

Drifting beyond Bayesics

A Bayesian Implementation of the Circular Drift Diffusion Model

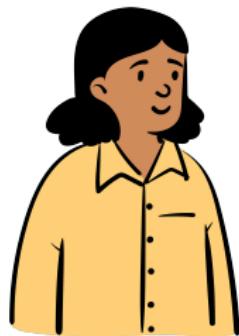
Adriana F. Chávez De la Peña, Manuel Villarreal,
Michael D. Lee, Joachim Vandekerckhove

University of California, Irvine

Some Circular Decisions

Indicate the Color

What is the color of the shirt?



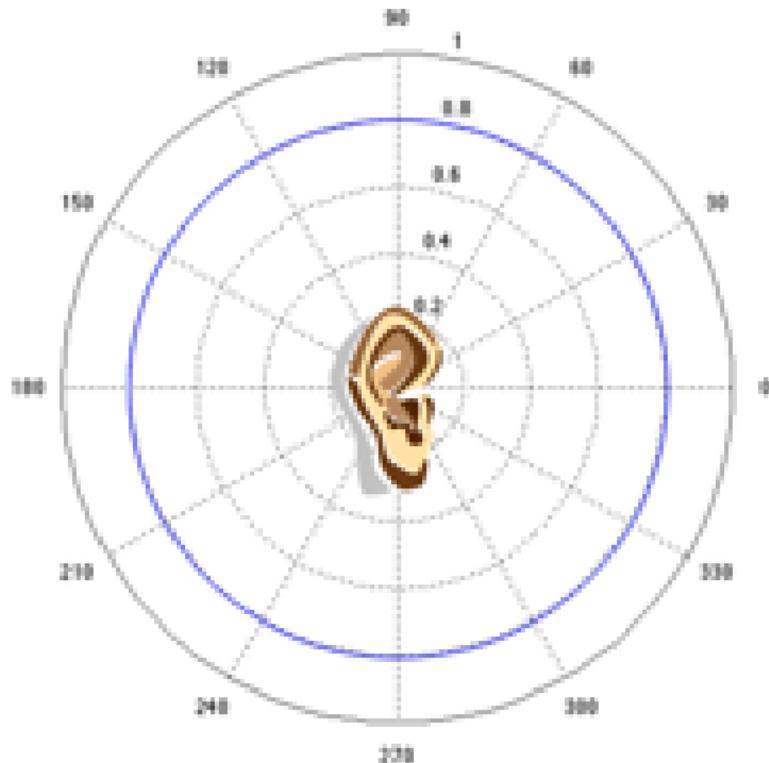
Did You Remember the Color?

What was the color of the shirt?



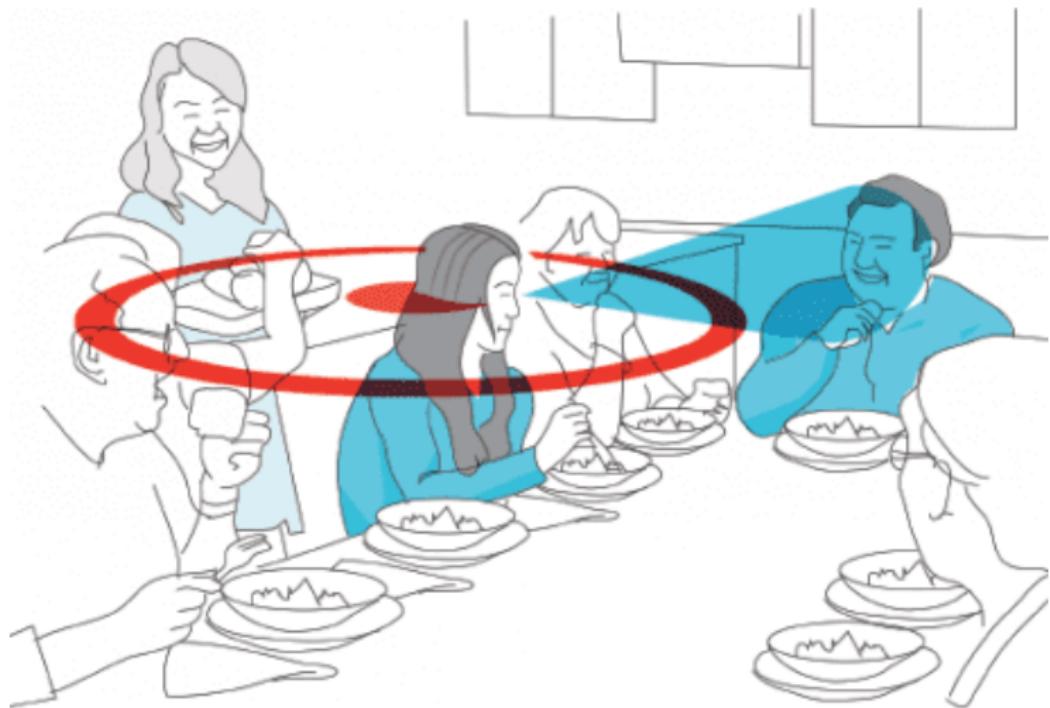
Spatial Identification of Sound

Testing a directional hearing aid



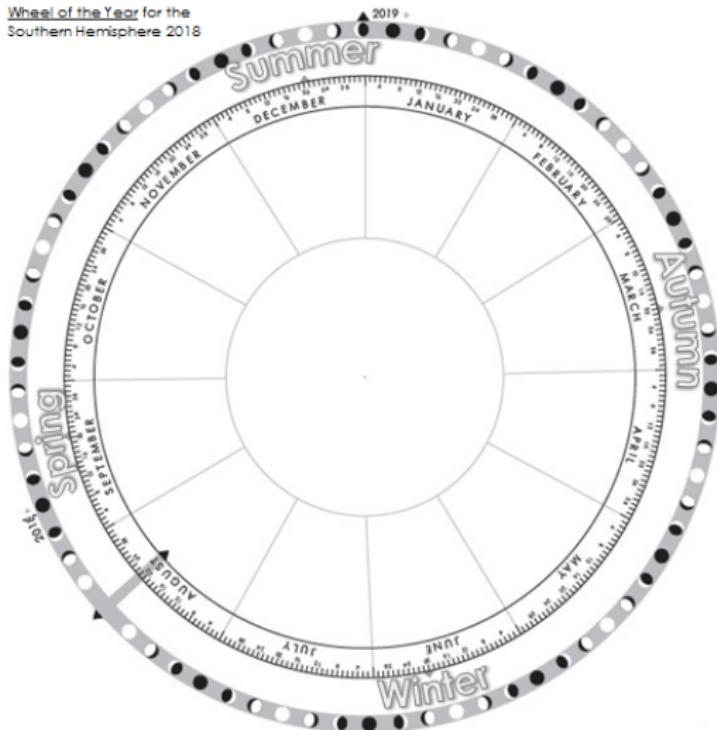
Conversation Source

Where is the conversation coming from?



