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MASTER OF ECONOMETRICS AND STATISTICS

Early warning signals of financial crises from critical slowing down theory

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1 Introduction

The prediction of financial crises has become a crucial research topic in recent years, as stock market crashes can have a highly negative impact on the rest of the economy. The number of crises and their impact have created a craze for finding elements that will allow us to know precisely when one will occur. Before trying to use a specific theory in order to forecast breaking points, we will try to understand what makes these phenomena so difficult to predict, and how these crises differ from classical prediction approaches.

In 2010, the "macro-models" used to predict crises were deemed ineffective.[2] They do not allow for the transcription of financial movements, which are too dynamic. The limits of these models were quickly discovered, and raised questions about the possibility of predicting crises and the methods to be used. On the other hand, the critical slowing down theory has been proven effective to forecast breaking points in complex systems of many fields. Initially created to measure critical ecological transitions, with indicators that display a decrease in resilience, this theory has since been applied to models in physics, engineering, biology and even psychology. We therefore have hope of understanding financial systems coming from these fields, where the theory has brought conclusive results.

In complex systems, the development of predictive tools is based on detecting signs of critical slowdowns, before critical transitions. These transitions from a state to another are called tipping points. The advantage of the leading indicators used in the critical slowing down theory, especially in economics and finance, is that they can be applied to univariate time series. This will be particularly useful for us, as we will study the performance of stock market indices over different time periods. We will therefore first consider financial dynamic systems as complex to allow us to establish a descriptive analysis of the indicators of our stock market indices according to the critical slowdown theory. This theory can allow us to predict the occurrence of financial crises in a complex system through the behaviour of certain components of our data, such as auto-correlation, variance or skewness. We will see later if this hypothesis can be validated based on how well these indicators predict tipping points in our series.

In the framework of our study, we will analyse four financial crises: Black Monday 1987, the Asian crisis of 1997, the collapse of the Internet bubble in 2000 and the financial crisis of 2008. In an attempt to predict critical transitions, we will look at the break-point corresponding to the date of the crisis. Thus we will observe whether in the period preceding the critical point, the first-order auto-correlation as well as the second and third-order moments have increased. The period in which these changes changes of our indicators are detected is defined as the critical downturn period. We will therefore obtain a comparative view, thanks to the tools used to predict crises and the interpretation of our results on the different indices of the different geographical areas.

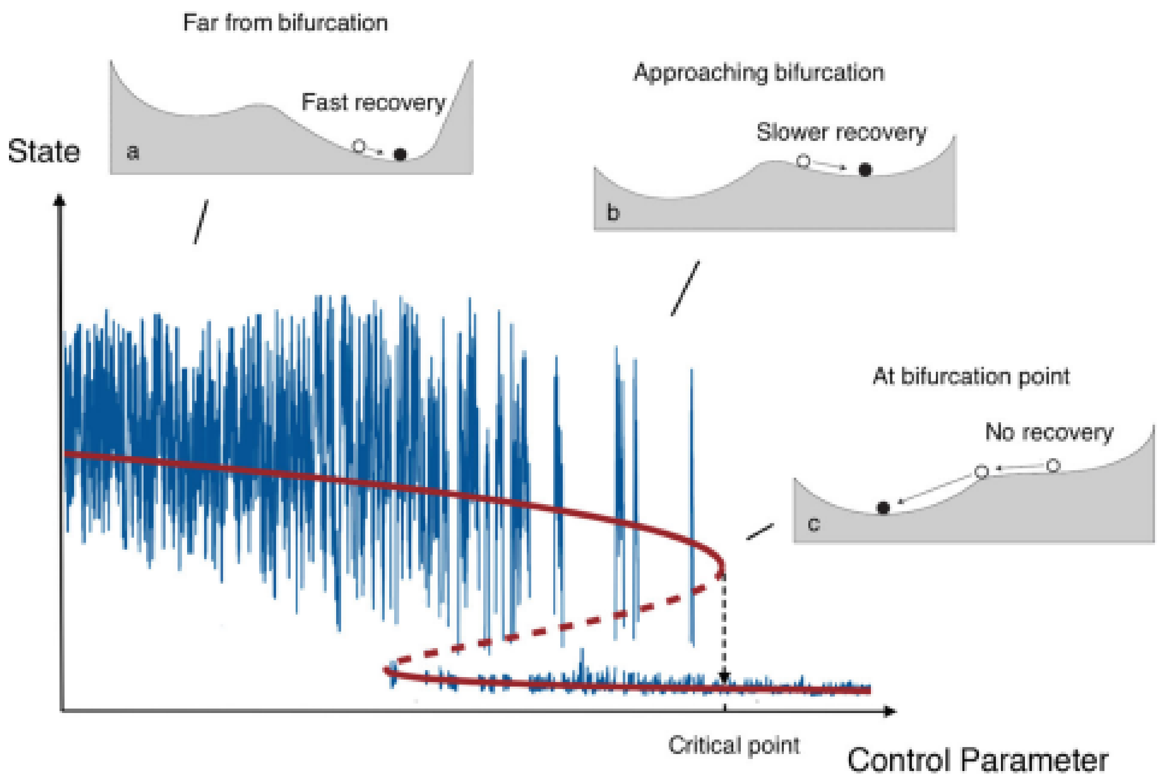
With the aim of detecting breakpoints in financial time series,and with an approach based on the critical slowing down theory, we will compute indicators to see if our series fit the particularities of complex systems. We will then make our data relevant and exploitable in order to be able to carry out different operations that will allow, according to established conditions, to be able to interpret our results and to draw potential conclusions. We will try to answer the following question, "Can financial breaking points be detected through early warning signals of the critical slowing down theory?" To answer this question, we will first analyse the theoretical content and then establish a methodology to exploit the data. The third step is to compute early warning indicators, and lastly we will analyse the empirical results and check the robustness of the indicators.

2 Explanation of the Theory and data exploitation

2.1 Critical Transitions

Critical transitions are the structural passage from a system to another, represented by tipping points.we wish to predict critical transitions in the financial system. in order to do so, let's first understand the functioning of the theory.

