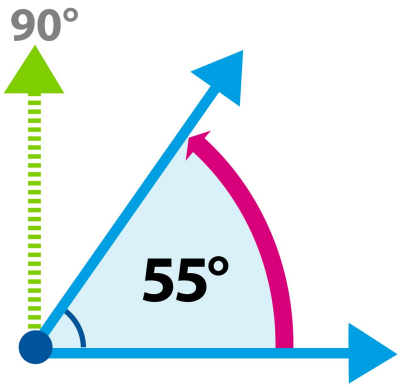
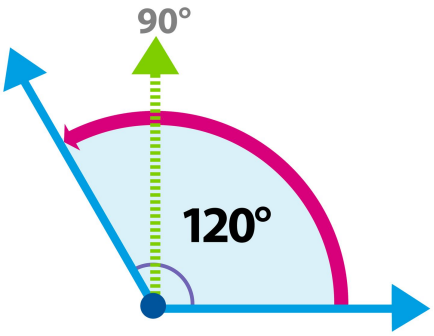
Plane and Solid Geometry

Angles

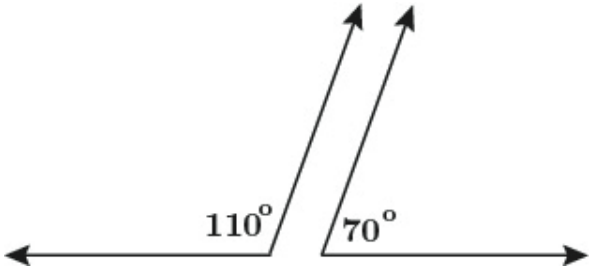
Acute angle < 90°



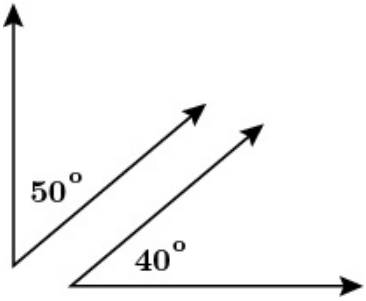
90 < Obtuse angle < 180°



Supplementary angles = 180°

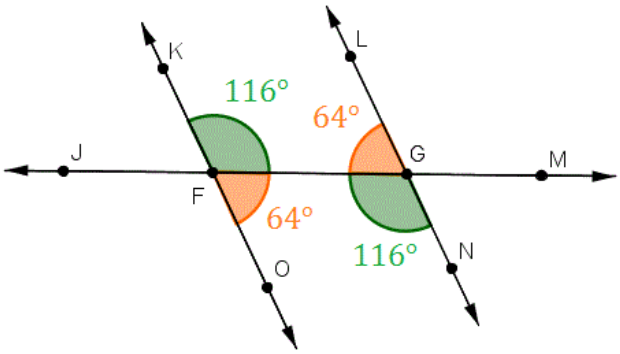


Complementary angles = 90°

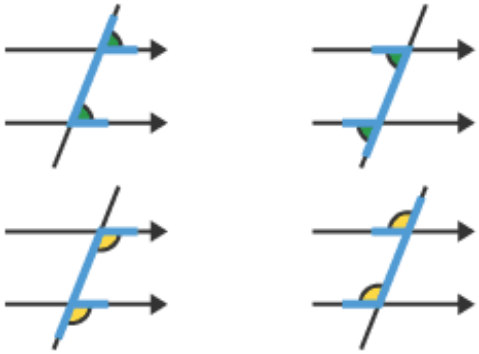


Parallel lines angles

Alternate angles



