

Tutorial 2, Advanced Topics in Markov-chain Monte Carlo

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1. Lifting for the V-shaped stationary distribution

In this exercise for the V-shaped stationary distribution on sites $i = \{1, \dots, n\}$, namely $(\pi_{2k-1} = \frac{2}{3n}, \pi_{2k} = \frac{4}{3n})$, you can either run the MCMC algorithm many times or else do the repeated matrix multiplication as $\pi^{\{t\}} = \pi^{\{t-1\}}P$ (in numpy: `pi = pi @ P`).

- (a) Program the time evolution of π , for different n , and track the TVD starting from $i = 1$ (suppose that this is the most unfavorable initial condition, that is, suppose that $\text{TVD} = d(t)$). Do this for the collapsed Markov chain (Metropolis algorithm), and for the lifted Markov chain (with transport and resampling).
- (b) For the collapsed MCMC, plot $d(t)$ vs t/n^2 and also vs $t/(n^2 \log n)$. The latter gives a better data collapse.
- (c) For the lifted MCMC, plot $d(t)$ vs t/n for small times (for a few values of n) and interpret the results.
- (d) For the lifted MCMC, plot $d(t)$ vs t/n^2 for all times (for a few values of n and interpret the results.

2. Lifting and global balance

In lecture 2, we discussed the lifting on a path graph P_n for $\pi_i = 1/n$ (NB: the one with transport + resampling)

- (a) Prove that it satisfies the global-balance condition for any lifted configuration.
- (b) Prove that it is really a lifting.

3. Conductance and correlation times (projects for next week)

- (a) In lecture 2, we discussed that a lifting cannot increase the conductance of a Markov chain. Prove this (and present it in week 3, if you want).

- (b) In lecture 2, we discussed an important achievement by Sinclair and Jerrum (1989), telling us that the conductance gives an upper bound for the correlation time. Look up the proof (for a reversible chain) in the original paper (Lemma 3.3, page 15-17), which consists in just two pages of basic linear algebra. Present this in week 3, if you want.