Data Driven Physics: Tutorial 10

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Bayesian inference and Gravitational Waves

Gravitational waves are traveling perturbations in the space-time, generated by cataclysmic events in distant galaxies. The detection of gravitational waves has been a long-held goal of the physics community, and represents a further confirmation of Einstein's Theory of General Relativity.

On September 14th 2015, the two detectors Laser Interferometer Gravitational-wave Observatory (LIGO) observed a gravitational wave signal generated by the merger of two black holes (BH). The LIGO detectors had been recently upgraded with new instrumentation and operating as Advanced LIGO, able to measure changes in the local shape of space-time with a precision better than one-thousandth the diameter of a proton. This discovery was awarded by the Physics Nobel prize in 2017.

The L-shaped LIGO instruments consist of two perpendicular arms, and each of them is 4 kilometers long. A passing gravitational wave will alternately stretch one arm and squeeze the other, and then vice-versa; this generates an interference pattern at the readout port of the interferometer, which is measured by photo-detectors (Fig.1). Excellent tutorials of vulgarization of this topic can be viewed in gw-openscience.

Since this first detection, different detectors operating in the word, including Ligo and Virgo, have detected an increasing number of events [4].

The goal of this tutorial is to use bayesian inference to measure the parameters of the binary black hole associated to the gravitational wave signal GW150914 that was first detected. We will focusing on the inference of the two component masses and the lumonisity distance of the source. We will use the data from the GW Open Science Center.

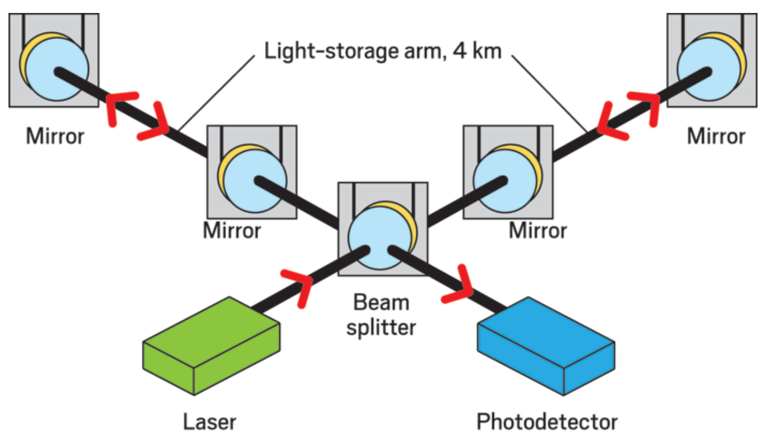

Figure 1: Sketch and description of the experiment from [5]. From the interference pattern of the laser light reflected by mirrors at each end of the interferometer arms, LIGO measures gravitational waves, which manifest themselves as minuscule fluctuations in the lengths of the arms.

Problem

Following [1] (see Appendix B of the paper), we introduce the Gaussian noise likelihood used in gravitational-wave astronomy. In this likelihood, $\mathbf{d}$ represent the data, more precisely the Fourier transform of the time series of the strain $d(t)$ measured by a gravitational-wave detector (see Fig. 2): $\mathbf{d} = fft(d(t))/f_s$, where f_s is the sampling frequency and fft stands for Fast Fourier transform . The fft signal can be decomposed in a sum of signals d_j for each frequency bin j. The signal is theoretically described by a template $\mu_j(\boldsymbol{\theta})$ based on Einstein's general relativity theory which describes the metric perturbation following a collision event [1].

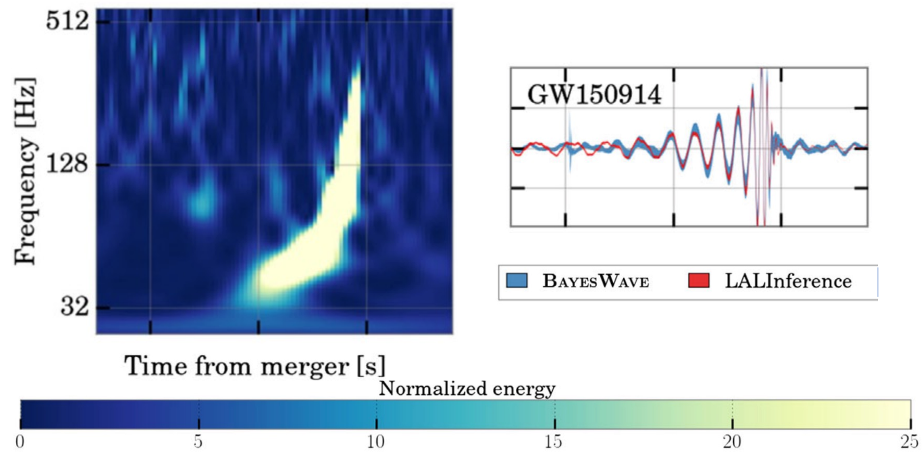

Figure 2: Detected signal for GW150914: The first panel shows a normalized time- frequency power map of the GW strain. The remaining pair of panels shows time-domain reconstructions of the whitened signal, in units of the standard deviation of the noise. The upper panels show the 90% credible intervals from the posterior probability density functions of the waveform time series, inferred using CBC waveform templates from Bayesian inference (LALINFERENCE) with the PhenomP model (red band) and by the BAYESWAVE wavelet model (blue band) [4].

