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A DECISION-MAKING THEORY OF VISUAL DETECTION [1]

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This paper is concerned with the human observer's behavior in detecting light signals in a uniform light background. Detection of these signals depends on information transmitted to cortical centers by way of the visual pathways. An analysis is made of the form of this information, and the types of decisions which can be based on information of this form. Based on this analysis, the expected form of data collected in "yes-no" and "forced-choice" psychophysical experiments is defined, and experiments demonstrating the internal consistency of the theory are presented.

As the theory at first glance appears to be inconsistent with the large quantity of existing data on this subject, it is wise to review the form of these data. The general procedure is to hold signal size, duration, and certain other physical parameters constant, and to observe the way in which the frequency of detection varies as a function of intensity of the light signal. The way in which data of this form are handled implies certain underlying theoretical viewpoints.

In Fig. 1 the dotted lines represent the form of the results of hypothetical experiments. Consider first a single dotted line. Any point on the line might represent an experimentally determined point. This point is corrected for chance by application of the usual formula:

$$p = \frac{p' - c}{1 - c}, \qquad [1]$$

where p' is the observed proportion of positive responses, p is the corrected proportion of positive responses, and c is the intercept of the dotted curve at $\Delta I = 0$.

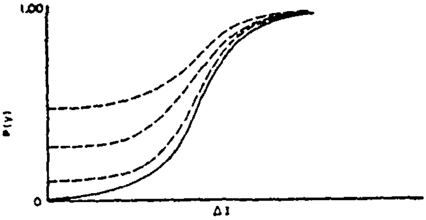

FIG. 1. Conventional seeing frequency or betting curve

Justification of this correction depends on the validity of the assumption that a "false alarm" is a guess, independent of any sensory activity upon which a decision might be based. For this to be the case it is necessary to have a mechanism which triggers when seeing occurs and which becomes incapable of discriminating between quantities of neural activity when seeing does not occur. Only under such a system would a guess be equally likely in the absence of seeing for all values of signal intensity. The application of the chance correction to data from both yes-no and forced-choice experiments is consistent with these assumptions.

The solid curve represents a "true" curve onto which each of the dotted, or experimental, curves can be mapped by using the chance correction and proper estimates of "c." The parameters of the solid curve are assumed to be characteristic of the physiology of the individual's sensory system, independent of psychological control. The assumption carries with it the

[1] This paper is based on work done for the U. S Army Signal Corps under Contract No. DA-36-039 sc-15358. The experiments reported herein were conducted in the Vision Research Laboratories of the University of Michigan.

notion that if some threshold of neural activity is exceeded, phenomenal seeing results.

To infer that the form of the curve representing the frequency of seeing as a function of light intensity is the same as the curve representing the frequency of seeing as a function of neural activity is to assume a linear relationship between neural activity and light intensity. Efforts to fit seeing frequency curves by normal probability functions suggest a predisposition toward accepting this assumption.

A New Theory of Visual Detection

The theory presented in this paper differs from conventional thinking about these assumptions. First, it is assumed that false-alarm rate and correct detection vary together. Secondly, neural activity is assumed to be a monotonically increasing function of light intensity, not necessarily linear. A more specific statement than this is left for experimental determination.

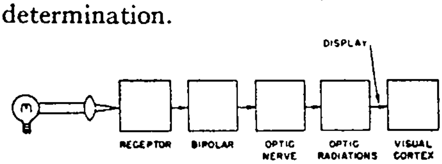

FIG. 2. Block diagram of the visual channel

Figure 2 is a block diagram of the visual pathways showing the major stages of transmission of visual information. All the stages prior to that labelled "cortex" are assumed to function only in the transmission of information, presenting to the cortex a representation of the environment. The function of interpreting this information is left to mechanisms at the cortical level.

In this simplified presentation, the displayed information consists of neural impulse activity. In the case under consideration, in which a signal is presented at a specified time in a known spatial location, the same restrictions are assumed to exist for the display. Thus, if the observer is asked to state whether a signal exists in location A at time B, he is assumed to consider only that information in the neural display which refers to location A at time B.

