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SHORT-TERM RECOGNITION MEMORY FOR SINGLE DIGITS AND PAIRS OF DIGITS[1]

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The operating characteristic is used to examine the relation between the recognition of a single item and the recognition of a pair of items. 29 Ss listened to a sequence of 5 digits, then copied a sequence of 8 digits, and then were given a test of recognition memory for 1 or 2 digits from the original sequence. The operating characteristic for single digits is a smooth function that is symmetrical about the major diagonal, whereas the curve for pairs is highly asymmetrical. False-recognition rates for test pairs containing 1 digit from the original sequence are only slightly greater than false-recognition rates for completely new pairs. Recognition of a pair does not appear to result from independent recognition of each digit. A mathematical model is developed in which the strength of the memory trace has a continuous distribution which is incremented in a probabilistic fashion upon presentation of an item or pair.

A recognition experiment is a choice experiment, for which powerful analytical tools exist (Bush, Galanter, & Luce, 1963; Swets, Tanner, & Birdsall, 1961). The analytical tools developed in psychophysics can be applied to the results of many substantive areas so long as the data come from experiments that meet certain formal requirements. Experiments in recognition memory, as opposed to experiments in recall, meet these requirements.

In an experiment in recognition memory *S* is asked to judge whether a test item appeared in the original list (and is an *old* item) or whether it is *new*. In a choice experiment such as this, changes in *S*'s response biases can have as much effect on his performance as changes in the stimulus conditions. A conservatively biased *S* might respond "yes" only if he were certain that the test item was old, whereas a liberally biased *S* might always respond yes unless certain that the test item was new.

We consider here a rather general two-stage model of the recognition process in which two systems operate to determine the response: a memory and a decision system. Stimulus items are represented in the memory system by any one of a number of forms—the actual process need not concern us at this time. When the recognition test takes place, the decision system must select a response using, as a basis, the information provided by the memory system. The output of the memory system can be characterized as a reflection of the strength of the test item in the storage system. (At this time it is important to note that even a new test item may have some strength in the memory system.) The decision function maps these strengths onto responses by comparing the strength of the test item with some criterion value. Items that exceed the criterion receive a response of yes, otherwise they are

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assumed to be unfamiliar (new) items and receive a no response. In this scheme, then, false recognitions contain valuable information. By forcing *S* to vary his biases (and thereby his criterion strength) while holding the stimulus conditions constant (and, hopefully, holding constant his memory of the items) we trace out the relative strengths of the distributions of old and new items. The relation between the relative amount of hits and false recognitions in psychophysics is called the receiver-operating characteristic (ROC curve). To emphasize the study of memory rather than signal detection, we call our curves the memory-operating characteristic (MOC curve).

In the binary-choice experiment just described each point must be obtained in what is essentially a separate experiment in which *S*'s biases are manipulated by changing his instructions or payoffs. Fortunately, however, a more economical technique has been developed (Egan, Schulman, & Greenberg, 1959; Pollack & Decker, 1958). After making his binary decision an *S* can indicate his confidence in that decision on a rating scale. We interpret his confidence as a direct reflection of the strength of the item along a unidimensional scale from a "most confident no" to a "most confident yes." This permits us to get several points of the operating characteristic in one experiment.

The present study applies the operating characteristic to examine the relation between the recognition of a single item and the recognition of a pair of items. In particular, we are concerned with the following problems:

1. Is the strength of the memory trace two valued, multivalued, or continuous? Is the number of values different for single items and pairs of items?

2. How does the presentation of an item or pair of items transform its strength in memory? That is to say, what is the relationship between the old and new distributions for single items and for pairs of items? Is the relation different for single items and pairs?

3. How does the nature of the test pair affect the recognition of pairs? In the case of single items there are only two types of test items: new and old. In the case of pairs there are many more possibilities: Test pairs might have both items old and in the original order, both items might be old but in the reversed order, test pairs might have only one item old, or neither item might be old. Does the false-recognition rate for pairs increase with increasing similarity of the test pair to a presented pair? If it does, then it would appear that recognition of a pair results from independent recognition of its components. If it does not, then perhaps what is recognized is the association between the items or a concept associated strongly to the pair, but weakly to each item.