In complex systems, a critical transition is led by an increase in more than one early warning indicator. There can still exist false alarms, meaning that indicators can show signs of an incoming tipping point even though this prediction isn't accurate (fraudulent early signals) [5]. But when indicators rightly predict the beginning of a crisis, it corresponds to an abrupt change in a dynamic system. A tipping point in a complex system is driven by a dangerous bifurcation, which mathematically means a sharp change of a dynamical system, leading to a loss of the system's equilibrium after a critical value is passed by some control parameters[2]. In their paper of 1994,Thompson et al. [7]look at different types of bifurcation in dynamic systems.
"Critical transitions are associated with a so-called dangerous bifurcation in which a stable equilibrium loses its stability when a control parameter passes a critical value" Sieber et Thompson (2012) [6] Empirically, a gradual increase of the control parameter towards the critical value is in principle detectable.



Following Lenton(2011)[4], the different panels describe critical slowdowns as an early warning indicator that the system has lost its resilience on the way to the critical point. The balls shown in planes a,b,c represent the current state of the system. In plane a we are far from the bifurcation, the fluctuations are fast and the variance is small, for b the bifurcation is approaching, the variance is increasing.In plane c we are at the bifurcation point, the transition to a new local minimum is irreversible.

2.2 Describing Data

Using this approach, we aim at focusing on analysing univariate time series that consist of observations recorded sequentially, over equal time increments (in this case, daily increments). We choose to use price returns of indices instead of simple returns,because it is easier to observe auto-correlation using price indices. In order for the analysis to be easier, the variables are used in logarithms, because in time series analysis, especially stock market prices, this transformation in logs contribute to stabilize the variance of the underlying series. In our study, four financial crises are studied: Black Monday (October 19th 1987); the 1997 Asian Crisis; the 2000 Dot-com Crash in and the 2008 Subprime Mortgage Crisis.

There were many reasons triggering these different crises, with specialists assigning different levels of blame to financial institutions, regulators, speculators, but we solely focus on the common characteristics of all of them, which is that during the respective crises, the corresponding stock prices for these events showed similar sudden collapses.

There can be two causes for a crisis: endogenous and exogenous causes. Exogenous shocks can't be detected through critical slowing down theory since, by construction, they are not brought up by a perturbation of the system. We will therefore merely focus on endogenously generated tipping points

For each crisis, several time series were studied, in order to have more accurate results. For Black Monday, the Standard and Poor 500 (SP 500) and the Dow Jones indices were used.The SET 50 and HANG SENG indices are used to analyse the Asian Crisis. The Dot-Com crash, caused by excessive speculation of Internet-related companies, is represented by the following indices : FTSE 100, SP SmallCap 600, SP MidCap 400, Dow Jones Industrial Average (Dow Jones IA), NASDAQ, CAC 40, Nikkei 225. The most recent one, the 2007 subprimes crisis is analyzed through the SP 500, we analyse critical slowing by the SP SmallCap 600, SP MidllCap 400, Dow Jones, DAX, IBEX 35, FTSE 100,and the FTSE MIB and CAC 40 indices. In Table 1 are provided Some further details on the events, such as the critical point in time associated with the crashes, and the justification of the chosen indices that ties in with georgraphical occurence of each crisis.

Table 1: Detected critical points of each crisis

Label	Crisis	Critical Point T1	Critical Point T2	Geographic Zone of indices
[1]	Black Monday	13 Oct 1987		USA
[2]	Asian Crisis	2 July 1997	1 Oct 1998	Thailand, Hong-Kong
[3]	Dot-com crash	10 March 2000	3 March 2003	USA, Japan, Europe
[4]	2008 Financial crisis	31 July 2007	08 July 2008	USA, Europe

Historical daily data of the stock indices' (log) price returns pertaining to this analysis were downloaded from universally available online databases (mainly yahoo finance) as well as the Macrobond database. Data was collected throughout two decades, in periods that correspond to analyzed crises, from october 1986 until March 2009. Since changes in indicators should be observed one or several months before bifurcation, we choose to extract 1 year samples, corresponding to T=250 daily observations before bifurcation point,on average, per index. We also decide to observe two bifurcation points per crisis: we therefore analyze early warning signals of the begining, and of the end of each crisis. An exception is made for the Black Monday crisis which starts and ends within a three months interval, and for which the detection of the crisis' ending would not be distinct. We use rolling windows of T/2, which translates into 130 observations per rolling window (bigger than 100 observations, as recommended by Dakos et al. (2008) [1])

We record the maximum value of the series' index in the corresponding period to identify the critical point over the time, where the decline began. For instance, this can show why in our thesis, 10 March 2000, is considered as a critical point for Dot-com crisis. The indices' growth is analyzed in relative terms in order to stabilize the variance of the residuals that will be estimate.

2.3 Detrending

To cope with non-stationarity of the data, and to solely focus on residuals, we need to detrend our time series. Doing so can be achieved using the moving average subtraction method, as used by Dakos et al (2008) [1]. This process consists of two main steps: the first step is to compute moving averages using Gaussian kernels. Once the data's underlying trend is detected, the second step is to

subtract the moving average from the overall time series to obtain stationary data.
Starting off with an initial series $\{z\}_{t=1}^{T}$, we will apply the MA process in order to obtain detrended utilizable data : $\{y\}_{t=1}^{T}$, where T denotes the window of pre-crisis observations.
Gaussian kernel function, which smooths data across time, is a function is given by:

$$G(s) = \frac{1}{\sqrt{2\pi\sigma}} e^{-\frac{s^2}{2\sigma^2}}$$

Using $\{z\}_{t=1}^{T}$ and G(.), we now compute the moving average, defined as follows:

$$MA_t = \frac{\sum_{r=1}^{T} G(r-t)zr}{\sum_{r=1}^{T} G(r-t)}, \qquad t = 1, ...T.$$

We lastly deduct our detrended series:

$$y_t = z_t - MA_t$$

The σ parameter used in the Gaussian kernel smoothing function represents the bandwidth parameter which has to be chosen carefully in order to get rid of the long term trend, but to also save short term fluctuations. If the bandwidth is too large, data will be over-smoothed which will provide a biased estimator of our series (fluctuations won't be as defined as they should be). Contrariwise, if the bandwidth chosen is too small, the Gaussian kernel won't filter enough information, and the detrended series will reflect some unneeded long term variation, which translates into an abnormally high variance. We therefore have a trade-off between high variance and high bias: we choose to apply $\sigma = 15$ which suits the conventional range of 10 to 20.

2.4 Early warning indicators

Indicators used on our detrended log-indices must be able to forecast an upcoming breaking point.To detect a potential slowdown in the series, on different levels, three indicators will be used: lag-1 autocorrelation, variance and skewness. An increase in these indicators should be detectable preceding the critical transition.

