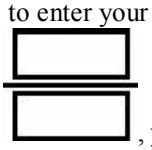
Probability, Combinatorics, and Overlapping Sets

For questions in the Quantitative Comparison format ("Quantity A" and "Quantity B" given), the answer choices are always as follows:

(A) Quantity A is greater.
(B) Quantity B is greater.
(C) The two quantities are equal.
(D) The relationship cannot be determined from the information given.

For questions followed by a numeric entry box  , you are to enter your own answer in the box. For questions followed by fraction-style numeric entry boxes  , you are to enter your answer in the form of a fraction. You are not required to reduce fractions. For example, if the answer is 1/4, you may enter 25/100 or any equivalent fraction.

All numbers used are real numbers. All figures are assumed to lie in a plane unless otherwise indicated. Geometric figures are not necessarily drawn to scale. You should assume, however, that lines that appear to be straight are actually straight, points on a line are in the order shown, and all geometric objects are in the relative positions shown. Coordinate systems, such as *xy*-planes and number lines, as well as graphical data presentations such as bar charts, circle graphs, and line graphs, *are* drawn to scale. A symbol that appears more than once in a question has the same meaning throughout the question.

1. A number is randomly chosen from a list of 10 consecutive positive integers. What is the probability that the number is greater than the mean?

(A) 3/10
(B) 2/5
(C) 1/2
(D) 7/10
(E) 4/5

2. A number is randomly chosen from the first 100 positive integers. What is the probability that it is a multiple of 3?

(A) 32/100
(B) 33/100
(C) 1/3
(D) 34/100
(E) 2/3

3. A restaurant menu has several options for tacos. There are 3 types of shells, 4 types of meat, 3 types of cheese, and 5 types of salsa. How many distinct tacos can be ordered assuming that any order contains exactly one of each of the above choices?

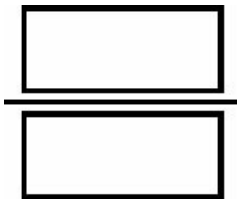
4. A history exam features 5 questions. 3 of the questions are multiple-choice with four options each. The other two questions are true or false. If Caroline selects one answer for every question, how many different ways can she answer the exam?

5. A certain company places a six-symbol code on each of their products. The first two symbols are one of the letters A–E and the last four symbols are digits. If repeats are allowed on both letters and numbers, how many such codes are possible?

6. The probability is 1/2 that a coin will turn up heads on any given toss and the probability is 1/6 that a number cube with faces numbered 1 to 6 will turn up any particular number. What is the probability of turning up a heads and a 6?

(A) 1/36
(B) 1/12
(C) 1/6
(D) 1/4
(E) 2/3

7. An integer is randomly chosen from 2 to 20 inclusive. What is the probability that the number is prime?



8. Five students in a classroom are lining up one behind the other for recess. How many different lines are possible?

(A) 5
(B) 10
(C) 24
(D) 25
(E) 120

9. An Italian restaurant boasts 320 distinct pasta dishes. Each dish contains exactly one pasta, one meat, and one sauce. If there are 8 pastas and 4 meats available, how many sauces are there to choose from?

10. A 10-student class is to choose a president, vice president, and secretary from the group. Assuming that no person can occupy more than one post, in how many ways can this be accomplished?

11.

Quantity A	**Quantity B**
The number of 4-digit positive integers where all 4 digits are less than 5	625

12. BurgerTown offers many options for customizing a burger. There are 3 types of meats and 7 condiments: lettuce, tomatoes, pickles, onions, ketchup, mustard, and special sauce. A burger must include meat, but may include as many or as few condiments as the customer wants. How many different burgers are possible?

(A) 8!
(B) (3)(7!)
(C) (3)(8!)
(D) $(8)(2^7)$
(E) $(3)(2^7)$

13. The probability of rain is 1/6 for any given day next week. What is the chance it rains on both Monday and Tuesday?

(A) 1/36
(B) 1/12
(C) 1/6
(D) 1/3
(E) 2/3

14. How many five-digit numbers can be formed using the digits 5, 6, 7, 8, 9, 0 if no digits can be repeated?

(A) 64
(B) 120
(C) 240
(D) 600
(E) 720

15. A bag contains 3 red, 2 blue, and 7 white marbles. If a marble is randomly chosen from the bag, what is the probability that it is NOT blue?