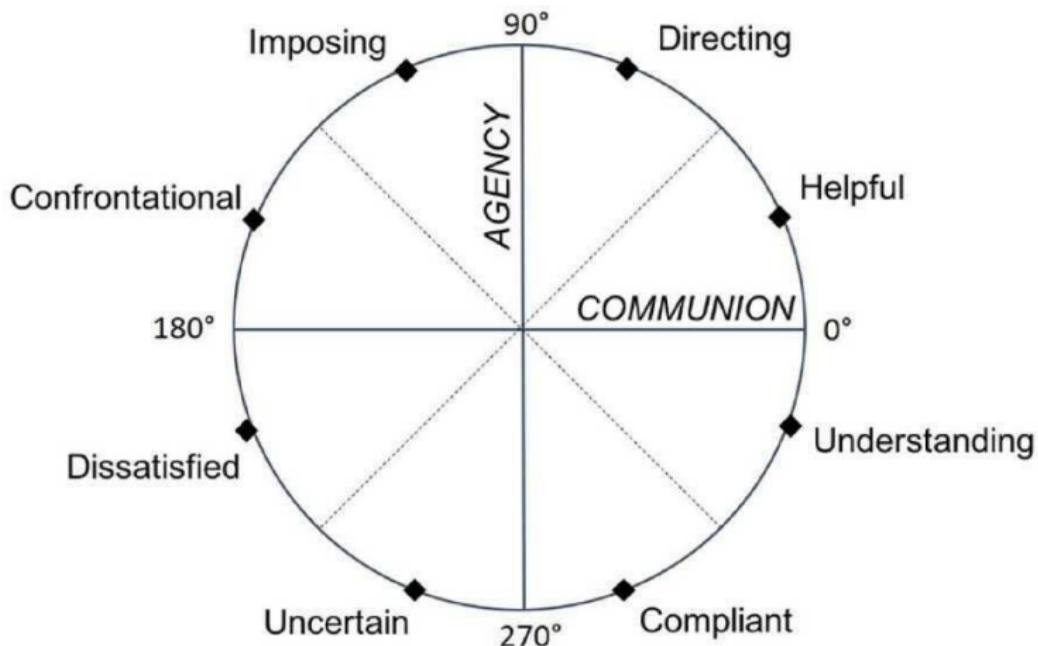
Predicting Weather

Which day will have the highest maximum temperature in Sydney?



Assessing Personalities

What is this person's personality?



The Circular Drift Diffusion Model (CDDM)

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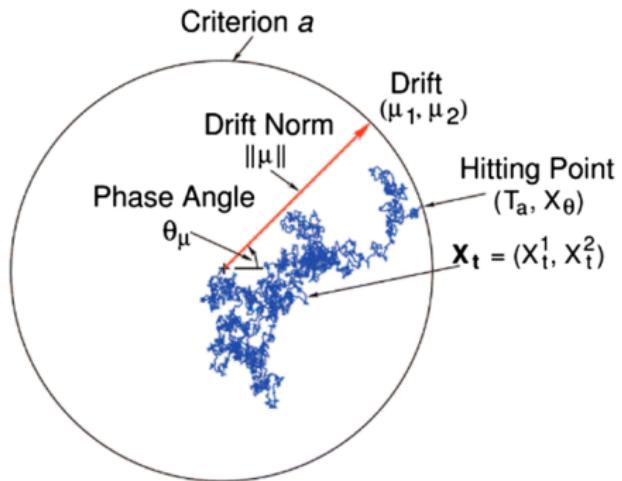
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extends binary choice to speeded continuous decisions on a circle
- Parameters are
 - **drift angle**: direction of stimulus evidence
 - **evidence threshold**: criterion to be reached to make a decision
 - **drift norm**: speed of information processing
 - **non-decision time**: visual encoding and motor movement

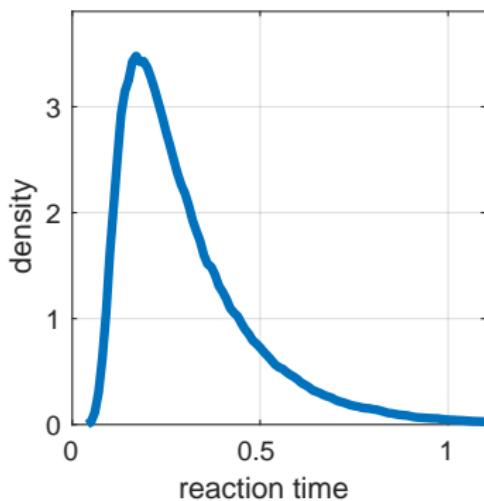
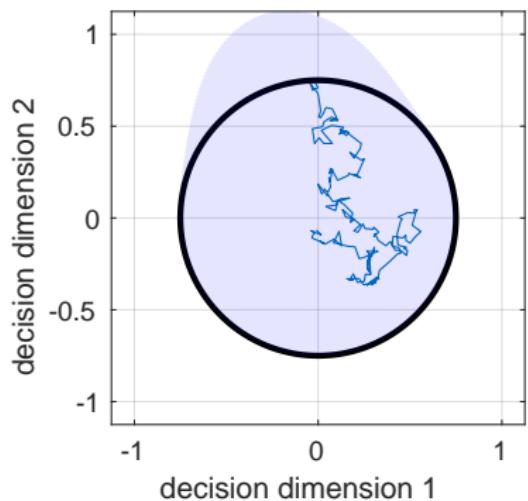
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The Circular Drift Diffusion Model (CDDM)

Given the parameters, CDDM predicts a distribution of angles and reaction times



JAGS Implementation

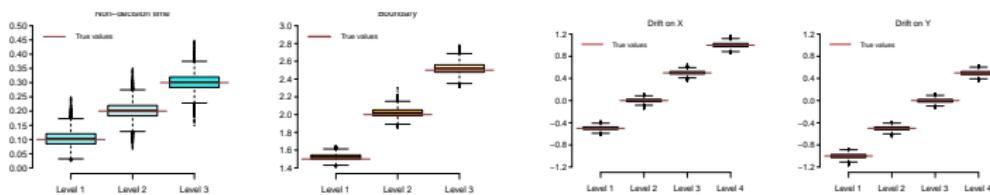
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- JAGS is a high-level scripting language for probabilistic generative models (Plummer, 2003)
 - allows for flexible and rapid model development, including hierarchical and latent-mixture structures
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- We conducted simulation studies and found good parameter recovery even with small sample sizes ($N = 80$)



JAGS Model Design Patterns

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- Likelihood function

```
y[time,1:2] ~ dcddm(delta[PERSON[time], DIFFICULTY[time]],  
                      eta[PERSON[time], SPEED_ACCURACY[time]],  
                      t0[PERSON[time]],  
                      theta[time, (latent_state[time] + 1)])
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- Prior distribution: latent mixture of angles

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latent_state[time] ~ dbern(omega[PERSON[time], CUE_DEFLECT[time]])  
theta[time,1]      ~ dnorm(POSITION[time], ... )  
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```

- Hierarchical distribution: drift