A judgment on the existence of a signal is presumably based on a

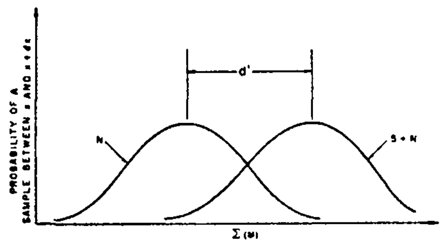

FIG. 3. Hypothetical distributions of noise and signal plus noise

measure of neural activity. There exists a statistical relationship between the measure and signal intensity. That is, the more intense the signal, the greater is the average of the measures resulting. Thus, for any signal there is a universe distribution which is in fact a sampling distribution. It includes all measures which might result if the signal were repeated and measured an infinite number of times. The mean of this universe distribution is associated with the intensity level of the signal. The variance may be associated with other parameters of the signal such as duration or size, but this is beyond the scope of this paper.

Figure 3 shows two probability distributions: N represents the case where noise alone is sampled—that is, no signal exists—and $S + N$, the case where signal plus noise exists. The mean of N depends on background intensity; the mean of $S + N$ on background plus signal intensity. The variance of N depends on signal parameters, not background parameters in the case considered here; that is, where the observer knows a priori that if a signal exists then it is a particular signal. From the way the diagram is conceptualized, the greater the measure, $\Sigma(M)$, the more likely it is that this sample represents a signal. But one can never be sure. Thus, if an observer is asked if a signal exists, he is assumed to base his judgment on the quantity of neural activity. He makes an observation, and then attempts to decide whether this observation is more representative of N or of $S + N$. His task is, in fact, the task of testing a statistical hypothesis.

The ideal behavior, that which makes optimum use of the information available in this task, is defined mathematically by Peterson and Birdsall (2). The mathematics and symbols used are theirs, unless otherwise stated. The first case considered is the yes-no psychophysical experiment in which a signal is presented at a known location during a well-defined interval in time. This corresponds to Peterson and Birdsall's case of the signal known exactly.

For mathematical convenience, it is assumed that the distributions shown in Fig. 3 are Gaussian, with variance equal for N and all values of $S + N$. Experimental results suggest that equal variance is not a true assumption, but that the deviations are not great enough to justify the inconvenience of a more precise assumption for the purpose of this analysis.

It is also assumed that there is a cutoff point such that any measure of neural activity which exceeds that cutoff is in the criterion; that is, any value exceeding cutoff is accepted as representing the existence of a signal,

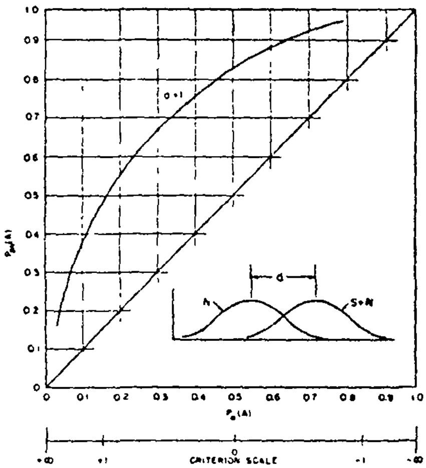

FIG. 4. $P_{SN}(A)$ vs. $P_N(A)$. The criterion scale shows the corresponding criteria expressed in terms of σ_N from M_N.

and any value less than the cutoff represents noise alone. Again, for mathematical convenience, the cutoff point is assumed to be well defined and stable. The justification for accepting this convenience is twofold: first, such behavior is statistically optimum, and second, if absolute stability is physically impossible, any lack of definition or random instability throughout an experiment has the same effect mathematically as additional variance in the sampling distributions.