Method

Procedure.—The procedure for each trial was as follows: *Presentation*: A random sequence of five different digits was presented at the rate of ½ sec/digit. The *S*s listened to these digits and attempted to remember them. *Interference*: A random sequence of eight different digits (though not all different from the first set of five digits) was presented at the rate of ½ sec/digit. The *S*s copied these digits as they were presented and covered with a blank card the digits they copied. *Probe*: A single digit or a pair of digits was presented. The *S*s decided whether this digit or pair of digits was in the original sequence of five digits and indicated their confidence in this decision on a 5-point rating scale. The *S*s were given 15 sec. on each trial to indicate their decision and confidence. A "ready" signal was given at the beginning of each trial, and a buzzer was sounded during the 1-sec. intervals between presentation and interference and between interference and probe. A trial required about 25 sec. altogether. The entire experiment was recorded on tape.

When the probe was a single digit, *S*'s decision was yes or no. When the probe was a pair of digits (e.g., 85), *S*s chose one of the following four alternatives: (85) both digits were in the original sequence in the same order as in the probe, (58) both digits were in the original sequence but in the opposite order, (5 or 8) only the one indicated digit was in the original sequence, (no) neither probe digit was in the original sequence. The *S*s were assured that if both digits were "old" (i.e., in the original sequence), then they had been immediately adjacent in the original sequence in one order or the other.

Design.—There were 20 conditions defined by the relation of the probe to the presented sequence. In 5 conditions the probe was a single digit that had occurred in the presented sequence, 1 condition for each serial position in the presented sequence. Since the probe consisted of one "old" digit, we refer to this as Cond. *o*. Condition *n* refers to cases where the probe was a single "new" digit (i.e., the digit did not occur in the original sequence of five digits). Condition *n* occurred as often as Cond. *o*.

In four conditions the probe was a pair of digits that had occurred in the presented sequence as a pair of adjacent digits in the same order as the probe; there is one condition for each serial position for a pair. Since the probe consisted of two old digits in the same order, we refer to this as Cond. *oos*. In four conditions the probe was a pair of old adjacent digits but in the reverse order from their order in the presented sequence: this is Cond. *oor*. In five conditions the probe was a pair of digits, one old and one new: this is Cond. *on*. In one condition the probe was a pair of new digits (Cond. *nn*); this condition occurred as often as Cond. *on*.

There were 18 conditions replicated once each per block of 28 trials and two control conditions (*n* and *nn*) replicated five times each per block. Thus, there were 28 trials per block, and three blocks in the experiment, for a total of 84 trials. Conditions were ordered randomly in each block. All three blocks were different. The *S*s were 29 M.I.T. undergraduates who participated in the experiment to fulfill a requirement of their psychology courses. The 29 *S*s were run in two groups of about equal size. Total time for the experiment was about 50 min.

Data analysis.—When the probe is a single digit (*o* or *n*) *S* must choose 1 of 10 decision-confidence pairs. Let $i = 1, 2, \cdots, 5, 6, \cdots, 10$ stand for the responses running from yes with confidence "5" (greatest confidence), through yes with confidence "1" (least confidence), no with confidence "1," and finally to no with confidence "5." Let $f_i(x)$ represent the total frequency (over all three blocks, all serial positions for that condition, and all 29 *S*s) with which response i occurred in Cond. x. Let $r_i(x) = f_i(x)/\sum_i f_i(x)$ represent the relative frequency with which response i occurred in Cond. x. Let $R_i(x) = \sum_{j=1}^{i} r_j(x)$ represent the cumulative relative frequency with which responses 1 through i occurred in Cond. x.

Using the above definitions it is easy to compute two functions that will be used in the analysis of the data, the MOC curve and the a posteriori probability of a response. The MOC curve is a plot of $R_i(x)$, the "hit rate" for some Experimental Cond. x, against $R_i(y)$, the "false-recognition" rate for some Control Cond. y. Since the MOC curve is a plot of two cumulative-probability functions, one against the other, the curve originates at (0, 0) and ends at (1, 1). The shape and the area under the curve are the principal properties of the MOC curve.