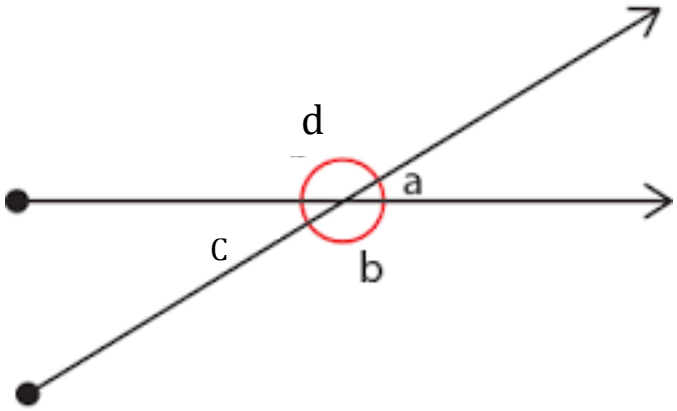
Corresponding Angles

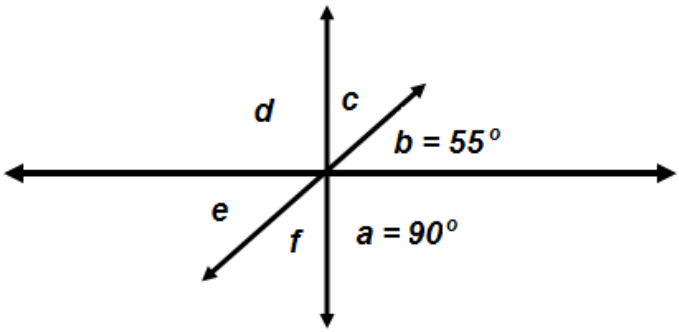


Intersecting lines

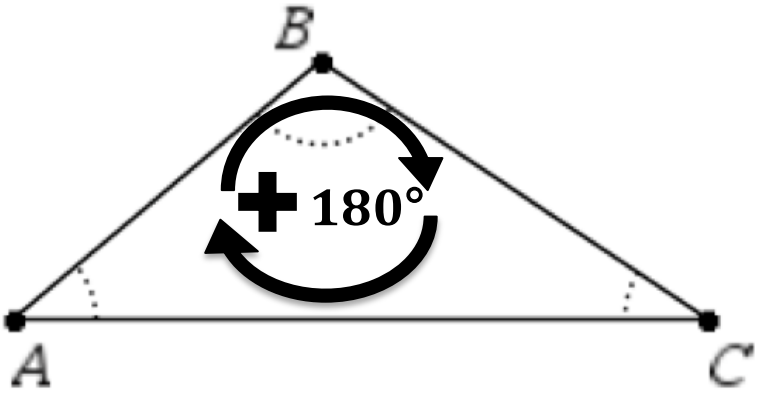
1. Interior angles = 360°
2. Interior angles that combine to form a line = 180° (a+b=b+c=c+d=d+a=180°)
3. Angles formed by the same two lines are equal (a=c and b=d)

All rules apply to more than two lines





Triangles



The sum of the angles = 180°

Angles correspond to its opposite side. This means:

