MAP 531: Homework

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You are asked to provided answers to all these exercises as both Rmd and pdf files. The two files should be uploaded on Moodle on the 15th of November (23h59 Paris time). This homework should be done by groups of 2. Only one submission per group on moodle, with both names indicated in the file. This homework is composed of 2 independent problems. Some of the questions are a bit more technical: they are marked by a * and are optional.

Problem 1: Estimating parameters of a Poisson distribution to model the number of goals scored in football

We recall that the Poisson distribution with parameter $\theta > 0$ has a pdf given by $(p(\theta, k), k \in \mathbb{N})$ w.r.t the counting measure on $\mathbb{N}$: $p(\theta, k) = exp(-\theta)\frac{\theta^k}{k!}$

Question 1: Is it a discrete or continuous distribution? Can you give 3 examples of phenomenons that could be modeled by such a distribution in statistics?

The poisson distribution is a discrete distribution since it has a countable number of possible values ($\mathbb{N}$).

In statistics, we use this distribution to compute the probability of a given number of events in a time period or the probability of waiting some time until the next event.

For examples: The number of patients arriving in an emergency room between 9 and 10am. The number of minutes we wait a bus at the bus stop.

Question 2: Compute the mean and the variance of this distribution.

We assume that $\mathbb{X}$ follows a Poisson distribution with parameter $\theta > 0$. $\mathbb{E}[\mathbb{X}] = \sum_{i=0}^{\infty}(i * p(\theta, i)) = \sum_{i=0}^{\infty}(i * exp(-\theta)\frac{\theta^i}{i!}) = \theta * exp(-\theta)\sum_{i=1}^{\infty}(\frac{\theta^{i-1}}{(i-1)!}) = \theta * exp(-\theta)\sum_{i=0}^{\infty}(\frac{\theta^i}{i!}) = \theta * exp(-\theta) * exp(\theta) = \theta$

$\mathbb{E}[\mathbb{X}^2] = \sum_{i=0}^{\infty}(i^2 * p(\theta, i)) = \sum_{i=0}^{\infty}(i^2 * exp(-\theta)\frac{\theta^i}{i!}) = \theta * exp(-\theta)\sum_{i=1}^{\infty}(i\frac{\theta^{i-1}}{(i-1)!}) = \theta * exp(-\theta)\sum_{i=0}^{\infty}((i+1)\frac{\theta^i}{i!})$
$= \theta * exp(-\theta)[\sum_{i=0}^{\infty}(i\frac{\theta^i}{i!}) + \sum_{i=0}^{\infty}(\frac{\theta^i}{i!})] = \theta * exp(-\theta)[\theta * exp(\theta) + exp(\theta)] = \theta(\theta + 1)$