```
for(dIdx in 1:nDifficulty){  
  mu_delta[dIdx] ~ dnorm(0, 1) # Prior on conditional means  
  for(pIdx in 1:nParticipants){  
    log_delta[pIdx, dIdx] ~ dnorm(mu_delta[dIdx], tau_delta)  
    delta[pIdx,dIdx]      = exp(log_delta[pIdx, dIdx])  
  }  
}
```

An Application

Kvam (2019) Task

- Perceptual study where participants produce orientation judgments

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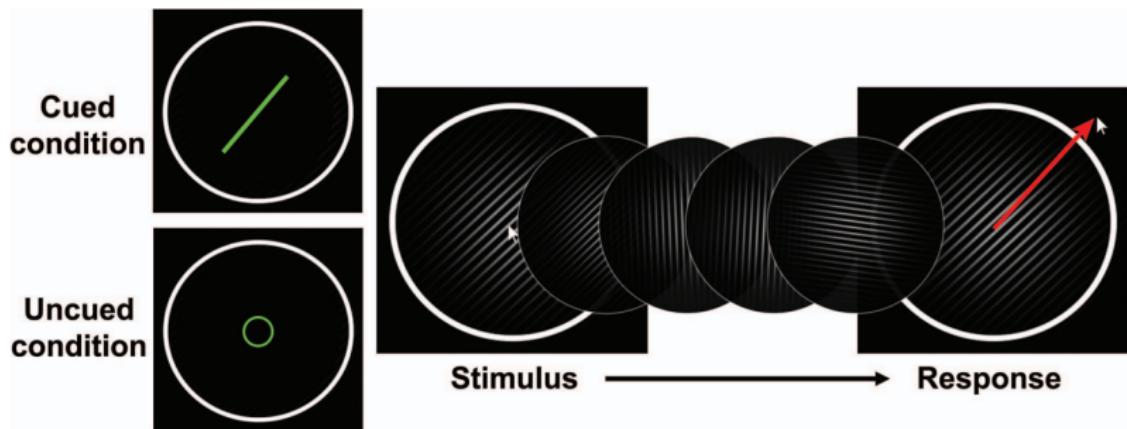
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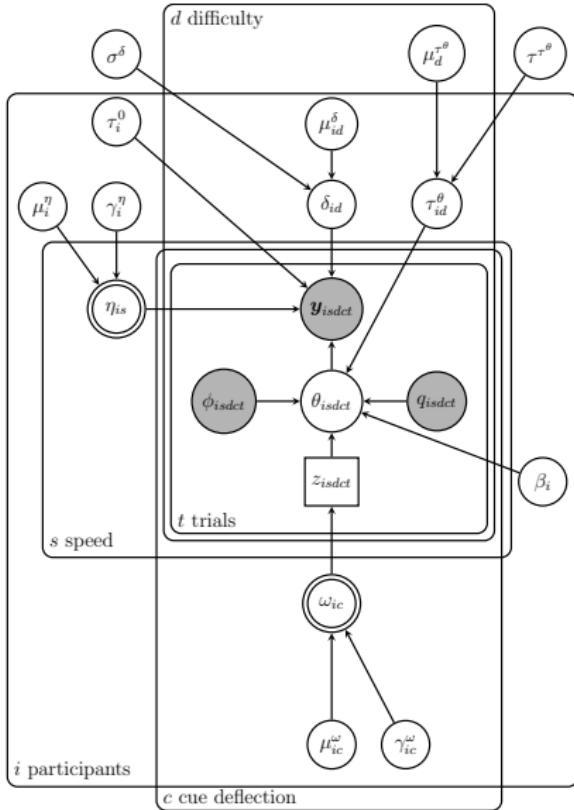
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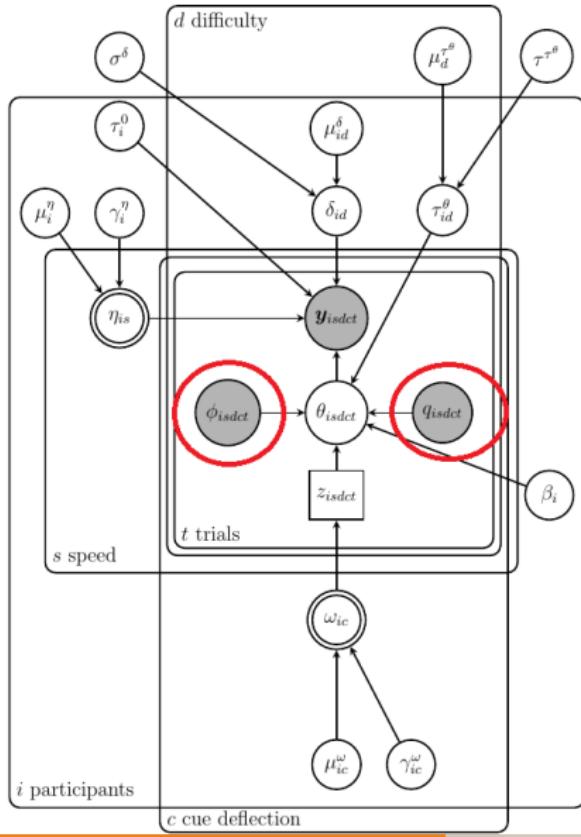
- ① Are people more **cautious** when they are instructed to prioritize accuracy over speed?
- ② Is the **speed of information processing** less for more variable stimuli?
- ③ Do people get **information less consistently** from more variable stimuli?
- ④ Are there differences in being **influenced by the cue** for different cue angles?

Model



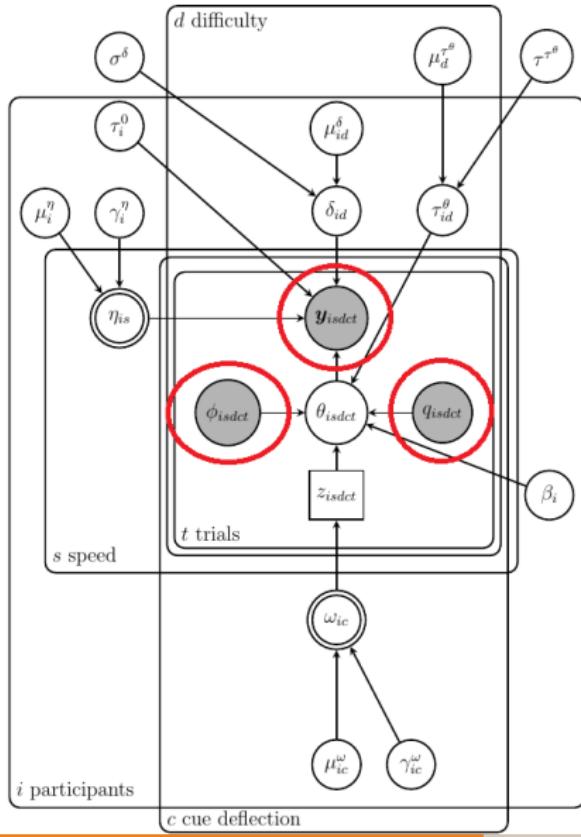
$$\begin{aligned}
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 \eta_{is} &= \begin{cases} \exp\left(\mu_i^\eta + \frac{\gamma_i^\eta}{2}\right) & \text{if } s = \text{accuracy} \\ \exp\left(\mu_i^\eta - \frac{\gamma_i^\eta}{2}\right) & \text{if } s = \text{speed} \end{cases} \\
 \tau_i^0 &\sim \text{uniform}(0, \min y_{i1}) \\
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 z_{isdet} &\sim \text{Bernoulli}(\omega_{ic}) \\
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Model