The ratio, $r_i(o)/[r_i(o) + r_i(n)]$, is the a posteriori probability that the probe item was old when the response of *S* was i. If the memory trace has many different degrees of strength over some range of values of i, then the a posteriori probability will be a monotonic decreasing function of i, for values of i in this range. If the memory trace has a more or less constant strength for several values of i, then the a posteriori probability for these values will be constant.

The analysis of single-digit conditions is easier than the analysis of pair conditions because with pairs there are four, instead of two, possible decisions. However, for our purposes it is possible to classify these four decisions into two categories (yes or no). Having done this, the data are in the same form as the data for single digits, and we can use the previously described analytic functions for both pairs and single digits. There are two meaningful ways of classifying these four decisions into two decisions, ordered-pair analysis and unordered-pair anlaysis. To analyze recognition memory of *ordered pairs* we count a decision as yes only if *S* responded that both digits were old and in the *same* order as the probe. This is called "*ordinary scoring.*" In "*reverse scoring,*" a decision is counted as a yes response only if *S reversed* the order of the probe. To analyze recognition memory of *unordered pairs,* we count a decision as yes if *S* responded that both probe digits were old, regardless of the order of the digits in his response.

Theory.—Aside from the purely empirical question of whether pairs are remembered differently from single digits, there are certain theories of the nature of the memory trace that can be evaluated using the data of the present experiment. One theoretical issue with which we are concerned in this paper is whether the memory trace is two valued, multivalued, or continuous. With a continuity model the memory trace may take on all degrees of strength over some range. If we assume that the memory traces for both old and new items have overlapping normal distributions, then we have a "normal-continuity model" formally identical to Thurstone's Law of Comparative Judgments (1927) and the decision aspects of signal-detectability theory in psychophysics. The normal-continuity model predicts MOC curves that are smooth functions with continually changing slope, which when plotted on normal-normal probability paper become straight lines. This model also predicts that the a posteriori probability function will be monotonic decreasing. Similar predictions would be made by any model which assumed that the strength distributions for old and new items were continuous and unimodal. Therefore, one could refer to this model as the "unimodal-continuity model."

The two-valued strength model of memory is formally equivalent to the two-state threshold model in psychophysics (Luce, 1963). The two strengths are "full strength" (above threshold) and "no strength" (below threshold). In the threshold model it is possible for some new items to have full strength. This model predicts MOC curves that consist of two intersecting straight-line segments ending at (0, 0) and (1, 1), where the probability that an old item exceeds the threshold (p) and the probability that a new item exceeds the threshold (q) defines the intersection of the lines at coordinates (q, p). If there are but two states for the memory trace, then *S*s cannot make the fine distinctions required of them by the category judgments, and the a posteriori probability function will consist of two horizontal straight-line segments, one above the threshold having the value $p/(p+q)$ and one below the threshold having the value $(1-p)/[(1-p)+(1-q)]$ (Nachmias & Steinman, 1963).

Results

Comparison of single digits and pairs.—The MOC curve for single digits is shown in Fig. 1A and 1B. The ordinate is the hit rate estimated by the average of all five serial positions of Cond. *o* $[R_i(o)]$; the abscissa is the false-recognition rate estimated from Cond. *n* $[R_i(n)]$. The data are shown on linear coordinates in Fig. 1A; Fig. 1B contains the same data plotted on normal-normal probability coordinates. Figure 1 also shows the two MOC curves of Cond. *oos* averaged over all four serial positions for pairs $[R_i(oos)]$, vs. the Control Cond. *nn* $[R_i(nn)]$, analyzed as both ordered and unordered pairs. In Fig. 1A the smooth curve is the best-fitting prediction of the normal-continuity model for the single digits; the two straight-line functions are the best-fitting predictions of the two-valued strength model to the ordered- and unordered-pair data. The normal-continuity curves were obtained by visually estimating the best-fitting straight lines to the data on normal-normal coordinates (Fig. 1B) and transferring the estimated function to Fig. 1A. The curves for the two-valued strength model were obtained by visually estimating the best-fitting pair of straight lines to the data of Fig. 1A, with the restriction that the lines pass through the coordinates (0, 0) and (1, 1).