Smallest angle shortest side

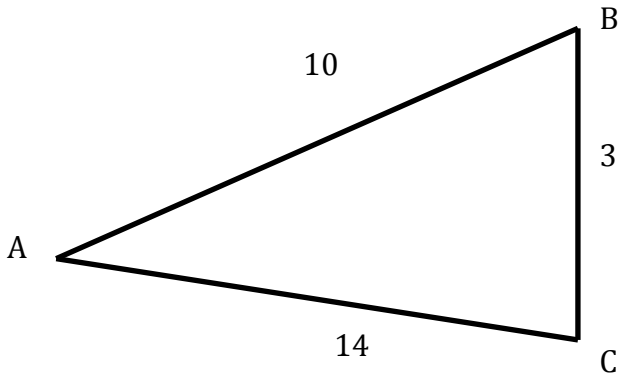
Largest angel longest side

The sides of a triangle

$\overline{AB} + \overline{BC} > \overline{CA}$

$\overline{BC} + \overline{CA} > \overline{AB}$

$\overline{CA} + \overline{AB} > \overline{BC}$

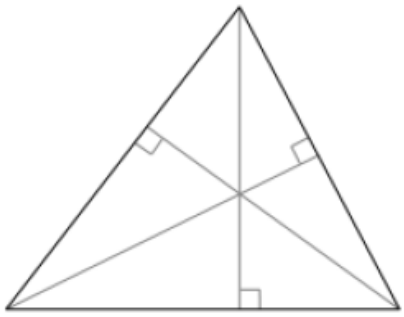


$\overline{AB} - \overline{BC} < \overline{CA}$

$\overline{BC} - \overline{CA} < \overline{AB}$

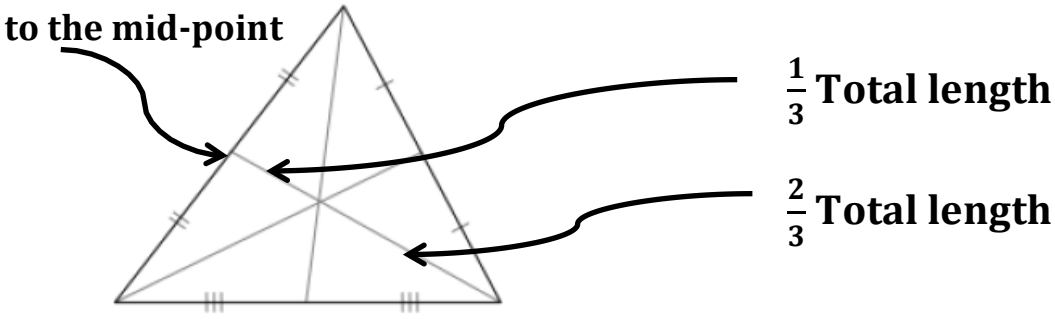
$\overline{CA} - \overline{AB} < \overline{BC}$

Altitude *The altitude of a triangle is the perpendicular from the base to the opposite vertex. (The base may need to be extended).*



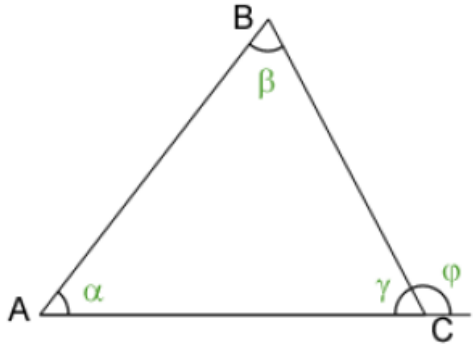
90° to the opposite side

Median *The median of a triangle is a line from a vertex to the midpoint of the opposite side.*



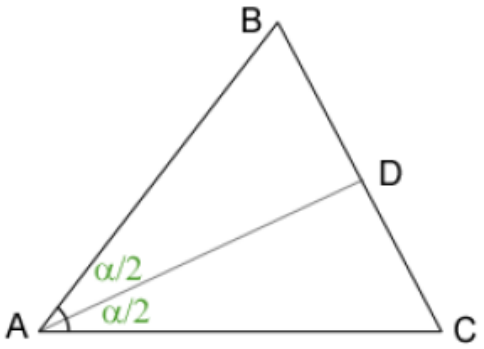
1. The medians intersect at a single point, called the **centroid** of the triangle.
2. **Two thirds** of the length of each median is between the **vertex** and the **centroid**, while **one third** is between the **centroid** and the **mid-point** of the opposite side.

Exterior angles *The angle between a side of a triangle and the extension of an adjacent side.*



$$\varphi = \alpha + \beta$$

Angle bisector *An angle bisector divides the angle into two angles with equal measures.*



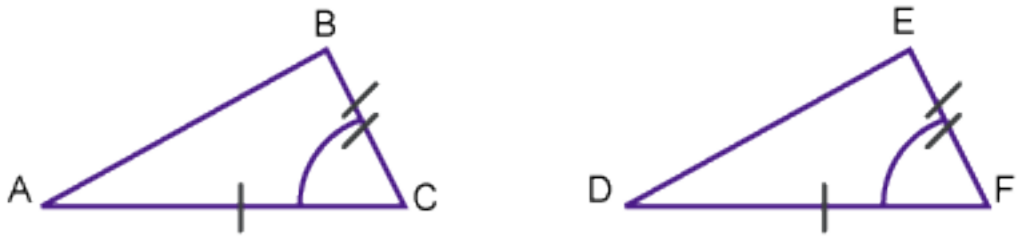
- An angle only has one bisector.
- **Each point of an angle bisector is equidistant from the sides of the angle.**
- **The angle bisector theorem states that the ratio of the length of the line segment BD to the length of segment DC is equal to the ratio of the length of side AB to the length of side AC:** $\frac{BD}{DC} = \frac{AB}{AC}$
- **The incenter is the point where the angle bisectors intersect. The incenter is also the center of the triangle's incircle - the largest circle that will fit inside the triangle.**