In order to scrutinize trends, these indicators are computed using moving windows across the entire time period T. The size of the k rolling window is crucial in order to obtain accurate estimations: if the window is too small, we won't capture enough information, but if it is too big, we will capture long term changes in the time varying indicator (Diks et al. (2018) [2]. Windows smaller than less than 100 observations don't capture enough information and should not be considered according to Dakos et al. (2008) [1] This parameter, as well as the bandwidth's size, will be further analyzed in a robustness test, in which we will observe variations of leading indicators according to different values of k and σ.

AR(1) Indicator

lag-1 auto-correlation reflects similarities between two observations, one at time t and one at t-1: if each observation is very similar to the one previously observed, the first order auto-correlation indicator will be high. According to the critical slowing down theory, slowdowns preceding a structural break are reflected by an increase in persistence of the data, and therefore a boost in the indicator. We estimate our first order auto-correlation using an AR(1) model (first-order autoregressive model)

Variance

The log-indice's outbreak should also be detectable through an increase in the variance (second moment of the detrended time series) of the series. Indeed, variance varies sharply right before a financial crisis, but there is also a slower increase that should be discernible several months in advance, and that we try to capture to validate the theory. We therefore use variance as the second early warning indicator of our system.

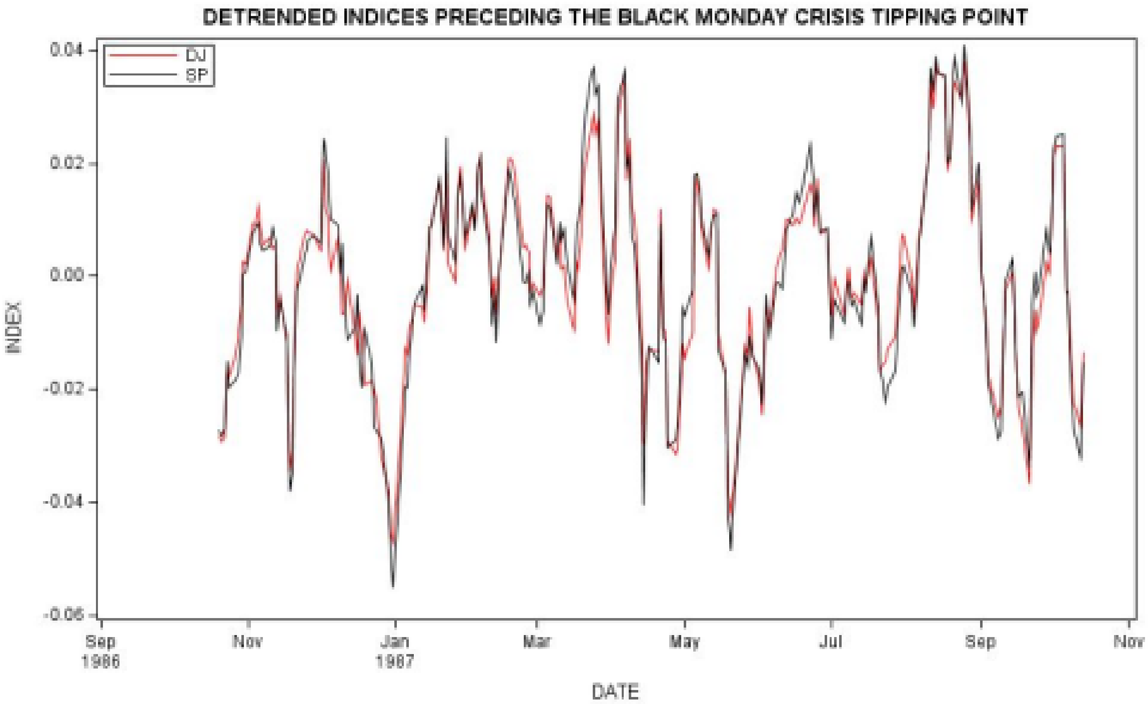
Skewness

The skewness, our last indicator displays how unstable the data is in the time series. Approaching a bifurcation point, data becomes lesser and lesser stable which drives the skewness away from zero. Graphically, this translates into an increased asymmetry in variations. Skewness will tend towards negative or positive values with regards to differences between the two possible states (Gatfaoui et al. (2016)) [3]

3 Empirical analysis and robustness of results

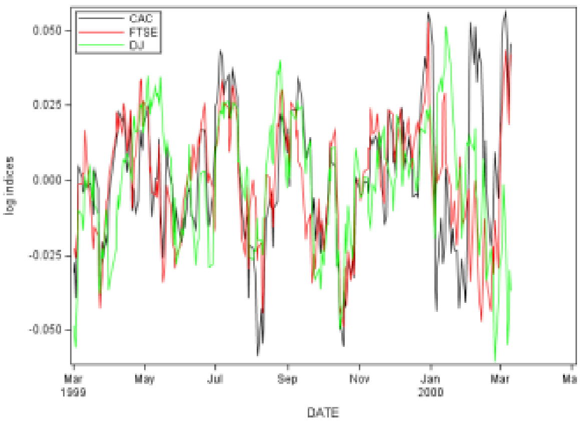
3.1 Detrended values representation

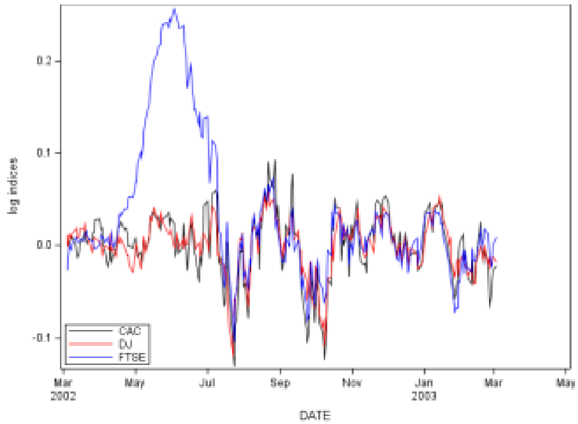
In the critical slowing down theory's framework, we have previously seen the necessary prerequisites for data exploitation. Using the moving average processthrough Gaussian smoothing, we obtained results of our detrended series. These series can be represented graphically, and will values close to 0 since they're logarithms.



