RATIOS AND PROPORTION

A *ratio* expresses proportion or comparative size—the size of one quantity *relative* to the size of another. As with fractions, you can simplify ratios by dividing common factors. For example, given a class of 28 students—12 freshmen and 16 sophomores:

- The ratio of freshmen to sophomores is 12:16, or 3:4.
- The ratio of freshmen to the total number of students is 12:28, or 3:7.
- The ratio of sophomores to the total number of students is 16:28, or 4:7.

Determining Quantities from a Ratio (Part-to-Whole Analysis)

You can think of any ratio as parts adding up to a whole. For example, in the ratio 5:6, 5 parts + 6 parts = 11 parts (the whole). If the actual total quantity were 22, you'd multiply each element by 2: 10 parts + 12 parts = 22 parts (the whole). Notice that the ratios are the same: 5:6 is the same ratio as 10:12.

You might be able to solve a GMAT ratio question using this part-to-whole approach.

> A class of students contains only freshmen and sophomores. If 18 of the students are sophomores, and if the ratio of the number of freshmen to the number of sophomores in the class is 5:3, how many students are in the class?
>
> **(A)** 30
> **(B)** 36
> **(C)** 40
> **(D)** 48
> **(E)** 56

Using a part-to-whole analysis, look first at the ratio and the sum of its parts: 5 (freshmen) + 3 (sophomores) = 8 (total students). These aren't the actual quantities, but they're proportionate to those quantities. Given 18 sophomores altogether, sophomores account for 3 parts—each part containing 6 students. Accordingly, the total number of students must be $6 \times 8 = 48$. **The correct answer is (D).**

Now let's focus on more advanced applications of fractions, percents, decimals, ratios, and proportion. We'll place special emphasis on how the test-makers incorporate algebraic features into GMAT questions covering these concepts:

- Altering fractions and ratios
- Ratios involving more than two quantities
- Proportion problems with variables

ALTERING FRACTIONS AND RATIOS

An average test-taker might assume that *adding* the same *positive* quantity to a fraction's numerator (p) and to its denominator (q) leaves the fraction's value $\left(\frac{p}{q}\right)$ unchanged. But this is true *if and only if* the original numerator and denominator were equal to each other. Otherwise, the fraction's value will change. Remember the following three rules, which apply to any positive numbers x, p, and q (the first one is the no-brainer you just read):

If $p = q$, then $\frac{p}{q} = \frac{p+x}{q+x}$. (The fraction's value remains unchanged and is always 1.)

If $p > q$, then $\frac{p}{q} > \frac{p+x}{q+x}$. (The fraction's value will *decrease*.)

If $p < q$, then $\frac{p}{q} < \frac{p+x}{q+x}$. (The fraction's value will *increase*.)

A GMAT question might ask you to alter a ratio by adding or subtracting from one (or both) terms in the ratio. The rules for altering ratios are the same as for altering fractions. In either case, set up a proportion and solve algebraically for the unknown term.

A drawer contains exactly half as many white shirts as blue shirts. If four more shirts of each color were to be added to the drawer, the ratio of white to blue shirts would be 5:8. How many blue shirts does the drawer contain?

(A) 14
(B) 12
(C) 11
(D) 10
(E) 9

Represent the original ratio of white to blue shirts by the fraction $\frac{x}{2x}$, where x is the number of white shirts, then add 4 to both the numerator and denominator. Set this fraction equal to $\frac{5}{8}$ (the ratio after adding shirts). Cross-multiply to solve for x:

$$\frac{x+4}{2x+4} = \frac{5}{8}$$
$$8x + 32 = 10x + 20$$
$$12 = 2x$$
$$x = 6$$

The original denominator is $2x$, or 12. **The correct answer is (B).**

Sometimes the GRE/GMAT provides you with two different ratios. However, one item is common in each of the ratios. For example, a jazz shop has 6 saxophones for every 5 drum kits and 2 drum kits for every 3 trombones. You can use the common item (drum kits) to combine the ratios for a single ratio of instruments at the jazz shop.

Combining ratios is a little like adding fractions. When adding fractions, you find the lowest common denominator. Ratios don't have numerators or denominators, but they do have something in common — in this case, drum kits. In order to combine ratios, follow these simple steps

Sam's jazz shop has 6 saxophones for every 5 drum kits and 2 drum kits for every 3 trombones. What's the ratio of saxophones to trombones?

1. Set up the ratios as A : B.

Place the item that the ratios have in common (drum kits) into a column.