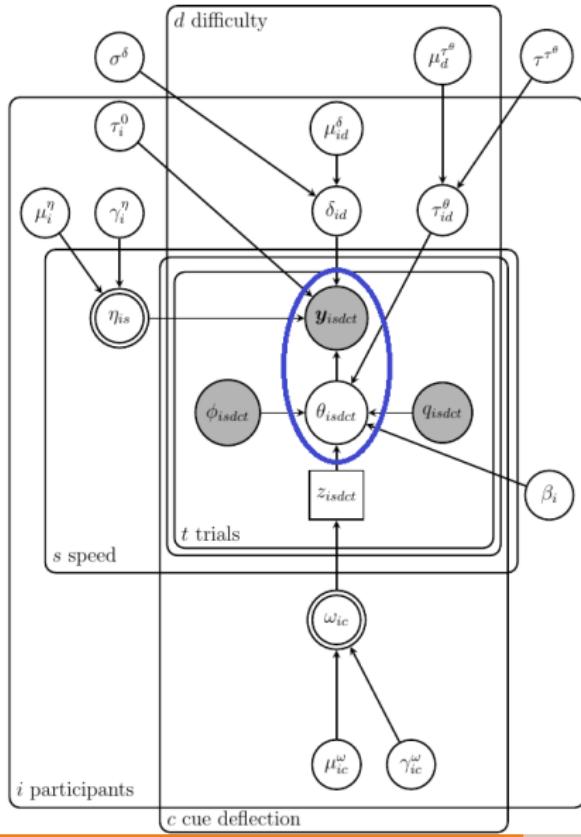
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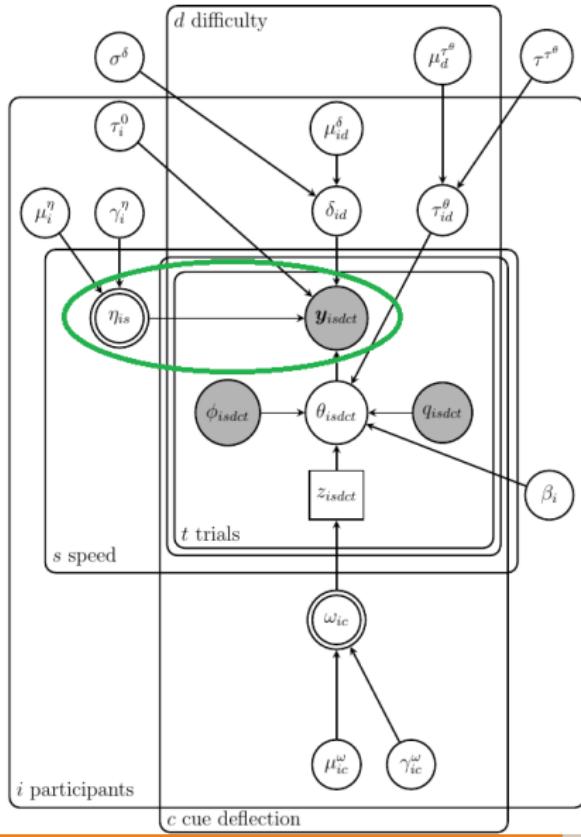
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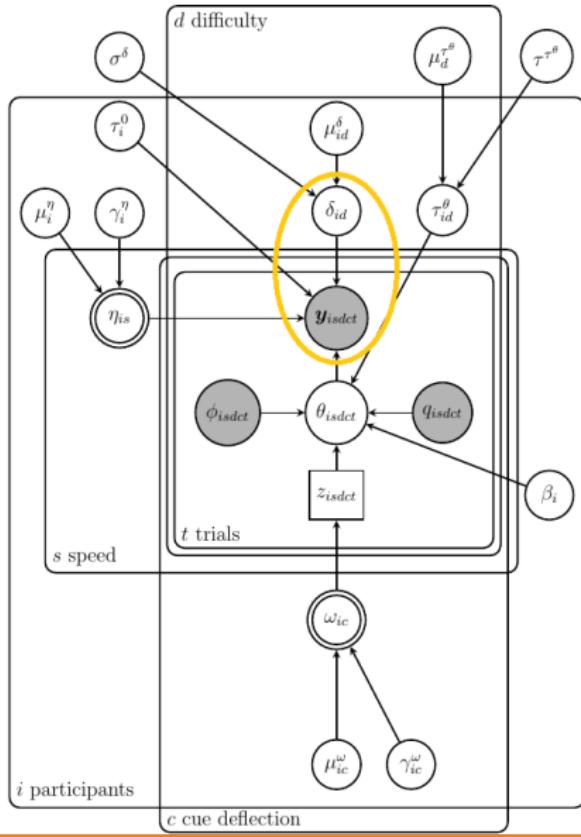
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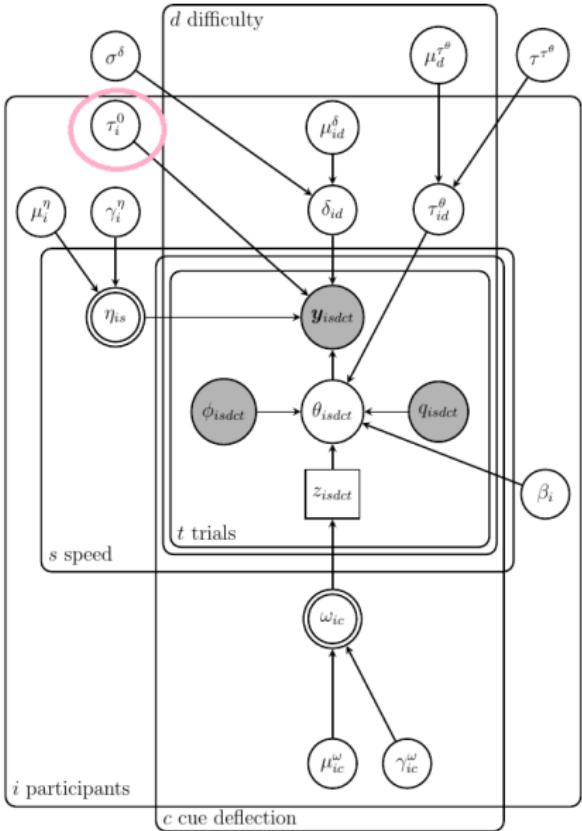
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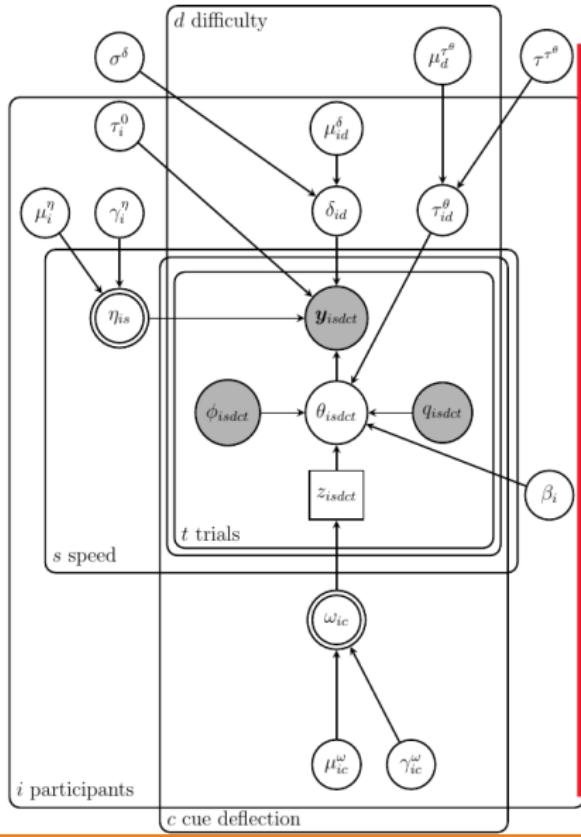
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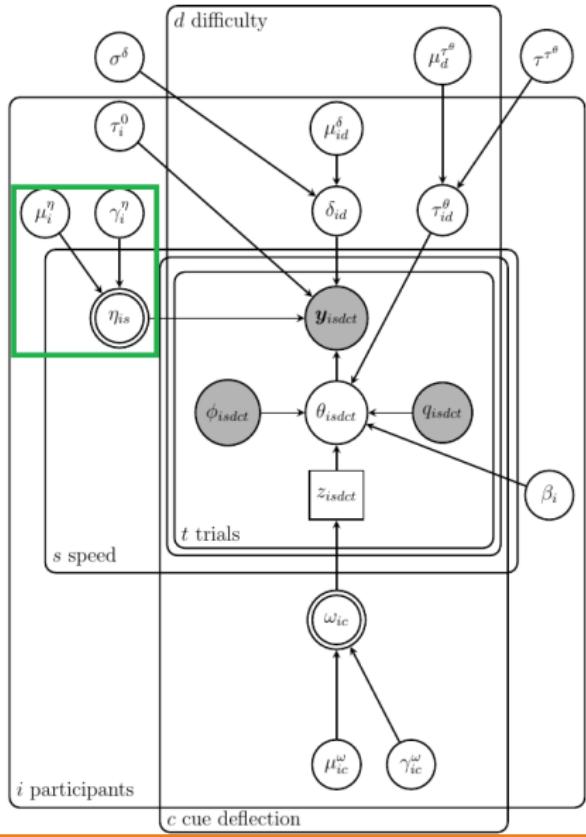
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Model



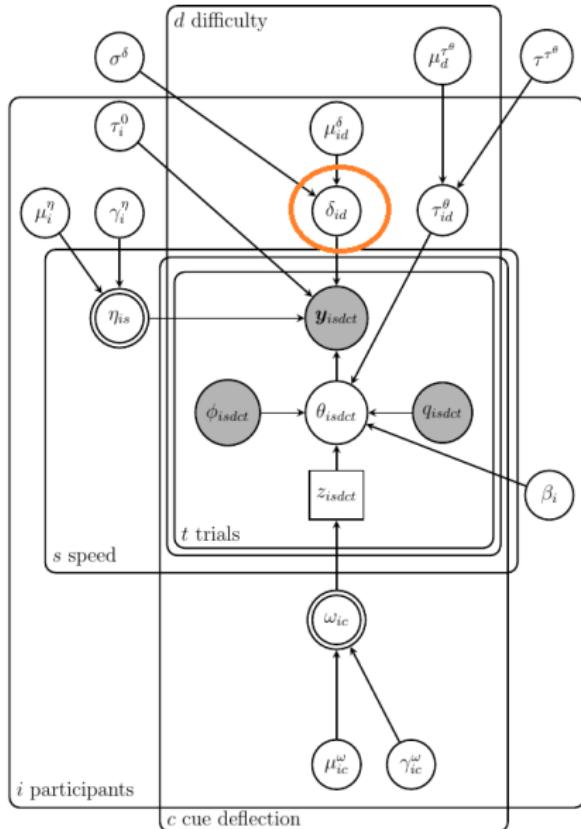
$$\begin{aligned} \mu_i^\eta &\sim \text{Gaussian}(0, 1) \\ \gamma_i^\eta &\sim \text{Gaussian}(0, 1)T(0, \infty) \\ \eta_{is} &= \begin{cases} \exp\left(\mu_i^\eta + \frac{\gamma_i^\eta}{2}\right) & \text{if } s = \text{accuracy} \\ \exp\left(\mu_i^\eta - \frac{\gamma_i^\eta}{2}\right) & \text{if } s = \text{speed} \end{cases} \\ \tau_i^0 &\sim \text{uniform}(0, \min y_{i1}) \\ \mu_{id}^\delta &\sim \text{Gaussian}(0, 1) \\ \sigma^\delta &\sim \text{uniform}(0, 1) \\ \delta_{id} &\sim \text{log-Gaussian}\left(\mu_{id}^\delta, \frac{1}{(\sigma^\delta)^2}\right) \\ \mu_d^{\tau^\theta} &\sim \text{Gaussian}(0, 1) \\ \tau^{\tau^\theta} &\sim \text{uniform}(0, 4) \\ \tau_{id}^\theta &\sim \text{log-Gaussian}\left(\mu_d^{\tau^\theta}, \frac{1}{(\sigma^{\tau^\theta})^2}\right) \\ \mu_{ic}^\omega &\sim \text{Gaussian}(0, 1) \\ \gamma_{ic}^\omega &\sim \text{Gaussian}(0, 1) \\ \omega_{ic} &= \begin{cases} \exp\left(\mu_{ic}^\omega + \frac{\gamma_{ic}^\omega}{2}\right) & \text{if } c > 0 \\ \exp\left(\mu_{ic}^\omega - \frac{\gamma_{ic}^\omega}{2}\right) & \text{if } c < 0 \\ \exp(\mu_{ic}^\omega) & \text{if } c = 0 \end{cases} \\ z_{isdct} &\sim \text{Bernoulli}(\omega_{ic}) \\ \beta_i &\sim \text{Gaussian}(0, 1) \\ \theta_{isdct} &\sim \begin{cases} \text{Gaussian}(\phi_{isdct}, \tau_{id}^\theta) & \text{if } z_{isdct} = 0 \\ \text{Gaussian}(q_{isdct}, \beta_i \tau_{id}^\theta) & \text{if } z_{isdct} = 1 \end{cases} \\ y_{isdct} &\sim \text{CDDM}(\delta_{id}, \eta_{is}, \tau_i^0, \text{mod}(\theta_{isdct}, 2\pi)) \end{aligned}$$

Model



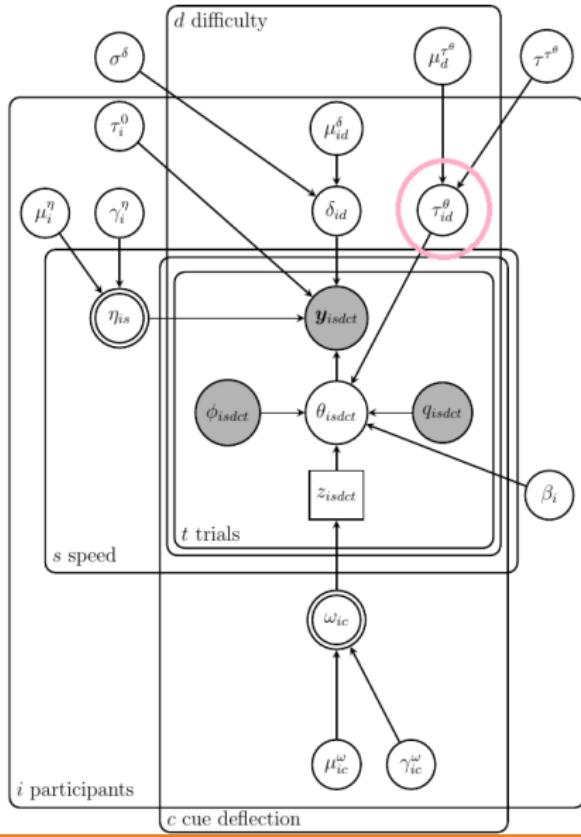
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Model



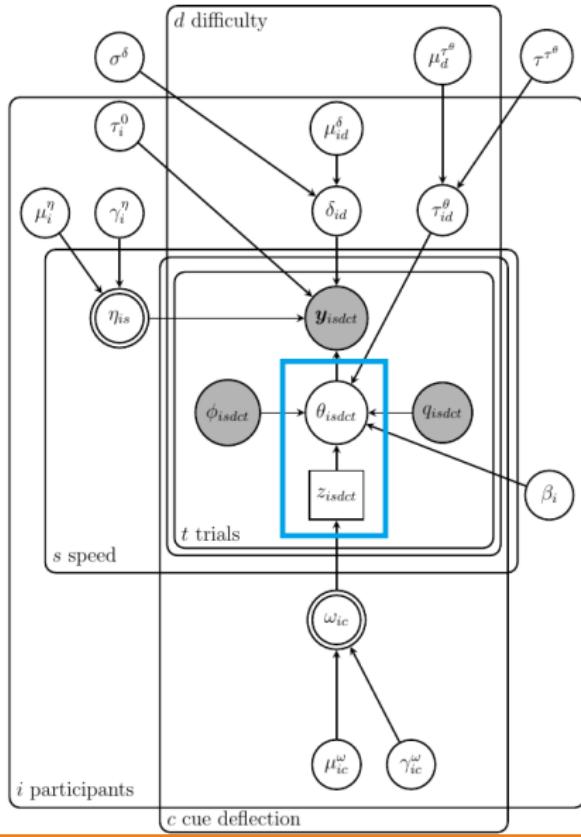
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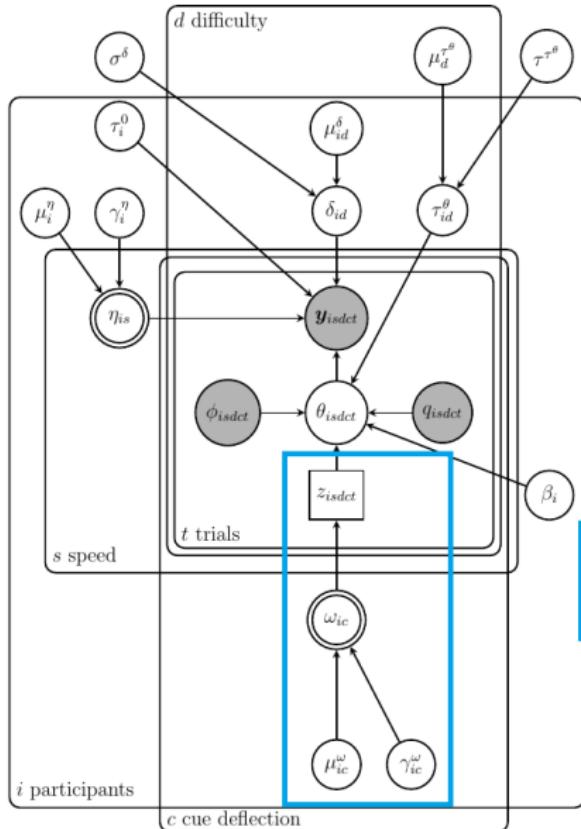
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Speed vs. Accuracy

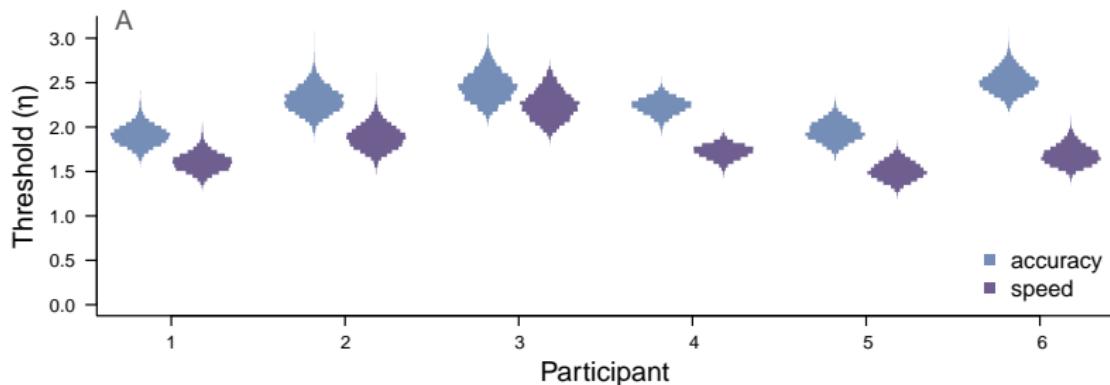
- Are people more cautious when they are instructed to prioritize accuracy over speed?

Speed vs. Accuracy

- Are people more cautious when they are instructed to prioritize accuracy over speed?
 - test for an increase in the evidence threshold η in the accuracy condition

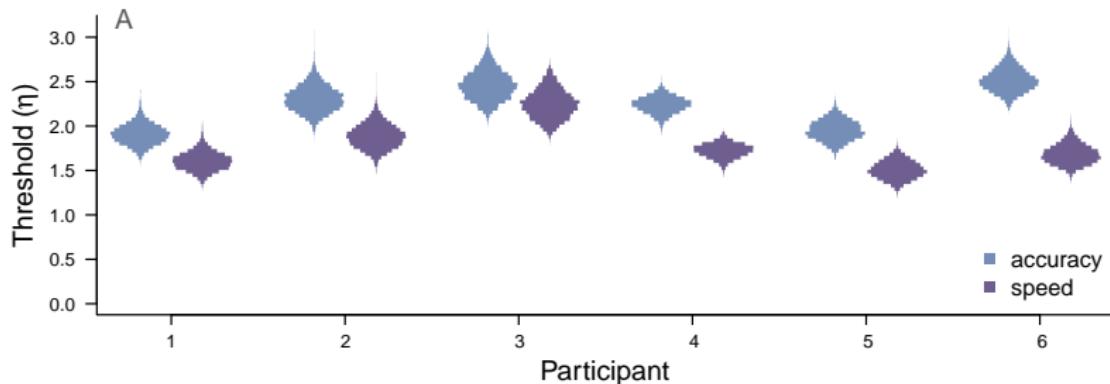
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Speed vs. Accuracy

- Are people more cautious when they are instructed to prioritize accuracy over speed?
