

**FAETERJ-Rio**  
**Cálculo I**  
**Professor DSc. Wagner Zanco**

**Solução dos Exercícios 2.13 – 2.23**

**Exercício 2.13:** Calcule as derivadas

a)  $(x^4 - 3x^2 + 5x - 2)^3$

$$= 3(x^4 - 3x^2 + 5x - 2)^2(4x^3 - 6x + 5)$$

b)  $\sqrt{7x^3 - 2x^2 + 5}$

$$\sqrt{7x^3 - 2x^2 + 5} = (7x^3 - 2x^2 + 5)^{\frac{1}{2}}$$

$$Dx\left((7x^3 - 2x^2 + 5)^{\frac{1}{2}}\right) = \frac{1}{2}(7x^3 - 2x^2 + 5)^{-\frac{1}{2}}(21x^2 - 4x)$$

$$= \frac{1}{2(7x^3 - 2x^2 + 5)^{\frac{1}{2}}}(21x^2 - 4x) = \frac{21x^2 - 4x}{2\sqrt{7x^3 - 2x^2 + 5}}$$

c)  $\frac{1}{(5x^2 + 4)^3}$

$$\frac{1}{(5x^2 + 4)^3} = (5x^2 + 4)^{-3}$$

$$Dx((5x^2 + 4)^{-3}) = -3(5x^2 + 4)^{-4}(10x)$$

$$= -30x \frac{1}{(5x^2 + 4)^4} = -\frac{30x}{(5x^2 + 4)^4}$$

d)  $\sqrt{1 + \sqrt{x + 1}}$

$$\sqrt{1 + \sqrt{x + 1}} = \sqrt{1 + (x + 1)^{\frac{1}{2}}} = \left(1 + (x + 1)^{\frac{1}{2}}\right)^{\frac{1}{2}}$$

$$Dx\left(\left(1 + (x + 1)^{\frac{1}{2}}\right)^{\frac{1}{2}}\right) = \frac{1}{2}\left(1 + (x + 1)^{\frac{1}{2}}\right)^{-\frac{1}{2}}\left(\frac{1}{2}(x + 1)^{-\frac{1}{2}} \cdot x\right)$$

$$\begin{aligned}
&= \frac{1}{2 \left( 1 + (x+1)^{\frac{1}{2}} \right)^{\frac{1}{2}}} \left( \frac{x}{2(x+1)^{\frac{1}{2}}} \right) \\
&= \frac{1}{2\sqrt{1 + \sqrt{x+1}}} \frac{x}{2\sqrt{x+1}} = \frac{x}{4\sqrt{x+1}\sqrt{1 + \sqrt{x+1}}} \\
&= \frac{x}{4\sqrt{1 + \sqrt{x+1}}(x+1)}
\end{aligned}$$

**Exercício 2.14:** Encontre os extremos absolutos de  $f(x) = x \cdot \sqrt{1 - x^2}$ , em  $[0, 1]$ .

$$f(x) = x \cdot \sqrt{1 - x^2} = \sqrt{(1 - x^2)x^2} = \sqrt{x^2 - x^4} = (x^2 - x^4)^{\frac{1}{2}}$$

$$f'(x) = \frac{1}{2}(x^2 - x^4)^{-\frac{1}{2}}(2x - 4x^3)$$

$$= \frac{1}{2(x^2 - x^4)^{\frac{1}{2}}}(2x - 4x^3) = \frac{2x - 4x^3}{2(x^2 - x^4)^{\frac{1}{2}}} = \frac{2x - 4x^3}{2\sqrt{x^2 - x^4}}$$

$$\frac{2x - 4x^3}{2\sqrt{x^2 - x^4}} = 0$$

$$2x - 4x^3 = 0$$

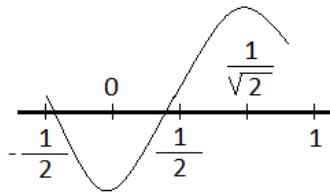
$$x' = 0; \quad x'' = \frac{1}{\sqrt{2}}; \quad x''' = -\frac{1}{\sqrt{2}}$$

$$f'\left(-\frac{1}{2}\right) = \frac{2\left(-\frac{1}{2}\right) - 4\left(-\frac{1}{2}\right)^3}{2\sqrt{\left(-\frac{1}{2}\right)^2 - \left(-\frac{1}{2}\right)^4}} = \frac{-1 + 4\frac{1}{8}}{2\sqrt{\frac{1}{4} - \frac{1}{16}}} = \frac{-1 + \frac{1}{2}}{2\sqrt{\frac{4-1}{16}}} = \frac{-\frac{1}{2}}{2\sqrt{\frac{3}{16}}} = -\frac{1}{2} \frac{1}{2\frac{\sqrt{3}}{\sqrt{16}}}$$

$$= -\frac{1}{4\frac{\sqrt{3}}{4}} = -\frac{1}{\sqrt{3}} \text{ (decrecente)}$$

$$f'\left(\frac{1}{2}\right) = \frac{2\left(\frac{1}{2}\right) - 4\left(\frac{1}{2}\right)^3}{2\sqrt{\left(\frac{1}{2}\right)^2 - \left(\frac{1}{2}\right)^4}} = \frac{1 - 4\frac{1}{8}}{2\sqrt{\frac{1}{4} - \frac{1}{16}}} = \frac{1 - \frac{1}{2}}{2\sqrt{\frac{4-1}{16}}} = \frac{\frac{1}{2}}{2\sqrt{\frac{3}{16}}} = \frac{1}{2} \frac{1}{2\frac{\sqrt{3}}{\sqrt{16}}} = \frac{1}{4\frac{\sqrt{3}}{4}}$$

$$= \frac{1}{\sqrt{3}} \text{ (crescente)}$$



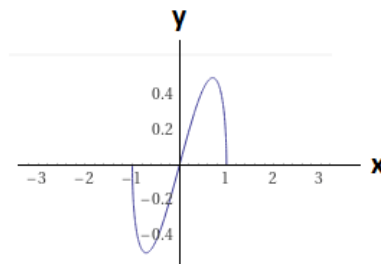
$$f(0) = 0.5 \cdot \sqrt{1 - 0^2} = 0$$

$$f\left(\frac{1}{\sqrt{2}}\right) = \frac{1}{\sqrt{2}} \cdot \sqrt{1 - \left(\frac{1}{\sqrt{2}}\right)^2} = \frac{1}{\sqrt{2}} \cdot \sqrt{1 - \frac{1^2}{\sqrt{2}^2}} = \frac{1}{\sqrt{2}} \cdot \sqrt{1 - \frac{1}{2}} = \frac{1}{\sqrt{2}} \cdot \sqrt{\frac{1}{2}}$$