Saxes		Drums		'Bones
6	:	5		
		2	:	3

2. Find a common multiple for the item that these ratios have in common.

In this instance, both ratios include drum kits. The least common multiple of 5 and 2 (the numbers of drum kits) is 10.

3. Multiply each term in the ratios so that the quantity of the item in common equals the common multiple (from Step 2).

Now, if you were adding these as fractions, you'd write the drums as the denominators, and your work would start out like this:

$$\frac{6}{5}+\frac{3}{2}=\frac{6(2)}{2(2)}+\frac{3(5)}{2(5)}=\frac{12}{10}+\frac{15}{10}$$

Of course, you aren't adding fractions, but you treat the ratios the same way: Multiply both terms of each ratio by the same number, as though you're getting a common denominator. Here, you want the number of drum kits to equal 10. Multiply both terms in the first ratio by 2, and multiply both terms in the second ratio by 5:

Saxes		Drums		'Bones
6(2)	:	5(2)		
		2(5)	:	3(5)

Saxes		Drums		'Bones
12	:	10		
		10	:	15

4. Write out a combined ratio.

Saxes		Drums		'Bones
12	:	10		
		10	:	15
12	:	10	:	15

The combined ratio of saxophones to drum kits to trombones is 12:10:15. To answer the question, give only the ratio of saxophones to trombones, which is 12:15, or 4:5.

Three lottery winners—X, Y, and Z—are sharing a lottery jackpot. X's share is $\frac{1}{5}$ of Y's share and $\frac{1}{7}$ of Z's share. If the total jackpot is \$195,000, what is the dollar amount of Z's share?

(A) \$15,000
(B) \$35,000
(C) \$75,000
(D) \$105,000
(E) \$115,000

PROPORTION PROBLEMS WITH VARIABLES

A GMAT proportion question might use *letters* instead of numbers—to focus on the process rather than the result. You can solve these problems algebraically or by using the plug-in strategy.

A candy store sells candy only in half-pound boxes. At c cents per box, which of the following is the cost of a ounces of candy? [1 pound = 16 ounces]

(A) $\frac{c}{a}$

(B) $\frac{a}{16c}$

(C) ac

(D) $\frac{ac}{8}$

(E) $\frac{8c}{a}$

This question is asking: "c cents is to one box as *how many cents* are to a ounces?" Set up a proportion, letting x equal the cost of a ounces. Because the question asks for the cost of ounces, convert 1 box to 8 ounces (a half pound). Use the cross-product method to solve quickly:

$$\frac{c}{8} = \frac{x}{a}$$

$$8x = ac$$

You can also use the plug-in strategy for this question, either instead of algebra or, better yet, to check the answer you chose using algebra. Pick easy numbers to work with, such as 100 for c and 16 for a. At 100 cents per 8-ounce box, 16 ounces of candy cost 200 cents. Plug your numbers for a and c into each answer choice. Only choice (D) gives you the number 200 you're looking for.

Inverse Variation

Two quantities are said to vary inversely if they change in opposite directions. As one increases, the other decreases.

For example, the number of workers hired to paint a house varies inversely with the number of days the job will take. A doctor's stock of flu vaccine varies inversely with the number of patients injected. The number of days a given supply of cat food lasts varies inversely with the number of cats being fed.

> **HINT:** Whenever two quantities vary inversely, you can find a missing term by using multiplication. Multiply the first quantity by the second and set the products equal.

Q If a case of cat food can feed 5 cats for 4 days, how long would it feed 8 cats?

A Since this is a case of inverse variation (the more cats, the fewer days), multiply the number of cats by the number of days in each instance and set them equal.

$$5 \times 4 = 8 \times x$$
$$20 = 8x$$
$$2\frac{1}{2} = x$$

Kahoot!!

A gear 50 inches in diameter turns a smaller gear 30 inches in diameter. If the larger gear makes 15 revolutions, how many revolutions does the smaller gear make in that time?

(A) 9

(B) 12

(C) 20

(D) 25

If x men can do a job in h days, how long would y men take to do the same job?

(A) $\frac{x}{h}$

(B) $\frac{xh}{y}$

(C) $\frac{hy}{x}$

(D) $\frac{xy}{h}$

(E) $\frac{x}{y}$

A school has enough bread to feed 30 children for 4 days. If 10 more children are added, how many days will the bread last?

(A) $1\frac{1}{3}$

(B) $2\frac{2}{3}$

(C) 3

(D) $5\frac{1}{3}$

(E) 12

Answer key: 1. D 2. B 3. C

Mixing It Up with Mixture Problems

A mixture problem looks much more confusing than it really is. Plan to encounter two types of mixture problems: Those in which the items remain separate (when you mix peanuts and raisins, you still have peanuts and raisins), and those in which the two elements blend (these elements are usually concentrations of chemicals, like percent of salt in a mixture).

