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Lista 2 de Exercícios

5.1) a) $2^{x^2-x-16} = 16$

$2^{x^2-x-16} = 2^4$

$\hookrightarrow x^2 - x - 16 = 4$

$x^2 - x - 20 = 0$

$(x+4)(x-5) = 0$

$x = -4 \ / \ x = 5$ ✓

c) $3 \cdot 2^x - 5 \cdot 2^{x+1} + 5 \cdot 2^{x+3} - 2^{x+5} = 2$

$2^x(3 - 5 \cdot 2^1 + 5 \cdot 2^3 - 2^5) = 2$

$2^x(3 - 10 + 40 - 32) = 2$

$2^x(3 - 2) = 2$

$2^x(1) = 2$

$\hookrightarrow x = 1$

b) $2^{3x+2} / 8^{2x-7} = 4^{x-1}$

$2^{3x+2} / 2^{3(2x-7)} = 2^{2x-2}$

$2^{[(3x+2)-(6x-21)]} = 2^{2x-2}$

$2^{23-3x} = 2^{2x-2}$

$\hookrightarrow 23 - 3x = 2x - 2$

$5x = 25$

$x = 5$ ✓

d) $2^{2x} + 2^{x+1} = 80$

$2^{x+1}(2^{x-1} + 1) = 2^4(2^2 + 1)$

$x + 1 = 4 \qquad x - 1 = 2$

$x = 3$ ✓ $\qquad x = 3$ ✓

e) $3^{x-1} - \frac{5}{3^{x+1}} = 4 \cdot 3^{1-3x}$ (3^{x+1})

$3^{2x} - 5 = 4 \cdot 3^{2-2x}$

$3^{2x} - 5 = \frac{4 \cdot 9}{3^{2x}}$

$3^{2x} - 5 = \frac{36}{3^{2x}}$ $\qquad k = 3^{2x}$

$k - 5 = \frac{36}{k}$ $\qquad \cancel{3^{2x} = -4}$

$k - 5 - \frac{36}{k} = 0$ $\qquad 3^{2x} = 9$

$k^2 - 5k - 36 = 0$ $\qquad 3^{2x} = 3^2$

$(k+4)(k-9) = 0$ $\qquad 2x = 2$

$k = -4 \mid k = 9$ $\qquad x = 1$ //

f) $x^{x^2-2} = 1$

$x^{x^2-2} = x^0$

$\hookrightarrow x^2 - 2 = 0$

$x = \pm\sqrt{2}$

g) $x^{4-2x} = x$

$4 - 2x = 1$

$-2x = -3$

$x = 3/2$

h) $4^x + 2 \cdot 14^x = 3 \cdot 49^x$ $\left(\cdot \frac{1}{14^x}\right)$ $\rightarrow \left(\frac{7}{2}\right)^x = t$

$\frac{4^x}{14^x} + 2 = 3 \cdot \frac{49^x}{14^x}$

$\left(\frac{2}{7}\right)^x + 2 = 3\left(\frac{7}{2}\right)^x$

$\frac{1}{\left(\frac{7}{2}\right)^x} + 2 = 3\left(\frac{7}{2}\right)^x$

spiral

$$\frac{1}{k} + 2 = 3k$$

$$1 + 2k = 3k^2$$

$$3k^2 - 2k - 1 = 0$$

$$\Delta = (-2)^2 - 4(3)(-1)$$

$$\Delta = 4 + 12 = 16$$

$$x = \frac{-(-2) \pm \sqrt{16}}{2 \cdot 3} = \frac{2 \pm 4}{6} \begin{cases} \to 6/6 = 1 \\ \to -2/6 = -1/3 \end{cases}$$

$$\cancel{\left(\frac{1}{2}\right)^x = -1/3} \qquad \left(\frac{1}{2}\right)^x = 1 \to x = 0$$

