Université catholique de Louvain
Louvain School of Statistics

LSTAT2130 - Bayesian Statistics

Project - Group Q

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Contents

1 Introduction

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## [1] 0.05892352
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2 Question 1

Let $\theta_k := (\mu_k, \phi_k)$ be the set of parameters for a HNI with respect to region k.

2.1 (a) Theoretical probability

Let X be the monthly net income of 1123 Belgian households net income (HNI) older than 30 years. Regardless the 2 regions ($k = \{1, 2\}$ wrt Flanders and Wallonnia, respectively), is assumed it follows a Gamma distribution. It can be reparametrised in terms of its mean μ and dispersion parameter ϕ with the following trick:

$$\kappa = \frac{1}{\phi}$$
$$\lambda = \frac{1}{\phi\,\mu}$$

For both regions $k = \{1, 2\}$: This gives

$$f(x_k) = \frac{1}{\Gamma(\phi_k^{-1})}\big(\phi_k\mu\big)^{\phi_k} x_k^{\frac{1}{\phi_k}-1} \exp\big(\frac{-x_k}{\phi_k\mu}\big)$$

Then, the probability to fall into a certain HNI interval is:

$$P(x_{j_1} < x < x_{j_2}) = \int_{x_{j_1}}^{x_{j_2}} \frac{1}{\Gamma(\phi_k^{-1})}\big(\phi_k\mu\big)^{\phi_k} x_k^{\frac{1}{\phi_k}-1} \exp\big(\frac{-x_k}{\phi_k\mu}\big)$$

2.2 (b) Theoretical expression for the likelihood

We have, writing $P := (p_{k,1}, .., p_{k,10})$ and $X_k := (x_{k,1}, ...x_{k,10})$

$$\begin{aligned} X_k|P \sim \text{Mul}(n_k, P) &= \frac{x_k!}{x_{k,1}! \dots x_{k,10}!} p_1^{n_{x,1}} \times ... \times p_1^{x_{k,10}} \text{ when } \sum_{j=1}^{10} x_j = x_k \\ &= 0 \text{ otherwise} \end{aligned}$$

$$L(\theta_k, D_k) = P(D_k|\theta_k) = \pi$$

A Appendix

A.1 Figures

A.2 Code

Note

For reproducibility purposes, the complete R project containing the source code and the results is available on https://github.com/.