A few Even-Odd questions may have scary-looking expressions. For example, consider this question

P1.1 **If j is a positive integer, is $(j^3-27)^2(j^3+1)^3$ odd?**

Did you feel a bit nervous reading this question? Well, that is the first pitfall that you have to guard against. Because, if you let yourself become nervous, you will:

i) Either leave the question without answering
ii) Or you will panic; panic clouds our ability to think rationally and so, increases our chances of making an error.
   - For example, in your panic, you may scramble to remember and apply the formula for $a^3 + b^3$ on the terms of this expression, and then, realize, much to your dismay, that you've complicated the question even further ☹

So, as you can see, 'getting intimidated by complex expressions' is indeed a dangerous pitfall.

What can you do to avoid this pitfall?

The next time you face such a question and notice your heartbeat increasing, take a deep breath and tell yourself,

Since this is a GMAT or GRE question, it can be simplified elegantly

Example

So, let's think through the question we posed above and see how it can be simplified.

1st Simplification

The given expression is $(j^3-27)^2(j^3+1)^3$

*You're probably familiar with the property that **the power of a number doesn't impact the even-odd nature of the number.***

- *$(Even)^n$, where n is a positive integer = Even*
- *Similarly, $(Odd)^n$ = Odd*

So,

i) $(j^3 – 27)^2$ will have the same even-odd nature as $(j^3 – 27)$. Similarly, $(j^3 + 1)^3$ will have the same even-odd nature as (j^3+1)

ii) j^3 will have the same even-odd nature as j itself.

So, using this property, we've done the first level of simplification: now, we only have to determine the even-odd nature of this, simpler expression: $(j-27)(j + 1)$

2nd Simplification

The simpler expression is a product of 2 terms: $(j – 27)$ and $(j+1)$

When will the product of 2 terms be odd? Only if each of the 2 terms are themselves odd. If even one of these terms is even, the product will be even.

So, to answer the question, we need to know: ***<u>are each of the 2 terms odd</u>****?*

So, from the earlier situation of dealing with the product as a whole, we are now dealing with individual terms only: (j – 27) and (j + 1)

<u>Getting to the answer</u>

Now, j can either be Even or Odd.

<u>Case 1: j is odd</u>

> *In this case, j + 1 = Odd + Odd = Even*
>
> *And j - 27 = Odd - Odd = Even*
>
> *Since both the terms are Even, the answer in this case will be **NO, the given expression in not odd.***

<u>Case 2: j is even</u>

> *In this case, j + 1 = Even + Odd = Odd*
>
> *And, j - 27 = Even - Odd = Odd*
>
> *Since both the terms are odd, the answer in this case will be **YES, the given expression is odd***

So, as you can see, using this step-wise approach, we've been able to simplify the question to this:

> ***Is j even?***

EXPONENTS

The mathematical expression $\mathbf{4^3}$ consists of a base (4) and an exponent (3).

The expression is read as "four to the third power." The base (4) is multiplied by itself as many times as the power requires (3).

Thus $4^3 = 4 \times 4 \times 4 = 64$.

Two exponents have special names: the exponent 2 is called the square, and the exponent 3 is called the cube.

5^2 can be read as five to the second power, or as five squared ($5^2 = 5 \times 5 = 25$).
5^3 can be read as five to the third power, or as five cubed ($5^3 = 5 \times 5 \times 5 = 125$).

Wow, That Increased Exponentially!

Have you ever heard the expression: "Wow, that increased exponentially!"? This phrase captures the essence of exponents. When a number (a positive number greater than 1) increases exponentially, it does not merely increase; it increases a whole lot in a short amount of time.

An important property of exponents is that the greater the exponent, the faster the rate of increase. Consider the following progression:

$5^1 = 5$	
$5^2 = 25$	Increased by 20
$5^3 = 125$	Increased by 100
$5^4 = 625$	Increased by 500

The important thing to remember is that, for positive bases bigger than 1, the greater the exponent, the faster the rate of increase.

All About the Base

THE SIGN OF THE BASE

The base of an exponential expression may be either positive or negative. With a negative base, simply multiply the negative number as many times as the exponent requires.

For example:

$$(-4)^2 = (-4) \times (-4) = 16 \qquad (-4)^3 = (-4) \times (-4) \times (-4) = -64$$

THE EVEN EXPONENT IS DANGEROUS: IT HIDES THE SIGN OF THE BASE!

One of the GMAT's most oft-used tricks involves the even exponent. In most cases, when an integer is raised to a power, the answer keeps the original sign of the base.

