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INFERRED COMPONENTS OF REACTION TIMES AS FUNCTIONS OF FOREPERIOD DURATION

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A distribution function representing simple reaction-time distributions was derived, assuming RT is the sum of 2 component variables with exponential and normal distributions. 4 Ss each gave 100 RTs to an auditory stimulus following each of 4 foreperiods, under each of 2 conditions: (a) foreperiod constant within sessions but varied over sessions, and (b) foreperiods appearing in a random sequence. The derived distribution function provided completely satisfactory representations of all 32 RT distributions, and the relations of the fitted parameters of this function to foreperiod suggest that the variation of RT as a function of foreperiod is due to variation in the normally distributed component.

Concern with the form of distributions of reaction times (RT) has been focused largely on the fact that the nonnormal character of these distributions tends to preclude application of the more common statistics in studies using RT as a dependent variable. Some attempts have been made recently, however, to relate certain properties of these distributions to hypothetical components of the observed response times.

Restle (1961), for example, observing an almost constant 10 to 1 ratio between the means and standard deviations of 15 distributions of RT (data from Chocolle, 1940), inferred that the gamma distribution function might describe these distributions. This would imply that an observed RT must consist of the sum of a fixed number of independent component times, each of which is a random variable with a common exponential distribution. From the observed ratio of means and standard deviations in Chocolle's data, Restle further inferred that the number of components must be on the order of 100. Unfortunately, as McGill (1963) has pointed out, this interesting result does not correspond to commonly observed data. According to the central-limit theorem, a gamma distribution with a parameter indicating more than, say, 10 or 15 components would be very nearly normal. But observed distributions of simple RTs obtained under constant conditions seem inevitably to be positively skewed.

McGill (1963) observed that the upper portions of plots of several cumulative distributions of simple RT to different auditory stimulus intensities all had the form of exponential distribution functions with similar time constants. This suggested that at least one component of the total RT was exponentially distributed; and since the time constants implied by the curves seemed to be nearly independent of stimulus intensity, McGill assumed that this component is the time required to make the required motor response. Furthermore, since the overall distributions clearly varied with intensity of the stimuli, it was assumed that the RTs contained a nonexponential component which might be interpreted as "decision time."

As part of a model intended to represent the structure of disjunctive RT, Christie and Luce (1956) assumed that an observed response time is composed of an exponentially distributed choice or decision time plus a variable "base" time or motor response time, the distribution of which was not specified. Both McGill and Christie and Luce thus assume that RT distributions result from convolutions of distributions of two component random variables, one of which has the exponential distribution. Both also hypothesize that the two components represent decision time and a residual latency which is primarily motor response time. But here the agreement ends: the two papers present directly opposite interpretations of the exponential component. McGill (1963) assumed it is the "movement response [p. 329]," whereas Christie and Luce (1956) assumed it to be "decision latency [p. 25]."

Clearly, interpretation of the component processes underlying reaction latencies would be facilitated by knowledge of a distribution function which accurately describes distributions of total observed reaction times. The aim of the present study is to provide evidence for such a function.

First of all, it is known that the distribution of the sum of a large number of random variables is asymptotically normal regardless of the distributions of the component variables, provided no one component contributes disproportionately to the variance of the sum (Cramer, 1946); therefore, if it is assumed that an RT is the sum of time intervals to complete a sequence of separate processes, then the distribution of this sum would necessarily approach normality unless one (or some very small number) of the components contribute substantially more variance than any of the others.

The simplest hypothesis seems to be that an observed RT results from summation of (*a*) a single dominant exponentially distributed variable as proposed by McGill (1963) and by Christie and Luce (1956), and (*b*) a normally distributed variable which may itself be the sum of a number of component variables with similar variances.

To test this hypothesis the probability density and cumulative distribution functions are needed for the variable $t = t_1 + t_2$, where t_1 and t_2 are the exponentially and normally distributed variables. Letting $a = E(t_1)$, $b = E(t_2)$, and $c = \sigma(t_2)$, and assuming t_1 and t_2 are independent, the cumulative distribution function for t was determined to be

$$H(t) \cong \frac{1}{\sqrt{2\pi}} \int_{-\frac{b}{c}}^{\frac{t-b}{c}} e^{-\frac{y^2}{2}}\, dy - \left[\frac{1}{\sqrt{2\pi}} \int_{-\frac{b}{c}-\frac{c}{a}}^{\frac{t-b}{c}-\frac{c}{a}} e^{-\frac{y^2}{2}}\, dy \right] e^{-\frac{t-b}{a}+\frac{c^2}{2a^2}} \qquad [1]$$

Strictly speaking, Equation 1 is an approximation: its derivation entailed the restriction of both t_1 and t_2 to positive values whereas the assumption of normality for t_2 requires that this variable can take any real value. The approximation is very close, however, as long as the ratio of the mean to the standard deviation of the normal variable is of an order of 3 or more.