Congruence of triangles

1. ASA (Angle-Side-Angle)

$< A \equiv < D$; $AC \equiv DF$; $< C \equiv < F$; luego $\triangle ABC \equiv \triangle DEF$

2. SAS (Side-Angle-Side)



AC ≡ DF ; < C ≡ < F ; CB ≡ FE ; luego △ ABC ≡ △ DEF

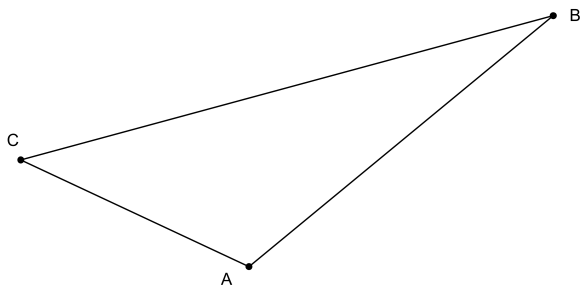
2. SSS (Side-Side-Side)

AB ≡ DE ; BC ≡ EF ; AC ≡ DF ; luego △ ABC ≡ △ DEF

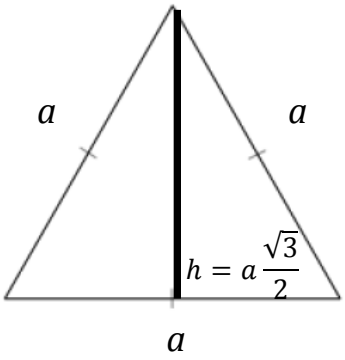
So:

1. Knowing SAS or ASA is **sufficient** to determine unknown angles or sides.
2. Knowing three angles is **not sufficient** to determine lengths of the sides.

Scalene triangle *all sides and angles are different from one another*



Equilateral triangle *all sides have the same length.*

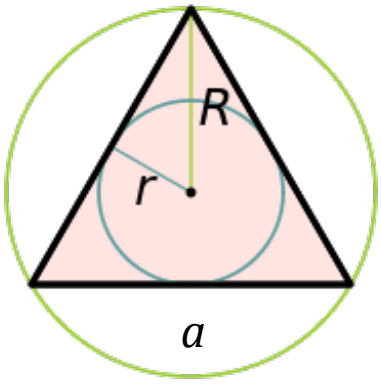


All angles = 60°

P = 3a $\qquad A = a^2\frac{\sqrt{3}}{4}$

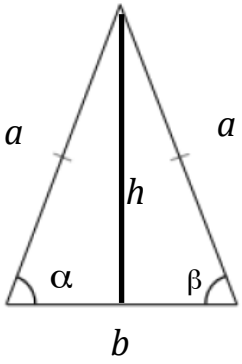
R of circunscribed circule = $a\frac{\sqrt{3}}{3}$

r of inscribed circle = $a\frac{\sqrt{3}}{6}$



In an **equilateral** triangle, the **radius** of an **inscribed** circle is exactly **half** the radius of the **circumscribed** circle.

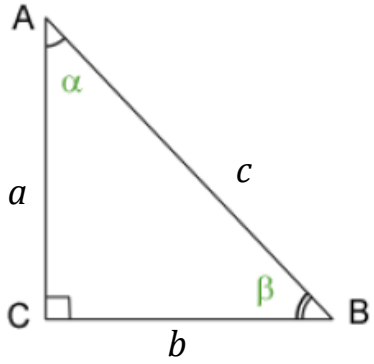
Isosceles triangle *two sides are equal in length.*



$\alpha = \beta$

Given "a" and "h", $b = 2\sqrt{a^2 - h^2}$

Right triangle *A triangle where one of its interior angles is a right angle (90 degrees)*