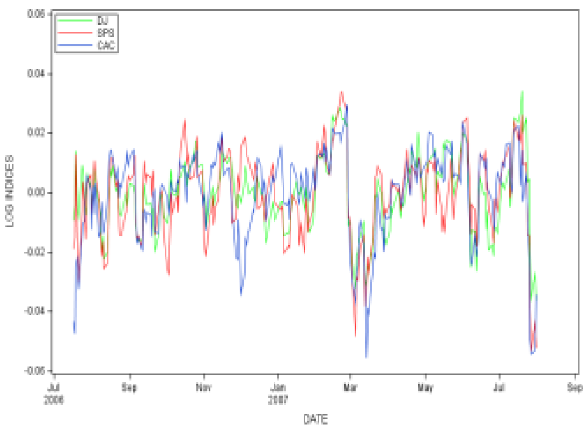
Looking at the graphs, we observe very similar levels and fluctuations, proving high correlation between european and american indices in the same time periods. We nevertheless notice differences for Asian markets, since financial markets aren't as developed in some countries, notably in Thailand where the asian crisis emerged.

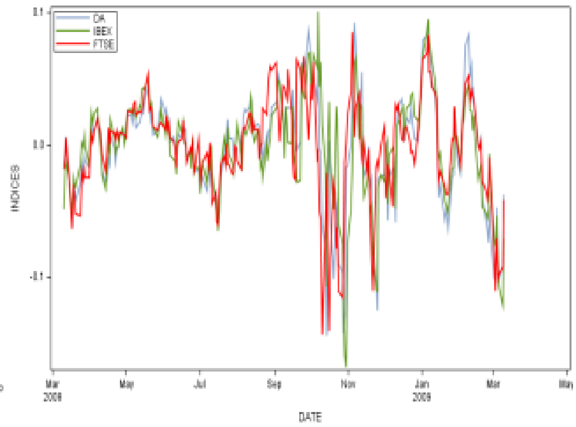
Detrended data for the beginning and end of the DOT-COM crisis



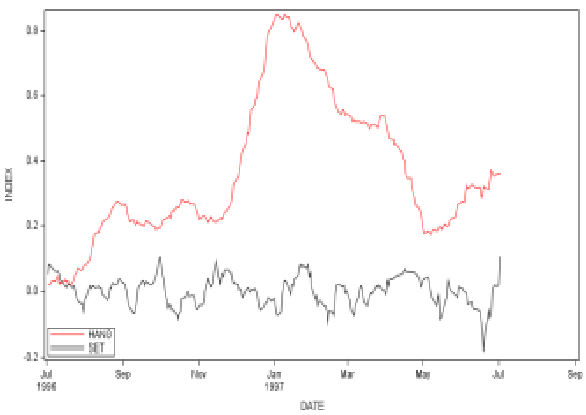


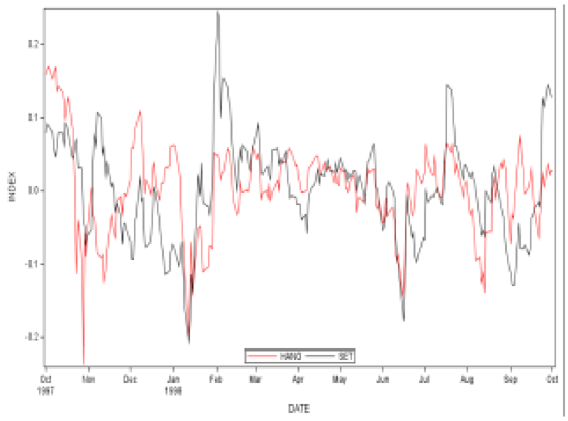
Detrended data for the beginning and end of the Subprimes crisis





Detrended data for the beginning and end of the Asian crisis

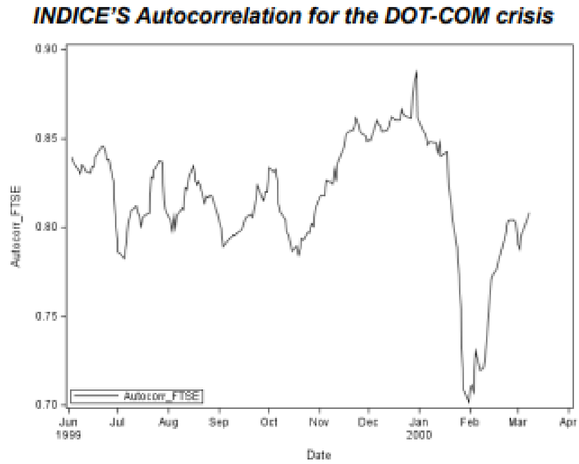


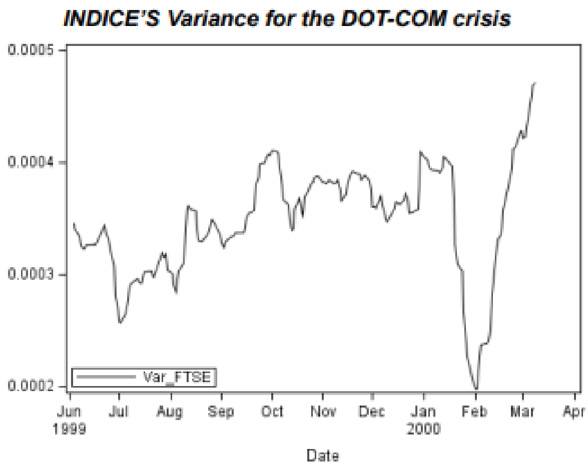


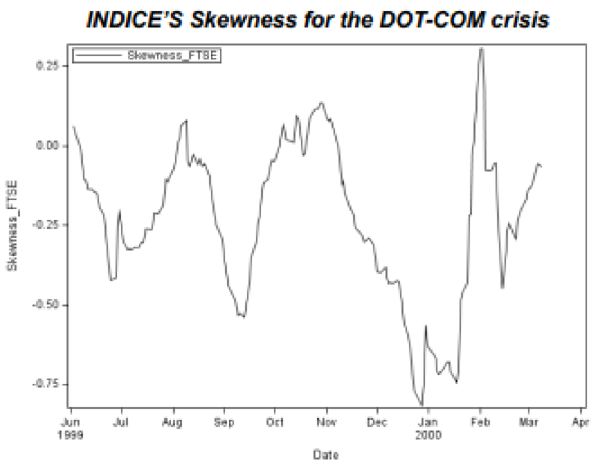
3.2 Indicators of the DOT-COM crisis

The bursting of the internet bubble occurred at the beginning of March 2000. Thanks to our data collected for this crisis, we can see that our series seems to follow a behaviour that can be associated to a critical slowing down. By taking into account certain indices such as the FTSE, the CAC 40 or the Dow Jones, we can perceive fluctuations. These are illustrated graphically with the FTSE index's indicators displayed above.