Examples:

$3^2 = 9$ (positive base, positive answer)	$(-3)^3 = -27$ (negative base, negative answer)	$3^3 = 27$ (positive base, positive answer)

However, any base raised to an even power will always result in a positive answer.

Examples:

$3^2 = 9$ (positive base, positive answer)	$(-3)^2 = 9$ (negative base, positive answer)	$(-3)^4 = 81$ (negative base, positive answer)

Therefore, when a base is raised to an even exponent, the resulting answer may either keep or change the original sign of the base. Whether $x = 3$ or -3, $x^2 = 9$. This makes even exponents extremely dangerous, especially in the hands of the GMAT test writers.

Consider this problem:

If $x^2 = 16$, is x equal to 4?

Your initial inclination is probably to say yes. However, x may not be 4; it may be -4! Thus, we cannot answer the question without additional information. Only if we are told that x is positive, can we answer affirmatively that x must be 4. Beware whenever you see an even exponent on the test.

Note that odd exponents are harmless, since they always keep the original sign of the base. For example, in the expression $x^3 = 64$, you can be sure that $x = 4$. You know that x is not -4 because $(-4)^3$ would yield -64.

A BASE OF 0 or 1

An exponential expression with a base of 0 always yields 0, regardless of the exponent. An exponential expression with a base of 1 always yields 1, regardless of the exponent.

For example, $0^3 = 0 \times 0 \times 0 = 0$ and $0^4 = 0 \times 0 \times 0 \times 0 = 0$.
Similarly, $1^3 = 1 \times 1 \times 1 = 1$ and $1^4 = 1 \times 1 \times 1 \times 1 = 1$.

Thus, if you are told that $x^6 = x^7 = x^{15}$, you know that x must be either 0 or 1.

Of course, if you are told that $x^6 = x^8 = x^{10}$, x could be 0, 1 *or* -1. (See why even exponents are so dangerous?)

A FRACTIONAL BASE

When the base of an exponential expression is a fraction between 0 and 1, an interesting thing occurs: as the exponent increases, the value of the expression decreases!

$$\left(\frac{3}{4}\right)^1 = \frac{3}{4} \quad \left(\frac{3}{4}\right)^2 = \left(\frac{3}{4}\right) \times \left(\frac{3}{4}\right) = \frac{9}{16} \quad \left(\frac{3}{4}\right)^3 = \left(\frac{3}{4}\right) \times \left(\frac{3}{4}\right) \times \left(\frac{3}{4}\right) = \frac{27}{64}$$

See that $\frac{3}{4} > \frac{9}{16} > \frac{27}{64}$. Powers operate as decreasing mechanisms on positive fractions.

All About the Exponent

THE SIGN OF THE EXPONENT

An exponent is not always positive. What happens if the exponent is negative?

$$5^{-1} = \frac{1}{5^1} = \frac{1}{5} \qquad 4^{-2} = \frac{1}{4^2} = \frac{1}{4 \times 4} = \frac{1}{16} \qquad (-2)^{-3} = \frac{1}{(-2)^3} = -\frac{1}{8}$$

Notice that an expression with a negative exponent yields the reciprocal of that expression with a positive exponent. When you see a negative exponent, think reciprocal!

AN EXPONENT OF 0 or 1

Any base raised to the exponent of 1 keeps the original base. This is obvious and true.

$$3^1 = 3 \qquad 4^1 = 4 \qquad (-6)^1 = -6 \qquad \left(-\frac{1}{2}\right)^1 = -\frac{1}{2}$$

By definition, any base raised to the 0 power yields 1. This is not intuitive but it is true.

$$3^0 = 1 \qquad 4^0 = 1 \qquad (-6)^0 = 1 \qquad \left(-\frac{1}{2}\right)^0 = 1$$

SUCCESSIVE EXPONENTS

A base raised to successive exponents means a base raised to one exponent and then that value raised to another exponent.

For example: $\left(3^2\right)^4$ Here, the base 3 is first squared, and then the result (which is 9) is raised to the fourth power.

However, there is a rule for combining successive exponents that can make the computation much easier and faster. Look at what happens when we write out each step of the previous example without doing any computation:

$$\begin{aligned} \left(3^2\right)^4 &= (3^2)(3^2)(3^2)(3^2) \\ &= (3 \times 3)(3 \times 3)(3 \times 3)(3 \times 3) \\ &= 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 = 3^8 \\ &= (3)^{2\times 4} \end{aligned}$$

Rule: When raising a power to a power, combine exponents by multiplying.

Combining Exponential Expressions

Now that you have the basics down for working with bases and exponents, what about working with more than one exponential expression at a time? If two exponential expressions have a base in common or an exponent in common, you can combine them.

