Empirical Review of Models used for Predicting Financial Market Crashes Using Market Data

COMP 451 Final Project: Final Report

By Ping-Chieh Tu, Adrien Bélanger, Inigo Torres

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Introduction Inigo

The volatility and complexity of financial markets have always been a significant challenge for the management of modern economies. For instance, the abrupt declines in the markets, often known in finance as crashes, can lead to widespread financial losses, economic recessions, and a loss of confidence in the stability of financial systems.

The idea of being able to anticipate and predict such market fluctuations is not new and has been widely studied from a mathematical point of view. However, conventional methods used by econometricians for time series analysis, such as Linear Trend Projection or Weighted Moving Average, may be effective for markets with general stationary trends but lack effectiveness for highly volatile ones. Modern machine learning techniques, particularly those involving neural networks and attention-based architectures, may provide a new pathway to surpassing these traditional techniques.

In this project, we propose to make an extensive comparative study of the effectiveness of 3 of the most widely used methods for Time series forecasting today: Recurrent Neural Networks (RNNs), Transformer-based architectures, and the Autoregressive Integrated Moving Average (ARIMA) model.

Each of these models is developed with a distinct methodological perspective. ARIMA models, based on classic statistics and linear algebra concepts, have long been a go-to tool for time series forecasting. RNNs, specifically Long Short-Term Memory (LSTM) networks, offer a non-linear and data-driven alternative that can capture long-range dependencies while reducing issues of vanishing or exploiting gradients. Transformers, which introduce attention mechanisms, aim to further improve predictive capabilities by focusing selectively on critical segments of past information, often achieving state-of-the-art performance in various sequential prediction tasks.

Our main objective is then to develop each of these models and subject them to a series of tests using real historical financial data to determine their effectiveness in predicting financial crashes.

Although the world of finance is changing, and there are no specific parameters since each financial market is governed by its own rules, we will test each of these models under the same conditions and thus establish an empirical review to provide clear insights into the strengths and limitations of each methodology. We aim to guide researchers and practitioners in selecting suitable models for short-term market crash prediction, ultimately contributing to more robust risk management strategies.

Litterature Review Adrien

We are not the first to attempt to compare ML models performance on their prediction of market crashes using SP500 historical data. Multiple approaches have been tried and tested. Time series analysis has used ARIMA and RNN based models, with the more recent addition of Transformers boosting and improving their performance [1]–[3]. Reviews of these models have been done before, but the comparison of these three models on short-term prediction using very recent market data is lacking in litterature.

One article compared Linear Regression and Autoregressive Moving Average (ARIMA) to predict the volatility and trend of SP500. [4]. Key-findings show that ARIMA struggled on short term predictions, particularly during the 1930s and 2020 volatile markets.

Zhou et al. demonstrated that transformers handle long-term dependencies well with the "Informer model," which efficiently handles long sequences and improves trend forecasting accuracy using a "self-attention" mechanism [5]. Moreover, Lim et al. were even able to outperform traditional methods (like ARIMA) by combining the Transformer architecture with recurrent layers [6].

Zhen Zeng studied the applicability of Transformers combined with RNNs in trend detection specifically for financial time series in his paper [7]. He concluded that combining these two architectures significantly improved forecasting accuracy for intraday stock price movements of the SP500.

Market Crashes do not hold a single definition. While historical data tags specific periods as depressions and bubbles, there are no specific metrics that are universally defined. Some have defined it as a rapid decline of 20% or more from a recent peak over a short period [8], [9]. Others still have defined them as a drawdown of 99.5% quantile [10].

Time series in the stock market often exhibit complex trends and non-linear patterns. Long Short-Term Memory (LSTM) Recurrent Neural Networks (RNN) are designed to learn long-term trends in sequential data, making them very good at modeling and forecasting financial market time series [11], [12].

LSTM networks address the vanishing gradient problem encountered in standard RNNs on this type of data by introducing specialized LSTM cells. These cells incorporate gates to control flow, allowing LSTMs to forget and remember information over long sequences [11].

The LSTM cell can be defined as follows [11]:

$$\begin{aligned}
i_t &= \sigma(W_i x_t + U_i h_{t-1} + b_i), \\
f_t &= \sigma(W_f x_t + U_f h_{t-1} + b_f), \\
o_t &= \sigma(W_o x_t + U_o h_{t-1} + b_o), \\
\tilde{c}t &= \tanh(W_c x_t + U_c ht - 1 + b_c), \\
c_t &= f_t \cdot c_{t-1} + i_t \cdot \tilde{c}_t, \\
h_t &= o_t \cdot \tanh(c_t),
\end{aligned}$$

where

- x_t is the input at time t, representing market data (e.g., stock price, volume),
- h_{t-1} is the hidden state (output) of the previous time step,
- i_t, f_t, o_t are the input gate, forget gate, and output gate
- $\tilde{c}_t$ is the candidate memory cell state,
- c_t is the cell state at time t, capturing long-term dependencies,
- W and U are the weight matrices and b is the bias vector,
- σ is the sigmoid activation function

The *forget* gate, f_t, determines how much of the previous cell c_{t-1} is retained. The *input* gate i_t updates the cell with new information. The *output* gate o_t controls the hidden state, which is the LSTM output at time t [11].