Now, consider the way in which the placing of the cutoff affects behavior in the case of a given signal. In the lower right-hand corner of Fig. 4 the distributions N and $S + N$ are reproduced for a value of $d' = 1$. The parameter d' is the square root of Peterson and Birdsall's d. The square root of d is more convenient here; d' is the difference between the means of N and $S + N$ in terms of the standard deviation of N. The criterion scale is also calibrated in terms of the standard deviation of N. On the abscissa there is $P_N(A)$, the probability that, if no signal exists, the measure will be in the criterion, and on the ordinate, $P_{SN}(A)$, the probability that if a signal exists, the measure will be in the criterion.

If the cutoff is at $-\infty$, all measures are in the criterion: $P_N(A) = P_{SN}(A) = 1$. At -1 standard deviation, $P_N(A) = .84$, and $P_{SN}(A) = .98$. At 0, $P_N(A) = .5$ and $P_{SN}(A) = .84$. At $+1$, $P_N(A) = .16$ and $P_{SN}(A) = .5$; and for $+\infty$ $P_N(A) = P_{SN}(A) = 0$. Thus, for $d' = 1$ this is the curve showing possible detections for each false-alarm rate. The curve represents the best that can be done with the information available, and the mirror image is the curve of worst possible behaviors.

The maximum behavior in any given experiment is a point on this curve at which the slope is β where

$$\beta = \frac{1-P(SN)}{P(SN)} \frac{(V_{N\cdot CA}+K_{N\cdot A})}{(V_{SN\cdot A}+K_{SN\cdot CA})}. \quad [2]$$

$P(SN)$ is the a priori probability that the signal exists, $V_{N\cdot CA}$ is the value of a correct rejection, $K_{N\cdot A}$ the cost of a false alarm, $V_{SN\cdot A}$ the value of a correct detection, and $K_{SN\cdot CA}$ is the cost of a miss. Thus, as $P(SN)$ or $V_{SN\cdot A}$ increases, or $K_{N\cdot A}$ decreases, β becomes smaller, and it is worth while to accept a higher false-alarm rate in the interest of achieving a greater percentage of correct decisions.

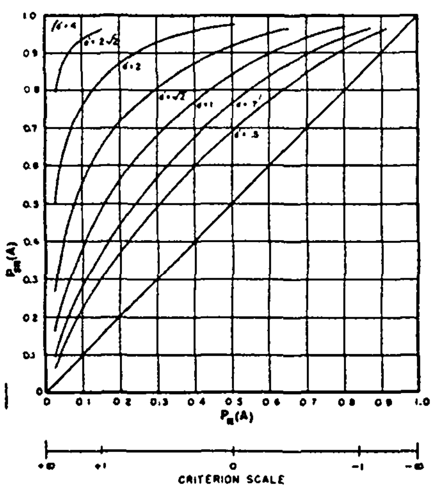

FIG. 5. $P_{SN}(A)$ vs. $P_N(A)$

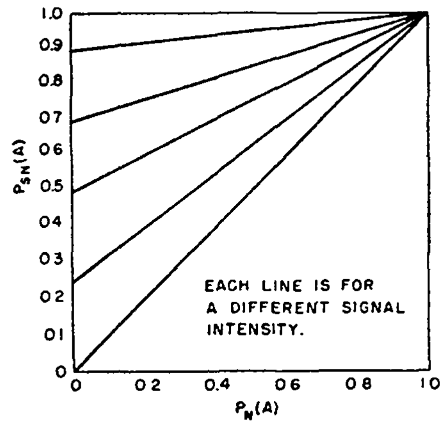

FIG. 6. $P_{SN}(A)$ vs. $P_N(A)$ as a function of d' assuming the guessing hypothesis

Figure 5 shows a family of curves of $P_{SN}(A)$ vs. $P_N(A)$ with d' as a parameter. For values of d' greater than 4, detection is very good. This is to be compared with the predictions of the conventional theory shown in Fig. 6 with $P_N(A)$ assumed to represent guesses. For each value of d' it is assumed that there is a true value of $P_{SN}(A)$ either for $P_N(A) = 0$ or for some very small value. The chance correction should transform each of these to horizontal lines.