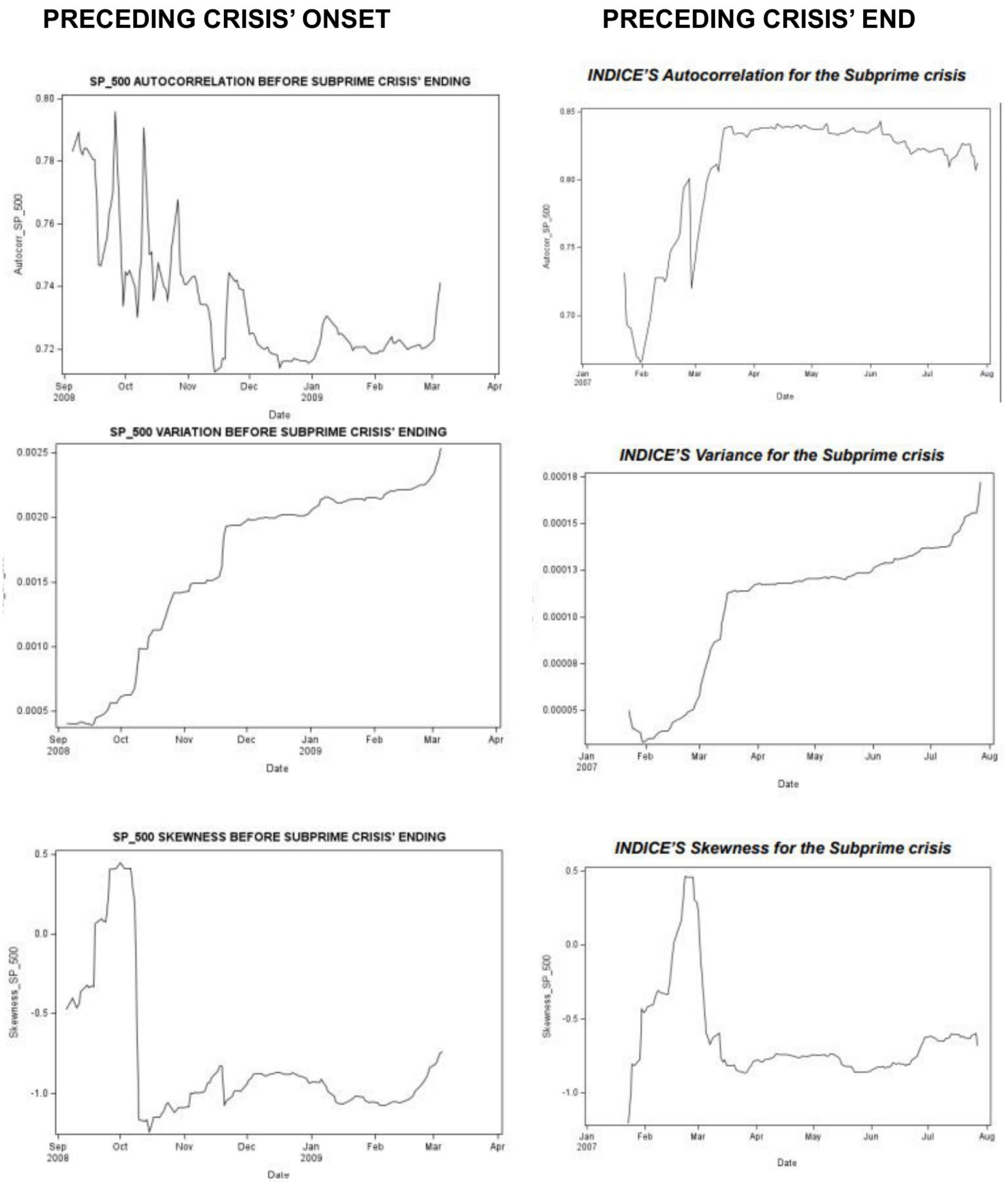
Indeed, one month before the onset of the crisis, we can see a significant increase in the first order autocorrelation as well as in the variance of the series. It is also relevant to look at its Skewness, which increases as well, about two weeks before the outbreak of the crisis, which is a short period of time, but it is still interesting to consider. For the study of the pre-crisis data, early warning signals do appear and we can therefore consider applicability of a complex system's critical slowing down. However, for the end-of-crisis transition,auto-correlations and variances of our indices do not provide any conclusive results. It is therefore necessary to be cautious in the conclusions that we could drawn from the study of this crisis, and it is necessary to compare these results with other cases.


INDICE'S Autocorrelation for the DOT-COM crisis
0.90
0.85
0.80
0.75
0.70
Autocorr_FTSE
Jun 1999
Jul
Aug
Sep
Oct
Nov
Dec
Jan 2000
Feb
Mar
Apr
Date


INDICE'S Variance for the DOT-COM crisis
0.0005
0.0004
0.0003
0.0002
Var_FTSE
Jun 1999
Jul
Aug
Sep
Oct
Nov
Dec
Jan 2000
Feb
Mar
Apr
Date


INDICE'S Skewness for the DOT-COM crisis
Skewness_FTSE
0.25
0.00
-0.25
-0.50
-0.75
Jun 1999
Jul
Aug
Sep
Oct
Nov
Dec
Jan 2000
Feb
Mar
Apr
Date