The curve for single digits appears to be a smooth function of continually changing slope that is symmetrical about the major diagonal. The fact that the MOC curve for single digits is a straight line with a slope of approximately 1 in Fig. 1B implies that these results could have been

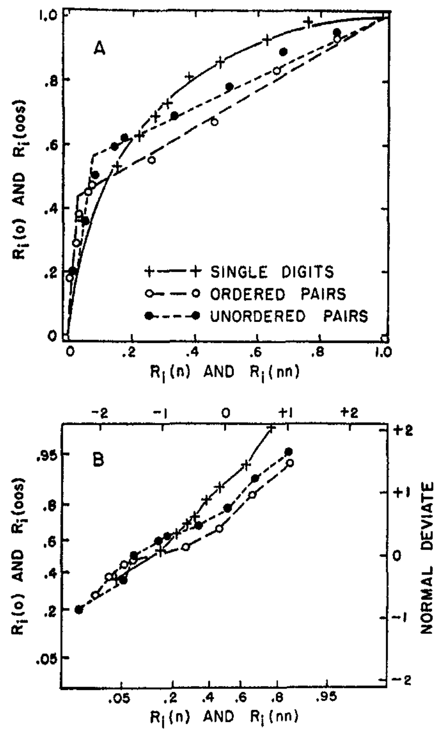

FIG. 1. MOC curves for single digits, ordered pairs, and unordered pairs. (The data are plotted on regular, linear coordinates in A and on normal-normal probability coordinates in B. In A the smooth curve is the best-fitting prediction of the normal-continuity theory to the data for single digits, and the straight-line functions are best-fitting predictions of the two-valued strength model to the data for pairs.)

obtained if the strengths of old and new items had overlapping normal distributions with equal variances.

The curves for pairs are highly asymmetrical and can be closely approximated by the two intersecting straight lines of the two-valued strength model. The data are fit very poorly by normal-continuity curves, even without the restriction that the variances of two underlying distributions be equal. It is difficult to assess visually the fit of the straight lines to the data, particularly above the threshold (the "lower limb" of the function) where the correct-recognition rate is changing very rapidly relative to the false-recognition rate. The a posteriori probability function provides a more sensitive test for the fit of the two-valued strength model.

The a posteriori probabilities for ordered and unordered pairs are shown in Fig. 2A and 2B, respectively. The straight horizontal lines are the theoretical predictions of the two-valued model; and it is clear that, above the threshold at least, they do not describe the data. The data do suggest, however, that although the response categories above threshold do convey information, those below the threshold do not. Thus, even though we are forced to discard the two-valued threshold model, we cannot discard the idea of a threshold.

MOC curves for ordered and unordered pairs, analogous to those of Fig. 1A, can be obtained for Cond.

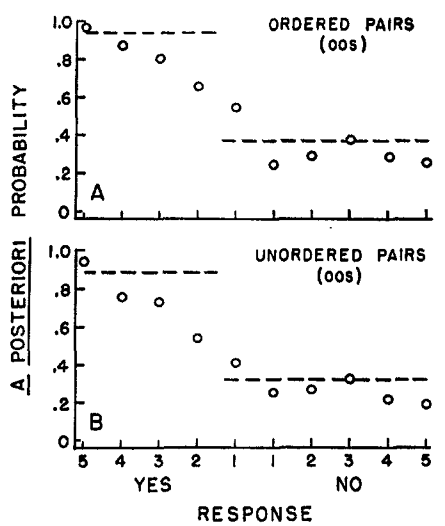

FIG. 2. A posteriori probability functions for ordered and unordered pairs. (The dotted lines are the theoretical predictions of the two-valued strength model.)

oor. The method of analyzing the unordered-pair data of Cond. *oor* is identical to the method of analyzing the unordered-pair data of Cond. *oos*. In the ordered-pair analysis for *oor*, however, a response is scored as yes, if and only if *S* correctly reverses the order of the probe (reverse scoring). Figure 3A compares the MOC curves for the ordered-pair analysis of Cond. *oos* and *oor*. MOC curves for the unordered-pair analysis of Cond. *oos* and *oor* are compared in Fig. 3B. The curves for the two conditions are very similar in shape and location, allowing us to draw the same conclusions about the underlying processes of recognition memory in the two conditions.