WHEN CAN YOU COMBINE THEM?

(1) You can only combine exponential expressions that are linked by multiplication or division. You can NEVER combine expressions linked by addition or subtraction.

(2) You can combine exponential expressions if they have either a base or an exponent in common.

HOW CAN YOU COMBINE THEM?

If you forget these rules, you can derive them on the test by writing out the exponential expressions, as we did above.

The GMAT will often try to trick you into adding or subtracting exponential expressions that have a base or an exponent in common. Remember that you can only simplify when multiplying or dividing exponential expressions!

	SAME BASE	**SAME EXPONENT**
MULTIPLY	When multiplying expressions with the same base, ADD the exponents first. $(3^2)(3^3) = (3)(3)(3)(3)(3) = 3^5$	When multiplying expressions with the same exponent, MULTIPLY the bases first. $(3^3)(5^3) = (3)(3)(3)(5)(5)(5)$ $= (15)(15)(15) = 15^3$
DIVIDE	When dividing expressions with the same base, SUBTRACT the exponents first. $\frac{3^5}{3^2} = \frac{(3)(3)(3)(3)(3)}{(3)(3)}$ $= (3)(3)(3) = 3^3$	When dividing expressions with the same exponent, DIVIDE the bases first. $\frac{9^3}{3^3} = \frac{(9)(9)(9)}{(3)(3)(3)} = (3)(3)(3) = 3^3$

These expressions CAN'T be simplified:	**These expressions CAN be simplified:**
$7^4 + 7^6$	$(7^4)(7^6)$
$3^4 + 12^4$	$(3^4)(12^4)$
$6^5 - 6^3$	$\frac{6^5}{6^3}$
$12^7 - 3^7$	$\frac{12^7}{3^7}$

Try using the rules outlined above to simplify the expressions in the right column.

EXPONENTS AND THE REAL NUMBER LINE

Raising bases to powers can have surprising effects on the magnitude and/or sign—negative vs. positive—of the base. You need to consider four separate regions of the real-number line:

1. Values greater than 1 (to the right of 1 on the number line)
2. Values less than –1 (to the left of –1 on the number line)
3. Fractional values between 0 and 1
4. Fractional values between –1 and 0

The next table indicates the impact of positive-integer exponent (x) on base (n) for each region.

$n > 1$	n raised to any power: $n^x > 1$ (the greater the exponent, the greater the value of n^x)
$n < -1$	n raised to even power: $n^x > 1$ (the greater the exponent, the greater the value of n^x) n raised to odd power: $n^x < -1$ (the greater the exponent, the lesser the value of n^x)
$0 < n < 1$	n raised to any power: $0 < n^x < 1$ (the greater the exponent, the lesser the value of n^x)
$-1 < n < 0$	n raised to even power: $0 < n^x < 1$ (the greater the exponent, the lesser the value of n^x, approaching 0 on the number line) n raised to odd power: $-1 < n^x < 0$ (the greater the exponent, the greater the value of n^x, approaching 0 on the number line)

The preceding set of rules are simple enough to understand. But when you apply them to a GMAT question, it can be surprisingly easy to confuse yourself, especially if the question is designed to create confusion.

Exponents You Should Know

For the GMAT, memorize the exponential values in the following table. You'll be glad you did, since these are the ones you're most likely to see on the exam.

	Power and Corresponding Value						
Base	2	3	4	5	6	7	8
2	4	8	16	32	64	128	256
3	9	27	81	243			
4	16	64	256				
5	25	125	625				
6	36	216					

Work out each problem.

1. If x and y are not equal to 0, then $x^{12}y^6$ must be

   I. Positive

   II. Negative

   III. An integer

   IV. A mixed fraction

   (A) I only

   (B) II only

   (C) III only

   (D) IV only

   (E) I and III

2. $(x^2y^3)^4 =$

   (A) x^6y^7

   (B) x^8y^{12}

   (C) $x^{12}y^8$

   (D) x^2y

   (E) x^6y^9

3. $\dfrac{x^{16}y^6}{x^4y^2} =$

   (A) $x^{20}y^8$

   (B) x^4y^3

   (C) x^5y^6

   (D) $x^{12}y^3$

   (E) $x^{12}y^4$

4. If $x^4 = 16$ and $y^2 = 36$, then the *maximum* possible value for $x - y$ is

   (A) –20

   (B) 20

   (C) –4

   (D) 6

   (E) 8

5. $p^8 \times q^4 \times p^4 \times q^8 =$

   (A) $p^{12}q^{12}$

   (B) p^4q^4

   (C) $p^{32}q^{32}$

   (D) $p^{64}q^{64}$

   (E) $p^{16}q^{16}$

ANSWER KEY AND EXPLANATIONS

1. A	2. B	3. E	4. E	5. A

1. **The correct answer is (A).** If x and y are not 0, then the even exponents would force x^{12} and y^6 to be positive.