Parameters $\boldsymbol{\theta}$ are both extrinsic depending on the detector, *e. g.* its position, and intrinsic depending on the collision event itself, *e. g.* the masses of the BHs and its distance to the earth. The parameters for a BBH are described below in more detail. The detection is affected by noise depending on the frequency as illustrated in Fig.3. This noise can be described by a Gaussian noise, with variance σ_j depending on the frequency bin. Given the model, described by the template $\mu(\boldsymbol{\theta})$, the likelihood for the data d_j in a single frequency bin j is:

$$\mathcal{L}(d_j|\boldsymbol{\theta}) = \frac{1}{2\pi\sigma_j} e^{-2\Delta f (d_j - \mu_j(\boldsymbol{\theta}))^2/\sigma_j} \tag{1}$$

where Δf is the frequency resolution. Note that the factor $2\Delta f$ comes from a factor of 1/2 in the normal distribution and a factor of $4\Delta f$ needed to convert the square of the Fourier transform into units of a one-sided (having only positive frequencies) power spectral density. Moreover, the normalization factor does not contain a square root because the data is complex-valued, and thus the Gaussian is a two-dimensional likelihood. Gravitational-wave signals are typically spread over many (M) frequency bins. Assuming the noise in each bin is independent, the combined likelihood is a product of the likelihoods for each bin, which is written after taking its logarithm, to avoid to multiply small numbers, as the following log-likelihood

$$\log \mathcal{L}(\{d_j|\boldsymbol{\theta})\} = \sum_{j=0}^{M} \log \mathcal{L}(d_j|\boldsymbol{\theta})\} = -\sum_j \log(2\pi\sigma_j) - 2\Delta f \sum_j (d_j - \mu_j(\boldsymbol{\theta}))^2/\sigma_j \tag{2}$$

Where the first term is a constant term.

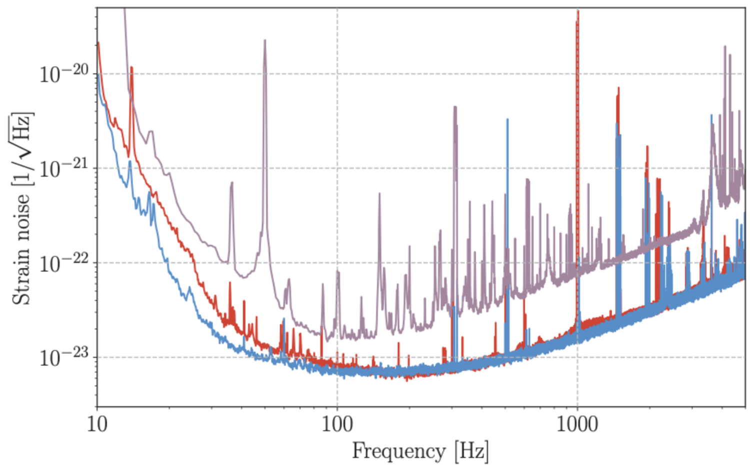

Figure 3: Amplitude spectral density of the total strain noise of the Virgo (violet), Ligo-Hanford LHO (red), and Ligo-Livingston LLO (blue) detectors. The curves are representative of the best performance of each detector during O2 [4]. Peaks correspond to known noises sources such as seismic (from ground vibrations), thermal noise (from the microscopic fluctuations of the individual atoms in the mirrors), quantum noise (from photon counting in the photodetector), laser noises (small variations in the laser intensity and frequency), beam jitter (misalignement of the laser beam with respect to the optical cavities), scattered light, generated by tiny imperfections in the mirrors of the interferometers, etc.. [4]

1 Questions

This tutorial is divided in two parts, the first on the analysis of artificial data and the second on the analysis of the event GW150914.

Artificial Data

We start by using artificial data, given in the starting Python notebook, to build and test our inference procedure. We take a simple linear model, where the variable d_n is a linear function of the frequency x, depending on two parameters: an intercept, called *distance* and a slope called m.

$$d_n = distance^* + m^*\, x_n + \epsilon_n \tag{3}$$

where (ϵ_n) is a Gaussian noise of variance σ_n. True parameters $\boldsymbol{\theta}^* = (m^*, distance^*)$ have been fixed to the values $distance^* = 600$, $m^* = 30$. To reproduce a large variability in the noise variance as a function of the frequency bin, we assume that the variance σ_n follows from a log-normal distribution : $\sigma_n = e^{\alpha}$ with α uniformly distributed in the interval $[-2, 6]$. The signal d_n sampled for frequencies x_n in $[0, 1]$ at intervals $dx = 5\,10^{-3}$ in $n = 1..M$ bins with $M = 200$, is given in the starting notebook for a random realization of the noise.