16. A man has 3 different suits, 4 different shirts, 2 different pairs of socks, and 5 different pairs of shoes. In how

many ways can the man dress himself if he must wear 1 suit, 1 shirt, 1 pair of socks, and 1 pair of shoes?

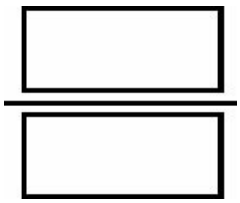
17. A state issues automobile license plates using two letters selected from a 26-letter alphabet, as well as four numerals selected from the digits 0 through 9, inclusive. Repeats are permitted. For example, one license plate combination could be GF3352.

Quantity A	**Quantity B**
The number of possible unique license plate combinations	6,000,000

18. A small nation issues license plates that consist of just one number (selected from the digits 0 through 9, inclusive) and four letters, selected from a 20-letter alphabet. Repeats are permitted. However, there is one four-letter combination that is not allowed to appear on license plates. How many allowable license plate combinations exist?

(A) 1,599,990
(B) 1,599,999
(C) 1,600,000
(D) 4,569,759
(E) 4,569,760

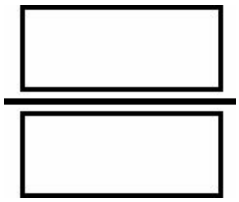
19. A bag contains 6 black chips numbered 1–6 respectively and 6 white chips numbered 1–6 respectively. If Pavel reaches into the bag of 12 chips and removes 2 chips, one after the other, without replacing them, what is the probability that he will pick black chip #3 and then white chip #3?



20. Tarik has a pile of 6 green chips numbered 1–6 respectively and another pile of 6 blue chips numbered 1–6 respectively. Tarik will randomly pick 1 chip from the green pile and 1 chip from the blue pile.

Quantity A	**Quantity B**
The probability that both chips selected by Tarik will display a number less than 4	1/2

21. A bag contains 6 red chips numbered 1–6 respectively and 6 blue chips numbered 1–6 respectively. If 2 chips are to be picked sequentially from the bag of 12 chips, without replacement, what is the probability of picking a red chip and then a blue chip with the same number?



22. In a school of 150 students, 75 take Latin, 110 take Spanish, and 11 take neither.

Quantity A	**Quantity B**
The number of students who take only Latin	46

23. How many 10-digit numbers can be formed using only the digits 2 and 5?

(A) 2^{10}
(B) (22)(5!)
(C) (5!)(5!)
(D) 10!/2
(E) 10!

24. A 6-sided cube has sides numbered 1 through 6. If the cube is rolled twice, what is the probability that the sum of the two rolls is equal to 8?

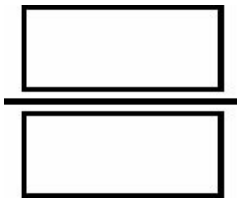
(A) 1/9
(B) 1/8
(C) 5/36
(D) 1/6
(E) 7/36

25. A coin with heads on one side and tails on the other has a 1/2 probability of landing on heads. If the coin is flipped 5 times, how many distinct outcomes are possible if the last flip must be heads? Outcomes are distinct if they do not contain exactly the same results in exactly the same order.

26. In a class of 25 students, every student takes either Spanish, Latin, or French, or two of the three, but no students take all three languages. 9 take Spanish, 7 take Latin and 5 take exactly two languages.

Quantity A	**Quantity B**
The number of students who take French	14

27. Bob has a 24-sided die with an integer between 1 and 24 on each face. Every number is featured exactly once. When he rolls, what is the probability that the number showing is a factor of 24?



28. A baby has x total toys. If 9 of the toys are stuffed animals, 7 of the toys were given to the baby by its grandmother, 5 of the toys are stuffed animals given to the baby by its grandmother, and 6 of the toys are neither stuffed animals nor given to the baby by its grandmother, what is the value of x?

29. How many integers between 2,000 and 3,999 have a ones digit that is a prime number?

30. How many integers between 2,000 and 6,999 are even and have a digit that is a prime number in the tens place?

31. A group of 12 people who have never met are in a classroom. How many handshakes are exchanged if each pair shakes hands exactly once?