2. **The correct answer is (B).** To raise a power to a power, multiply the exponents. $x^{(2)(4)}y^{(3)(4)} = x^8y^{12}$

3. **The correct answer is (E).** All fractions are implied division. When dividing terms with a common base and different exponents, subtract the exponents. $16 - 4 = 12$ and $6 - 2 = 4$.

4. **The correct answer is (E).** x could be positive 2 or negative 2. y could be positive 6 or negative 6. The four possible values for $x - y$ are as follows:

$$2 - 6 = -4$$
$$2 - (-6) = 8$$
$$-2 - 6 = -8$$
$$-2 - (-6) = 4$$

The maximum value would be 8.

5. **The correct answer is (A).** The multiplication signs do not change the fact that this is the multiplication of terms with a common base and different exponents. Solve this kind of problem by adding the exponents.

$p^{8+4} \times q^{4+8} = p^{12}q^{12}$

Simplifying a Root

Sometimes there are two numbers inside the radical sign. In order to simplify this type of root, it is often helpful to split up the numbers into two roots and then solve. Other times, the opposite is true: there are two roots which you would like to simplify by combining them under one radical sign.

<u>WHEN CAN YOU COMBINE THEM?</u>

You can only combine roots in multiplication and division. You can NEVER combine roots in addition or subtraction.

<u>HOW CAN YOU COMBINE THEM?</u>

When multiplying roots, you can split up a larger product into its separate factors. Creating two radicals and solving each individually before multiplying can save you from long computation. Similarly, you can also combine two roots that are being multiplied together into a single root of the product.

EX. $\sqrt{25 \times 16} = \sqrt{25} \times \sqrt{16} = 5 \times 4 = 20$
$\sqrt{25} \times \sqrt{16} = \sqrt{25 \times 16} = \sqrt{400} = 20$

Dividing roots works the same way. You can split a larger quotient into the dividend and divisor. You can also combine two roots that are being divided into a single root of the quotient.

EX. $\sqrt{144 \div 16} = \sqrt{144} \div \sqrt{16} = 12 \div 4 = 3$
$\sqrt{144} \div \sqrt{16} = \sqrt{144 \div 16} = \sqrt{9} = 3$

The GMAT will often try to trick you into splitting the sum or difference of two numbers inside a radical into two individual roots. Or they will trick you into combining the sum or difference of two roots inside one radical sign. **Remember that you may only split or combine the <u>product</u> or <u>quotient</u> of two roots.**

WRONG: $\sqrt{16 + 9} = \sqrt{16} + \sqrt{9} = 4 + 3 = 7$
CORRECT: $\sqrt{16 + 9} = \sqrt{25} = 5$

Imperfect vs. Perfect Squares

Not all square roots yield an integer. For example: $\sqrt{52}$ does not yield an integer answer because no integer multiplied by itself will yield 52. $\sqrt{52}$ is an example of an imperfect square because it does not yield an integer answer.

$\sqrt{52}$ is between what two integers?

Find the closest square roots that you do know. We know that $\sqrt{49} = 7$ and $\sqrt{64} = 8$. Thus we can estimate that $\sqrt{52}$ is between 7 and 8. A good estimate might be 7.2. (Notice that we estimated the number to be closer to 7 than to 8, because $\sqrt{52}$ is closer to $\sqrt{49}$ than it is to $\sqrt{64}$.)

Simplifying an Imperfect Square

If we do not want to estimate an imperfect square such as $\sqrt{52}$, there is a more accurate method of simplifying it. Simply rewrite $\sqrt{52}$ as a product of primes inside the radical.

$$\sqrt{52} = \sqrt{2 \times 2 \times 13}$$

We can simplify any pairs inside the radical. In this case, there is a pair of 2's. Since $\sqrt{2 \times 2} = \sqrt{4} = 2$, we can rewrite $\sqrt{52}$ as follows:

$$\sqrt{52} = \sqrt{2 \times 2 \times 13} = 2 \times \sqrt{13}$$

This is often written as $2\sqrt{13}$. Let's look at another example:

Find $\sqrt{72}$.

