Tutorial 4, Advanced MCMC
2020/21 ICFP Master (second year)

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1. **Symmetric simple exclusion process**

   For a system of two (indistinguishable) hard spheres on 4 lattice sites:

   (a) Determine the sample space Ω.

   (b) Describe the Metropolis algorithm both for periodic boundary conditions and for hard-wall boundary conditions.

   (c) Show that this algorithm satisfies detailed balance for both conditions.

   (d) Write down (on paper, or on the blackboard) the transition matrices for both conditions.

2. **Totally asymmetric simple exclusion process**

   Proceed as in exercise 1., but for the TASEP. For periodic boundary conditions, you can restrict yourself to the forward-moving part of the lifted sample space. In addition to the above:

   (a) Show that the TASEP (for hard-wall conditions) is a lifting of the SSEP.

3. **Lifted TASEP**

   Proceed as in exercise 1., but for the lifted TASEP. Restrict yourself to the case of periodic boundary conditions. In addition to the above:

   (a) Show that the lifted TASEP (for periodic boundary conditions) is a lifting of the TASEP

   (b) Can you show that the same holds in the presence of periodic boundary conditions?

4. **Perfect card shuffling through the top-to-random shuffle**

   In lecture 4 (part 4.1), we discussed a card-shuffling algorithm that produced a perfect sample.

(a) Write a simple MCMC algorithm. Test it for 5 cards, and show (by simulation) that all the 120 possible states of the deck come up with the same probability.

5. **Particle diffusion on the path graph**

   In lecture 4 (part 4.1), we flashed a perfect-sampling algorithm for the diffusion of a particle on the path graph P_5.

   (a) Implement this algorithm and show that, indeed, it realizes a perfect sampling.

6. **Perfect sampling for the Ising model**

   In lecture 4 (part 4.2), we sketched a perfect-sampling algorithm for the Ising model, that relied on the heat-bath algorithm.

   (a) Implement the heat-bath algorithm for the Ising model, and run it in parallel (with identical choices of i (spin to be updated) and Υ (random number needed for the heat-bath). Show that, after some time, the two configurations are identical.

   (b) Sketch how this could be used to implement the coupling-from-the-past algorithm and, if you have time, implement it in Python.