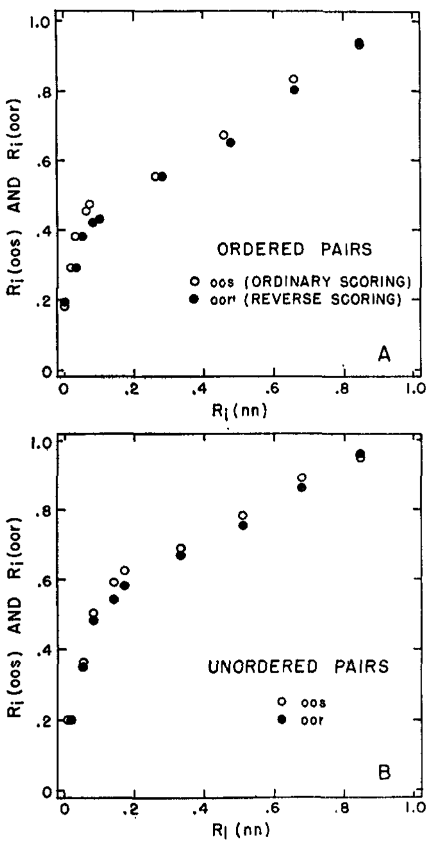

FIG. 3. MOC curves for Cond. *oos* and *oor*.

Correct recognition of pairs, either ordered or unordered, is superior in Cond. *oos*, but the difference between the functions for *oos* and *oor* is very small. It is rather surprising that it makes so little difference for the recognition of pairs whether the probe is in the same or reversed order as the initial presentation.

Degree of oldness and false-recognition rates.—In the preceding section the false-recognition rate for pairs was determined for Cond. *nn* using *ordinary* scoring. Using ordinary scoring in the analysis of ordered-pair recognition, false-recognition rates can also be determined for Cond. *on* and *oor*. Using *reverse* scoring in the analysis of ordered-pair recognition, false-recognition rates can be determined for *on* and *oos*. In the analysis of unordered pairs, the false-recognition rates can also be determined for Cond. *on*.

Using ordinary scoring and ordered-pair analysis, the pair conditions were ranked as follows according to increasing "degree of oldness": *nn*, *on*, *oor*, and *oos*. In Cond. *nn*, neither digit is old; in *on* one of the digits is old; in *oor* both digits are old but in a new order; in *oos* both digits are old and in the old order. Using reverse scoring and ordered-pair analysis the pair conditions were ranked in increasing degree of oldness in the following way: *nn*, *on*, *oos*, and *oor*. The ranking of degree of oldness depends on the oldness of the response that is scored as a yes, not on the oldness of the probe pair. With reverse scoring the inversion between *oor* and *oos* occurs because the probe pair must be reversed to qualify as a yes response.

The lower three curves in Fig. 4A show the cumulative false-recognition rates for each response category using ordinary scoring and ordered-pair