We can rewrite $\sqrt{72}$ as a product of primes: $\sqrt{72} = \sqrt{2 \times 2 \times 2 \times 3 \times 3}$. Since there are a pair of 2's and a pair of 3's inside the radical, we can simplify them. We are left with: $\sqrt{72} = 2 \times 3 \times \sqrt{2} = 6\sqrt{2}$.

Memorize: Squares and Square Roots

You should memorize the following squares and square roots as they appear often on the GMAT.

$1^2 = 1$	$\sqrt{1} = 1$
$1.4^2 \approx 2$	$\sqrt{2} \approx 1.4$
$1.7^2 \approx 3$	$\sqrt{3} \approx 1.7$
$2^2 = 4$	$\sqrt{4} = 2$
$3^2 = 9$	$\sqrt{9} = 3$
$4^2 = 16$	$\sqrt{16} = 4$
$5^2 = 25$	$\sqrt{25} = 5$
$6^2 = 36$	$\sqrt{36} = 6$
$7^2 = 49$	$\sqrt{49} = 7$
$8^2 = 64$	$\sqrt{64} = 8$
$9^2 = 81$	$\sqrt{81} = 9$
$10^2 = 100$	$\sqrt{100} = 10$
$11^2 = 121$	$\sqrt{121} = 11$
$12^2 = 144$	$\sqrt{144} = 12$
$13^2 = 169$	$\sqrt{169} = 13$

Memorize: Cubes and Cube Roots

You should memorize the following cubes and cube roots as they appear often on the GMAT.

$1^3 = 1$	$\sqrt[3]{1} = 1$
$2^3 = 8$	$\sqrt[3]{8} = 2$
$3^3 = 27$	$\sqrt[3]{27} = 3$
$4^3 = 64$	$\sqrt[3]{64} = 4$
$5^3 = 125$	$\sqrt[3]{125} = 5$

ROOTS AND THE REAL NUMBER LINE

As with exponents, the root of a number can bear a surprising relationship to the magnitude and/or sign (negative vs. positive) of the number (another of the test-makers' favorite areas). Here are three rules you should remember:

1. If $n > 1$, then $1 < \sqrt[3]{n} < \sqrt{n} < n$ (the greater the root, the lesser the value). However, if n lies between 0 and 1, then $n < \sqrt{n} < \sqrt[3]{n} < 1$ (the greater the root, the greater the value).

$n = 64$ $1 < \sqrt[3]{64} < \sqrt{64} < 64$ $1 < 4 < 8 < 64$	$n = \frac{1}{64}$ $\frac{1}{64} < \sqrt{\frac{1}{64}} < \sqrt[3]{\frac{1}{64}} < 1$ $\frac{1}{64} < \frac{1}{8} < \frac{1}{4} < 1$

2. Every negative number has exactly one cube root, and that root is a negative number. The same holds true for all other odd-numbered roots of negative numbers.

$\sqrt[3]{-27} = -3$ $(-3)(-3)(-3) = -27$	$\sqrt[5]{-32} = -2$ $(-2)(-2)(-2)(-2)(-2) = -32$

3. Every positive number has only one cube root, and that root is always a positive number. The same holds true for all other odd-numbered roots of positive numbers.

10. $\sqrt{\dfrac{28a^6b^4}{36a^4b^6}} =$

(A) $\dfrac{a}{b}\sqrt{\dfrac{a}{2b}}$

(B) $\dfrac{a}{2b}\sqrt{\dfrac{a}{b}}$

(C) $\dfrac{|a|}{3|b|}\sqrt{7}$

(D) $\dfrac{a^2}{3b^2}\sqrt{2}$

(E) $\dfrac{2a}{3b}$

Divide a^4 and b^4 from the numerator and denominator of the fraction. Also, factor out 4 from 28 and 36. Then, remove perfect squares from under the radical sign:

$$\sqrt{\frac{28a^6b^4}{36a^4b^6}} = \sqrt{\frac{7a^2}{9b^2}} = \frac{|a|\sqrt{7}}{3|b|}, \text{ or } \frac{|a|}{3|b|}\sqrt{7}$$

The correct answer is (C).

In GMAT questions involving radical terms, you might want to remove a radical term from a fraction's denominator to match the correct answer. To accomplish this, multiply both numerator and denominator by the radical value. (This process is called "rationalizing the denominator.") Here's an example of how to do it:

$$\frac{3}{\sqrt{15}} = \frac{3\sqrt{15}}{\sqrt{15}\sqrt{15}} = \frac{3\sqrt{15}}{15} \text{ or } \frac{1}{5}\sqrt{15}$$

11. $\sqrt{24} - \sqrt{16} - \sqrt{6} =$

(A) $\sqrt{6} - 4$

(B) $4 - 2\sqrt{2}$

(C) 2

(D) $\sqrt{6}$

(E) $2\sqrt{2}$

Although the numbers under the three radicals combine to equal 2, you cannot combine terms this way. Instead, simplify the first two terms, then combine the first and third terms:

$$\sqrt{24} - \sqrt{16} - \sqrt{6} = 2\sqrt{6} - 4 - \sqrt{6} = \sqrt{6} - 4$$

The correct answer is (A).

