat{y} \quad -6\hat{z}}_{6\hat{y}-6\hat{z}}$$

$= 6\hat{y} - 6\hat{z} - (9\hat{x} + 4\hat{y})$

$= 6\hat{y} - 6\hat{z} - 9\hat{x} - 4\hat{y}$

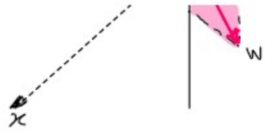
$= -9\hat{x} + 2\hat{y} - 6\hat{z}$

$N = (-9, 2, -6)$



EQUAÇÃO DO PLANO

$9\hat{x}+4\hat{y} \quad 6\hat{y}-6\hat{z}$ $\quad N=(-9,2,-6)$



Equação do plano

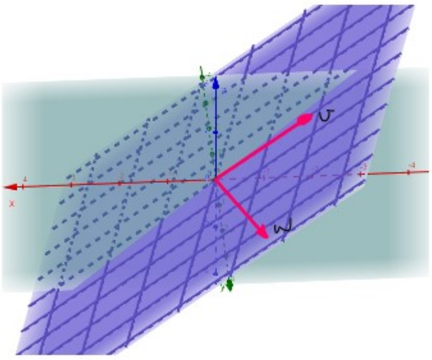
$Ax+By+Cz+D=0$

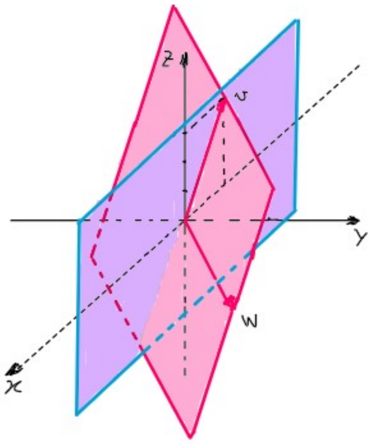
$-9x+2y-6z+D=0$ $\quad \boxed{e_{PLANO}=9x+2y-6z}$

$-9(-2)+2(0)-6(3)+D=0$

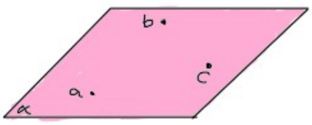
$+18-18+D=0$

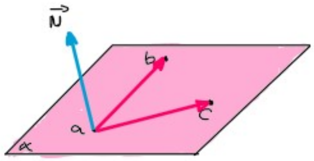
$D=0$





Exercício 11.24: Determine a equação do plano que passa pelos pontos $a=(0,1,1)$, $b=(1,0,1)$ e $c=(1,1,0)$.





$\vec{ab}=(1,0,1)-(0,1,1)=(1,-1,0)$

$\vec{ac}=(1,1,0)-(0,1,1)=(1,0,-1)$

$\vec{N}=\vec{ac}\times\vec{ab}=(\hat{x},0,-\hat{z})\times(\hat{x},-\hat{y},0)$

$=\cancel{\hat{x}\times\hat{x}}^{\,0}-\hat{x}\times\hat{y}-\hat{z}\times\hat{x}+\hat{z}\times\hat{y}$

$=-\hat{z}-\hat{y}-\hat{x}$

$\vec{N}=-\hat{x}-\hat{y}-\hat{z}=(-1,-1,-1)$

Equação do plano

$Ax+By+Cz+D=0$

$-x-y-z+D=0$

Substituindo o ponto a na expressão, temos que

$-1-0-1+D=0$

$-2+D=0$

$D=2$

$\boxed{e_{PLANO}=-x-y-z+2=0}$