

LSTAT2130BayesianProject

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<code>library(kableExtra)</code>	

Introduction

```
Table <- matrix(data= c(25,69, 65,106, 80,106,136,94,76,46,
                        17,36, 47,58 , 47,53 ,59 ,54,33,21),
                nrow = 2, byrow = T)
rownames(Table) <- c("Flanders", "Wallonia")
colnames(Table) <- c("<1200", "[1200-1500]", "1500-1800", "1800-2300", "2300-2700",
                    "2700-3300", "3300-4000", "4000-4900", "4900-6000", ">6000")

NbFlemish <- sum(Table[1,])
NbWaWalloons <-sum(Table[2,])

kappaFct <- function(phi){1/phi}
lambdaFct <- function(phi,mu){1/(phi*mu)}
```

```
## [1] 0.05892352
```

Question 1

Let $\theta_k := (\mu_k, \phi_k)$ be the set of parameters for a HNI with respect to region k .

(a) Theoretical probability

Let X be the monthly net income of 1123 Belgian households net income (HNI) older than 30 years. Regardless the 2 regions ($k = \{1, 2\}$ wrt Flanders and Wallonia, respectively), is assumed it follows a Gamma distribution. It can be reparametrised in terms of its mean μ and dispersion parameter ϕ with the following trick:

$$\begin{aligned}\kappa &= \frac{1}{\phi} \\ \lambda &= \frac{1}{\phi \mu}\end{aligned}\tag{1}$$

For both regions $k = \{1, 2\}$: This gives

$$f(x_k) = \frac{1}{\Gamma(\phi_k^{-1})} (\phi_k \mu)^{\phi_k} x_k^{\frac{1}{\phi_k} - 1} \exp\left(\frac{-x_k}{\phi_k \mu}\right)\tag{2}$$

Then, the probability to fall into a certain HNI interval is:

$$P(x_{j_1} < x < x_{j_2}) = \int_{x_{j_1}}^{x_{j_2}} \frac{1}{\Gamma(\phi_k^{-1})} (\phi_k \mu)^{\phi_k} x_k^{\frac{1}{\phi_k} - 1} \exp\left(\frac{-x_k}{\phi_k \mu}\right) dx_k\tag{3}$$

(b) Theoretical expression for the likelihood

$$L(\theta_k, D_k) = P(D_k | \theta_k) \propto\tag{4}$$

First analyses