$$= \frac{1}{\sqrt{2}} \cdot \sqrt{\frac{1}{2}} = \frac{1}{\sqrt{2}} \cdot \frac{1}{\sqrt{2}} = \frac{1}{2}$$

$$f(1) = 1 \cdot \sqrt{1 - 1^2} = 0$$

Máximo  $(\frac{1}{\sqrt{2}}, \frac{1}{2})$ ; Mínimo  $(0, 0)$  e  $(1, 0)$



**Exercício 2.15:** Para cada par de funções  $f(x)$  e  $g(x)$ , encontre fórmulas para  $(f \circ g)(x)$  e  $(g \circ f)(x)$ .

a)  $f(x) = \frac{2}{x+1}$ ,  $g(x) = 3x$

$$(f \circ g)(x) = \frac{2}{3x+1}$$

$$(g \circ f)(x) = 3\left(\frac{2}{x+1}\right) = \frac{6}{x+1}$$

b)  $f(x) = x^2 + 2x - 5$ ,  $g(x) = x^3$

$$(f \circ g)(x) = (x^3)^2 + 2(x^3) - 5 = x^6 + 2x^3 - 5$$

$$(g \circ f)(x) = (x^2 + 2x - 5)^3$$

$$c) f(x) = 2x^3 - x^2 + 4, g(x) = 3$$

$$(f \circ g)(x) = 2 \cdot 3^3 - 3^2 + 4 = 54 - 9 + 4 = 49$$

$$(g \circ f)(x) = 3$$

$$d) f(x) = x^3, g(x) = x^2$$

$$(f \circ g)(x) = (x^2)^3 = x^6$$

$$(g \circ f)(x) = (x^3)^2 = x^6$$

$$e) f(x) = \frac{1}{x}, g(x) = \frac{1}{x}$$

$$(f \circ g)(x) = \frac{1}{\frac{1}{x}} = x$$

$$(g \circ f)(x) = \frac{1}{\frac{1}{x}} = x$$

$$f) f(x) = x, g(x) = x^2 - 4$$

$$(f \circ g)(x) = x^2 - 4$$

$$(g \circ f)(x) = x^2 - 4$$

**Exercício 2.16:** Para cada par de funções encontre o conjunto de soluções da equação  $(f \circ g)(x) = (g \circ f)(x)$ .

$$a) f(x) = x^3; g(x) = x^6$$

$$(f \circ g)(x) = (x^6)^3 = x^{18}$$

$$(g \circ f)(x) = x^{18}$$

$$S = \{x \in \mathbb{R}\}$$

$$b) f(x) = \frac{2}{x+1}; g(x) = 3x$$

$$(f \circ g)(x) = \frac{2}{3x+1}$$

$$(g \circ f)(x) = 3 \frac{2}{x+1} = \frac{6}{x+1}$$

$$\frac{2}{3x+1} = \frac{6}{x+1}$$

$$2(x+1) = 6(3x+1)$$

$$x + 1 = \frac{6(3x + 1)}{2} = 3(3x + 1)$$

$$x + 1 = 9x + 3$$

$$1 - 3 = 9x - x$$

$$-2 = 8x$$

$$x = -\frac{1}{4}$$

$$\text{c) } f(x) = 2x; \quad g(x) = \frac{1}{x-1}$$

$$(f \circ g)(x) = \frac{2}{x-1}$$

$$(g \circ f)(x) = \frac{1}{2x-1}$$

$$\frac{2}{x-1} = \frac{1}{2x-1}$$

$$2(2x-1) = x-1$$

$$4x-2 = x-1$$

$$4x-x = 2-1$$

$$3x = 2$$

$$x = \frac{1}{3}$$

$$\text{d) } f(x) = x^2; \quad g(x) = \frac{1}{x+1}$$

$$(f \circ g)(x) = \left(\frac{1}{x+1}\right)^2 = \frac{1}{(x+1)^2} = \frac{1}{x^2 + 2x + 1}$$

$$(g \circ f)(x) = \frac{1}{x^2 + 1}$$

$$\frac{1}{x^2 + 2x + 1} = \frac{1}{x^2 + 1}$$

$$x^2 + 1 = x^2 + 2x + 1$$

$$1 = 2x + 1$$

$$2x = 0$$

$$x = 0$$

$$\text{e) } f(x) = x^2; \quad g(x) = \frac{1}{x^3-3}$$

$$(f \circ g)(x) = \left( \frac{1}{x^3-3} \right)^2 = \frac{1}{(x^3-3)^2} = \frac{1}{x^6-6x^3+9}$$

$$(g \circ f)(x) = \frac{1}{(x^2)^3-3} = \frac{1}{x^6-3}$$

$$\frac{1}{x^6-6x^3+9} = \frac{1}{x^6-3}$$

$$x^6-3 = x^6-6x^3+9$$

$$-3 = -6x^3+9$$

$$6x^3 = 3+9$$

$$6x^3 = 12$$

$$x^3 = 2$$

$$x = \sqrt[3]{2}$$

**Exercício 2.17:** Expresse cada uma das seguintes funções como a composição  $(g \circ f)(x)$  de duas funções mais simples.

