Examples of LMIs

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In this document, we provide explicit examples of LMIs that are considered in our work. For their context and motivation, we refer to the documents provided in the same repository.

- MKN11 denotes the LMI defined by

$$A = \begin{pmatrix} \varepsilon & \frac{1}{2}+\frac{3\varepsilon}{2} & 0 & 0 & -x_{24} & 0 \\ \frac{1}{2}+\frac{3\varepsilon}{2} & 1+3\varepsilon & 0 & x_{24} & 0 & 0 \\ 0 & 0 & \varepsilon & 0 & 0 & 0 \\ 0 & x_{24} & 0 & 3\varepsilon & -\frac{3}{2}+3\varepsilon & \frac{3\varepsilon}{2} \\ -x_{24} & 0 & 0 & -\frac{3}{2}+3\varepsilon & 3\varepsilon & \frac{3\varepsilon}{2} \\ 0 & 0 & 0 & \frac{3\varepsilon}{2} & \frac{3\varepsilon}{2} & 1+\varepsilon \end{pmatrix}$$

  $$\boldsymbol{y} = \varepsilon, \quad \boldsymbol{x} = x_{24}.$$

- RBN11 denotes the LMI defined by

$$A = \begin{pmatrix} 1 & -\frac{1}{2}+2\epsilon & 0 & 0 & -x_{24} & 0 \\ -\frac{1}{2}+2\epsilon & -1+4\epsilon & 0 & x_{24} & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & x_{24} & 0 & -1+4\epsilon & \frac{3}{2}+\frac{3}{2}\epsilon & -\frac{1}{2}+2\epsilon \\ -x_{24} & 0 & 0 & \frac{3}{2}+\frac{3}{2}\epsilon & -1+4\epsilon & -\frac{1}{2}+2\epsilon \\ 0 & 0 & 0 & -\frac{1}{2}+2\epsilon & -\frac{1}{2}+2\epsilon & 1 \end{pmatrix}$$

  $$\boldsymbol{y} = \varepsilon, \quad \boldsymbol{x} = x_{24}.$$

- GRD24 denotes the LMI defined by

$$A = \begin{pmatrix} \frac{\mu L(\lambda_1+\lambda_3+\lambda_5+\lambda_6)}{L-\mu} & \star & \star \\ -\frac{\lambda_5\mu+L(\gamma\mu(\lambda_1+\lambda_6)+\lambda_3)}{L-\mu} & \frac{\lambda_2+\lambda_3+\lambda_4+\lambda_5+\gamma\mu(\gamma L(\lambda_1+\lambda_2+\lambda_4+\lambda_6)-2\lambda_2)-2\gamma\lambda_4 L}{L-\mu} & \star \\ -\frac{\lambda_6\mu+\lambda_1 L}{L-\mu} & \frac{\gamma\lambda_4\mu+\gamma\lambda_6\mu-\lambda_2-\lambda_4+\gamma L(\lambda_1+\lambda_2)}{L-\mu} & \frac{\lambda_1+\lambda_2+\lambda_4+\lambda_6}{L-\mu} \end{pmatrix},$$

  and $\boldsymbol{y} = (\mu, \tau)$, $\boldsymbol{x} = (\lambda_1, \ldots, \lambda_6)$, with the constraints $\lambda_1, \ldots, \lambda_6 \geq 0$, $\gamma = \frac{2}{L+\mu}$, $L = 1$, $0 < \mu < 1$, and

$$\begin{cases} -\lambda_2 + \lambda_3 + \lambda_4 - \lambda_5 + \tau & = 0 \\ \lambda_1 + \lambda_2 - \lambda_4 - \lambda_6 - 1 & = 0, \end{cases}$$

  so that the LMI has 4 free variables, say $\lambda_1, \lambda_4, \lambda_5, \lambda_6$, and 2 parameters.

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- GRD23 denotes the LMI obtained by setting $\lambda_6 = 0$ in GRD24.
- GRD14 denotes the LMI obtained by moving τ from $\boldsymbol{y}$ to $\boldsymbol{x}$ in GRD21.
- GRD22 denotes the LMI obtained by setting $\lambda_5 = 0$ in GRD23.
- GRD13 denotes the LMI obtained by moving τ from $\boldsymbol{y}$ to $\boldsymbol{x}$ in GRD21.
- GRD21 denotes the LMI obtained by setting $\lambda_4 = 0$ in GRD22.
- GRD12 denotes the LMI obtained by moving τ from $\boldsymbol{y}$ to $\boldsymbol{x}$ in GRD21.
- PPM31 denotes the LMI defined by

$$A = \begin{pmatrix} 2\lambda\mu + 2\tau - 2 & -\gamma(2\lambda\mu - 2) - \lambda \\ -\gamma(2\lambda\mu - 2) - \lambda & \gamma(\gamma(2\lambda\mu - 2) + 2\lambda) \end{pmatrix},$$

  and $\boldsymbol{y} = (\mu, \gamma, \tau)$, $\boldsymbol{x} = \lambda$, with the constraints $\mu > 0$, $\gamma > 0$, $\tau > 0$.
- PPM21 denotes the LMI obtained by setting $\gamma = 1$ in PPM31.
- DRS42 denotes the LMI proposed in [1, SM3.1.1.], defined by

$$A = \begin{pmatrix} \rho^2 + \beta\lambda_\beta^B - 1 & -\theta + \frac{\lambda_\mu^A}{2} & \theta - (\frac{1}{2} + \beta)\lambda_\beta^B \\ -\theta + \frac{\lambda_\mu^A}{2} & -\theta^2 + (1+\mu)\lambda_\mu^A & \theta^2 - \lambda_\mu^A \\ \theta - (\frac{1}{2} + \beta)\lambda_\beta^B & \theta^2 - \lambda_\mu^A & -\theta^2 + (1+\beta)\lambda_\beta^B \end{pmatrix},$$

  and $\boldsymbol{y} = (\mu, \beta, \rho, \theta)$, $\boldsymbol{x} = (\lambda_\mu^A, \lambda_\beta^B)$, with the constraints $\mu > 0$, $0 < \theta < 2$, $\beta - \mu \geq 0$.
- DRS32 denotes the LMI obtained by setting $\theta = 1$ in DRS42.
- DRS43 denotes the LMI proposed in [1, SM3.2.2.], defined by

$$A = \begin{pmatrix} \rho^2 + \lambda_L^B - 1 & \frac{\lambda_\mu^A}{2} - \theta & \theta - \lambda_L^B - \frac{\lambda_\mu^B}{2} \\ \frac{\lambda_\mu^A}{2} - \theta & -\theta^2 + \lambda_\mu^A + \lambda_\mu^A\mu & \theta^2 - \lambda_\mu^A \\ \theta - \lambda_L^B - \frac{\lambda_\mu^B}{2} & \theta^2 - \lambda_\mu^A & -\lambda_L^B L^2 - \theta^2 + \lambda_L^B + \lambda_\mu^B \end{pmatrix},$$

  and $\boldsymbol{y} = (\mu, L, \rho, \theta)$, $\boldsymbol{x} = (\lambda_\mu^A, \lambda_L^B, \lambda_\mu^B)$, with $\mu > 0$, $L - \mu > 0$.
- DRS33 denotes the LMI obtained by setting $\theta = 1$ in DRS43.

References

[1] E. K. Ryu, A. B. Taylor, C. Bergeling, and P. Giselsson. Operator splitting performance estimation: tight contraction factors and optimal parameter selection. *SIAM Journal on Optimization*, 30(3):2251–2271, 2020.