Equation 1 provided good representations of 12 distributions of RT to onset of low-intensity visual stimuli (.05 and .125 footlambert (ftL.) against a 1.8-ftL. background).[1] In these data the fitted parameter a was markedly different for the two intensities, but the fitted b was not related to this variable. A tentative interpretation of these results was that RTs do contain an exponentially distributed component and that this

[1] Data presented at the Midwestern Psychological Association meetings at Chicago, May 1963.

component is the decision or "perception" portion of an RT. The remaining portion, the mean of which is represented by the parameter b, was assumed to be the time required for organization and execution of the motor response.

The present study was designed to provide a further test of Equation 1 and to attempt to manipulate the normally distributed component of simple RTs. It has been generally assumed that effects of different foreperiod durations on simple RT are due to changes in *S*'s preparatory set during the prestimulus delay. And evidence seems to indicate that, for clearly suprathreshold stimuli, the preparatory set is primarily muscular (Teichner, 1954). If this is indeed the case, and if the assumption is correct that a normally distributed component of RT includes motor response time, then it should be possible to account for variation in RT due to foreperiod in terms of variation of the b parameter in Equation 1 as a function of foreperiod.

Method

Distributions of simple RT to an auditory stimulus were obtained with four different foreperiods under two conditions: (*a*) constant foreperiod within sessions, and (*b*) random sequences of foreperiods within sessions.

Each trial began with onset of four panel lights surrounding a 4-in. speaker. After a delay (foreperiod) of 1.60, 3.20, 4.65, or 6.13 sec., a 62-db. (re .0002 dynes/cm^2), 1,000-cycle tone appeared from the speaker, which was *S*'s signal to press a microswitch button as quickly as possible. The response terminated both the lights and the tone. There was a constant 1-sec. interval between *S*'s response and the beginning of the next trial. Ambient background noise in the experimental room was 35 db.

Four 14-yr.-old female *S*s each participated in a practice session followed by eight 130-trial sessions. Only the last 100 trials from a session were retained for analysis, the first 30 being treated as warm-up trials. The eight sessions were spread over 10 days with no *S* participating in more than one session on any one day.

During the first four sessions the foreperiods were presented in a random sequence for all *S*s; during the last four the foreperiod was constant within a session but varied over sessions. Two *S*s received the constant foreperiods in descending order, the other two in ascending order. A distribution of 100 responses was thus obtained from each *S* for each foreperiod under each of the two foreperiod conditions.

For each of the 32 distributions, 10 intervals of response times were selected such that 10% of the observations fell in each interval. Parameters in Equation 1 were then determined by successive approximations such that this distribution function assigned as nearly as possible 10% of the probability to each of the 10 intervals. The criterion of closeness of fit was a chi-square statistic based on deviations of observed from expected frequencies in the 10 intervals.

Results and Conclusions

The correspondence between Equation 1 and the four *S*s' distributions of RTs following the shortest foreperiod used is indicated in Fig. 1. The upper four distributions are those obtained with this foreperiod occurring in a random sequence; the lower four were obtained with the foreperiod constant. That this generally excellent correspondence was uniform over the 32 sets of data is indicated by the distribution of the goodness-of-fit statistic: Under the null hypothesis—i.e., the hypothesis of perfect fit of Equation 1 except for random sampling error—this statistic would be distributed as a chi-square variable with $df = 6$. Thus if a perfect fit is assumed, the expected value of the statistic for any one distribution would be 6, and the expected value of the sum of the 32 (independent) statistics would be $(32)(6) = 192$. Obtained values of the 32 statistics ranged from .65 to 11.85, and their sum was 172.17. The median value was 4.92 which may be compared to an expected median of 5.35 on the basis of random sampling.

Figure 2 shows the obtained relationships of mean RT and its inferred components to foreperiod duration