Time series in the stock market are non-stationary, which means that their statistical properties (Avg, median) change over time. ARIMA models can be adapted to capture the behavior of non-stationary time-series [13]. A mehodology formalized by Box et Al. in 2015 to apply the ARIMA model with a moving average component [14]. This makes ARIMA suitable for predicting non-stationary time series, such as financial markets, as demonstrated by M. K. Ho et al. in their paper [13]. The ARIMA model is defined mathematically as [14]:

$$\varphi(B)z_t = \phi(B)\nabla^d z_t = \theta_0 + \theta(B)a_t$$

where

- $\phi(B)$ is the autoregressive part with p degrees,
- $\theta(B)$ is the moving average part with q degrees,
- ∇ is the integrated (degree of differencing) part with d degrees,
- and θ_0 is the constant term.

The use of ACF (autocorrelation function) and PACF (partial ACF) will help us decide the hyperparameters p, d, q for building our model [15], and performing Grid-Search on the specific hyperparameters will help us pin point the exact best accuracy we can get.

Transformers are based on self-attention mechanisms. They are able to weigh the importance of input elements by computing *attention scores*. While transformers were originally developed for natural language processing tasks, they have become popular for time-series forecasting due to their ability to handle long-range dependencies [16].

Unlike RNNs, Transformers process all steps at the same time, enabling parallel computation and improved efficiency for long-range dependency learning [16].

The core attention scoring by scaled dotv product is defined as [16]:

$$\text{Attention}(Q, K, V) = \text{softmax}\left(\frac{QK^T}{\sqrt{d_k}}\right)V,$$

where:

- Q (queries), K (keys), and V (values) are linear projections of the input data,
- is the dimension of the key vectors,
- The softmax function ensures that attention scores are normalized.

Transformers eliminate the need for recurrent connections by relying entirely on attention mechanisms. This allows them model long-range dependencies more effectively than RNNs, which sometimes struggle with sequential bottlenecks and vanishing gradients [16].

While originally developed for natural language processing tasks, Transformers have been increasingly applied to time-series forecasting [7]. Given the volatility and complexity of financial markets, their ability to capture complex patterns makes them an ideal choice for financial market forecasting [17].

Drastic market crashes are rare. Models can achieve extremely high accuracy by simply predicting no market crash for every datapoint [18]. Other methods are thus needed to evaluate the models. Others in litterature have used many methods, such as evaluating true positives and true negatives [19]. Other have used mean absolute error (MAE) to assess prediction accuracy [15]. Finally, some have used runtime and resource usage for practical feasibility to assess their performance [6].

Methodology Adrien for market crash and Experiment choice Everyone for their assigned model

Note: All code can be found in the gitHub repository linked in the Appendix.

Definitions and Metrics used in our project

For our project, we use an LSTM-based RNN, Transformers and ARIMA to predict whether there will be a crash in the next x days based on the past y days. We defined crash as a 99.5% quantile drawdown within a day. This gave us around 26 crashes in the test data.

To evaluate our models, we will measure the number of Predicted crashes against the real number of crashes. We will also check the number of false alarms and missed crashes. Finally, we will evaluate the training and resource usage time of all three models for each experiment.

Database and preprocessing

We pulled the code from the SP500 public data available on Yahoo Finance financial data library. From there, we preprocessed the code by labelling crashes in the database.

Our code made use of a sliding window for the sequence length. Then, we created batches of datapoints of size 20. We then tried to predict if there would be a crash in the following shift days, which was a hyperparameter. For example, we could have a 15 day sliding window to predict if there would be a crash in the 3 days after those 15 days. This made the problem one of binary classification, which was simple and efficient to process, and made for easy comparison.

RNN Implementation

Our RNN code was based on the *Keras* Sequential model, using the pre-made LSTM layers [20], [21]. This was used since our project aimed to be a comparison, and not one of implementation. The code used two LSTM layers of 50 units each, then a dense layer with a sigmoid activation layer for binary classification. We then used the adam optimizer with a binary cross-entropy loss function to train our binary model.

ARIMA Implementation

Transformers Implementation

Experiments

1. 30 days sliding window, 14 days shift days

Hypothesis: Here, we expect Transformers to profit from the larger sliding window, surpassing the performance of RNNs [4], [12]. We expect ARIMA to have the best performance of all three experiments in this one, as it has the most context and does better with more data [13].

2. Oscar - 14 days sliding window, 7 days shift days

3. Inigo - 7 days sliding window, 3 days shift days

Overall, we expect ARIMA to do good on longer sequences [13]. We expect RNNs to do the best with the low context of the third experiment [22]. Transformers should excell in higher context windows, and fall slightly behind RNNs in the third experiment. Because of the short-term nature of the experiments, we expect ARIMA to be behind the other two models in all three experiment, given the low context.

Empirical Evaluation Oscar

Discussion Everyone discusses their experiment

Conclusion Everyone does the conclusion for their own experiments

Future Directions Inigo

Self-Assessment Oscar

Contributions Everyone writes their own contribution

1. Adrien - For the report, my main contributions were for writing the Litterature Review and parts of the Methodology, Discussion and Conclusion. I've also contributed to the code by implementing the RNN model, consolidating the three models in a notebook and helping design the experiments.

2. Ping-Chieh

3. Inigo

Appendix

1. Link to Github with the code: https://github.com/AdrienBelanger/451-Project

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