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- Accuracy thresholds are significantly different (and larger), with Bayes factors above 1,000 for all but participant 3, who has a Bayes factor favoring 'different' of 9

Speed of Information Processing

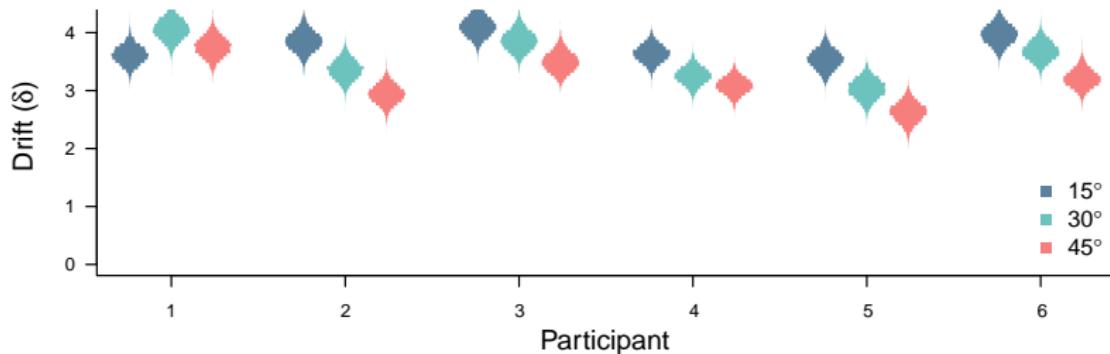
- Is the speed of information processing less for more variable stimuli?

Speed of Information Processing

- Is the speed of information processing less for more variable stimuli?
 - a decrease in the drift norm parameter δ as stimuli become more difficult because of increased variability

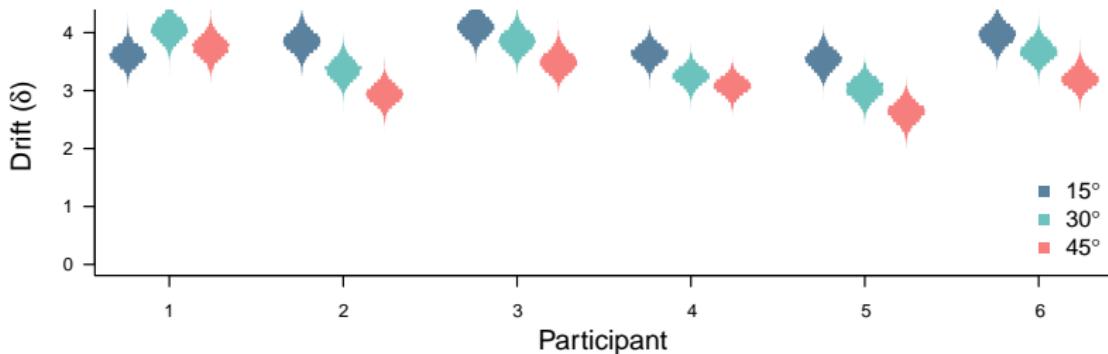
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Speed of Information Processing

- Is the speed of information processing less for more variable stimuli?
 - a decrease in the drift norm parameter δ as stimuli become more difficult because of increased variability



- Ordering of δ generally shows greater difficulty with more variability
 - participant 1 has lower δ than is expected for the easiest 15° stimuli

Consistency of Information Processing

- Do people get **information less consistently** from more variable stimuli?

Consistency of Information Processing

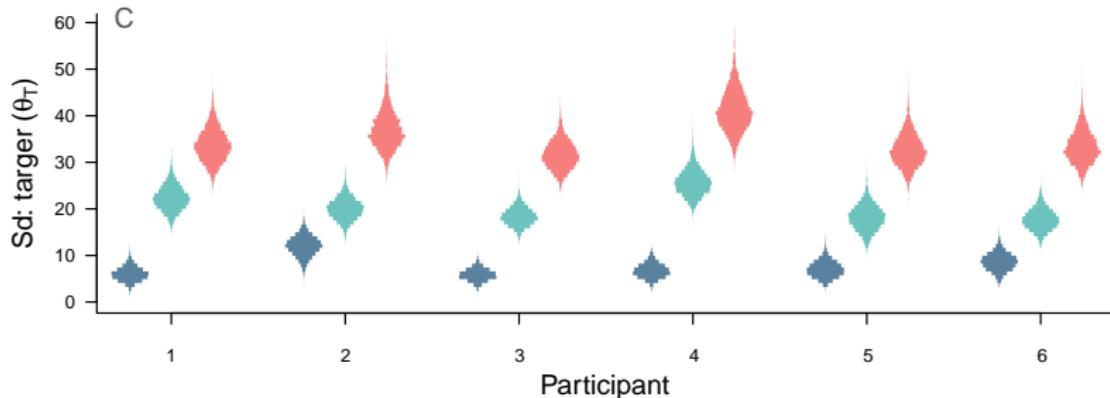
- Do people get information less consistently from more variable stimuli?
 - measured by the trial-to-trial variability in the drift angle θ

Consistency of Information Processing

- Do people get information less consistently from more variable stimuli?
 - measured by the trial-to-trial variability in the drift angle θ