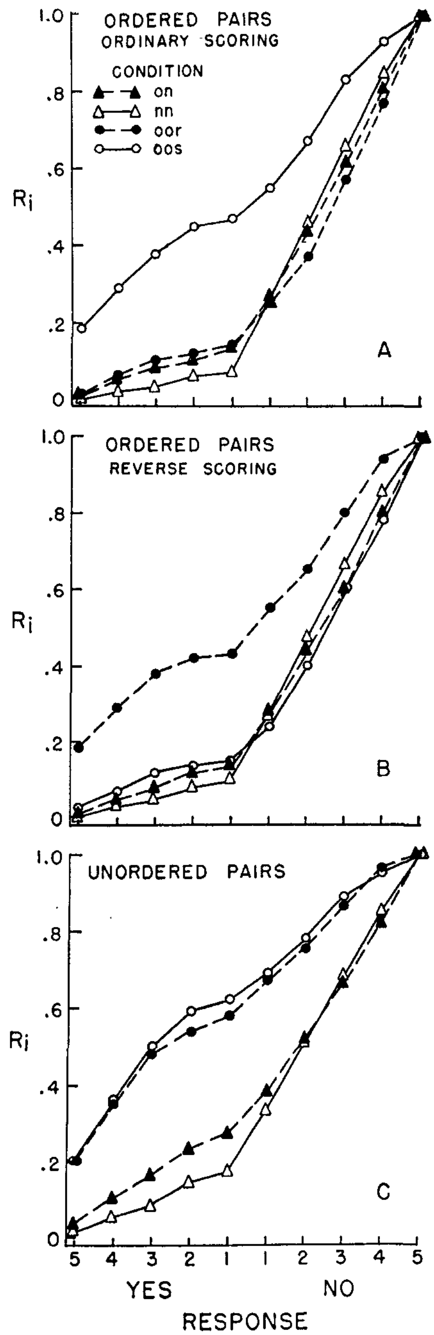

FIG. 4. Recognition rates for different degrees of oldness of the test pair.

analysis. The false-recognition rates are determined for conditions where the yes responses have three different degrees of oldness—namely, *oor*, *on*, and *nn*. If *S*s used partial cues to help in the recognition task we might expect the results to be ordered by degree of oldness; the greater the degree of oldness, the more false recognitions. The results of the ordered-pair analysis show that this effect does indeed occur, although the magnitude of the effect is very small. On the bottom portion of the curves (where *S*s are responding yes with various degrees of confidence), the greater the degree of oldness, the higher the curve. On the upper portions of the curves (where *S*s are responding no with various degrees of confidence) the situation is reversed: the higher the degree of oldness, the lower the curves. These results indicate the degree of oldness does not help much in deciding whether to say yes or no, but *S*s generally are more confident of either a yes or a no decision as the degree of oldness increases.

Figures 4B and 4C present data analogous to that in Fig. 4A. In Fig. 4B we used reverse scoring for ordered-pair analysis. In Fig. 4C we used unordered-pair analysis. The relationship between degree of oldness and false-recognition rate in Fig. 4B and 4C is the same as that in Fig. 4A. No statistical test has been developed to determine if two operating characteristics are significantly different. However, for the curves in Fig. 4A, 4B, and 4C it is possible to use the Kolmogorov-Smirnov two-sample test to determine if the maximum difference between any two cumulative-probability functions is significant. In this case a maximum difference of .10 between two false-recognition curves will be significant at the .05

level. The maximum differences in Fig. 4A, 4B, and 4C are .09, .07, and .10, respectively. Only the latter difference is significant at the .05 level, though the other differences are close to significance.

The upper curve in Fig. 4A and 4B and the upper two curves in Fig. 4C are the correct-recognition rates for ordered-ordinary, ordered-reverse, and unordered analyses, respectively. It is clear that the differences between the correct-recognition curves and any of the false-recognition curves are far greater than any of the differences among the various false-recognition curves. The fact that we did not obtain a gradual change in recognition rate with degree of oldness indicates that *S*s do not remember a pair of digits by the first digit, the second digit, and the order independently. Memory for a pair is clearly not an additive function of the memory for its parts.

Practice and serial-position effects.—To examine the effects of practice, we computed separate MOC curves for each of the three blocks of the experiment. The results show a definite improvement in recognition memory for single digits over the course of the experiment. However, the opposite practice effect is obtained in recognition memory of ordered pairs. There is no obvious interpretation of this difference. We can only suggest that the practice effects may result from the differential operation of proactive inhibition or a change in strategy of *S*s during the experiment. The reliability of the average MOC curve for a condition is underscored by the fact that the MOC curves for a given condition in each of the three blocks of the experiment are similar in shape and differ only in height.

The conventional serial-position effect was obtained for both pairs and single digits, but the effect was much more pronounced for pairs.