Multiplication and Division: Terms under different radicals can be combined under a common radical if one term is multiplied or divided by the other, but only if the radical is the same.

$$\sqrt{x}\sqrt{x} = (\sqrt{x})^2, \text{ or } x$$

$$\sqrt{x}\sqrt{y} = \sqrt{xy}$$

$$\frac{\sqrt{x}}{\sqrt{y}} = \sqrt{\frac{x}{y}}$$

$$\sqrt[3]{x}\sqrt{x} = ?$$

You cannot easily combine $\sqrt[3]{x}\sqrt{x} = x^{\frac{1}{3}}x^{\frac{1}{2}} = x^{\frac{1}{3}+\frac{1}{2}} = x^{\frac{5}{6}}$.

You cannot easily combine $\sqrt[3]{x}\sqrt{x} = x^{\frac{1}{3}}x^{\frac{1}{2}} = x^{\frac{1}{3}+\frac{1}{2}} = x^{\frac{5}{6}}$.

12. $\left(2\sqrt{2a}\right)^2 =$

- **(A)** $4a$
- **(B)** $4a^2$
- **(C)** $8a$
- **(D)** $8a^2$
- **(E)** $6a$

Square each of the two terms, 2 and $\sqrt{2a}$, separately. Then combine their squares by multiplication: $\left(2\sqrt{2a}\right)^2 = 2^2 \times \left(\sqrt{2a}\right)^2 = 4 \times 2a = 8a$. **The correct answer is (C).**

13. Which of the following inequalities, if true, is sufficient alone to show that $\sqrt[3]{x} < \sqrt[5]{x}$?

- **(A)** $-1 < x < 0$
- **(B)** $x > 1$
- **(C)** $|x| < -1$
- **(D)** $|x| > 1$
- **(E)** $x < -1$

If $x < -1$, then applying a greater root yields a lesser negative value—farther to the left on the real number line. **The correct answer is (E).**

Work out each problem.

1. What is the sum of $\sqrt{12} + \sqrt{27}$?
   (A) $\sqrt{29}$
   (B) $3\sqrt{5}$
   (C) $13\sqrt{3}$
   (D) $5\sqrt{3}$
   (E) $7\sqrt{3}$

2. What is the difference between $\sqrt{150}$ and $\sqrt{54}$?
   (A) $2\sqrt{6}$
   (B) $16\sqrt{6}$
   (C) $\sqrt{96}$
   (D) $6\sqrt{2}$
   (E) $8\sqrt{6}$

3. What is the product of $\sqrt{18x}$ and $\sqrt{2x}$?
   (A) $6x^2$
   (B) $6x$
   (C) $36x$
   (D) $36x^2$
   (E) $6\sqrt{x}$

4. If $\frac{1}{x} = \sqrt{.25}$, what does x equal?
   (A) 2
   (B) .5
   (C) .2
   (D) 20
   (E) 5

5. If $n = 3.14$, find n^3 to the nearest hundredth.
   (A) 3.10
   (B) 30.96
   (C) 309.59
   (D) 3095.91
   (E) 30959.14

6. The square root of 24336 is exactly
   (A) 152
   (B) 153
   (C) 155
   (D) 156
   (E) 158

7. The square root of 306.25 is exactly
   (A) .175
   (B) 1.75
   (C) 17.5
   (D) 175
   (E) 1750

8. Divide $6\sqrt{45}$ by $3\sqrt{5}$.
   (A) 9
   (B) 4
   (C) 54
   (D) 15
   (E) 6

9. $\sqrt{\frac{y^2}{25}+\frac{y^2}{16}} =$
   (A) $\frac{2|y|}{9}$
   (B) $\frac{9|y|}{20}$
   (C) $\frac{|y|}{9}$
   (D) $\frac{|y|\sqrt{41}}{20}$
   (E) $\frac{41|y|}{20}$

10. $\sqrt{a^2+b^2}$ is equal to
   (A) $a + b$
   (B) $a - b$
   (C) $(a + b)(a - b)$
   (D) $\sqrt{a^2} + \sqrt{b^2}$
   (E) None of these