-Hypotenuse: The side opposite to the rigth angle

$-a^2 + b^2 = c^2$ Pythagoras theorem

-It can be Isosceles but it cannot be equilateral

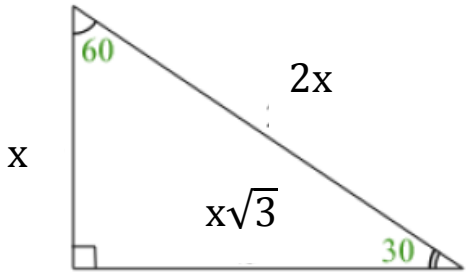
Perfect rigth triangles

Position	Side 1(a)	Side 2(b)	Hypotenuse(c)
1)	3	4	5
2)	5	12	13
3)	7	24	25
4)	9	40	41
5)	11	60	61

If (a,b,c) is a perfect rigth triangle then so is (ka,kb,kc) for any positive integer k.

Right triangles special cases.

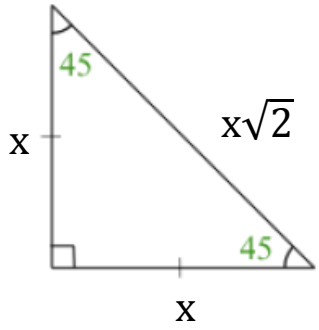
Right triangle, 30°, 60°:



Sides are always in ratio $x : x\sqrt{3} : 2x$

Be careful;

The smallest side (1) is opposite the smallest angle (30°)



Sides are always in ratio $x : x : x\sqrt{2}$

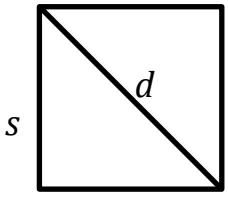
Isosceles triangle

$A = \frac{s^2}{2}$

Diagonals of other polygons

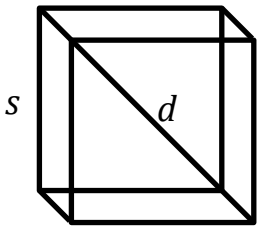
The diagonal can be found as:

$d = s\sqrt{2}$, where "s" is the side of the square

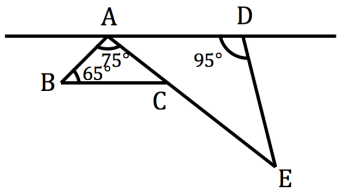


The diagonal of a cube can be found as:

$d = s\sqrt{3}$, where "s" is the edge of the cube.



Kahoot!!

1. 

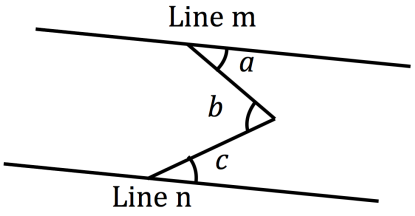
In the diagram above, angle measures in degrees are marked as shown, and segment BC is parallel to line AD. What is the measure of angle E?

A) 30

B) 35

C) 40

D) 45

2. 

In the diagram above, **line m** and **line n** are parallel. Given that angle a = 40° and angle c = 55°, what is the measure of angle b?

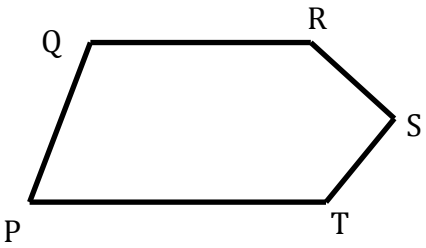
A) 40

B) 85

C) 95

D) 75

3. Pentagon PQRST where PQ=3 QR=2 RS=4 ST=5. Which of the lengths 5, 10 & 15 could be the value of PT?



A) 5 only B) 15 only C) 5 and 10 only D) 10 and 15 only

4. If two sides of a triangle have lengths 2 and 5, which of the following could be the perimeter of the triangle?

I. 9
II. 15
III. 19

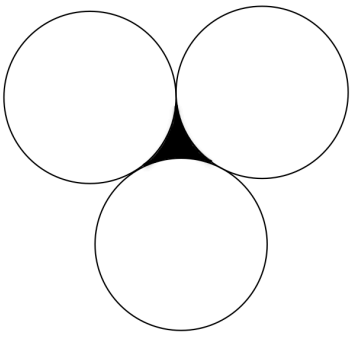
A) None

B) I only

C) II only

D) II and III only

5. 3 identical circles, tangent to each other. The area of the shaded region is $64\sqrt{3} - 32\pi$. What is the radius of each circle?