Discussion

The findings for recognition memory of pairs of digits are rather different from the corresponding findings for recognition memory of single digits. There is a bigger serial-position effect for pairs than for single digits, the practice effects are opposite in the two cases, and the MOC curves have very different shapes. Logically, the process of recognizing ordered pairs must involve more than the single-digit recognition process applied to each of the two digits, viz., it must involve the recognition of order. However, the recognition of unordered pairs could result from the independent recognition of each digit of the pair. It is possible to test this hypothesis using the MOC curves for single digits and unordered pairs by making some assumptions about the way an *S* combines the recognition of single digits into the recognition of pairs. One reasonable assumption is that *S* assigns to the pair the decision-confidence rating that reflects the strength of the weakest of the two digits. This model is wrong. It predicts an MOC curve for pairs that is similar in shape to the curve for single digits and substantially above it. This means that the predicted curve has a shape and location entirely different from the empirically obtained curve for pairs. Moreover, several other models of the way that *S* might combine single-digit recognition into pair recognition fail in similar ways. The conclusion that pair recognition is not based on single-digit recognition is further supported by the fact that degree of oldness of a pair has so little effect on the false-recognition rate.

Granted that pair recognition is a different process from single-digit recognition, what relation obtains between the recognition of a pair and the single digits that make up the pair? Do pairs containing common digits (94, 49, 91) differ from each other in the same way as pairs with no digits in common (94, 83)? The present findings indicate that they do, almost. The false-recognition rates *for two old items in reverse order*, one old item and one new item, and two new items are almost identical, although the ordering of these conditions is consistent in the direction of higher false-recognition rate for higher degrees of oldness. The consistency of the ordering should not detract from the more important fact that the differences in false-recognition rate are extremely small.

It has already been shown that the normal-continuity model is consistent with the MOC curves for single digits, although other continuous distribution functions for old and new digits might also describe these data. The important result is that the memory trace for a *single digit may have one or many* different degrees of strength rather than only two or three. It is completely consistent with the present findings to assume that the distribution of single-digit trace strengths is a continuous function of a real variable. It must be noted, however, that the present findings are obtained by averaging over all *S*s. If there are substantial individual differences in the trace distributions and the characteristics of the decision systems, then it is quite possible to obtain smooth average MOC curves from individual MOC curves of very different shape and consistent with other models.

There is less need for caution in interpreting the MOC curve for pairs because such a curve is highly unlikely to result from averaging individual curves of very different shape. The straight-line portion of the MOC curve for pairs is inconsistent with the normal-continuity model, no matter what one assumes about individual differences. However, as we shall show later, it is the unimodal-distribution assumption, not the continuous nature of the memory trace that is contraindicated by the data. The a posteriori probability results for pairs are incompatible with the two-valued strength model, but this incompatibility is just the kind that could result from averaging individuals. Hence we must be cautious in deciding that the memory trace has more than two values in either single-digit or pair *recognition, though the group data are* certainly not well described by the two-valued strength model in either case.

Although we cannot conclusively establish whether the memory trace is two valued, multivalued, or continuous for either single digits or pairs, we can reach a definite conclusion about the process of incrementing the strength of digit pairs upon presentation of the pair: Some pairs are not incremented by presentation. With a two-valued or multivalued strength model, this amounts to assuming that presentation increments the strength of a pair with some probability, $\pi < 1$. This probability cannot be unity because in that case there would be no *old pairs in the category with the lowest* strength. The deterministic-increment model predicts that the straight-line portion of the MOC curve would be parallel to the horizontal axis, running through the point (1, 1). This model is clearly wrong. With a continuous-strength model, the MOC curve for pairs clearly requires that the strength distribution for old pairs be bimodal, with one portion of the distribution being directly beneath the distribution for new pairs and proportional to it. The most reasonable way for this to occur is for some pairs not to be incremented in strength by presentation, which is a formal way of stating one consequence of not "attending" to a pair. Thus, whether trace strengths are two valued, multivalued, or continuous, it is clearly necessary to assume that pair strengths are incremented with probability, $\pi < 1$. On the other hand it is perfectly reasonable to conclude from the present data that the trace strength for a single digit is incremented with probability, $\pi = 1$. This result is quite plausible on intuitive grounds if we assume that *S*s code and rehearse the digits in nonoverlapping pairs. This would result in attending to every digit, but only about half of the pairs (since there are five items in a list). This intuitive notion is confirmed by the fact that the best estimate of π for ordered pairs is .42.