5.2) a) $\begin{cases} 4^x = 16y \\ 2^{x+1} = 4y \end{cases}$

$$2^{2x} = 4(4y)$$

$$2^{2x} = 2^2 \cdot 2^{x+1}$$

$$2^{2x} = 2^{x+3}$$

$$\to 2x = x + 3$$

$$x = 3 \;//$$

$$4^3 = 16y$$

$$64 = 16y$$

$$y = 4 \;//$$

b) $\begin{cases} x^{x+y} = y^{x-y} \\ x^2 y = 1 \end{cases}$

$$y = \frac{1}{x^2}$$

$$\to x^{x + \frac{1}{x^2}} = \left(\frac{1}{x^2}\right)^{x - \frac{1}{x^2}}$$

$$x^{\left(\frac{x^3+1}{x^2}\right)} = \left(x^{-2}\right)^{\left(\frac{x^3-1}{x^2}\right)}$$

$$x^{\left(\frac{x^3+1}{x^2}\right)} = x^{\left(\frac{2-2x^3}{x^2}\right)}$$

$$\to \frac{x^3+1}{\cancel{x^2}} = \frac{2-2x^3}{\cancel{x^2}}$$

$$3x^3 = 1$$

$$x^3 = \frac{1}{3}$$

$$x = \frac{1}{3^{1/3}}$$

$$y = \frac{1}{x^2} = \frac{1}{\left(\frac{1}{3^{1/3}}\right)^2} = 3^{2/3}$$

5.3) a) $2^{5x-1} > 8$

$2^{5x-1} > 2^3$

$\hookrightarrow 5x-1 > 3$

$5x > 4$

$x > 4/5$

b) $4^{x^2+1} < 32^{1-x}$

$2^{2(x^2+1)} < 2^{5(1-x)}$

$\hookrightarrow 2x^2+2 < 5x-5$

$2x^2-5x+7 < 0$

$\Delta = (-5)^2 - 4(2)(7)$

$\Delta = 25-56 = -31$ $\quad \nexists x \in \mathbb{R}$

c) $\left(\frac{2}{3}\right)^{3x-2} \cdot \left(\frac{4}{9}\right)^{2x+1} < \left(\frac{8}{27}\right)^{x-1}$

$\left(\frac{2}{3}\right)^{3x-2} \cdot \left(\frac{2}{3}\right)^{2(2x+1)} < \left(\frac{2}{3}\right)^{3(x-1)}$

$\left(\frac{2}{3}\right)^{[3x-2+4x+2]} < \left(\frac{2}{3}\right)^{3x-3}$

$\left(\frac{2}{3}\right)^{(7x)} < \left(\frac{2}{3}\right)^{3x-3}$

$7x \geq 3x-3$

$4x \geq -3$

$x \geq -3/4$

d) $(0,3)^{x-5} \leqslant (0,09)^{2x+3} \leqslant (0,3)^{x+6}$

① ② ③

1-2: $(0,3)^{x-5} \leqslant (0,09)^{2x+3}$

$(0,3)^{x-5} \leqslant (0,3)^{2(2x+3)}$

$(0,3)^{x-5} \leqslant (0,3)^{4x+6}$

$x-5 \geqslant 4x+6$

$-3x \geqslant 11$

$3x \leqslant -11$

$x \leqslant -11/3$

2-3: $(0,09)^{2x+3} \leqslant (0,3)^{x+6}$

$(0,3)^{2(2x+3)} \leqslant (0,3)^{x+6}$

$(0,3)^{4x+6} \leqslant (0,3)^{x+6}$

$4x+6 \geqslant x+6$

$3x \geqslant 0$

$x \geqslant 0$

1-3: $(0,3)^{x-5} \leqslant (0,3)^{x+6}$

$x-5 \geqslant x+6$

$-5 \geqslant 6$

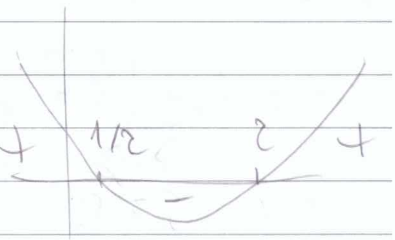
$\nexists x \in \mathbb{R}$

e) $4^{\left(x+\frac{1}{2}\right)} - 5\cdot 2^{x} + 2 > 0$

$2^{2\left(\frac{2x+1}{2}\right)} - 5\cdot 2^{x} + 2 > 0$

$(2^{x})^{2}\cdot 2^{1} - 5\cdot 2^{x} + 2 > 0$ $\qquad k = 2^{x}$

$2k^{2} - 5k + 2 > 0$



$\Delta = (-5)^{2} - 4(2)(2)$

$\Delta = 25 - 16 = 9$

$x = \frac{5 \pm \sqrt{9}}{2\cdot 2}$ $\nearrow \frac{5+3}{4} = 2$ $\searrow \frac{5-3}{4} = 1/2$

