

All operations on the following equations are modulo-3 operations:https://en.wikipedia.org/wiki/Modular_arithmetic

Let Red = 0, Green = 1, Blue = 2.

Let A be the first row and B the second row.

Let a_0, a_1, a_2 the number associated with the colour of the hat of peoples in A row

Let b_0, b_1 the number associated with the colour of the hat of peoples in B row

Let a A-function be a function of what people of the A column see

Let a B-function be a function of what people of the B column see

To solve the problem, We need 2 A-functions:

$$A_0(b_0, b_1) = x_0b_0 + y_0b_1$$

$$A_1(b_0, b_1) = x_1b_0 + y_1b_1$$

and 2 B-functions:

$$B_0(a_0, a_1) = x_2a_0 + y_2a_1$$

$$B_1(a_0, a_1) = x_3a_0 + y_3a_1$$

such that:

$$A_0(B_0(a_0, a_1), B_1(a_0, a_1)) = a_0$$

$$A_1(B_0(a_0, a_1), B_1(a_0, a_1)) = a_1$$

$$B_0(A_0(b_0, b_1), A_1(b_0, b_1)) = b_0$$

$$B_1(A_0(b_0, b_1), A_1(b_0, b_1)) = b_1$$

where all x_0, x_1, x_2, x_3 and all y_0, y_1, y_2, y_3 are in $[1, 2]$. There is no need to consider 0 as each result must carry as much informations as possible and there is no need to consider factor greater than 2 as all operations are modulo-3

First 2 peoples of the A row will each say the colour associated with result of these A-functions

First 2 peoples of the B row will each say the colour associated with result of these B-functions

From each of these equations, we deduce 8 equalities:

$$A_0(B_0(a_0, a_1), B_1(a_0, a_1)) = x_0(x_2a_0 + y_2a_1) + y_0(x_3a_0 + y_3a_1) = (x_0x_2 + y_0x_3)a_0 + (x_0y_2 + y_0y_3)a_1$$

so,

$$(x_0x_2 + y_0x_3)a_0 + (x_0y_2 + y_0y_3)a_1 = a_0$$

so,

$$1) x_0x_2 + y_0x_3 = 1$$

$$2) x_0y_2 + y_0y_3 = 0$$

$$A_1(B_0(a_0, a_1), B_1(a_0, a_1)) = x_1(x_2a_0 + y_2a_1) + y_1(x_3a_0 + y_3a_1) = (x_1x_2 + y_1x_3)a_0 + (x_1y_2 + y_1y_3)a_1$$

so,

$$(x_1x_2 + y_1x_3)a_0 + (x_1y_2 + y_1y_3)a_1 = a_1$$

so,

$$3) x_1x_2 + y_1x_3 = 0$$

$$4) x_1y_2 + y_1y_3 = 1$$

$$B_0(A_0(a_0, a_1), A_1(a_0, a_1)) = x_2(x_0b_0 + y_0b_1) + y_2(x_1b_0 + y_1b_1) = (x_0x_2 + x_2y_0)b_0 + (x_1y_2 + y_1y_2)b_1$$

so,

$$(x_0x_2 + x_2y_0)b_0 + (x_1y_2 + y_1y_2)b_1 = b_0$$

so,

$$5) x_0x_2 + x_2y_0 = 1$$

$$6) x_1y_2 + y_1y_2 = 0$$

$$B_1(A_0(a_0, a_1), A_1(a_0, a_1)) = x_3(x_0b_0 + y_0b_1) + y_3(x_1b_0 + y_1b_1) = (x_0x_3 + x_1y_3)b_0 + (x_3y_0 + y_1y_3)b_1$$

so,

$$(x_0x_3 + x_1y_3)b_0 + (x_3y_0 + y_1y_3)b_1 = b_1$$

so,

$$7) x_0x_3 + x_1y_3 = 1$$

$$8) x_3y_0 + y_1y_3 = 0$$

From 1) and 5):

$$x_0x_2 + y_0x_3 = 1$$

$$x_0x_2 + x_2y_0 = 1$$

so,

$$y_0(x_3 - x_2) = 0$$

as y_0 is in $[1,2]$,

$$\mathbf{x_3=x_2}$$

From 3):