$$\text{a) } (x^3 - 2x + 2)^4$$

$$g(f(x)) = (x^3 - 2x + 2)^4$$

$$f(x) = x^3 - 2x + 2$$

$$g(x) = x^4$$

$$\text{b) } (8 - x)^4$$

$$g(f(x)) = (8 - x)^4$$

$$f(x) = 8 - x$$

$$g(x) = x^4$$

c)  $\sqrt{1 + x^2}$

$$g(f(x)) = \sqrt{1 + x^2}$$

$$f(x) = 1 + x^2$$

$$g(x) = \sqrt{x}$$

d)  $\frac{1}{x^2 - 4}$

$$g(f(x)) = \frac{1}{x^2 - 4}$$

$$f(x) = x^2 - 4$$

$$g(x) = \frac{1}{x}$$

**Exercício 2.18:** Encontre as derivadas das seguintes funções

a)  $(x^3 - 2x^2 + 7x - 3)^4$

$$Dx((x^3 - 2x^2 + 7x - 3)^4) = 4(3x^2 - 4x + 7)^3(3x^2 - 4x + 7)$$

b)  $(7 - 3x)^5$

$$Dx((7 - 3x)^5) = 5(7 - 3x)^4(-3) = -15(7 - 3x)^4$$

c)  $(2x - 3)^{-2}$

$$Dx((2x - 3)^{-2}) = -2(2x - 3)^{-3}(2)$$

$$= -\frac{4}{(2x - 3)^3}$$

d)  $(3x^2 + 5)^{-3}$

$$Dx((3x^2 + 5)^{-3}) = -2(3x^2 + 5)^{-3}(6x)$$

$$= -\frac{12x}{(3x^2 + 5)^4}$$

e)  $(4x^2 - 3)^2 \cdot (x + 5)^3$

$$Dx((4x^2 - 3)^2 \cdot (x + 5)^3) = Dx((4x^2 - 3)^2)(x + 5)^3 + (4x^2 - 3)^2 Dx((x + 5)^3)$$

$$Dx((4x^2 - 3)^2) = 2(4x^2 - 3)(8x) = 16x(4x^2 - 3)$$

$$Dx((x + 5)^3) = 3(x + 5)^2 \cdot (1) = 3(x + 5)^2$$

$$Dx((4x^2 - 3)^2 \cdot (x + 5)^3) = 16x(4x^2 - 3)(x + 5)^3 + (4x^2 - 3)^2 3(x + 5)^2$$

$$= 16x(4x^2 - 3)(x + 5)^3 + 3(4x^2 - 3)^2(x + 5)^2$$

f)  $\left(\frac{x^2+2}{x-3}\right)^3$

$$Dx\left(\left(\frac{x^2+2}{x-3}\right)^3\right) = 3\left(\frac{x^2+2}{x-3}\right)^2 Dx\left(\frac{x^2+2}{x-3}\right)$$

$$Dx\left(\frac{x^2+2}{x-3}\right) = \frac{(x-3)(2x) - (1)(x^2+2)}{(x-3)^2} = \frac{2x(x-3) - (x^2+2)}{(x-3)^2}$$

$$= \frac{2x^2 - 6x - x^2 - 2}{(x-3)^2} = \frac{x^2 - 6x - 2}{(x-3)^2}$$

$$Dx\left(\left(\frac{x^2+2}{x-3}\right)^3\right) = 3\left(\frac{x^2+2}{x-3}\right)^2 \left(\frac{x^2 - 6x - 2}{(x-3)^2}\right)$$

$$= \frac{3(x^2+2)^2(x^2 - 6x - 2)}{(x-3)^4}$$

g)  $\left(\frac{x^2-2}{2x^2+1}\right)^2$

$$Dx\left(\left(\frac{x^2-2}{2x^2+1}\right)^2\right) = 2\left(\frac{x^2-2}{2x^2+1}\right) Dx\left(\frac{x^2-2}{2x^2+1}\right)$$

$$Dx\left(\frac{x^2-2}{2x^2+1}\right) = \frac{(2x^2+1)(2x) - (4x)(x^2-2)}{(2x^2+1)^2}$$

$$= \frac{4x^3 + 2x - 4x^3 + 8x}{(2x^2+1)^2} = \frac{10x}{(2x^2+1)^2}$$

$$Dx\left(\left(\frac{x^2-2}{2x^2+1}\right)^2\right) = 2\left(\frac{x^2-2}{2x^2+1}\right) \left(\frac{10x}{(2x^2+1)^2}\right)$$

$$= 2\left(\frac{(x^2-2)10x}{(2x^2+1)^3}\right) = \frac{20(x^2-2)}{(2x^2+1)^3}$$

h)  $\frac{4}{3x^2-x+5}$

$$Dx\left(\frac{4}{3x^2-x+5}\right) = \frac{(3x^2-x+5)(0) - (6x-1)(4)}{(3x^2-x+5)^2}$$



$$= \frac{4(-6x + 1)}{(3x^2 - x + 5)^2}$$

i)  $\sqrt{1+x^3}$

$$\begin{aligned} Dx \left( (1+x^3)^{\frac{1}{2}} \right) &= \frac{1}{2} (1+x^3)^{-\frac{1}{2}} (3x^2) \\ &= \frac{3x^2}{2(1+x^3)^{\frac{1}{2}}} = \frac{3x^2}{2\sqrt{1+x^3}} \end{aligned}$$

**Exercício 2.19:** Encontre as derivadas das seguintes funções

a)  $2x^{\frac{3}{4}}$

$$Dx \left( 2x^{\frac{3}{4}} \right) = 2 \frac{3}{4} x^{-\frac{1}{4}} = \frac{3}{2x^{\frac{1}{4}}} = \frac{3}{2\sqrt[4]{x}}$$

b)  $x^2(1-3x^3)^{\frac{1}{3}}$

$$\begin{aligned} Dx \left( x^2(1-3x^3)^{\frac{1}{3}} \right) &= x^2 Dx \left( (1-3x^3)^{\frac{1}{3}} \right) + 2x(1-3x^3)^{\frac{1}{3}} \\ Dx \left( (1-3x^3)^{\frac{1}{3}} \right) &= \frac{1}{3} (1-3x^3)^{-\frac{2}{3}} (-9x^2) = \frac{-9x^2}{3(1-3x^3)^{\frac{2}{3}}} \\ &= -\frac{3x^2}{(1-3x^3)^{\frac{2}{3}}} \\ Dx \left( x^2(1-3x^3)^{\frac{1}{3}} \right) &= x^2 \left( \frac{-3x^2}{(1-3x^3)^{\frac{2}{3}}} \right) + 2x(1-3x^3)^{\frac{1}{3}} \\ &= \frac{-3x^4}{(1-3x^3)^{\frac{2}{3}}} + 2x(1-3x^3)^{\frac{1}{3}} \\ &= \frac{-3x^4 + 2x(1-3x^3)^{\frac{1}{3}}(1-3x^3)^{\frac{2}{3}}}{(1-3x^3)^{\frac{2}{3}}} = \frac{-3x^4 + 2x(1-3x^3)^{\frac{1}{3}+\frac{2}{3}}}{(1-3x^3)^{\frac{2}{3}}} \\ &= \frac{-3x^4 + 2x(1-3x^3)}{(1-3x^3)^{\frac{2}{3}}} = \frac{-3x^4 + 2x - 6x^4}{(1-3x^3)^{\frac{2}{3}}} \\ &= \frac{2x - 9x^4}{\sqrt[3]{(1-3x^3)^2}} \end{aligned}$$