You can solve both types in the same general way, setting up tables that account for both the total mix and the component parts. Check out the separate mixture first.

Example:

Carolyn wants to mix 40 pounds of beads selling for 30 cents a pound with a quantity of sequins selling for 80 cents a pound. She wants to pay 40 cents per pound for the final mix. How many pounds of sequins should she use?

The hardest part for most test-takers is knowing where to begin. Stick to these steps to solve problems like this one

1. Make a chart and start with the bare essentials — the labels for all the data you have to work with.

	Pounds	Price	Total
Beads			
Sequins			
Mixture			

2. Fill in the values that the test gives you.

Here, beads are 40 pounds at 30 cents a pound. Sequins are 80 cents per pound. You want the mixture to cost 40 cents a pound.

	Pounds	Price	Total
Beads	40	$0.30	
Sequins		$0.80	

Mixture		$0.40	

3. Use a variable to stand in for your unknown value.

	Pounds	Price	Total
Beads	40	$0.30	
Sequins	X	$0.80	
Mixture	40+X	$0.40	

4. Multiply across the rows to fill in the total column.

	Pounds	Price	Total
Beads	40	$0.30	$12.00
Sequins	X	$0.80	$0.80X
Mixture	40+X	$0.40	$0.40(40+X)

5. Solve for x.

In the Total column, the total cost of beads plus sequins equals the total cost of the mixture, so the equation looks like this:

$\$12.00 + \$0.80x = \$0.40(40 + x)$

You know from the table that x is going to be in pounds, because it's in the Pounds column, so do the math:

$$\begin{aligned} 12.00 + 0.80x &= 0.40(40 + x) \\ 12.00 + 0.80x &= 16.00 + 0.40x \\ 0.40x &= 4.00 \\ x &= 10 \end{aligned}$$

Go back and double-check the answer by plugging this value into the equation. You already know that Carolyn spent \$12 on beads. If she buys 10 pounds of sequins for 80 cents, she spends \$8, for a total of \$20. She spends that \$20 on 50 pounds: 2,000 ÷ 50 = 40. Easy as that.

A mixture question based on a concentration of chemicals works the same way. The only difference is that the total column represents amounts of substances (5 liters of a 40 percent saline solution contains 2 liters of salt) rather than cost.

Kahoot!!

1. At a certain college the ratio of freshmen to sophomores is 2:3, and the ratio of sophomores to juniors is 5:6. If the ratio of juniors to seniors is 3:5, what is the ratio of freshmen to seniors?

A) 1:3 B) 5:9 C) 3:5 D) 4:7 E) 5:6

2. The present ratio of almonds to cashews in a certain can of nuts is 2 to 3. If 45 almonds and 30 cashews were to be added to the can, the ratio of almonds to cashews would be 7 to 8. What is the present number of nuts in the can?

A) 30 B) 60 C) 90 D) 120 E) 150

3. The numbers of stamps that Kaye and Alberto had were in the ratio 5:3 respectively. After Kaye gave Alberto 10 of her stamps, the ratio went to 7 : 5. As a result of this gift, Kaye had how many more stamps than Alberto?

A) 30 B) 40 C) 50 D) 60 E) 70

4. The ratio of buses to cars on River Road is 2 to 23. If there are 630 fewer buses than cars on River Road, how many cars are on River Road?

A) 30 B) 60 C) 660 D) 690 E) 750

5. The ratio, by volume, of salt to pepper to oregano in a certain recipe is 4: 5: 6. The recipe will be altered so that the ratio of salt to pepper is doubled while the ratio of salt to oregano is halved. If the altered recipe will contain 2.5 teaspoons of pepper, how many teaspoons of oregano will it contain?

A) 2 B) 4 C) 6 D) 12 E) 24

6. A chemistry student has one solution that's 25% acid and another that's 15% acid. Approximately how many liters of the 25% solution must be added to the 15% solution to make 10 liters of a solution that's 20% acid?

A) 2.5
B) 3.3
C) 5.0
D) 6.7
E) 7.5

Kahoot Answer key

1. A
2. E
3. B
4. D
5. D
6. C

Homework

Ratios

Appendix A: 18, 50, 52, 61, 63, 73, 76, 97, 106, 118, 163, 168, 170, 181, 193, 196

Appendix B: 71, 82