$2^{x} > 2 \qquad 2^{x} < 2^{-1}$

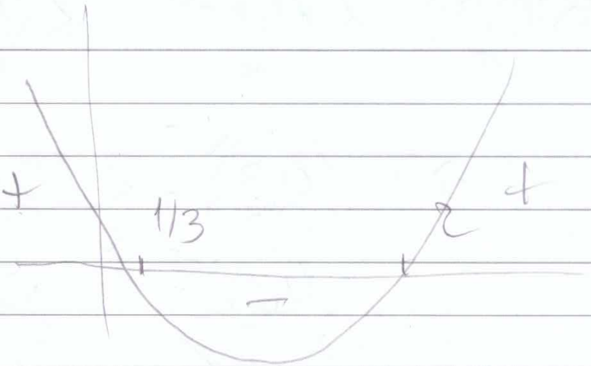
$x > 1 \qquad x < -1$

R: $\{x \in \mathbb{R} \mid]-\infty, -1[\cup]1, +\infty[\}$

f) $x^{3x^2-7x+2} < 1$

$x^{3x^2-7x+2} < x^0$

$\Rightarrow 3x^2-7x+2 < 0$



$\Delta = (-7)^2 - 4(3)(2)$

$= 49 - 24 = 25$

$x = \frac{7 \pm \sqrt{25}}{2 \cdot 3}$ → $\frac{7+5}{6} = 2$; → $\frac{7-5}{6} = 1/3$

$R: \{x \in \mathbb{R} \mid 1/3 < x < 2\}$

g) $a^{x^2} > a^9$

$\Rightarrow x^2 > 9$

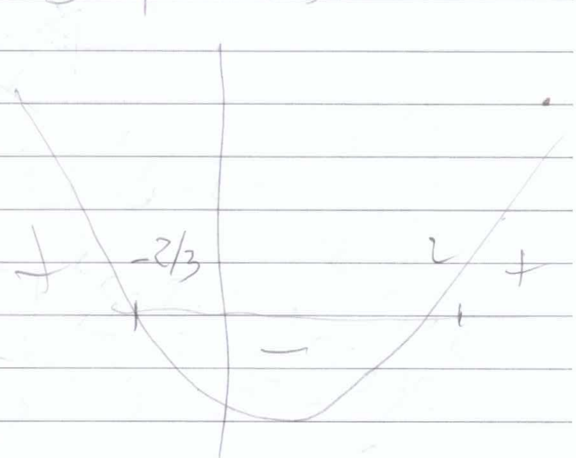
$x > \sqrt{9} \rightarrow x > 3 \cup x < -3$

$R: \{x \in \mathbb{R} \mid]-\infty, -3[\cup]3, +\infty[\}$

h) $|x|^{3x^2-4x-4} > 1$

$|x|^{3x^2-4x-4} > x^0$

$\Rightarrow 3x^2-4x-4 > 0$



$\Delta = (-4)^2 - 4(3)(-4)$

$\Delta = 16 + 48 = 64$

$x = \frac{4 \pm \sqrt{64}}{2 \cdot 3}$ → $\frac{4+8}{6} = 2$; → $\frac{4-8}{6} = -2/3$

$R: \{x \in \mathbb{R} \mid]-\infty, -2/3[\cup]2, +\infty[\}$

5.4) a) $\log_{2x} x = -2$

$\Leftrightarrow (2x)^{-2} = x$

$\frac{1}{4x^2} = x$

$4x^3 = 1$

$x^3 = \frac{1}{4} \rightarrow x = 2^{-2/3}$

$\log_a b = x$

$a^x = b$

b) $\log_{x-3} 4x = 2$

$\Leftrightarrow (x-3)^2 = 4x$

$x^2 - 6x + 9 = 4x$

$x^2 - 10x + 9 = 0$

$(x-9)(x-1) = 0$

$\cancel{x=1}$ / $x = 9$

$x - 3 > 0$

$x > 3$

R: $x = 9$

c) $\log_{27} \log_2 \log_2 (x-1) = \frac{1}{3}$

$\Leftrightarrow 27^{1/3} = \log_2 \log_2 (x-1)$

$(3^3)^{1/3} = \log_2 \log_2 (x-1)$

$\log_2 \log_2 (x-1) = 3$

$2^3 = \log_2 (x-1)$

$8 = \log_2 (x-1)$

$2^8 = x - 1$

$256 = x - 1 \rightarrow x = 257$ //

5.5) a) $\log_9 (x-1) = \log_3 (\sqrt{10x-4}) - \log_3 (x+2)$

$\frac{\log_3 (x-1)}{\log_3 9 = 2} = \log_3 \left(\frac{\sqrt{10x-4}}{x+2}\right)$