c)  $\frac{x}{\sqrt{x^2+1}}$

$$Dx \left( \frac{x}{(x^2+1)^{\frac{1}{2}}} \right) = \left( \frac{(x^2+1)^{\frac{1}{2}}(1) - Dx \left( (x^2+1)^{\frac{1}{2}} \right) x}{\left( (x^2+1)^{\frac{1}{2}} \right)^2} \right)$$

$$Dx \left( (x^2+1)^{\frac{1}{2}} \right) = \frac{1}{2} (x^2+1)^{-\frac{1}{2}} \cdot (2x) = \frac{2x}{2(x^2+1)^{\frac{1}{2}}} = \frac{x}{(x^2+1)^{\frac{1}{2}}}$$

$$Dx \left( \frac{x}{(x^2+1)^{\frac{1}{2}}} \right) = \left( \frac{(x^2+1)^{\frac{1}{2}}(1) - \frac{x}{(x^2+1)^{\frac{1}{2}}} x}{\left( (x^2+1)^{\frac{1}{2}} \right)^2} \right)$$

$$= \left( \frac{(x^2+1)^{\frac{1}{2}} - \frac{x^2}{(x^2+1)^{\frac{1}{2}}}}{\left( (x^2+1)^{\frac{1}{2}} \right)^2} \right) = \left( \frac{\frac{\left( (x^2+1)^{\frac{1}{2}} \right)^2 - x^2}{(x^2+1)^{\frac{1}{2}}}}{\left( (x^2+1)^{\frac{1}{2}} \right)^2} \right)$$

$$= \left( \frac{\frac{(x^2+1) - x^2}{(x^2+1)^{\frac{1}{2}}}}{x^2+1} \right) = \frac{1}{(x^2+1)^{\frac{1}{2}}} \frac{1}{(x^2+1)} = \frac{1}{(x^2+1)^{\frac{3}{2}}} = \frac{1}{\sqrt{\sqrt{(x^2+1)^3}}}$$

d)  $(7x^3 - 4x^2 + 2)^{\frac{1}{4}}$

$$Dx \left( (7x^3 - 4x^2 + 2)^{\frac{1}{4}} \right) = \frac{1}{4} (7x^3 - 4x^2 + 2)^{-\frac{3}{4}} (21x^2 - 8x)$$

$$= \frac{21x^2 - 8x}{4(7x^3 - 4x^2 + 2)^{\frac{3}{4}}} = \frac{21x^2 - 8x}{4\sqrt[4]{(7x^3 - 4x^2 + 2)^3}}$$

e)  $\frac{\sqrt{x+2}}{\sqrt{x-1}}$

$$Dx \left( \frac{(x+2)^{\frac{1}{2}}}{(x-1)^{\frac{1}{2}}} \right) = \frac{(x-1)^{\frac{1}{2}} Dx \left( (x+2)^{\frac{1}{2}} \right) - Dx \left( (x-1)^{\frac{1}{2}} \right) (x+2)^{\frac{1}{2}}}{(x-1)^{\frac{1}{2}}^2}$$

$$Dx \left( (x+2)^{\frac{1}{2}} \right) = \frac{1}{2} (x+2)^{-\frac{1}{2}} (1) = \frac{1}{2(x+2)^{\frac{1}{2}}}$$

$$Dx \left( (x-1)^{\frac{1}{2}} \right) = \frac{1}{2} (x-1)^{-\frac{1}{2}} (1) = \frac{1}{2(x-1)^{\frac{1}{2}}}$$

$$\begin{aligned} Dx \left( \frac{(x+2)^{\frac{1}{2}}}{(x-1)^{\frac{1}{2}}} \right) &= \frac{(x-1)^{\frac{1}{2}} \frac{1}{2(x+2)^{\frac{1}{2}}} - \frac{1}{2(x-1)^{\frac{1}{2}}} (x+2)^{\frac{1}{2}}}{x-1} \\ &= \frac{\frac{(x-1)^{\frac{1}{2}}}{2(x+2)^{\frac{1}{2}}} - \frac{(x+2)^{\frac{1}{2}}}{2(x-1)^{\frac{1}{2}}}}{x-1} = \frac{\frac{2(x-1)^{\frac{1}{2}}(x-1)^{\frac{1}{2}} - 2(x+2)^{\frac{1}{2}}(x+2)^{\frac{1}{2}}}{2(x+2)^{\frac{1}{2}}2(x-1)^{\frac{1}{2}}}}{x-1} \\ &= \frac{\frac{2(x-1) - 2(x+2)}{4 \left( (x+2)^{\frac{1}{2}}(x-1)^{\frac{1}{2}} \right)}}{x-1} = \frac{\frac{2x-2-2x-4}{4 \left( (x+2)^{\frac{1}{2}}(x-1)^{\frac{1}{2}} \right)}}{x-1} = \frac{\frac{-6}{4 \left( (x+2)^{\frac{1}{2}}(x-1)^{\frac{1}{2}} \right)}}{x-1} \\ &= \frac{-6}{4(x+2)^{\frac{1}{2}}(x-1)^{\frac{1}{2}} x-1} = -\frac{3}{2(x+2)^{\frac{1}{2}}(x-1)^{\frac{1}{2}}(x-1)} \\ &= -\frac{3}{2(x-1)^{\frac{3}{2}}(x+2)^{\frac{1}{2}}} = \frac{3}{2\sqrt{(x-1)^3}\sqrt{x+2}} \end{aligned}$$

