LSTAT2130BayesianProject

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Contents


```
library(kableExtra)
```

Introduction

```
Table <- matrix(data= c(25,69, 65,106, 80,106,136,94,76,46,
                        17,36, 47,58 , 47,53 ,59 ,54,33,21),
                nrow = 2, byrow = T)
rownames(Table) <- c("Flanders", "Wallonia")
colnames(Table) <- c("<1200", "[1200-1500)", "1500-1800", "1800-2300", "2300-2700",
                     "2700-3300", "3300-4000", "4000-4900", "4900-6000", ">6000")

NbFlemish <-  sum(Table[1,])
NbWaWalloons <-sum(Table[2,])
```

```
kappaFct <- function(phi){1/phi}
lambdaFct <- function(phi,mu){1/(phi*mu)}
```

```
## [1] 0.05892352
```

Question 1

Let $\theta_k := (\mu_k, \phi_k)$ be the set of parameters for a HNI with respect to region k.

(a) Theoretical probability

Let X be the monthly net income of 1123 Belgian households net income (HNI) older than 30 years. Regardless the 2 regions ($k = \{1, 2\}$ wrt Flanders and Wallonnia, respectively), is assumed it follows a Gamma distribution. It can be reparametrised in terms of its mean μ and dispersion parameter ϕ with the following trick:

$$
\begin{aligned}
\kappa &= \frac{1}{\phi} \\
\lambda &= \frac{1}{\phi\ \mu}
\end{aligned}
\tag{1}
$$

For both regions $k = \{1, 2\}$: This gives

$$f(x_k) = \frac{1}{\Gamma(\phi_k^{-1})}\big(\phi_k \mu\big)^{\phi_k} x_k^{\frac{1}{\phi_k}-1} \exp\big(\frac{-x_k}{\phi_k \mu}\big) \tag{2}$$

Then, the probability to fall into a certain HNI interval is:

$$P(x_{j_1} < x < x_{j_2}) = \int_{x_{j_1}}^{x_{j_2}} \frac{1}{\Gamma(\phi_k^{-1})}\big(\phi_k \mu\big)^{\phi_k} x_k^{\frac{1}{\phi_k}-1} \exp\big(\frac{-x_k}{\phi_k \mu}\big) \tag{3}$$

(b) Theoretical expression for the likelihood

We have, writing $P := (p_{k,1}, .., p_{k,10})$ and $X_k := (x_{k,1}, ...x_{k,10})$

$$
\begin{aligned}
X_k|P \sim \mathrm{Mul}(n_k, P) &= \frac{x_k!}{x_{k,1}! \dots x_{k,10}!} p_1^{n_{x,1}} \times ... \times p_1^{x_{k,10}} \text{ when} \sum_{j=1}^{10} x_j = x_k \\
&= 0 \text{ otherwise}
\end{aligned}
$$

$$L(\theta_k, D_k) = P(D_k|\theta_k) = \pi \tag{4}$$