 - implemented hierarchically in our model with standard deviation θ_τ

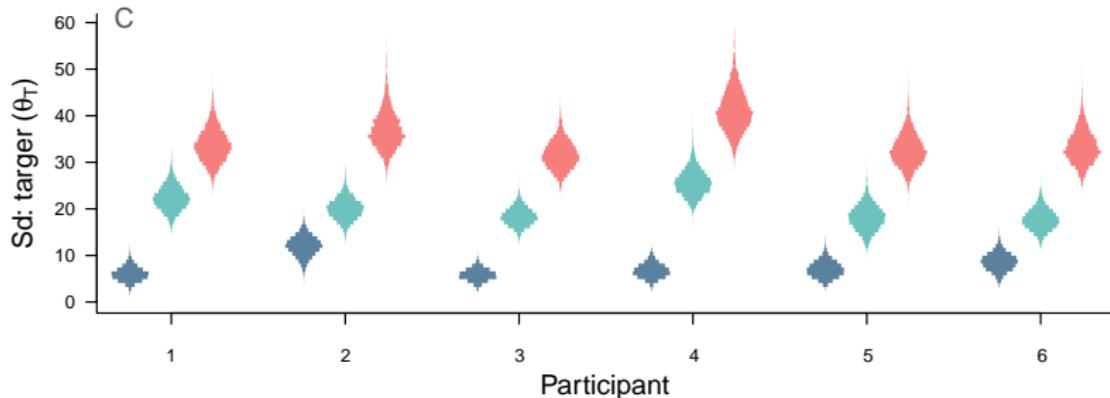
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Consistency of Information Processing

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- Ordering shows less drift rate consistency as stimuli become more difficult via increased variability

Influence of Cues

- Are there differences in being influenced by the cue for different cue angles?

Influence of Cues

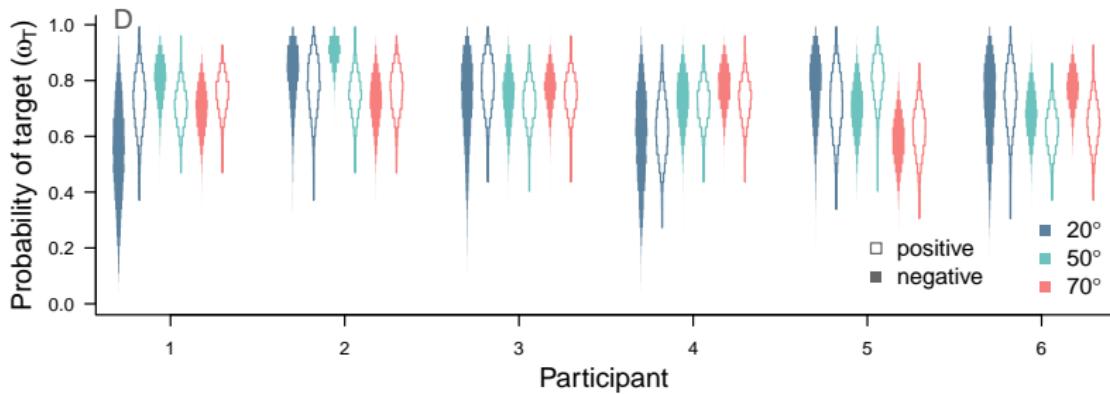
- Are there differences in being influenced by the cue for different cue angles?
 - measured by how often the cue angle determines the drift

Influence of Cues

- Are there differences in being influenced by the cue for different cue angles?
 - measured by how often the cue angle determines the drift
 - implemented as a hierarchical base-rate ω_τ over a trial-by-trial latent mixture in our model

Influence of Cues

- Are there differences in being influenced by the cue for different cue angles?
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 - implemented as a hierarchical base-rate ω_T over a trial-by-trial latent mixture in our model

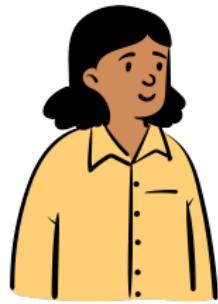


- Participants mostly ignore the cue and there are no significant differences in the base-rate for different (positive and negative) cue angle displacements

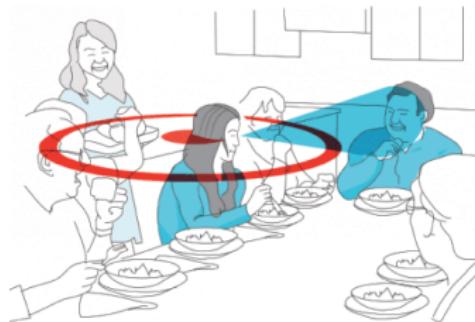
Discussion

Future work

Name the color of the shirt?

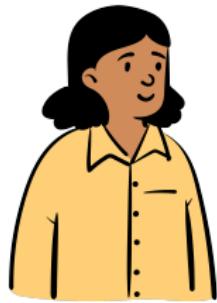