f)  $8x^{\frac{3}{4}} + 4x^{\frac{1}{4}} - x^{-\frac{1}{3}}$

$$\begin{aligned} Dx \left( 8x^{\frac{3}{4}} + 4x^{\frac{1}{4}} - x^{-\frac{1}{3}} \right) &= 8 \frac{3}{4} x^{\frac{-1}{4}} + 4 \frac{1}{4} x^{\frac{-3}{4}} + \frac{1}{3} x^{\frac{-4}{3}} \\ &= \frac{6}{x^{\frac{1}{4}}} + \frac{1}{x^{\frac{3}{4}}} + \frac{1}{3x^{\frac{4}{3}}} = \frac{6}{\sqrt[4]{x}} + \frac{1}{\sqrt[4]{x^3}} + \frac{1}{3\sqrt[4]{x^3}} \end{aligned}$$

g)  $\sqrt[3]{(4x^2 + 3)^2}$

$$Dx \left( (4x^2 + 3)^{\frac{2}{3}} \right) = \frac{2}{3} (4x^2 + 3)^{-\frac{1}{3}} (8x) = \frac{16x}{3(4x^2 + 3)^{\frac{1}{3}}} = \frac{16x}{3\sqrt[3]{4x^2 + 3}}$$

$$\text{h) } \sqrt{\frac{4}{x}}$$

$$f(x) = \sqrt{\frac{4}{x}} = \left(\frac{4}{x}\right)^{\frac{1}{2}} = (4x^{-1})^{\frac{1}{2}} = 4^{\frac{1}{2}}x^{-\frac{1}{2}}$$

$$Dx\left(4^{\frac{1}{2}}x^{-\frac{1}{2}}\right) = 4^{\frac{1}{2}}Dx\left(x^{-\frac{1}{2}}\right)$$

$$Dx\left(x^{-\frac{1}{2}}\right) = -\frac{1}{2}x^{-\frac{3}{2}} = -\frac{1}{2x^{\frac{3}{2}}}$$

$$Dx\left(4^{\frac{1}{2}}x^{-\frac{1}{2}}\right) = 4^{\frac{1}{2}}\left(-\frac{1}{2x^{\frac{3}{2}}}\right) = -\frac{\sqrt{4}}{2\sqrt{x^3}} = -\frac{2}{2\sqrt{x^3}} = -\frac{1}{\sqrt{x^3}}$$

$$\text{i) } \sqrt{4 - \sqrt{4+x}}$$

$$f(x) = \sqrt{4 - \sqrt{4+x}} = (4 - \sqrt{4+x})^{\frac{1}{2}}$$

$$Dx\left((4 - \sqrt{4+x})^{\frac{1}{2}}\right) = \frac{1}{2}(4 - \sqrt{4+x})^{-\frac{1}{2}}Dx(4 - \sqrt{4+x})$$

$$= \frac{1}{2(4 - \sqrt{4+x})^{\frac{1}{2}}}Dx(4 - \sqrt{4+x})$$

$$Dx(4 - \sqrt{4+x}) = Dx(4) - Dx\left((4+x)^{\frac{1}{2}}\right)$$

$$Dx\left((4+x)^{\frac{1}{2}}\right) = \frac{1}{2}(4+x)^{-\frac{1}{2}}(1) = \frac{1}{2(4+x)^{\frac{1}{2}}}$$

$$Dx(4 - \sqrt{4+x}) = 0 - \frac{1}{2(4+x)^{\frac{1}{2}}} = -\frac{1}{2(4+x)^{\frac{1}{2}}}$$

$$Dx\left((4 - \sqrt{4+x})^{\frac{1}{2}}\right) = \frac{1}{2}(4 - \sqrt{4+x})^{-\frac{1}{2}}\left(-\frac{1}{2(4+x)^{\frac{1}{2}}}\right)$$

$$= \frac{1}{2(4 - \sqrt{4+x})^{\frac{1}{2}}}\left(-\frac{1}{2(4+x)^{\frac{1}{2}}}\right) = -\frac{1}{2(4 - \sqrt{4+x})^{\frac{1}{2}}2(4+x)^{\frac{1}{2}}}$$

$$= -\frac{1}{4\sqrt{4 - \sqrt{4+x}}\sqrt{4+x}}$$

$$j) \frac{(1+x^3)^{\frac{2}{3}}}{1-2x}$$

$$Dx \left( \frac{(1+x^3)^{\frac{2}{3}}}{1-2x} \right) = \frac{(1-2x)Dx \left( (1+x^3)^{\frac{2}{3}} \right) - Dx(1-2x)(1+x^3)^{\frac{2}{3}}}{(1-2x)^2}$$

$$Dx \left( (1+x^3)^{\frac{2}{3}} \right) = \frac{2}{3}(1+x^3)^{-\frac{1}{3}}(3x^2) = \frac{6x^2}{3(1+x^3)^{\frac{1}{3}}}$$

$$Dx(1-2x) = -2$$

$$Dx \left( \frac{(1+x^3)^{\frac{2}{3}}}{1-2x} \right) = \frac{(1-2x) \left( \frac{6x^2}{3(1+x^3)^{\frac{1}{3}}} \right) - (-2)(1+x^3)^{\frac{2}{3}}}{(1-2x)^2}$$

$$= \frac{\frac{6x^2(1-2x)}{3(1+x^3)^{\frac{1}{3}}} + 2(1+x^3)^{\frac{2}{3}}}{(1-2x)^2} = \frac{\frac{6x^2(1-2x) + 2(1+x^3)^{\frac{2}{3}}3(1+x^3)^{\frac{1}{3}}}{3(1+x^3)^{\frac{1}{3}}}}{(1-2x)^2}$$

$$= \frac{6x^2(1-2x) + 2(1+x^3)^{\frac{2}{3}}3(1+x^3)^{\frac{1}{3}}}{3(1+x^3)^{\frac{1}{3}}(1-2x)^2}$$

$$= \frac{6x^2(1-2x) + 6(1+x^3)}{3(1+x^3)^{\frac{1}{3}}(1-2x)^2} = \frac{6[x^2(1-2x) + (1+x^3)]}{3(1+x^3)^{\frac{1}{3}}(1-2x)^2}$$

$$= \frac{2[x^2(1-2x) + (1+x^3)]}{(1+x^3)^{\frac{1}{3}}(1-2x)^2} = \frac{2(x^2 - 2x^3 + 1 + x^3)}{(1+x^3)^{\frac{1}{3}}(1-2x)^2}$$

$$= \frac{2(-x^3 + x^2 + 1)}{(1+x^3)^{\frac{1}{3}}(1-2x)^2} = \frac{2(-x^3 + x^2 + 1)}{\sqrt[3]{1+x^3}(1-2x)^2}$$