ANSWER KEY AND EXPLANATIONS

1. D	3. B	5. B	7. C	9. D
2. A	4. A	6. D	8. E	10. E

1. **The correct answer is (D).**

$\sqrt{12} = \sqrt{4}\ \sqrt{3} = 2\sqrt{3}$

$\sqrt{27} = \sqrt{9}\ \sqrt{3} = 3\sqrt{3}$

$2\sqrt{3} + 3\sqrt{3} = 5\sqrt{3}$

2. **The correct answer is (A).**

$\sqrt{150} = \sqrt{25}\ \sqrt{6} = 5\sqrt{6}$

$\sqrt{54} = \sqrt{9}\ \sqrt{6} = 3\sqrt{6}$

$5\sqrt{6} - 3\sqrt{6} = 2\sqrt{6}$

3. **The correct answer is (B).**

$\sqrt{18x} \cdot \sqrt{2x} = \sqrt{36x^2} = 6x$

4. **The correct answer is (A).**

$$\sqrt{.25} = .5$$
$$\frac{1}{x} = .5$$
$$1 = .5x$$
$$10 = 5x$$
$$2 = x$$

5. **The correct answer is (B).** $(3)^3$ would be 27, so the answer should be a little larger than 27.

6. **The correct answer is (D).** The only answer that will end in 6 when squared is (D).

7. **The correct answer is (C).** The square root of this number must have two digits before the decimal point.

8. **The correct answer is (E).**

$$\frac{6\sqrt{45}}{3\sqrt{5}} = 2\sqrt{9} = 2 \cdot 3 = 6$$

9. **The correct answer is (D).**

$$\sqrt{\frac{y^2}{25} + \frac{y^2}{16}} = \sqrt{\frac{16y^2 + 25y^2}{400}}$$
$$= \sqrt{\frac{41y^2}{400}} = \frac{y\sqrt{41}}{20}$$

10. **The correct answer is (E).** Never take the square root of a sum separately. There is no way to simplify $\sqrt{a^2 + b^2}$.

Standard Deviation

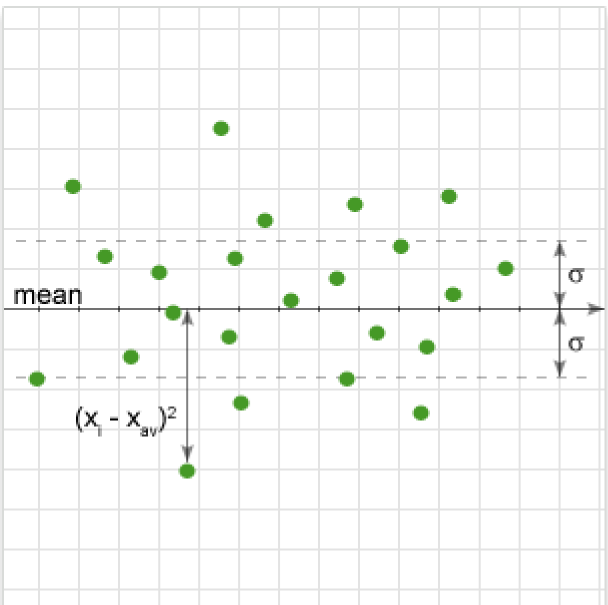
Definition

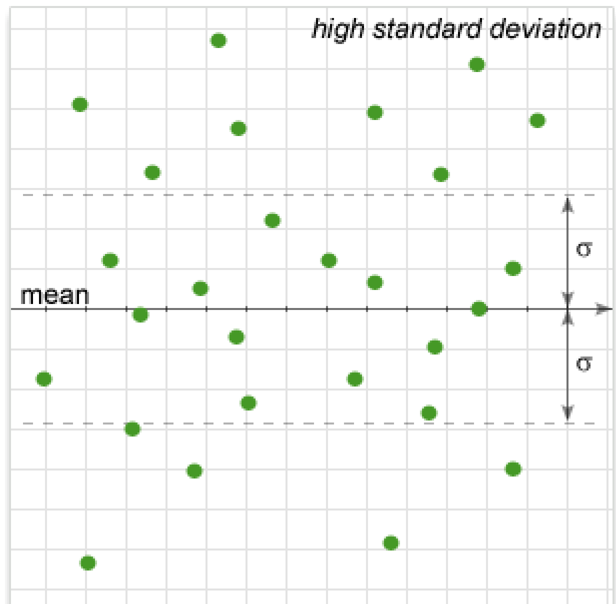
Standard Deviation (SD, or STD or σ) - a measure of the dispersion or variation in a distribution, equal to the square root of variance or the arithmetic mean (average) of squares of deviations from the arithmetic mean.