Two formal models can describe

the paired-digits results: a multivalue strength model and a continuous-strength model. Both must assume a probabilistic, rather than a deterministic, incrementing process. The multivalue strength model is very similar to the multistate threshold model developed by Norman (1964) for psychophysical-detection experiments and will not be presented here. The probabilistic increment, continuous-strength model has not been stated previously and, for this reason, we now present a brief summary of the properties of such a model.

Assume some initial distribution of strengths (s) for all new items (or pairs of items), $f_n(s)$. We make no assumptions of f_n except that it be a continuous, unimodal probability-density function, where s has a lower bound of 0, so that $f_n(s) = 0,\ s < 0$, and

$$\int_0^\infty f_n(s)ds = 1.$$

When an item is presented, its strength in memory receives some increment with probability π (and no increment with probability $1 - \pi$). Let the stochastic operator T reflect the transformation of new strengths into incremented (old) strengths. The probability distribution of the strengths of old items is then simply

$$f_0(s) = \pi T[f_n(s)] + (1 - \pi)f_n(s).$$

The probability of a response of i or less to an old item is

$$p(i|oo) = \pi \int_{c_i}^\infty T[f_n(s)]ds + (1 - \pi)\int_{c_i}^\infty f_n(s)ds \quad [1]$$

and of a response of i or less to a new item is

$$p(i|nn) = \int_{c_i}^\infty f_n(s)ds, \quad [2]$$

where c_i is the decision cutoff for this criterion.

Solving Equations 1 and 2 for the equation of the MOC curve yields

$$p(i|oo) = (1 - \pi)p(i|nn) + \pi \int_{c_i}^\infty T[f_n(s)]ds. \quad [3]$$

If the increment in strengths governed by the operator T is sufficiently large to preclude much overlapping of the new distribution with the incremented part of the old distribution, then for strengths between zero and the region where the overlap begins, the value of the integral in Equation 3 is unity, so that

$$p(i|oo) = (1 - \pi)p(i|nn) + \pi. \quad [4]$$

Equation 4 describes an MOC curve which is a straight line of slope $1 - \pi$ that passes through point (1, 1). This line describes the linear part of our curves and allows us to estimate π as .42 for ordered pairs.

The model can be made more specific by assuming that, with probability π, a presentation of an item causes its strength

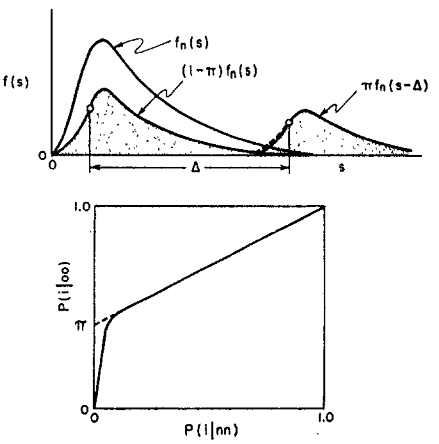

FIG. 5. MOC curve and theoretical distributions of trace strength for the probabilistic-increment, continuous-strength model. (The shaded area represents the old distribution, given by the sum of its two component distributions. An item, or pair, presentation is assumed to increment the strength of the trace in memory by a constant amount, Δ; the parameters are chosen to illustrate the ordered-pair data.)

in memory to be incremented by a constant amount of size Δ. In other words,

$$T[f_n(s)]=\begin{cases} f_n(s-\Delta), & 0 \leqq (s-\Delta), \\ 0, & (s-\Delta)<0. \end{cases} \qquad [5]$$

The model described by Equations 3, 4, and 5 is illustrated in Fig. 5A and 5B with the parameters chosen to illustrate the ordered-pair data.

This model has one important virtue over the multivalued strength model: It has the proper form for both the single- and paired-digit results. If we let π equal unity, the model contains two overlapping continuous distributions of the form required by the single-digit results. Further development and specification of the functions and operator cannot be performed with the limited data of this study.

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