**Exercício 2.20:** Determine a equação reduzida da reta tangente ao gráfico  $\frac{\sqrt{x-1}}{x^2+1}$  no ponto  $(2, \frac{1}{5})$

Cálculo o coeficiente angular ( $m$ )

$$m = Dx \left( \frac{\sqrt{x-1}}{x^2+1} \right) = \frac{(x^2+1)Dx \left( (x-1)^{\frac{1}{2}} \right) - 2x \left( (x-1)^{\frac{1}{2}} \right)}{(x^2+1)^2}$$

$$\begin{aligned}
Dx \left( (x-1)^{\frac{1}{2}} \right) &= \frac{1}{2} (x-1)^{-\frac{1}{2}} (1) = \frac{1}{2(x-1)^{\frac{1}{2}}} \\
m = Dx \left( \frac{\sqrt{x-1}}{x^2+1} \right) &= \frac{(x^2+1) \frac{1}{2(x-1)^{\frac{1}{2}}} - 2x \left( (x-1)^{\frac{1}{2}} \right)}{(x^2+1)^2} \\
&= \frac{\frac{(x^2+1)}{2(x-1)^{\frac{1}{2}}} - 2x \left( (x-1)^{\frac{1}{2}} \right)}{(x^2+1)^2} = \frac{\frac{(x^2+1) - 2(x-1)^{\frac{1}{2}} 2x \left( (x-1)^{\frac{1}{2}} \right)}{2(x-1)^{\frac{1}{2}}}}{(x^2+1)^2} \\
&= \frac{(x^2+1) - 2(x-1)^{\frac{1}{2}} 2x \left( (x-1)^{\frac{1}{2}} \right)}{2(x-1)^{\frac{1}{2}}} \frac{1}{(x^2+1)^2} \\
&= \frac{(x^2+1) - 2(x-1)^{\frac{1}{2}} 2x \left( (x-1)^{\frac{1}{2}} \right)}{2(x-1)^{\frac{1}{2}}(x^2+1)^2} = \frac{(x^2+1) - 4x(x-1)}{2(x-1)^{\frac{1}{2}}(x^2+1)^2} \\
&= \frac{x^2+1-4x^2+4x}{2(x-1)^{\frac{1}{2}}(x^2+1)^2} = \frac{-3x^2+4x+1}{2(x-1)^{\frac{1}{2}}(x^2+1)^2} \\
m &= \frac{-3x^2+4x+1}{2(x-1)^{\frac{1}{2}}(x^2+1)^2}
\end{aligned}$$

Em  $x = 2$ , temos que

$$m = \frac{-3 \cdot 2^2 + 4 \cdot 2 + 1}{2(2-1)^{\frac{1}{2}}(2+1)^2} = \frac{-12 + 8 + 1}{2 \cdot 9} = \frac{-3}{18} = -\frac{1}{6}$$

$$y = \frac{\sqrt{2-1}}{2^2+1} = \frac{1}{5}$$

Equação reduzida da reta tangente ao ponto  $\left(2, \frac{1}{5}\right)$ .

$$y = mx + b$$

$$\frac{1}{5} = -\frac{1}{6} \cdot 2 + b$$

$$\frac{1}{5} = -\frac{1}{3} + b$$

$$\frac{1}{5} + \frac{1}{3} = b$$

$$\frac{3+5}{15} = b = \frac{8}{15}$$

$$y = -\frac{1}{6}x + \frac{8}{15}$$

**Exercício 2.21:** Determine a equação reduzida da reta normal ao gráfico  $\sqrt{x^2 + 16}$  nos pontos (3, 5).

$$m = Dx \left( (x^2 + 16)^{\frac{1}{2}} \right) = \frac{1}{2} (x^2 + 16)^{-\frac{1}{2}} (2x) = \frac{2x}{2(x^2 + 16)^{\frac{1}{2}}}$$

Em  $x = 3$ , temos que

$$m = \frac{2 \cdot 3}{2\sqrt{3^2 + 16}} = \frac{6}{2\sqrt{25}} = \frac{6}{10} = \frac{3}{5}$$

$$y = \sqrt{3^2 + 16} = \sqrt{25} = 5$$

Coefficiente angular da reta normal ( $m_2$ ) ao ponto (3, 5)

$$m \cdot m_2 = -1$$

$$m_2 = -\frac{1}{\frac{3}{5}} = -\frac{5}{3}$$

$$y = mx + b$$

$$5 = -\frac{5}{3} \cdot 3 + b$$

$$5 + 5 = b = 10$$

$$y = -\frac{5}{3}x + 10$$

**Exercício 2.22:** Sejam  $g(x) = x^2 - 4$  e  $f(x) = \frac{x+2}{x-2}$ , determine  $(f \circ g)(x)$  e  $(g \circ f)(x)$ .

$$(f \circ g)(x) = \frac{x^2 - 4 + 2}{x^2 - 4 - 2} = \frac{x^2 - 2}{x^2 - 6}$$

$$(g \circ f)(x) = \left( \frac{x+2}{x-2} \right)^2 - 4$$

$$= \frac{(x+2)^2}{(x-2)^2} - 4 = \frac{x^2 + 4x + 4}{x^2 - 4x + 4} - 4 = \frac{x^2 + 4x + 4 - 4x^2 + 16x - 16}{x^2 - 4x + 4}$$

$$= \frac{-3x^2 + 20x - 12}{x^2 - 4x + 4}$$

**Exercício 2.23:** Encontre os pontos críticos das seguintes funções:

a)  $f(x) = \frac{x}{\sqrt{1+x^2}}$ , em  $[-1, 1]$

$$f'(x) = \frac{(1+x^2)^{\frac{1}{2}}(1) - Dx\left((1+x^2)^{\frac{1}{2}}\right)x}{(1+x^2)^{\frac{1}{2}^2}}$$