1. Write the Gaussian log-likelihood for the data $\mathbf{d}$ given the model.
2. Deduce the posteriori probability $P_{post}\ (\boldsymbol{\theta}|\mathbf{d}, \boldsymbol{\sigma})$ given the data $\mathbf{d}$. Use two type of priors: a non-negative prior and a more stringent uniform prior in a given interval. More precisely, the latter prior should enforce the mass m to lie in the interval between 20 and 100 and the distance *distance* to lie in the interval between 300 and 3000 (in appropriate units).
3. Write a Monte-Carlo algorithm to sample the probability distribution for the parameters. The log-probability, given by the sum of the log posterior and the log prior is the energy in the Monte-Carlo

algorithm at the inverse temperature $\beta = 1$. Start the algorithm with the minimal possible values of the parameters and then use Monte-Carlo steps in the parameter space, with amplitudes chosen from a Gaussian distribution of variance $\sigma_m = 1$ for the mass and $\sigma_d = 10$ for the distance (such a choice only depends on the order of magnitude chosen for such parameters). Plot the log-probability as a function of the number of Monte-Carlo steps.

4. Once a stationary distribution for the log-probability has been reached, deduce the optimal values of the parameters; plot their distribution and compare their average value and standard variation with their true values. Use a corner plot (thanks to the Python module corner.py) to represent the distribution of the obtained parameters as function of the true ones.

5. Compare the results for the non-negative prior and for the more stringent uniform prior in the given interval

Analysis of the GW150914

The second part of the tutorial consists in analyzing the first Binary Black hole detection, GW150914 with the Template IMRPhenomPv2 and SEOBNRv4 and to infer the masses of the two black holes and their luminosity distance from earth.

Parameters $\boldsymbol{\theta}$ for the BBH coalescence: The GW signal emitted from a BBH coalescence depends on several intrinsic parameters that directly characterize the dynamics of the two objects and their emitted waveform, and extrinsic parameters that encode the relation of the source to the detector network.

In general relativity, an isolated BH is uniquely described by its mass, spin, and electric charge. For astrophysical BHs, we assume the electric charge to be negligible. A BBH undergoing quasi-circular spiraling motion can be described by eight intrinsic parameters : the two masses m_1, m_2 and the two three-dimensional spin vectors $\mathbf{s}_i$ characterizing the BHs in a reference frequency. Seven additional extrinsic parameters are needed to describe a BH binary: the sky location (right ascension and declination), the luminosity distance, the orbital inclination and polarization angle, the time, and phase at coalescence.

We will focus here on the inference of the masses $m_1(M_{sun})$ and $m_2(M_{sun})$, in units of solar mass (M_{sun}) and the the luminosity distance *distance* in MegaParsecs (Mpc) (where $1Mpc \sim 3\,10^{22}$ meters). As explained in detail in [1], the posterior for such parameters can be obtained by marginalizing over all the other parameters (e. g. see Appendix C.6 for the reconstruction of the luminosity distance parameter D_L). For the sake of simplicity, we will fix all parameters to the optimal ones except the 3 parameters we are interested in and to further simplify, we will assume that the two masses are equal $m_1 = m_2 = m$ and we will only infer theref 2 parameters, the mass m and the distance *distance*. Moreover we will only use the signal from a single detector, In this case the precise distance D_L cannot be determined, our inferred is proportional to the true one $distance = D_L/F$ by a factor $F \approx 0.5$ which depends on the direction of the antenna. Two detectors are necessary to uniquely determine the distance D_L.

Question 6. Look at the notebook Tutorial 10. The starting code for signal pre-treatment is taken from gwodw

Follow the previous used in the analysis of artificial data for the real signal, in particular use your own Monte-Carlo to sample the posterior probability, starting from initial parameter $\theta = 30, 300$ (for smaller masses the phase parameter should be probably re-set) and find the distribution of the optimal parameters $\theta = \{m, distance\}$ and their average values and variance. Build the corner plot for the distribution of the parameters during the MC sampling.

Include both a non-negative prior and a prior enforcing parameter intervals from BBH detected in the first and second run of the gravitational waves detections [4], where BBH masses between 20 and 100 M_{sun} and distances between 300 and 3000 M_{pc} were found.

The treatment of signal from two detectors can be found in gwodw.

References

[1] Eric Thrane and Colm Talbot An introduction to Bayesian inference in gravitational-wave astronomy: parameter estimation, model selection, and hierarchical models Astronomical Society of Australia (PASA) doi: 10.1017/pas.2020.xxx. (2020)

[2] Anderson W. G., Brady P. R., Creighton J. D. E., Flana- gan E. E., 2001, Phys. Rev. D, 63, 042003

[3] Codes available at bilby example and gwodw

[4] B. P. Abbott et al. GWTC-1: A Gravitational-Wave Transient Catalog of Compact Binary Mergers Observed by LIGO and Virgo during the First and Second Observing Runs. arXiv. This should lead to discussion and interpretation. The data used in these tutorials will be downloaded from the public DCC page LIGO-P1800370. Phys.Review X 9, 031040 (2019)

[5] see ligo.org and Nobel prize description for a brief description of the experiment.