Who is talking?

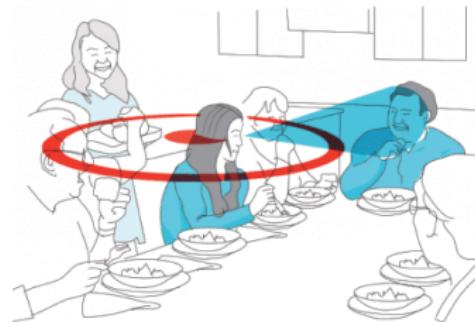


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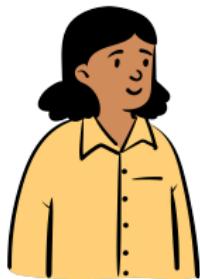
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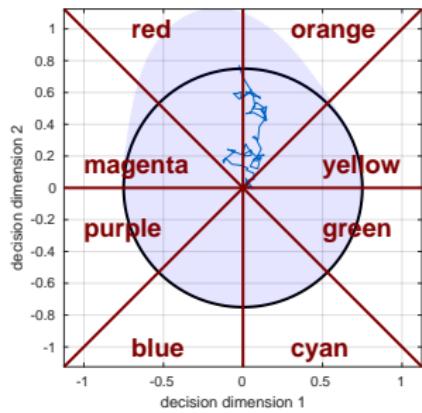
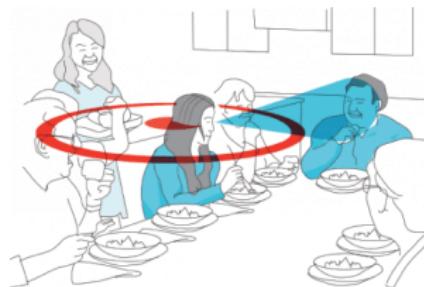
In practice, speeded orientation responses are often recorded with a discrete set of response options

Future work: A Thurstonian extension

Name the color of the shirt?

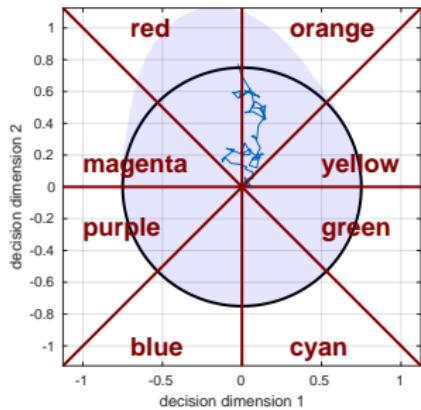
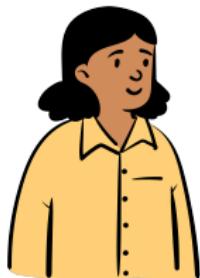


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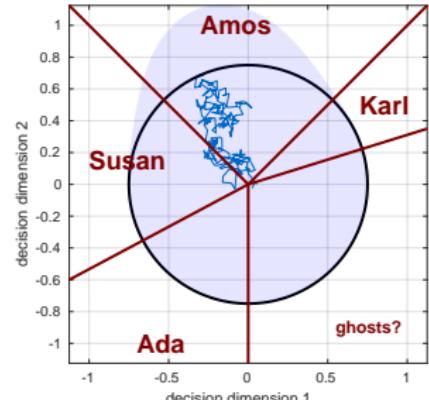
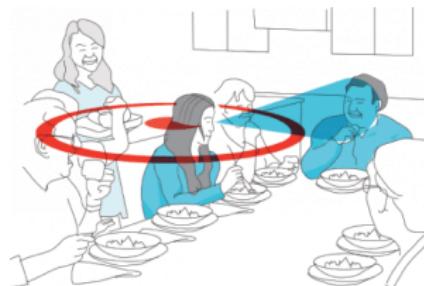


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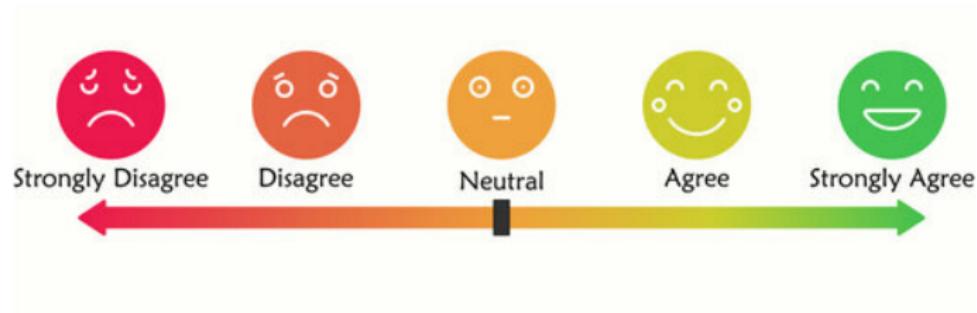
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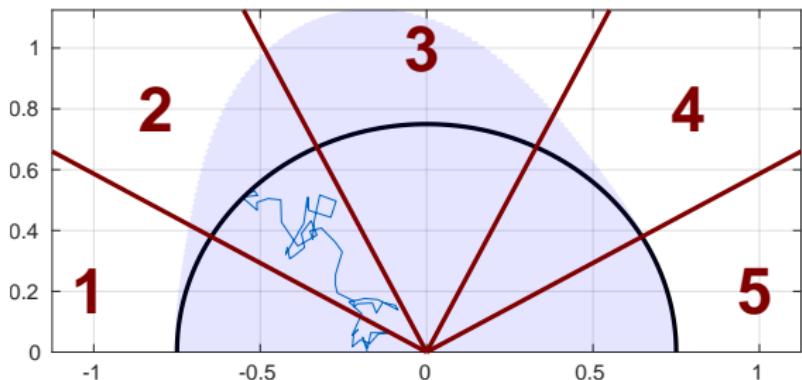
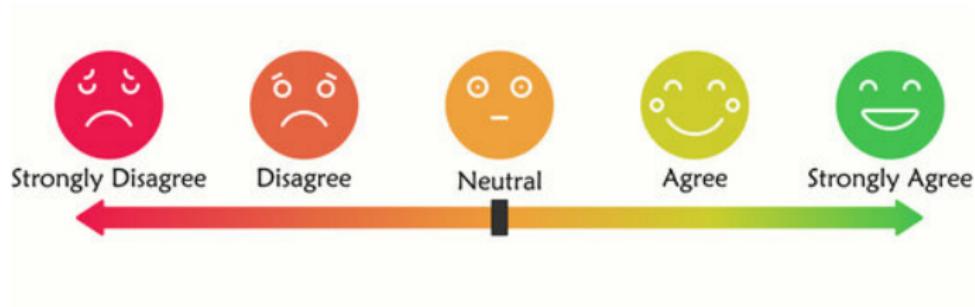
Who is talking?



A Likert extension



A Likert extension



References

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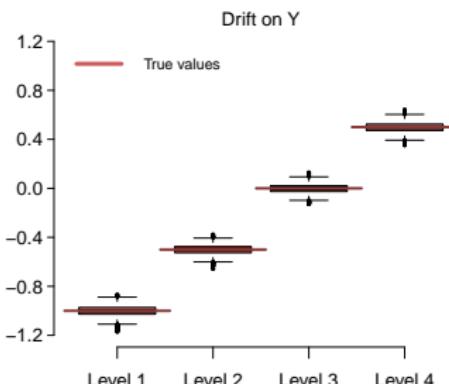
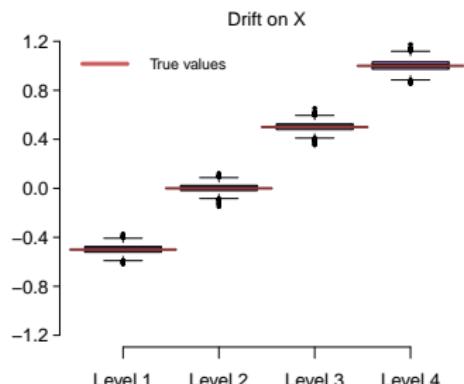
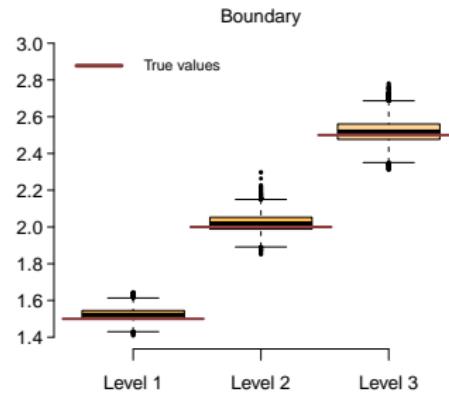
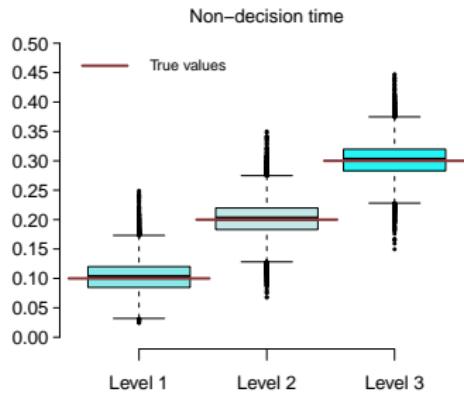
Drifting beyond Bayesics

A Bayesian Implementation of the Circular Drift Diffusion Model

Adriana F. Chávez De la Peña, Manuel Villarreal,
Michael D. Lee, Joachim Vandekerckhove

University of California, Irvine

Recovery Study for Cartesian Implementation



Recovery Study for Polar Implementation