3.3 Indicators for Subprime crisis



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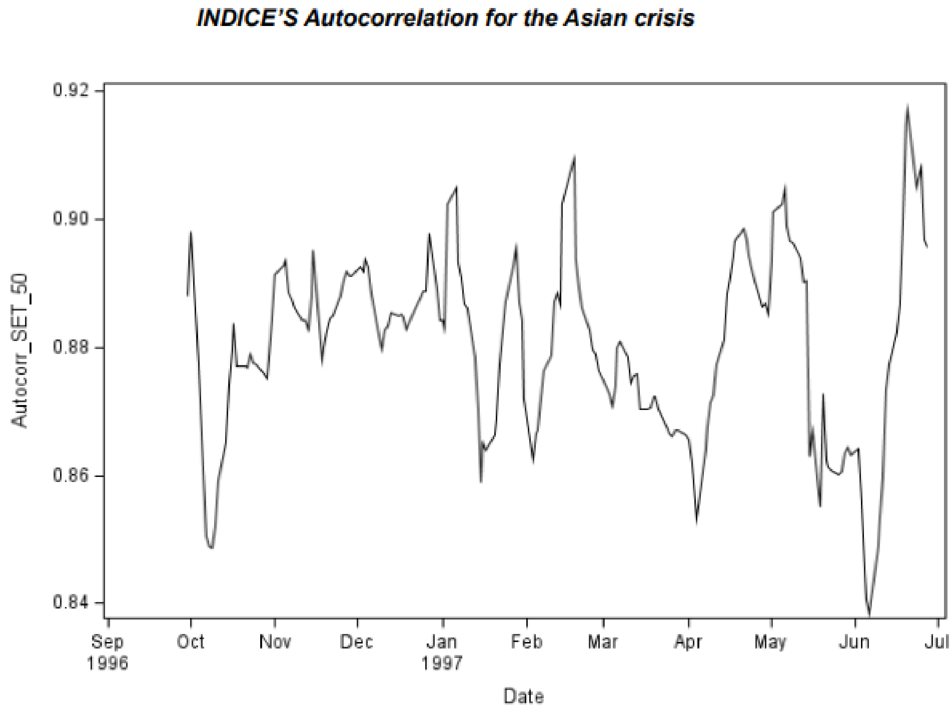
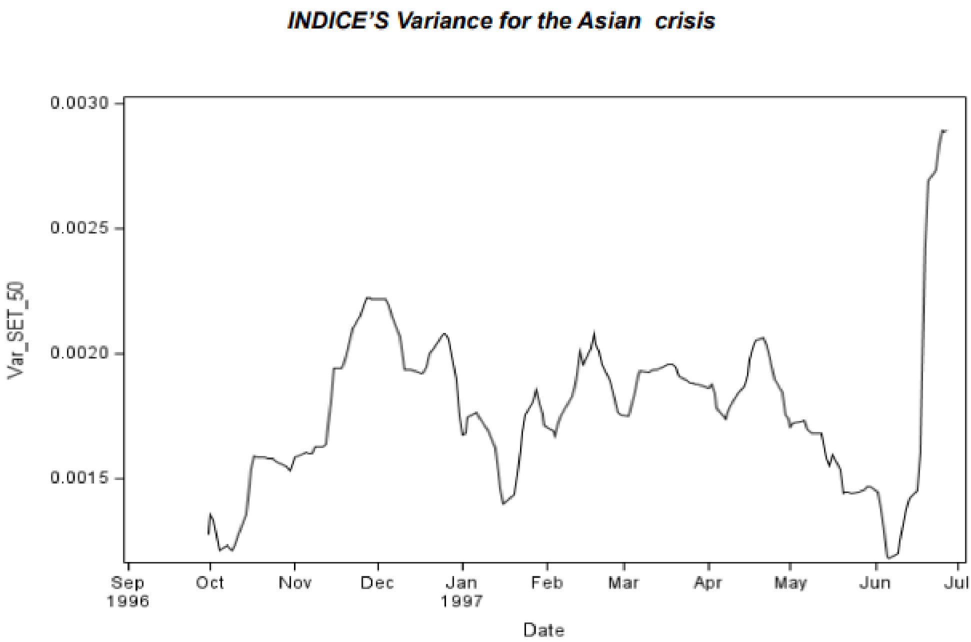
The Subprime crisis 2008 is considered to be one of the most felt financial crises in the world during this century, recording the most severe consequences in the financial sphere as well as in the economy. The crisis burst in the summer of 2007, which is why we use July 2007 as our beginning point for the crisis.

The graphs above represent our three early warning indicators in the year preceding bifurcation point, for each, the beginning and the end of the subprime crisis. Data is thereby represented by the SP 500's indicators, which are very similar to the other ones studied in the frame of this analysis. For instance, the SP Small Cap, SP MidCap, Dow Jones and FTSE all displayed similar indicators. We can observe in the above figures, a sharp increase in the SP 500's auto-correlation and variance, before the critical transition, from February 2007 onward. This strong upward trend in the AR(1) indicator, could refer to a slowing down since it appears nearly two months before the beginning of the crisis.Per contra, skewness variations seem harder to analyse. Indeed, even if a slight increase is observed over a month before each bifurcation point, we also observe a really abrupt decrease several month back.

Nonetheless, auto-correlation and variance increases confirm that two leading indicators show patterns of an upcoming tipping point before actual transition, which represents a pertinent explanation to validate our theory.