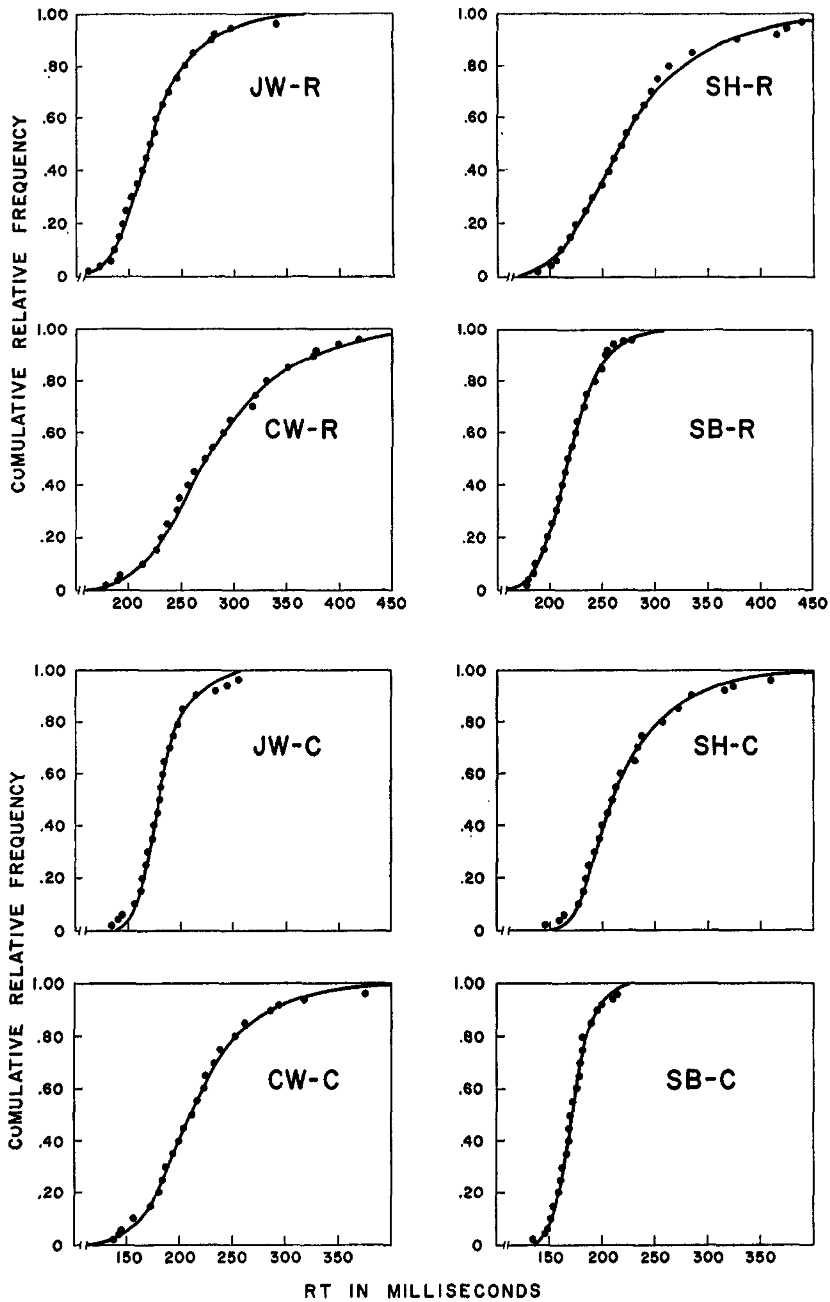

FIG. 1. Equation 1 fitted to the four *Ss'* cumulative distributions of RTs to a 62-db., 1,000-cycle tone with a foreperiod of 1.60 sec. (The plotted points are the second, fourth, sixth, ninety-second, ninety-fourth, and ninety-sixth percentiles, and every fifth percentile from the tenth to the ninetieth.)

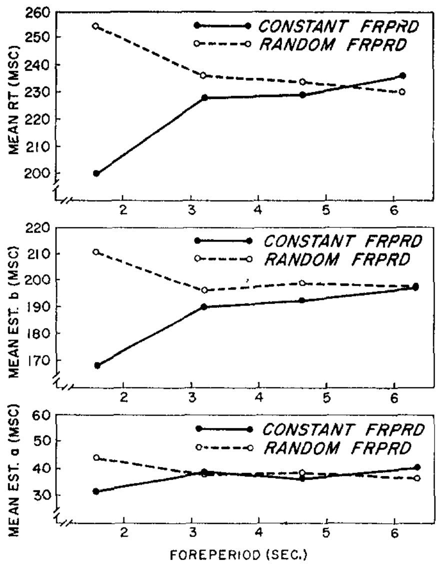

FIG. 2. Relations of mean RT and its inferred components to foreperiod duration. (The parameter a is the mean of a hypothetical exponentially distributed component; b is the mean of a hypothetical normally distributed component.)

under the two conditions. The mean RT curves are very similar to those reported in other studies (e.g., Karlin, 1959); the shortest foreperiod appears to have been disruptive when it occurred in a random sequence including longer foreperiods, but resulted in the shortest RTs when it was constant from trial to trial. Figure 2 suggests that most of this effect can be accounted for as differential variation of b, the normal component, with foreperiod under the two conditions.

A Foreperiod × Constant-vs.-Random × *S*s analysis of variance for mean RT and for each of the fitted parameters indicated that (*a*) the Foreperiod × Constant vs. Random interaction effect on mean RT was significant at the .05 level, F (3, 9) = 6.61; (*b*) the corresponding obtained F for the b parameter was 3.81, which is very close to the critical value (3.86) required for significance at the .05 level; while (*c*) this interaction effect on the parameter a did not approach significance (F = 1.46); and (*d*) although the parameter c, interpreted as the standard deviation of the normally distributed component, showed considerable variation among *S*s, its variations due to Foreperiod and Constant vs. Random conditions were not statistically significant. The range of estimated c's over the 32 sets of data was from 10 to 42 msec.

The distributions of RTs to low-intensity visual stimuli mentioned earlier and the auditory RT data reported here were thus both consistent with the hypothesis that an observed distribution of simple RTs may be described by a convolution of a normal and an exponential distribution. Moreover, the apparent differential dependence of the means of these inferred component distributions on stimulus intensity and foreperiod duration strongly suggests that the component distributions are in fact associated with times to complete different processes underlying observed RTs.

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