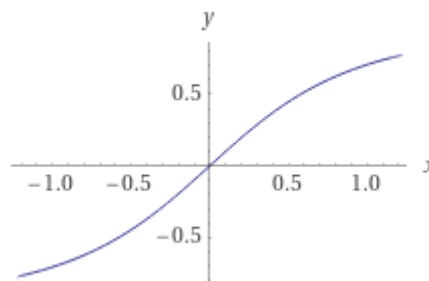
$$Dx\left((1+x^2)^{\frac{1}{2}}\right) = \frac{1}{2}(1+x^2)^{-\frac{1}{2}}(2x) = \frac{2x}{2(1+x^2)^{\frac{1}{2}}} = \frac{x}{\sqrt{1+x^2}}$$

$$f'(x) = \frac{(1+x^2)^{\frac{1}{2}} - \frac{x}{(1+x^2)^{\frac{1}{2}}}x}{1+x^2} = \frac{(1+x^2)^{\frac{1}{2}}(1+x^2)^{\frac{1}{2}} - x^2}{(1+x^2)^{\frac{1}{2}}} = \frac{1+x^2-x^2}{1+x^2}$$

$$= \frac{1+x^2-x^2}{(1+x^2)^{\frac{1}{2}}} = \frac{1}{(1+x^2)^{\frac{1}{2}}} \frac{1}{1+x^2} = \frac{1}{(1+x^2)^{\frac{3}{2}}} = \frac{1}{\sqrt{(1+x^2)^3}}$$

$$(1+x^2)^{-\frac{3}{2}} = 0$$

Não existe ponto crítico no intervalo  $[-1, 1]$ .



b)  $f(x) = (x-2)^2(x+3)^3$ ,  $[-4, 3]$

$$f'(x) = Dx(x-2)^2(x+3)^3 + (x-2)^2Dx(x+3)^3$$

$$Dx(x-2)^2 = 2(x-2)(1) = 2x-4$$

$$Dx(x+3)^3 = 3(x+3)^2(1) = 3(x^2+6x+9) = 3x^2+18x+27$$



$$\begin{aligned}
 f'(x) &= (2x - 4)(x + 3)^3 + (x - 2)^2(3x^2 + 18x + 27) \\
 &= (2x - 4)(x + 3)^2(x + 3) + (x^2 - 4x + 4)(3x^2 + 18x + 27) \\
 &= (2x - 4)(x^2 + 6x + 9)(x + 3) + (x^2 - 4x + 4)(3x^2 + 18x + 27)
 \end{aligned}$$

$$\begin{aligned}
 (2x - 4)(x^2 + 6x + 9)(x + 3) &= (2x^3 + 12x^2 + 18x - 4x^2 - 24x - 36)(x + 3) \\
 &= 2x^4 + 12x^3 + 18x^2 - 4x^3 - 24x^2 - 36x + 6x^3 + 36x^2 + 54x - 12x^2 - 72x - 108 \\
 &= 2x^4 + 14x^3 + 18x^2 - 54x - 108
 \end{aligned}$$

$$\begin{aligned}
 (x^2 - 4x + 4)(3x^2 + 18x + 27) &= 3x^4 + 18x^3 + 27x^2 - 12x^3 - 72x^2 - 108x + 12x^2 + 72x + 108 \\
 &= 3x^4 + 6x^3 - 33x^2 - 36x + 108
 \end{aligned}$$

$$\begin{aligned}
 f'(x) &= 2x^4 + 14x^3 + 18x^2 - 54x - 108 + 3x^4 + 6x^3 - 33x^2 - 36x + 108 \\
 &= 5x^4 + 20x^3 - 15x^2 - 90x \\
 &= 5x^4 + 20x^3 - 15x^2 - 90x \\
 &= 5x(x^3 + 4x^2 - 3x - 18)
 \end{aligned}$$

$$\frac{x^3 + 4x^2 - 3x - 18}{x - 2} = (x + 3)^2$$

$$5x(x^3 + 4x^2 - 3x - 18) = 5x(x - 2)(x + 3)^2$$

$$5x(x - 2)(x + 3)^2 = 0$$

$$5x = 0$$

$$x' = 0$$

$$x - 2 = 0$$

$$x'' = 2$$

$$(x + 3)^2 = 0$$

$$x^2 + 6x + 9 = 0$$

$$x''' = x'''' = -3$$

$$x' = 0; \quad x'' = 2; \quad x''' = x'''' = -3$$

$$f(-3) = (-3 - 2)^2(-3 + 3)^3 = 25 \cdot 0 = 0$$

$$f(0) = (0 - 2)^2(0 + 3)^3 = 4.27 = 108$$

$$f(2) = (2 - 2)^2(2 + 3)^3 = 0.125 = 0$$

$$R = (-3, 0); (0, 108); (2, 0)$$

c)  $f(x) = \sqrt{5 - 4x}$ , em  $[-1, 1]$

$$f'(x) = \frac{1}{2}(5 - 4x)^{-\frac{1}{2}}(-4) = -\frac{4}{2(5 - 4x)^{\frac{1}{2}}} = -\frac{2}{\sqrt{5 - 4x}}$$

$$-\frac{2}{\sqrt{5 - 4x}} = 0$$

Não existe ponto crítico no intervalo  $[-1, 1]$

d)  $f(x) = \frac{2}{3}x - x^{\frac{2}{3}}$ , em  $[0, 8]$

$$f'(x) = \frac{2}{3} - \frac{2}{3}x^{-\frac{1}{3}} = \frac{2}{3} - \frac{2}{3x^{\frac{1}{3}}} = \frac{2}{3} - \frac{2}{3^{\frac{1}{3}}\sqrt[3]{x}}$$