$\log_3 (x-1) = 2\log_3 \left(\frac{\sqrt{10x-4}}{x+2}\right)$

$\log_3 (x-1) = \log_3 \left[\left(\frac{\sqrt{10x-4}}{x+2}\right)^2\right]$

$x - 1 = \left(\frac{\sqrt{10x-4}}{x+2}\right)^2$

/ /

$x-1 = \left(\sqrt{\frac{10x-4}{x+2}}\right)^2$

$(x-1)(x+2) = 10x-4$

$(x-1)(x^2-2x+4) = 10x-4$

$x^3-2x^2+4x-x^2+2x-\cancel{4} = 10x-\cancel{4}$

$x^3-3x^2+6x = 10x$

$x^3-3x^2-4x = 0$

$x(x^2-3x-4) = 0$ R: $x=4$

$x=0 \mid x^2-3x-4=0$

$(x-4)(x+1)=0$

$x=4 \mid \cancel{x=-1}$

$\sqrt{10x-4} \geq 0$

$10x-4 \geq 0$

$10x \geq 4$

$x \geq 2/5$

b) $x^{\log x} = \frac{x^4}{1000}$

$\log\left(x^{\log x}\right) = \log\left(\frac{x^4}{1000}\right)$

$\log x \cdot \log x = \log x^4 - \log 10^3$

$[\log x]^2 = 4\log x - 3$ $\log x = k$

$\hookrightarrow k^2 = 4k-3$

$k^2-4k+3=0$ R: $x=10$ ou $x=3$

$(k-3)(k-1)=0$

$k=1 \mid k=3$

spiral

c) $(\log x)^{\log x} = x^2$

$\hookrightarrow \log\left[(\log x)^{\log x}\right] = \log x^2$

$\cancel{\log x} \cdot \log(\log x) = 2\cancel{(\log x)}$

$\log(\log x) = 2$

$10^2 = \log x$

$x = 10^{100}$

d) $\log_a ax \cdot \log_x ax = 4$

$\hookrightarrow \frac{\log_x ax}{\log_x a} \cdot (\log_x a + \log_x x) = 4$

$\frac{(\log_x a + \cancel{\log_x x}^{1})(\log_x a + \cancel{\log_x x}^{1})}{\log_x a} = 4$

$(\log_x a + 1)^2 = 4\log_x a$ $\qquad \log_a b = 1$

$(\log_x a)^2 + 2\log_x a + 1 = 4\log_x a$

$(\log_x a)^2 - 2\log_x a + 1 = 0$

$(\log_x a - 1)^2 = 0$

$\log_x a - 1 = 0$

$\log_x a = 1 \rightarrow x^1 = a$

$x = a$

e) $5^x + 5^{x+1} = 3^x + 3^{x+1} + 3^{x+2}$

$5^x(1+5) = 3^x(1+3+9)$

$5^x \cdot 6 = 3^x \cdot 13$

$\frac{5^x}{3^x} = \frac{13}{6}$

$\left(\frac{5}{3}\right)^x = \frac{13}{6}$

$\hookrightarrow \log\left(\frac{5}{3}\right)^x = \log\frac{13}{6}$

$x \log\left(\frac{5}{3}\right) = \log\frac{13}{6}$

$x(\log 5 - \log 3) = \log 13 - (\log 2 + \log 3)$

$x = \frac{\log 13 - (\log 2 + \log 3)}{\log 5 - \log 3} \approx 1{,}514$

f) $2^{3x+2} \cdot 3^{2x-1} = 8$

$2^{3x+2} \cdot 3^{2x-1} = 2^3$

$3^{2x-1} = \frac{2^3}{2^{3x+2}}$

$3^{2x-1} = 2^{1-3x}$

$\log(3^{2x-1}) = \log(2^{1-3x})$

$(2x-1)\log 3 = (1-3x)\log 2$

$(2\log 3)x - \log 3 = \log 2 - (3\log 2)x$

$[(2\log 3) + (3\log 2)]x = \log 2 + \log 3$

$x = \frac{\log 2 + \log 3}{2\log 3 + 3\log 2} \approx 0{,}419$

spiral

5.6) a) $\begin{cases} xy = 16 \\ \log_2 x = 2 + \log_2 y \end{cases}$

$y = \frac{16}{x}$

$\to \log_2 x = 2 + \log_2 \frac{16}{x}$

$\log_2 x = 2 + \log_2 16 - \log_2 x$

$2\log_2 x = 2 + 4$

$2\log_2 x = 6$

$\log_2 x = 3$

$x = 8 \quad y = 2$

b) $\begin{cases} \log\sqrt{x} - \log\sqrt{y} = \log 3 \\ 9y^3 - x^2 = 90y \end{cases}$ $\quad \sqrt{y} > 0$