3.4 Indicators for Asian crisis

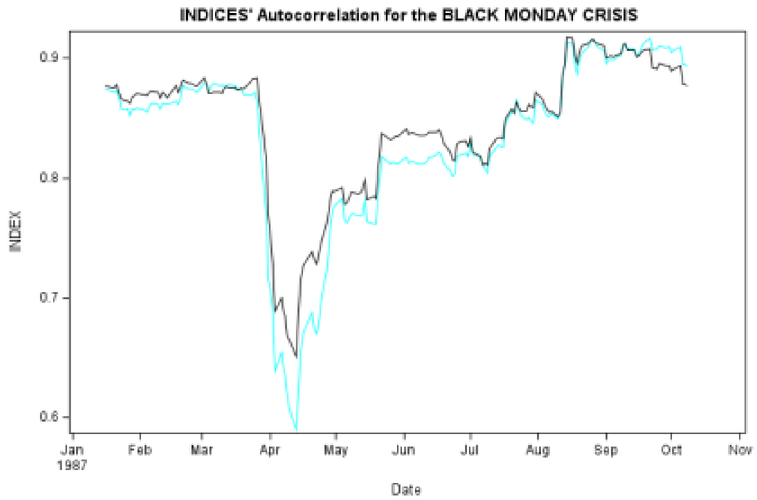


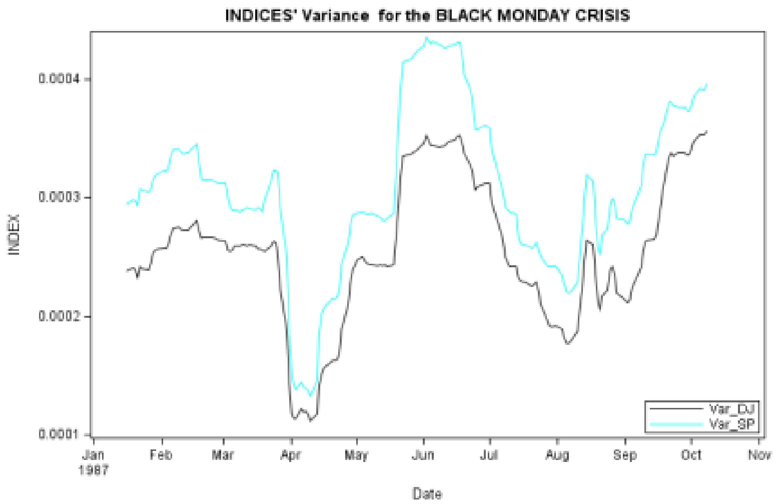


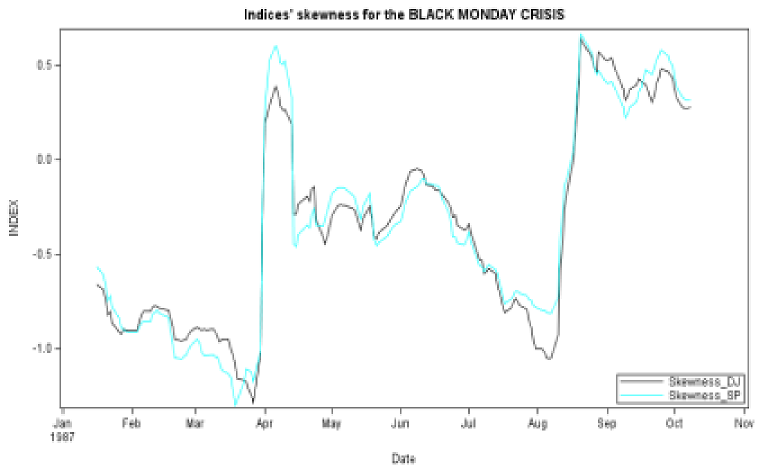
The Asian crisis appeared at the beginning of July 1997 in Thailand and affected several Asian countries thereafter. We therefore use the first crisis bursting as our breaking point (corresponding to the SET 50). Even if this period of time might be early to detect rising indicators in the Hang-Seng index, (for which the breakin point arrives after) some information might still be detected. We have chosen to collect data on the Thai stock market indices, with the SET 50, and on the Hong Kong index with the HANG SENG. It seems risky to consider the applicability of the critical slowing down theory in the case of the Asian crisis. The first-order autocorrelation and the variance of the SET 50 both increased before the crisis over a period of one month before the explosion, which is short but interpretable. But this increase is difficult to accept because of its sharpness and suddenness.Even in Thailand, the country in which the crisis bursts, interpretaability of our indicators as early warning signals is not relevant. The autocorrelation and variance of the HANG SENG do not increase before the crisis, and nor do they reflect any information that would allow the critical slowing down theory to be established. It is perhaps too early to interpret this index. The inability to demonstrate critical slowing down for these indices may be due to the context of Asian financial markets in the 1990s, which were not highly developed and could lead to lower impacts of the bifurcation point. This may also affect the reliability of the data on stock market indices, making it difficult to identify the tipping point.

3.5 Indicators for Black Monday crisis

For the crash of October 19, 1987, also called "Black Monday", we chose to take into account the Dow Jones and SP500 stock market indices. Thanks to the data we collected for this crisis, it is possible to observe clear patterns of early warning signals associated with a critical slowdown. Our two chosen indices are modelled graphically. First looking at the autocorrelation for our two stock market indices, it is possible to conduct a joint analysis because of the strong similarity of the first order autocorrelation graphs for our two indices. We can clearly identify an increase in autocorrelation starting in April 1987, and continuing until the onset of the crisis. Concerning the variance, we can also visualise an increase from August 1987, 2 months before the crisis, followed by a slight decrease, before increasing again in September until the explosion of the crisis. These


INDICES' Autocorrelation for the BLACK MONDAY CRISIS
INDEX
0.9
0.8
0.7
0.6
Jan 1987
Feb
Mar
Apr
May
Jun
Jul
Aug
Sep
Oct
Nov
Date


INDICES' Variance for the BLACK MONDAY CRISIS
INDEX
0.0004
0.0003
0.0002
0.0001
Var_DJ
Var_SP
Jan 1987
Feb
Mar
Apr
May
Jun
Jul
Aug
Sep
Oct
Nov
Date

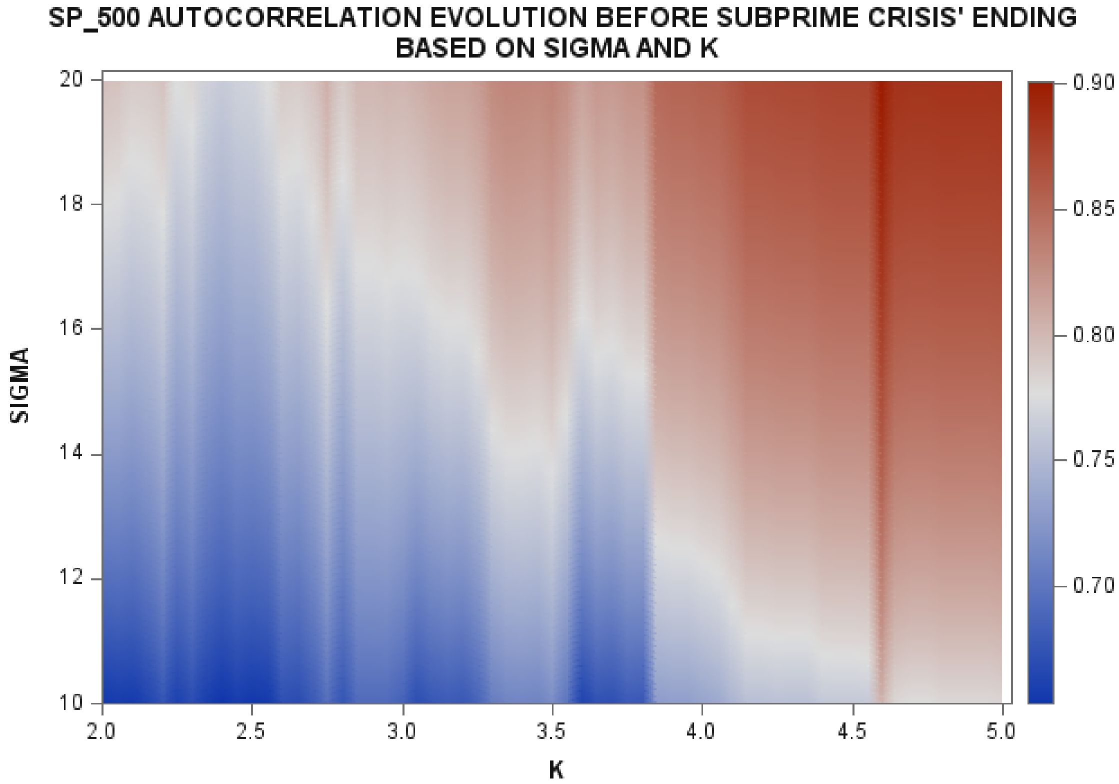

Indices' skewness for the BLACK MONDAY CRISIS
INDEX
0.5
0.0
-0.5
-1.0
Skewness_DJ
Skewness_SP
Jan 1987
Feb
Mar
Apr
May
Jun
Jul
Aug
Sep
Oct
Nov
Date

