Exercícios 11.18 - 11.19

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Exercício 11.18: Sendo $u = (3, -1, -2), v = (2,4,-1)$ e $w = (-1,0,1)$, calcule:

a) $[u, u, v]$
b) $[u, v, \omega]$
c) $[v, \omega, u]$
d) $u \cdot (\omega \times v)$
e) $(\omega \times u) \cdot v$

a) $[u, u, v] = (u \times u) \cdot v$

sabendo que

$u \times u = \vec{0} = (0,0,0)$

temos que

$(0,0,0) \cdot (2,4,1) = 0.2 + 0.4 + 0.1 = 0$

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b) $= [u, v, w] = (u \times v) \cdot w$

$$u \times v = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} & \hat{x} & \hat{y} \\ 3 & -1 & -2 & 3 & -1 \\ 2 & 4 & -1 & 2 & 4 \end{vmatrix} = \hat{x} - 4\hat{y} + 12\hat{z} + 2\hat{z} + 8\hat{x} + 3\hat{y}$$

$$= 9\hat{x} - \hat{y} + 14\hat{z}$$

$+2\hat{z} + 8\hat{x} + 3\hat{y} + \hat{x} - 4\hat{y} + 12\hat{z}$

$(u \times v) \cdot w = (9, -1, 14) \cdot (-1, 0, 1)$

$= 9.(-1) + 1(0) + 14.1 = -9 + 14 = 6$

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c) $[v, w, u] = (v \times w) \cdot u$

$$v \times w = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} & \hat{x} & \hat{y} \\ 2 & 4 & -1 & 2 & 4 \\ -1 & 0 & 1 & -1 & 0 \end{vmatrix} = 4\hat{x} + \hat{y} + 4\hat{z} - 2\hat{y}$$

$$= 4\hat{x} - \hat{y} + 4\hat{z}$$

$+4\hat{z} + 0 - 2\hat{y} + 4\hat{x} + \hat{y} + 0$

$(v \times w) \cdot u = (4, -1, 4) \cdot (3, -1, -2)$

$= 4.3 + (-1)(-1) + 4(-2) = 12 + 1 - 8 = 5$

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d) $u \cdot (w \times v)$

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d) $u \cdot (w \times v)$

$$w \times v = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} & \hat{x} & \hat{y} \\ -1 & 0 & 1 & -1 & 0 \\ 2 & 4 & -1 & 2 & 4 \end{vmatrix} = 2\hat{y} - 4\hat{z} - 4\hat{x} - \hat{y}$$

$$= -4\hat{x} + \hat{y} - 4z$$

$0 - 4\hat{x} - \hat{y} + 0 + 2\hat{y} - 4\hat{z}$

$$u \cdot (w \times v) = (3,-1,-2) \cdot (-4,1,-4)$$

$$= 3(-4) + (-1).1 + (-2)(-4) = -12 - 1 + 8 = -5$$

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e) $(w \times u) \cdot v$

$$w \times v = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} & \hat{x} & \hat{y} \\ -1 & 0 & 1 & -1 & 0 \\ 3 & -1 & -2 & 3 & -1 \end{vmatrix} = 3\hat{y} + \hat{z} + \hat{x} - 2\hat{y}$$

$$= \hat{x} + \hat{y} + \hat{z}$$

$+0 + \hat{x} - 2\hat{y} + 0 + 3\hat{y} + \hat{z}$

$$(w \times v) \cdot v = (1,1,1) \cdot (2,4,-1)$$

$$= 2 + 4 - 1 = 5$$

Exercício 11.19: Prove que os vetores $u = (1,0,0)$, $v = (0,2,0)$ e $w = (2,4,0)$ são coplanares por meio do produto misto $(u \times v) \cdot w$.

$$u \times v = \hat{x} \times 2\hat{y} = 2\,\hat{x} \times \hat{y} = +2\hat{z}$$

$$(u \times v) \cdot w = (0,0,2) \cdot (2,4,0)$$

$$= 0.2 + 0.4 + 0 = 0$$