$y > 0$

$\log\sqrt{x} - \log\sqrt{y} = \log 3$

$\log\sqrt{\frac{x}{y}} = \log 3$

$\sqrt{\frac{x}{y}} = 3 \to \frac{x}{y} = 9 \to x = 9y$

$\to 9y^3 - (9y)^2 = 90y$

$9y^3 - 81y^2 - 90y = 0$

$9y(y^2 - 9y - 10) = 0$

$y = 0 \quad | \quad y^2 - 9y - 10 = 0$

$(y-10)(y+1) = 0$

$y = 10 \quad | \quad \cancel{y = -1}$

R: $y = 10$

5.7) a) $\log_3 (x^2-4x) < \log_{\sqrt{3}} (3\sqrt{5})$

$\log_3(x^2-4x) < \dfrac{\log_3(3\sqrt{5})}{\log_3(3^{1/2})}$

$\log_3(x^2-4x) < \dfrac{\log_3(3\sqrt{5})}{\frac{1}{2}(\cancel{\log_3 3})}$

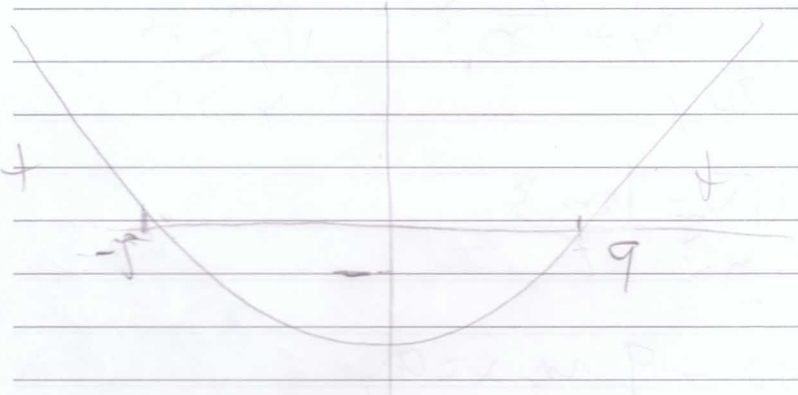
$\log_3(x^2-4x) < 2\log_3(3\sqrt{5})$

$\log_3(x^2-4x) < \log_3(45)$

↳ $x^2-4x < 45$

$x^2-4x-45 < 0$

$(x-9)(x+5) < 0$



R: $\{x \in \mathbb{R} \mid -5 < x < 9\}$

b) $\log_{1/3}(x^2-13) \geq -1$

$\dfrac{\log_3(x^2-13)}{\log_3\left(\frac{1}{3}\right)} \geq -1$

$\dfrac{\log_3(x^2-13)}{\cancel{\log_3 1} - \cancel{\log_3 3}} \geq -1$

$\log_3(x^2-13) \leq 1$

$\log_3(x^2-13) \leq \log_3 3$

↳ $x^2-13 \leq 3$

$x^2 \leq 16$

$x \leq 4$ ou $x \geq -4$

R: $\{x \in \mathbb{R} \mid [-4, 4]\}$

Cond. Existência

$x^2-13 > 0$

$x^2 > 13$

$x > \sqrt{13}$ ou

$x < -\sqrt{13}$

12/14

c) $\log(x+2) + \log(x+3) > \log 12$

$\log[(x+2)(x+3)] > \log 12$

$\log(x^2+5x+6) > \log 12$

$\to x^2+5x+6 > 12$

$x^2+5x-6 > 0$

$\Delta = 5^2 - 4(1)(-6)$

$\Delta = 49$

$x = \dfrac{-5 \pm \sqrt{49}}{2} \to \dfrac{-5-7}{2} = -6$

$\to \dfrac{-5+7}{2} = 1$