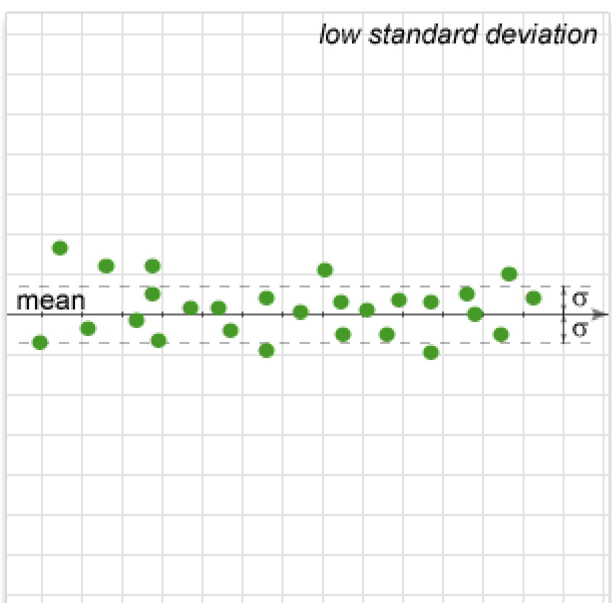
$$variance = \frac{\sum (x_i - x_{av})^2}{N}$$

$$\sigma = \sqrt{variance} = \sqrt{\frac{\sum (x_i - x_{av})^2}{N}}$$

In simple terms, it shows how much variation there is from the "average" (mean). It may be thought of as the average difference from the mean of distribution, how far data points are away from the mean. A low standard deviation indicates that data points tend to be very close to the mean, whereas high standard deviation indicates that the data are spread out over a large range of values.







Properties

$\sigma \geq 0$;

$\sigma = 0$ only if all elements in a set is equal;

Let standard deviation of $\{x_i\}$ be σ and mean of the set be μ:

Standard deviation of $\left\{\frac{x_i}{a}\right\}$ is $\sigma' = \frac{\sigma}{a}$. Decrease/increase in all elements of a set by a constant percentage will decrease/increase standard deviation of the set by the same percentage.

Standard deviation of $\{x_i + a\}$ is $\sigma' = \sigma$. Decrease/increase in all elements of a set by a constant value DOES NOT decrease/increase standard deviation of the set.

if a new element y is added to $\{x_i\}$ set and standard deviation of a new set $\{\{x_i\}, y\}$ is σ', then:

1) $\sigma' > \sigma$ if $|y - \mu| > \sigma$

2) $\sigma' = \sigma$ if $|y - \mu| = \sigma$

3) $\sigma' < \sigma$ if $|y - \mu| < \sigma$

4) σ' is the lowest if $y = \mu$

Tips and Tricks

GMAC in majority of problems doesn't ask you to calculate standard deviation. Instead it tests your intuitive understanding of the concept. In 90% cases it is a faster way to use just average of $|x_i - x_{av}|$ instead of true formula for standard deviation, and treat standard deviation as "average difference between elements and mean". Therefore, before trying to calculate standard deviation, maybe you can solve a problem much faster by using just your intuition.

Advance tip. Not all points contribute equally to standard deviation. Taking into account that standard deviation uses sum of squares of deviations from mean, the most remote points will essentially contribute to standard deviation. For example, we have a set A that has a mean of 5. The point 10 gives $(10-5)^2 = 25$ in sum of squares but point 6 gives only $(6-5)^2 = 1$. 25 times the difference! So, when you need to find what set has the largest standard deviation, always look for set with the largest range because remote points have a very significant contribution to standard deviation.

Example #1

Q: There is a set $\{67,32,76,35,101,45,24,37\}$. If we create a new set that consists of all elements of the initial set but decreased by 17%, what is the change in standard deviation?

Example #2
Q: There is a set of consecutive even integers. What is the standard deviation of the set?
(1) There are 39 elements in the set.
(2) the mean of the set is 382.

Example #3

Q: Standard deviation of set $\{23,31,76,45,16,55,54,36\}$ is 18.3. How many elements are 1 standard deviation above the mean?

Example #4
Q: There is a set A of 19 integers with mean 4 and standard deviation of 3. Now we form a new set B by adding 2 more elements to the set A. What two elements will decrease the standard deviation the most?

Homework

Exponent & Roots

Appendix A: D17, 9, 29, 43, 49, 56, 57, 137, 233

Appendix B: 34, 45, 64, 81, 97, 117, 145, 147, 149, 152, 163, 170