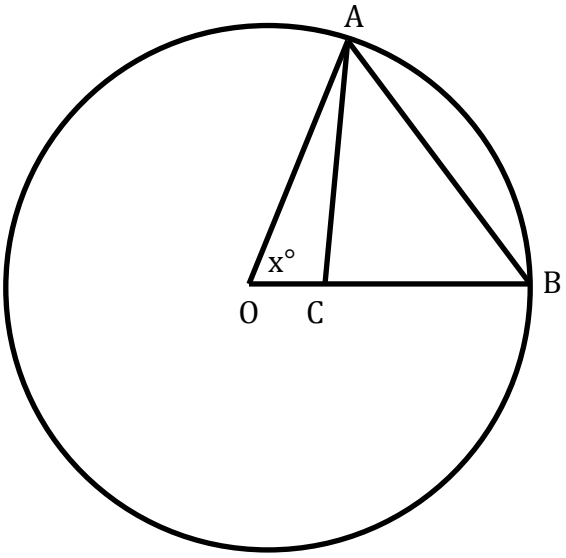
A) 4

B) 8

C) 16

D) 24

6. Point O is the center of the circle and OC=AC=AB. What is the value of X° ?



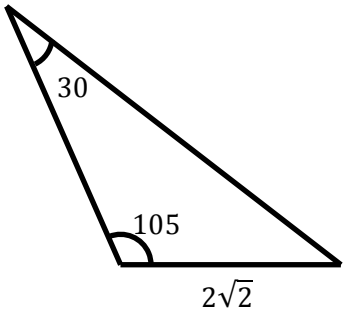
A) 32

B) 34

C) 36

D) 40

7. What is the area of the triangle?



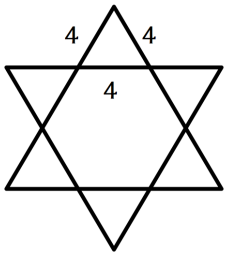
A) $2 + 2\sqrt{3}$

B) $4\sqrt{3}$

C) $8\sqrt{3}$

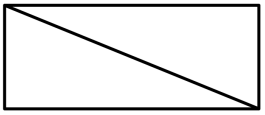
D) 8

8. The points of a six-point star consist of six identical equilateral triangles, with each side 4cm. What is the area of the entire star, including the center?



A) $6\sqrt{3}$ B) $12\sqrt{3}$ C) $36\sqrt{3}$ D) $48\sqrt{3}$

9.



If the rectangle above has with of 6, and the ratio of the width to the length is 3:4, what is the diagonal?

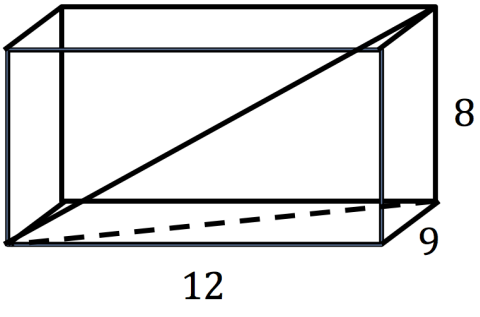
A) 12

B) 9

C) 18

D) 10

10. What is the length of the diagonal of this rectangular solid?



A) 15

B) 17

C) 16

D) 18

Homework

Lines and angles
Appendix A: D10, 51, 60, 226
Appendix B: 28
Appendix C: 6, 23, 72

Triangles and diagonals
Appendix A: D19, 45, 145, 147, 152, 176, 222
Appendix C: 27, 32, 51, 108, 113, 152

Answert key Kahoot Geometry

1	45
2	95
3	5 and 19 only
4	None
5	8
6	36
7	$2+2\sqrt{3}$
8	$48\sqrt{3}$
9	10
10	17