$(x+6)(x-1) > 0$

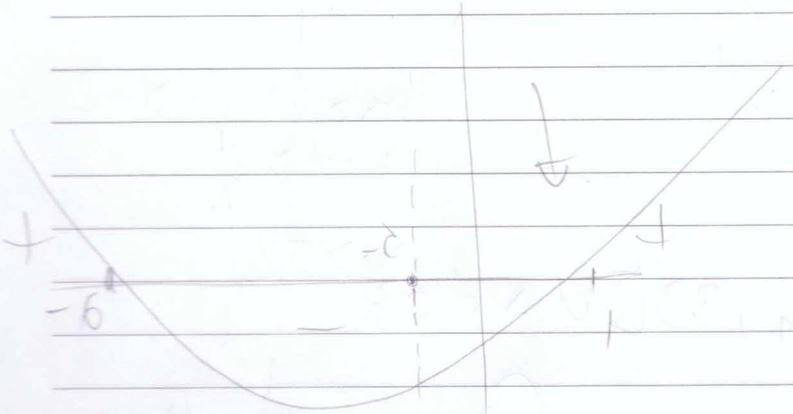
Cond. Existência

$x+2>0$

$x>-2$ ✓

$x+3>0$

$x>-3$ ✓



R: $\{x \in \mathbb{R} \mid x > 1\}$

d) $\log_{(x^2-8)} 11 < \log_{(x^2-8)} 21$

Para $x>3$ ou $x<-3$: OK

Para $\sqrt{8} < x < 3$ ou $-\sqrt{8} > x > -3$: $\nexists x$ que dá

Para $x \leq \sqrt{8}$ e $x \geq -\sqrt{8}$: log não existe!

Cond. Existência:

$x^2-8>0$

$x^2>8$

$x>\sqrt{8}$ ou $x<-\sqrt{8}$

$x^2-8 \neq 1$

$x^2 \neq 9$

$x \neq 3$ ou $x \neq -3$

R: $\{x \in \mathbb{R} \mid x>3 \text{ ou } x<-3\}$

e) $\log_2(x-1) < 3 + 10\log_{(x-1)} 2$

Cond. existência
$x-1>0$
$x>1$
$x-1 \neq 1$
$x \neq 2$

$\log_2(x-1) < 3\cancel{\log_2 2} + 10\dfrac{\cancel{\log_2 2}}{\log_2(x-1)}$

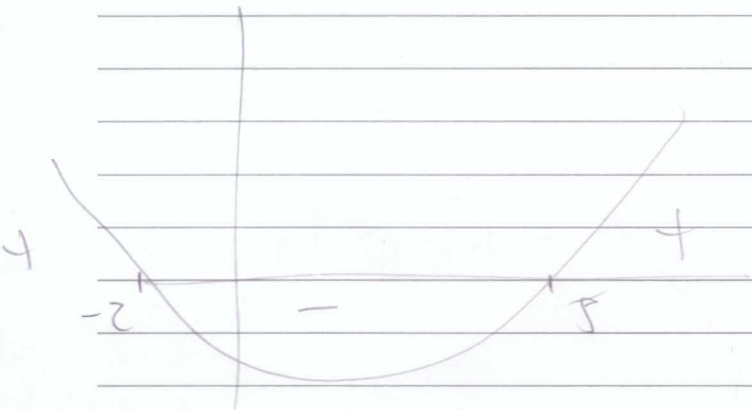
$\log_2(x-1) = K$

$K < 3 + \dfrac{10}{K}$

$K^2 < 3K + 10$

$K^2 - 3K - 10 < 0$

$(K-5)(K+2) < 0$



$\log_2(x-1) < 5$
$x-1 < 2^5$
$x-1 < 32$
$x < 33$

$\log_2(x-1) > -2$
$x-1 > \dfrac{1}{4}$
$x > 5/4$

R: $\{x \in \mathbb{R} \mid]5/4, 33[\cup x \neq 2\}$

f) $x^{\log_3 x} > x^2$

Cond. Existência
$x > 0$

$\log(x^{\log_3 x}) > \log(x^2)$

$\log_3 x \cdot \cancel{\log x} > 2\cancel{\log x}$

$\log_3 x > 2$
$x > 3^2$
$x > 9$

Para $0 < x < 1$

$x^{-\log_3 x} < x^2$

$\dfrac{1}{x^{\log_3 x}} < x^2$

$1 < x^{(\log_3 x + 2)}$

$x^{(\log_3 x + 2)} > 1$
$x^{(\log_3 x + 2)} > x^0$
$\log_3 x + 2 > 0$
$\log_3 x > -2$
$x > 1/9$

R: $\{x \in \mathbb{R} \mid 1/9 < x < 1 \cup x > 9\}$