results are very relevant, especially thanks to the first-order autocorrelation, which increases over a period of 6 months for our two indices. The decision to take the critical slowing down theory even more seriously in the context of financial systems is brought further with the indicators of the Black Monday crsis.This is reinforced by the increase in skewness from August 1987 onwards, despite a small decline in September 1987.

3.6 Robustness of parameters

In order to make sure that our results don't only depend on our choice of control parameters, we establish a robustness test. The two parameters that were chosen to compute indicators are the bandwidth σ used to smooth data in the context of data detrending, and the size of the rolling window (k) used to compute our three indicators. To see if results are robust and generalizable, we decide to apply to our autocorrelation indicator several values of σ and k. By using a loop, we compute each autocorrelation indicator of the last observation of the rolling window for the SP 500 index, before the end of the Subprime crisis. We therefore vary σ by a step of 0.1 between the values of 10 and 20, and we do the same with k by using a step of 0.05 and for values fluctuating from 2 to 5.

The graph below provides different values of the autotcorrelation index according to different values of k and σ. We can observe that this indicator increases with the increase of k (or with the diminution of the sliding window). That means, in other words, that the smaller the rolling window is, the bigger the autocorrelation indicator of our last value will increase For σ, the autocorrelation values also increase as the value of the bandwodth parameter increases.That seems plausible as over-smoothed data will induce an artificially high autocorrelation([3]. Overall, results seem to be pretty robust to parameters as the autocorrelation only goes from 0.7 to 0.9, meaning that a change in σ and k won't lead to significantly different results.


SP_500 AUTOCORRELATION EVOLUTION BEFORE SUBPRIME CRISIS' ENDING
BASED ON SIGMA AND K
SIGMA
K
20
18
16
14
12
10
2.0
2.5
3.0
3.5
4.0
4.5
5.0
0.90
0.85
0.80
0.75
0.70

4 Conclusion

In order to address the issue of detecting breakpoints through critical slowing down indicators, we first presented the reasons for considering financial systems as complex. Following this hypothesis, we applied a methodology on our data in order to obtain series allowing us to visualise possible indicators of the critical slowing down theory for different financial crises. Through the modelling of our series, we can reach conclusions around the detection of breakpoints. For the majority of the financial crises modelling carried out, results allowing the detection of these breaking points are present. The Black Monday, the subprime crisis and the Dot-Com bubble all have indicators that signal the imminence of breaking points, while the results for the Asian crisis do not seem to be conclusive. However, the analysis must be qualified, by underlining, for example, the fact that the results of the series for the end of the crises provide results that are more difficult to interpret and that may sometimes deviate from the theory we are trying to demonstrate. Nevertheless, by applying the robustness test we observe that control parameters don't have exacerbated effets on our indicators, and while analysis of financial time series through this theory can't always display early warning signals, indicators used can be generalized.

References

[1] Vasilis Dakos, Marten Scheffer, Egbert H van Nes, Victor Brovkin, Vladimir Petoukhov, and Hermann Held. Slowing down as an early warning signal for abrupt climate change. *Proceedings of the National Academy of Sciences*, 105(38):14308–14312, 2008.

[2] Cees Diks, Cars Hommes, and Juanxi Wang. Critical slowing down as an early warning signal for financial crises? *Empirical Economics*, 57(4):1201–1228, 2019.

[3] Hayette Gatfaoui, Isabelle Nagot, and Philippe De Peretti. Are critical slowing down indicators useful to detect financial crises? In *Systemic Risk Tomography*, pages 73–93. Elsevier, 2017.

[4] Timothy M Lenton. Early warning of climate tipping points. *Nature climate change*, 1(4):201–209, 2011.

[5] Marten Scheffer, Stephen R Carpenter, Timothy M Lenton, Jordi Bascompte, William Brock, Vasilis Dakos, Johan Van de Koppel, Ingrid A Van de Leemput, Simon A Levin, Egbert H Van Nes, et al. Anticipating critical transitions. *science*, 338(6105):344–348, 2012.

[6] Jan Sieber and J Michael T Thompson. Nonlinear softening as a predictive precursor to climate tipping. *Philosophical Transactions of the Royal Society A: Mathematical, Physical and Engineering Sciences*, 370(1962):1205–1227, 2012.

[7] J Michael T Thompson, HB Stewart, and Y Ueda. Safe, explosive, and dangerous bifurcations in dissipative dynamical systems. *Physical Review E*, 49(2):1019, 1994.

Diks, C., Hommes, C., Wang, J. (2018). Critical slowing down as an early warning signal for financial crises ? Empirical Economics, 57(4), 1201-1228. https://doi.org/10.1007/s00181-018-1527-3

Kéfi, S., Dakos, V., Scheffer, M., van Nes, E. H., Rietkerk, M. (2012). Early warning signals also precede non-catastrophic transitions. Oikos, 122(5), 641-648. https://doi.org/10.1111/j.1600-0706.2012.20838.x

Gatfaoui, H., de Peretti, P. (2019). Flickering in Information Spreading Precedes Critical Transitions in Financial Markets. Scientific Reports, 9(1). https://doi.org/10.1038/s41598-019-42223-9

Scheffer, M., Carpenter, S. R., Lenton, T. M., Bascompte, J., Brock, W., Dakos, V., van de Koppel, J., van de Leemput, I. A., Levin, S. A., van Nes, E. H., Pascual, M., Vandermeer, J. (2012). Anticipating Critical Transitions. Science, 338(6105), 344-348. https://doi.org/10.1126/science.1225244