$$x_1x_2 + y_1x_3 = x_1x_2 + y_1x_2 = x_2(x_1 + y_1)=0$$

as x_2 is in $[1,2]$,

$$\mathbf{x_1=3-y_1}$$

From 1)

$$x_0x_2 + y_0x_3 = 1 \Rightarrow x_0x_2 + y_0x_2 = 1 \Rightarrow x_2(x_0 + y_0) = 1$$

so either:

$$\mathbf{x_2 = 1, x_0 = 2, y_0 = 2}$$

or:

$$\mathbf{x_2 = 2, x_0 = 1, y_0 = 1}$$

From 4)

$$x_1y_2 + y_1y_3 = 1 \Rightarrow x_1y_2 + (3 - x_1)y_3 = 1 \Rightarrow x_1y_2 - x_1y_3 = 1 \Rightarrow x_1(y_2 - y_3) = 1$$

so either:

$$\mathbf{x_1 = 1, y_2 = 2, y_3 = 1}$$

or:

$$\mathbf{x_1 = 2, y_2 = 1, y_3 = 2}$$

Now, we have 4 equivalent solutions for the factors - which is expected as we can assign any of the 2 sets of functions to any of the 2 rows and any of the 2 functions of the set to any of the 2 peoples of the row:

$$\mathbf{x_0 = 1 \ y_0 = 1}$$

$$\mathbf{x_1 = 2 \ y_1 = 1}$$

$$\mathbf{x_2 = 2 \ y_2 = 1}$$

$$\mathbf{x_3 = 2 \ y_3 = 2}$$

or:

$$x_0 = 1 \ y_0 = 1$$

$$x_1 = 2 \ y_1 = 1$$

$$x_2 = 2 \ y_2 = 2$$

$$x_3 = 2 \ y_3 = 1$$

or:

$$x_0 = 2 \ y_0 = 2$$

$$x_1 = 2 \ y_1 = 1$$

$$x_2 = 2 \ y_2 = 1$$

$$x_3 = 1 \ y_3 = 1$$

or:

$$x_0 = 2 \ y_0 = 2$$

$$x_1 = 2 \ y_1 = 1$$

$$x_2 = 1 \ y_2 = 1$$

$$x_3 = 2 \ y_3 = 1$$

Let consider the first set of factors:

$$A0(b0, b1)=b0+b1$$

$$A1(b0, b1)=2b0+b1$$

$$B0(a0, a1)=2a0+a1$$

$$B1(a0, a1)=2a0+2a1$$

Now let's prove that at least one of them is correct.

Let say the peoples of the A row are all wrong:

It means it exists $n0, n1$ in $[1,2]$ such that:

$$A0(b0, b1) = a0 + n0$$

$$A1(b0, b1) = a1 + n1$$

$$A0(b0, b1) = b0 + b1 = a0 + n0$$

so,

$$a0 = b0 + b1 - n0$$

$$A1(b0, b1) = 2b0 + b1 = a1 + n1$$

so,

$$a1 = 2b0 + b1 - n1$$

Now, we inject that in the B-functions:

$$B0(a0, a1) = 2(b0 + b1 - n0) + (2b0 + b1 - n1) = 4b0 + 3b1 - 2n0 - n1$$

$$B1(a0, a1) = 2(b0 + b1 - n0) + 2(2b0 + b1 - n1) = 6b0 + 4b1 - 2n0 - 2n1$$

Which can be simplify as we are using modulo-3 operations:

$$B0(a0, a1) = b0 - (2n0 + n1)$$

$$B1(a0, a1) = b1 - 2(n0 + n1)$$

$$\text{if } n0 = n1, 2n0 + n1 = 0 \quad ((2*1 + 1)\%3 = 3\%3 = 0 \text{ or } (2*2 + 2)\%3 = 6\%3 = 0)$$

then:

$$B0(a0, a1) = b0$$

$$\text{if } n0 \neq n1, n0 + n1 = 0 \quad ((2 + 1)\%3 = 3\%3 = 0)$$

then:

$$B1(a0, a1) = b1$$

So, if the people of the A row are wrong, one of the people of the B row will be right

In conclusion at least one of them is right.

A2 can say whatever he or she want.