$$\frac{2}{3} - \frac{2}{3^{\frac{1}{3}}\sqrt[3]{x}} = 0$$

$$\frac{2}{3} = \frac{2}{3^{\frac{1}{3}}\sqrt[3]{x}}$$

$$6^{\frac{1}{3}}\sqrt[3]{x} = 6$$

$$\sqrt[3]{x} = 1$$

$$x = 1^3 = 1$$

$$f(1) = \frac{2}{3}1 - 1^{\frac{2}{3}} = \frac{2}{3} - 1 = -\frac{1}{3}$$

$$R = (1, -\frac{1}{3})$$

e)  $f(x) = x^{\frac{2}{5}} - \frac{1}{9}x^{\frac{7}{5}}$ , em  $(-1, 1)$

$$f'(x) = \frac{2}{5}x^{-\frac{3}{5}} - \frac{1}{9}\frac{7}{5}x^{\frac{2}{5}} = \frac{2}{5x^{\frac{3}{5}}} - \frac{7}{45}x^{\frac{2}{5}} = \frac{2 - \left(5x^{\frac{3}{5}}\frac{7}{45}x^{\frac{2}{5}}\right)}{5x^{\frac{3}{5}}}$$

$$\begin{aligned}
&= \frac{2 - \left(\frac{7}{9}x^{\frac{3}{5}}x^{\frac{2}{5}}\right)}{5x^{\frac{3}{5}}} = \frac{2 - \left(\frac{7}{9}x^{\frac{5}{5}}\right)}{5x^{\frac{3}{5}}} = \frac{2 - \frac{7}{9}x}{5x^{\frac{3}{5}}} = \frac{\frac{18-7x}{9}}{5x^{\frac{3}{5}}} = \frac{18-7x}{9} \frac{1}{5x^{\frac{3}{5}}} \\
&= \frac{18-7x}{45x^{\frac{3}{5}}} \\
&\frac{18-7x}{45x^{\frac{3}{5}}} = 0 \\
&18-7x = 0 \\
&18 = 7x \\
&x = \frac{18}{7}
\end{aligned}$$

Não existe ponto crítico no intervalo  $(-1, 1)$ .

### Gabarito:

2.13a)  $3(x^4 - 3x^2 + 5x - 2)^2(4x^3 - 6x + 5)$ . 2.13b)  $\frac{x(21x-4)}{2\sqrt{7x^3-2x^2+5}}$ . 2.13c)  $\frac{-30x}{(5x^2-4)^4}$ .

2.13d)  $\frac{1}{4\sqrt{1+\sqrt{x+1}}(x+1)}$ . 2.14) Mínimos  $(0, 0)$ ,  $(1, 0)$  e máximo  $(\frac{1}{\sqrt{2}}, \frac{1}{2})$ . 2.15a)  $(f \circ g)(x) = \frac{2}{3x+1}$  e  $(g \circ f)(x) = \frac{6}{x+1}$ .

2.15b)  $(f \circ g)(x) = x^6 + 2x^3 - 5$  e  $(g \circ f)(x) = (x^2 + 2x - 5)^3$ .

2.15c)  $(f \circ g)(x) = 49$  e  $(g \circ f)(x) = 3$ .

2.15d)  $(f \circ g)(x) = x^6$  e  $(g \circ f)(x) = x^6$ .

2.15e)  $(f \circ g)(x) = x$  e  $(g \circ f)(x) = x$ .

2.15f)  $(f \circ g)(x) = x^2 - 4$  e  $(g \circ f)(x) = x^2 - 4$ .

2.16a)  $x \in \mathbb{R}$ . 2.16b)  $-\frac{1}{4}$ . 2.16c)  $\frac{1}{3}$ . 2.16d)  $0$ . 2.16e)  $\sqrt[3]{2}$ .

2.17a)  $f(x) = (x^3 - 2x + 2)^4$  e  $g(x) = x^4$ .

2.17b)  $f(x) = 8 - x$  e  $g(x) = x^4$ .

2.17c)  $f(x) = 1 + x^2$  e  $g(x) = \sqrt{x}$ .

2.17d)  $f(x) = x^2 - 4$  e  $g(x) = \frac{1}{x}$ .

2.18a)  $4(x^3 - 2x^2 + 7x - 3)^3(3x^2 - 4x + 7)$ .

2.18b)  $-15(7 - 3x)^4$ .

2.18c)  $-\frac{4}{(2x-3)^3}$ .

2.18d)  $\frac{18x}{(3x^2+5)^4}$ .

2.18e)  $16x(4x^2 - 3)(x + 5)^3 + 3(4x^2 - 3)^2(x + 5)^2$ .

2.18f)  $\frac{3(x^2+2)^2(x^2-6x-2)}{(x-3)^4}$ . 2.18g)  $\frac{20x(x^2-2)}{(2x^2+1)^3}$ . 2.18h)  $\frac{4(-6x+1)}{(3x^2-x+5)^2}$ . 2.18i)  $\frac{3x}{2\sqrt{1+x^3}}$ .

$$2.19a) \frac{3}{2\sqrt[4]{x}}. \quad 2.19b) \frac{2x-9x^4}{\sqrt[3]{(1-3x^3)^2}}. \quad 2.19c) \frac{1}{\sqrt{(x^2+1)^3}}. \quad 2.19d) \frac{21x^2-8x}{4\sqrt[4]{(7x^3-4x^2+2)^3}}.$$

$$2.19e) \frac{3}{2\sqrt{(x-1)^3}\sqrt{x+2}}. \quad 2.19f) \frac{6}{\sqrt[4]{x}} + \frac{1}{\sqrt[4]{x^3}} + \frac{1}{3\sqrt[4]{x^3}}.$$

$$2.19g) \frac{16x}{3\sqrt[3]{4x^2+3}}. \quad 2.19h) -\frac{1}{\sqrt{x^3}}. \quad 2.19i) -\frac{1}{4\sqrt{4-\sqrt{4+x}}\sqrt{4+x}}. \quad 2.18j) \frac{2(-x^3+x^2+1)}{\sqrt[3]{1+x^3}(1-2x)^2}.$$

$$2.20) y = -\frac{1}{6}x + \frac{8}{15}. \quad 2.21) y = -\frac{5}{3}x + 10.$$

$$2.22) \frac{x^2-2}{x^2-6} e.$$

$$2.23a) \text{ Não existe. } \quad 2.23b) (-3, 0); (0, 108); (2, 0). \quad 2.23c) \text{ Não existe.}$$

$$2.23d) (1, -\frac{1}{3}).$$

$$2.23e) \text{ Não existe.}$$