(A) 12
(B) 22
(C) 66
(D) 132
(E) 244

32. A classroom has 12 girls and 20 boys. One quarter of the girls in the class have blue eyes. If a child is selected at random from the class, what is the probability that he/she is a girl who does not have blue eyes?

(A) 3/32
(B) 9/32
(C) 3/8
(D) 23/32
(E) 29/32

33. A coin with heads on one side and tails on the other has a 1/2 probability of landing on heads. If the coin is flipped three times, what is the probability of flipping 2 tails and 1 head, in any order?

(A) 1/8
(B) 1/3
(C) 3/8
(D) 5/8

(E) 2/3

34. A 6-sided cube has sides numbered 1 through 6. If the cube is rolled twice, what is the probability that at least one of the rolls will result in a number higher than 4?

(A) 2/9
(B) 1/3
(C) 4/9
(D) 5/9
(E) 2/3

35. Tiles are labeled with the integers from 1 to 100 inclusive; no numbers are repeated. If Alma chooses one tile at random, replaces it in the group, and chooses another tile at random, what is the probability that the product of the two integer values on the tiles is odd?

(A) 1/8
(B) 1/4
(C) 1/3
(D) 1/2
(E) 3/4

36. If the word "WOW" can be rearranged in exactly 3 ways (WOW, OWW, WWO), in how many ways can the word "MISSISSIPPI" be rearranged?

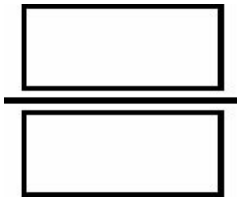
37. If a, b, and c are integers randomly chosen from the set of prime numbers greater than 2 and less than 30, what is the probability that $ab + c$ is equal to 23?

38.

The probability of rain is 1/2 on any given day next week.

Quantity A	**Quantity B**
The probability that it rains on AT LEAST one out of the 7 days next week	127/128

39. Two fair dice with sides numbered 1 to 6 are tossed. What is the probability that the sum of the exposed faces on the dice is a prime number?



40. Jack has a cube with 6 sides numbered 1 through 6. He rolls the cube repeatedly until the first time that the sum of all of his rolls is even, at which time he stops. (Note: it is possible to roll the cube just once.) What is the probability that Jack will need to roll the cube more than 2 times in order to get an even sum?

(A) 1/8
(B) 1/4
(C) 3/8
(D) 1/2
(E) 3/4

41. Jan and 5 other children are in a classroom. The principal of the school walks in and chooses two children at random. What is the probability that Jan is chosen?

(A) 4/5
(B) 1/3
(C) 2/5
(D) 7/15
(E) 1/2

42. The probability that Gary will eat eggs for breakfast on any given day is 3/7. The probability that Gary will eat cereal for breakfast on any given day is 4/7. Gary never has both eggs and cereal for breakfast on the same day.

Quantity A	**Quantity B**
Probability that Gary eats eggs or cereal for breakfast on a particular day	1

43. The probability that Maria will eat breakfast on any given day is 0.5. The probability that Maria will wear a sweater on any given day is 0.3. The two probabilities are independent of each other.

Quantity A	**Quantity B**
The probability that Maria eats breakfast or wears a sweater	0.8

44. The probability of rain in Greg's town on Tuesday is 0.3. The probability that Greg's teacher will give him a pop quiz on Tuesday is 0.2. The events occur independently of each other.

Quantity A	**Quantity B**
The probability that either or both events occur	The probability that neither event occurs

45. The probability of event *X* occurring is the same as the probability of event *Y* occurring. The events occur independently of each other.

Quantity A	**Quantity B**
The probability that both events occur	The probability that neither event occurs.

46. A certain city has a 1/3 chance of rain occurring on any given day. In any given 3-day period, what is the probability that the city experiences rain?

(A) 1/3
(B) 8/27
(C) 2/3
(D) 19/27

(E) 1

47. Five students, Adnan, Beth, Carol, Dan, and Edmund are to be arranged in a line. How many such arrangements are possible if Beth is not allowed to stand next to Dan?

