What is the drawdown of a portfolio ?

The formula is given by

$$D(T) = \max_{t \in [0,T]} (X_t - X_T)$$

Last peak

Present value

More concretely, the drawdown of a portfolio indicates a moment of decline, which is psychologically significant and relevant. A portfolio with a low drawdown, regardless of its performance, will not have a period of intense stress, which is easier to bear.

Since to recover from a 50% loss, you need to make a 100% return, the idea is to minimise the drawdown in order to avoid this type of situation, and thus avoid periods of crisis with large losses.

Drawdown graphical view

Null drawdown

$$D(T) = \max_{t \in [0,T]} (X_t - X_T) = X_T - X_T = 0$$

Positive drawdown

$$D(T) = \max_{t \in [0,T]} (X_t - X_T) \approx 1{,}09 - 0{,}0017 = 1{,}0883$$

Drawdown is used in passive and diversified investments.

Historically, one of the largest portfolios that has achieved a fairly high rate of return without using sophisticated techniques such as high-frequency trading, machine learning, etc., is that of **Ray Dalio**, who built the All Weather, reputed to have one of the lowest drawdowns, and consequently, among the fastest recovery rates.

The portfolio is constructed to provide stable returns regardless of economic trends. It does, however, include bonds, which is not the case with the club.

He founded Bridgewater Associates in 1975 which had **$125 billion** in assets under management in 2023.
Ray Dalio's fortune reached **$20 billion** in 2022 according to Forbes.

Concerns about implementation for our portfolio

- Drawdown is all very well in theory, but in practice, when we want to choose the distribution of a portfolio, we are faced with an optimization problem that has to be solved numerically.
- Each decision variable is the weight to be assigned to each action.
- Unfortunately, by means of mathematical justifications, we need to reformulate our problem so that our objective function has good properties and our program (solver) has tools that lead to the solution.

Solution

By introducing another value, linked to drawdown, called the **Conditional Drawdown-at-Risk (CDaR)**, we can theoretically prove that our problem can be solved (with a chosen objective function to minimize).
We have already mentioned Value at Risk (**VaR**) and Expected Shortfall (average of the worst portfolio returns), which is also called Conditional Value-at-Risk (**CVaR**), hence the parallel.

Conditional Drawdown-at-Risk (CDaR)

(The frightening formula)

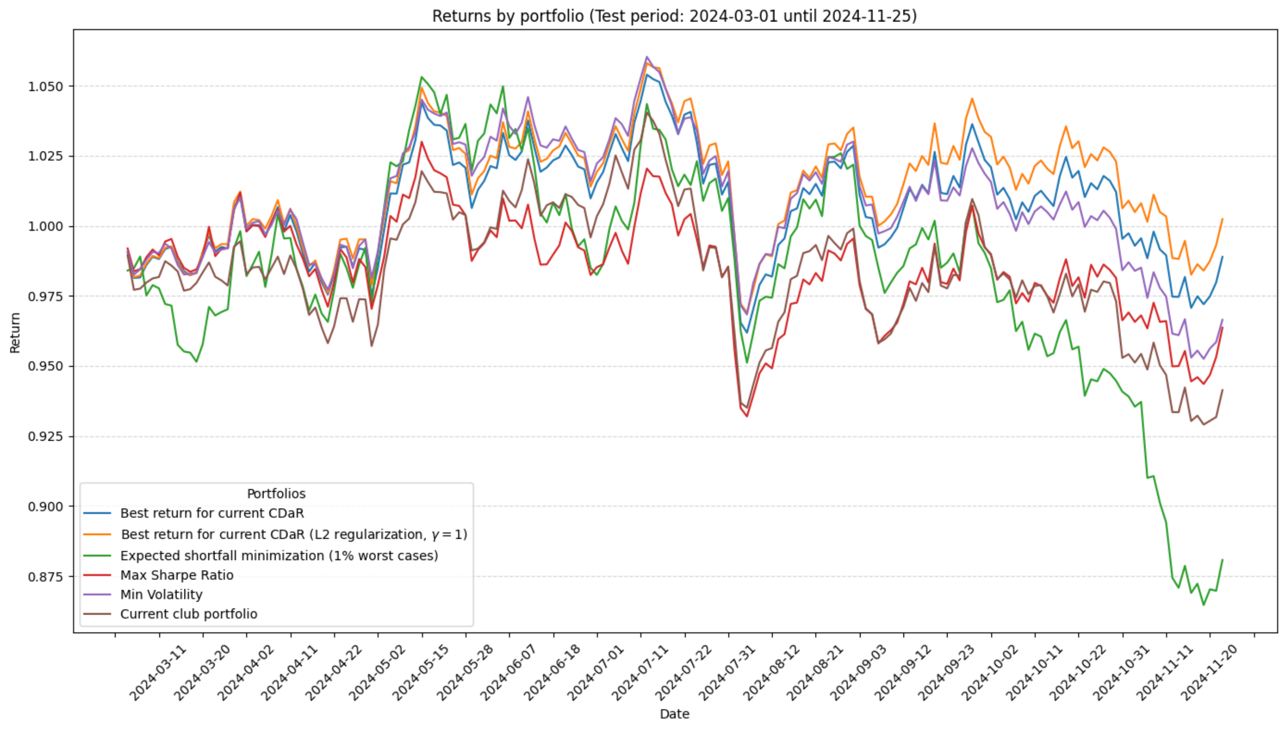
$$CDaR(w,\beta) = \frac{1}{1-\beta}\int_{D(w,r,t)\geq\alpha(w)} D(w,r,t)p(r(t))dr(t).$$

Where β indicates the β% worst situations for the average drawdown.

This is in fact an average with a threshold on the worst values that the drawdown can take (bearing in mind that we want to minimise the drawdown in our initial problem).

- It was introduced in « Portfolio optimization with drawdown constraints » by Cheklov et al. published in April 2000.
- You shouldn't try to apply new measures or models just to show off. On the other hand, the fact that the model isn't older could indicate that some analysts haven't adopted it, which is a technological and scientific advantage sought in quantitative. This is how Jim Simons, the father of quantitative finance, made his billions.

Portfolio return simulation over 9 months



- Weights were choosen according to information available on 2024-03-01.
- We then compute the returns day by day without adjusting the weights of the differents portfolios.
- CDaR seems to perform better than other methods in the long run.

Optimal portfolio allocation by CDaR optimization