All operations on the following equations are modulo-3 operations:<https://en.wikipedia.org/wiki/Modular_arithmetic>

Let Red = 0, Green = 1, Blue = 2.

Let A be the first row and B the second row.

Let a0, a1, a2 the number associated with the colour of the hat of peoples in A row

Let b0, b1 the number associated with the colour of the hat of peoples in B row

Let a A-function be a function of what people of the A column see

Let a B-function be a function of what people of the B column see

To solve the problem, We need 2 A-functions:

A0(bo, b1) = x0b0 + y0b1

A1(bo, b1) = x1b0 + y1b1

and 2 B-functions:

B0(ao, a1) = x2a0 + y2a1

B1(ao, a1) = x3a0 + y3a1

such that:

A0(B0(a0,a1), B1(a0,a1)) = a0

A1(B0(a0,a1), B1(a0,a1)) = a1

B0(A0(b0,b1), A1(b0,b1)) = b0

B1(A0(b0,b1), A1(b0,b1)) = b1

where all x0,x1,x2,x3 and all y0,y1,y2,y3 are in [1,2]. There is no need to consider 0 as each result must carry as much informations as possible and there is no need to consider factor greater than 2 as all operations are modulo-3

First 2 peoples of the A row will each say the colour associated with result of these A-functions

First 2 peoples of the B row will each say the colour associated with result of these B-functions

From each of these equations, we deduce 8 equalities:

A0(B0(a0,a1),B1(a0,a1)) = x0(x2a0 + y2a1) + y0(x3a0 + y3a1) = (x0x2 + y0x3)a0 + (x0y2 + y0y3)a1

so,

(x0x2 + y0x3)a0 + (x0y2 + y0y3)a1 = a0

so,

1) x0x2 + y0x3 = 1

2) x0y2 + y0y3 = 0

A1(B0(a0,a1),B1(a0,a1)) = x1(x2a0 + y2a1) + y1(x3a0 + y3a1) = (x1x2 + y1x3)a0 + (x1y2 + y1y3)a1

so,

(x1x2 + y1x3)a0 + (x1y2 + y1y3)a1 = a1

so,

3) x1x2 + y1x3 = 0

4) x1y2 + y1y3 = 1

B0(A0(a0,a1),A1(a0,a1)) = x2(x0b0 + y0b1) + y2(x1b0 + y1b1) = (x0x2 + x2y0)b0 + (x1y2 + y1y2)b1

so,

(x0x2 + x2y0)b0 + (x1y2 + y1y2)b1 = b0

so,

5) x0x2 + x2y0 = 1

6) x1y2 + y1y2 = 0

B1(A0(a0,a1),A1(a0,a1)) = x3(x0b0 + y0b1) + y3(x1b0 + y1b1) = (x0x3 + x1y3)b0 + (x3y0 + y1y3)b1

so,

(x0x3 + x1y3)b0 + (x3y0 + y1y3)b1 = b1

so,

7) x0x3 + x1y3 = 1

8) x3y0 + y1y3 = 0

From 1) and 5):

x0x2 + y0x3 = 1

x0x2 + x2y0 = 1

so,

y0(x3 - x2) = 0

as y0 is in [1,2],

**x3=x2**

From 3):

x1x2 + y1x3 = x1x2 + y1x2 = x2(x1 + y1)=0

as x2 is in [1,2],

**x1=3-y1**

From 1)

x0x2 + y0x3 = 1 => x0x2 + y0x2 = 1 => x2(x0 + y0) = 1

so either:

**x2 = 1, x0 = 2, y0 = 2**

**or:**

**x2 = 2, x0 = 1, y0 = 1**

From 4)

x1y2 + y1y3 = 1 => x1y2 + (3 - x1)y3 = 1 => x1y2 - x1y3 = 1 => x1(y2 - y3) = 1

so either:

**x1 = 1, y2 = 2, y3 = 1**

**or:**

**x1 = 2, y2 = 1, y3 = 2**

Now, we have 4 equivalent solutions for the factors - which is expected as we can assign any of the 2 sets of functions to any of the 2 rows and any of the 2 functions of the set to any of the 2 peoples of the row:

**x0 = 1 y0 = 1**

**x1 = 2 y1 = 1**

**x2 = 2 y2 = 1**

**x3 = 2 y3 = 2**

or:

x0 = 1 y0 = 1

x1 = 2 y1 = 1

x2 = 2 y2 = 2

x3 = 2 y3 = 1

or:

x0 = 2 y0 = 2

x1 = 2 y1 = 1

x2 = 2 y2 = 1

x3 = 1 y3 = 1

or:

x0 = 2 y0 = 2

x1 = 2 y1 = 1

x2 = 1 y2 = 1

x3 = 2 y3 = 1

Let consider the first set of factors:

**A0(bo, b1)=b0+b1**

**A1(b0, b1)=2b0+b1**

**B0(a0, a1)=2a0+a1**

**B1(a0, a1)=2a0+2a1**

Now let’s prove that at least one of them is correct.

Let say the peoples of the A row are all wrong:

It means it exists n0,n1 in [1,2] such that:

A0(bo, b1) = a0 + n0

A1(bo, b1) = a1 + n1

A0(bo, b1) = b0 + b1=a0 + n0

so,

a0 = b0 + b1 - n0

A1(bo, b1) = 2b0 + b1 = a1 + n1

so,

a1 = 2b0 + b1 - n1

Now, we inject that in the B-functions:

B0(a0,a1) = 2(b0 + b1 - n0) + (2b0 + b1 - n1) = 4b0 + 3b1 - 2n0 - n1

B1(a0,a1) = 2(b0 + b1 - n0) + 2(2b0 + b1 - n1) = 6b0 + 4b1 - 2n0 - 2n1

Which can be simplify as we are using modulo-3 operations:

B0(a0,a1) = b0 - (2n0 + n1)

B1(a0,a1) = b1 - 2(n0 + n1)

if n0 = n1, 2n0 + n1 = 0 ((2\*1 + 1)%3 = 3%3 = 0 or (2\*2 + 2)%3 = 6%3 = 0)

then:

B0(a0,a1) = b0

if n0 != n1, n0 + n1 = 0 ((2 + 1)%3 = 3%3 = 0)

then:

B1(a0,a1) = b1

So, if the people of the A row are wrong, one of the people of the B row will be right

In conclusion at least one of them is right.

A2 can say whatever he or she want.