(A) 24
(B) 48
(C) 72
(D) 96
(E) 120

48. A polygon has 12 edges. How many different diagonals does it have? (A diagonal is a line drawn from one vertex to any other vertex inside the given shape. This line cannot touch or cross any of the edges of the shape. For example, a triangle has zero diagonals and a rectangle has two.)

(A) 54
(B) 66
(C) 108
(D) 132
(E) 144

49. A student council is to be chosen from a class of 12 students consisting of a president, a vice president, and 3 committee members. How many such councils are possible?

(A) $\frac{12!}{7!5!}$
(B) $\frac{12!}{7!3!}$
(C) $\frac{12!}{5!3!}$
(D) $\frac{12!}{7!}$
(E) 12!

50.

Quantity A	**Quantity B**
The number of possible pairings of 2 colors that can be selected from 5 possible options	The number of possible pairings of 8 colors that can be selected from 9 possible options

51.

Quantity A	**Quantity B**
The number of possible 4-person teams that can be selected from 6 people	The number of possible 2-person teams that can be selected from 6 people

52.

Quantity A	**Quantity B**

The number of ways 1st, 2nd, and 3rd place prizes could be awarded to 3 out of 6 contestants	The number of ways 1st, 2nd, 3rd, 4th, and 5th place prizes could be awarded to 5 contestants

53. An inventory of coins contains 100 different coins.

Quantity A	**Quantity B**
The number of possible collections of 56 coins that can be selected where the order of the coins does not matter	The number of possible collections of 44 coins that can be selected where the order of the coins does not matter

54. An office supply store carries an inventory of 1,345 different products, all of which it categorizes as "business use," "personal use," or both. 740 products are categorized as "business use" ONLY and 520 products are categorized as both "business use" and "personal use."

Quantity A	**Quantity B**
The number of products characterized as "personal use"	600

55. How many distinct 4-letter "words" can be made from the name "CHRISTYNA"? (A "word" is any arrangement of 4 letters regardless of whether it can be found in a dictionary.)

(A) 9
(B) 24
(C) 36
(D) 504
(E) 3,024

56. Seiko has a 6-sided number cube with sides labeled 1 through 6. If she rolls the cube twice, what is the probability that the product of the two rolls is less than 36?

(A) 1/6
(B) 1/3
(C) 2/3
(D) 5/6
(E) 35/36

57. There is an 80% chance David will eat a healthy breakfast and a 25% chance that it will rain. If these events are independent, what is the probability that David will eat a healthy breakfast OR that it will rain?

(A) 20%
(B) 80%
(C) 85%
(D) 95%
(E) 105%

58. The probability of rain is 1/2 for every day next week. What is the chance that it rains on at least one day during the workweek (Monday through Friday)?

(A) 1/2
(B) 31/32
(C) 63/64
(D) 127/128

(E) 5/2

59. Eight women and two men are available to serve on a committee. If three people are picked, what is the probability that the committee includes at least one man?

(A) 1/32
(B) 1/4
(C) 2/5
(D) 7/15
(E) 8/15

60. At Lexington High School, everyone takes at least one language — Spanish, French, or Latin — but no one takes all three languages. If 100 students take Spanish, 80 take French, 40 take Latin, and 22 take exactly two languages, how many students are there?

(A) 198
(B) 220
(C) 242
(D) 264
(E) 286

61.

Of 60 birds found in a certain location, 20 are songbirds and 23 are migratory. (It is possible for a songbird to be migratory, or not.)

Quantity A	**Quantity B**
The number of the 60 birds that are neither migratory nor songbirds	16

Probability, combinators & Overlapping Answer Key

Question	Answer	Question	Answer
1	C	40	B
2	B	41	B
3	180	42	C
4	256	43	B
5	250	44	B
6	B	45	D
7	8/19	46	D
8	E	47	C
9	10	48	A
10	720	49	B
11	B	50	A
12	E	51	C
13	A	52	C
14	D	53	C
15	5/6	54	A
16	120	55	E
17	A	56	E
18	A	57	C
19	1/121	58	B
20	B	59	E
21	1/22	60	A
22	B	61	A
23	A		
24	C		
25	16		
26	C		
27	1/3		
28	17		
29	800		
30	1000		
31	C		
32	B		
33	C		
34	D		
35	B		
36	34650		
37	0		
38	C		
39	5/12		