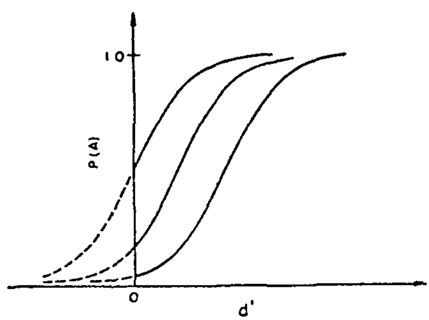

FIG. 7. $P(A)$ as a function of d' assuming the theory

Another way of comparing the predictions of this theory with those of conventional theory is to construct the so-called betting curves, or curves showing the predicted shape of the psychophysical function. These are shown in Fig. 7, where $P(A)$, the probability of acceptance, is plotted as a function of d'. These curves will not map onto the same curve by the application of the chance correction. The shift is horizontal rather than vertical. The dotted portions of the curve show that we are dealing with only a part of the curve, and thus, in the terms of this theory, it is improper to apply a normalizing procedure such as the chance-correction formula to that part of the curve.

In the forced-choice psychophysical experiment, maximum behavior is defined in a different way. In the general forced-choice experiment, the observer knows that the signal will occur in one of n intervals, and he is forced to choose in which of these intervals it occurs. The information upon which his decision is based is contained in the same display as in the case of the yes-no experiment, and, presumably, the values of d' for any given light intensity must be the same. While the solution of this problem is not contained in their study, Peterson and Birdsall have

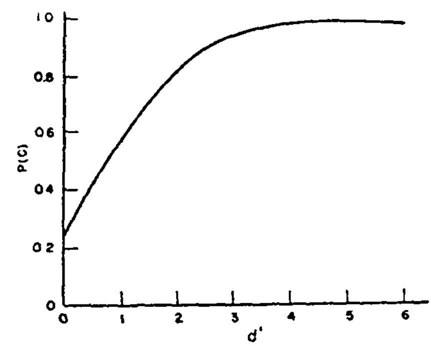

FIG. 8. $P(C)$ as a function of d'. A theoretical curve.

assisted greatly in determining this solution. The probability that a correct answer $P(C)$ will result for a given value of d' is the probability that one sample from the $S + N$ distribution is greater than the greatest of $n - 1$ samples from the distribution of noise alone. The case in which four intervals are used is the basis for Fig. 8. This figure shows the probability of one sample from $S + N$ being greater than the greatest of three from N. For a given value of d' this is

$$P(C) = \int_{x=-\infty}^{+\infty} F(x)^3 g(x)dx, \qquad [3]$$

where $F(x)$ is the area of N and $g(x)$ is the ordinate of $S + N$. In Fig. 8 $P(C)$, as determined by this integration, is plotted as a function of d' for the equal-variance case.

CRITERION OF INTERNAL CONSISTENCY

These two sets of predictions are for the standard experimental situations. They are based on the same neurological parameters. Thus, if the parameters, that is, d's, are estimated from one of the experiments, these estimates should furnish a basis for predicting the data for the other experiment if the theory is internally consistent. An equivalent criterion of internal consistency is that both experiments yield the same estimates of d'.

EXPERIMENTAL DESIGN

Experiments were conducted to test this internal consistency, using three Michigan sophomores as observers. All the experiments employed a circular target 30 minutes in diameter, 1/100 second in duration, on a 10-foot-lambert background. Details of the experimental procedure and the laboratory have been published by Blackwell, Pritchard, and Ohmart (1).

The observers were trained in the temporal forced-choice experiment. The signal appeared in a known location at one of four specified times, and the observers were forced to choose the time at which they thought the signal occurred Five light intensity increments were used here, with 50 observations per point per experimental session. The last two of these sessions were the test sessions, so that each forced-choice point in the analysis is based on 100 experimental observations.

Following the forced-choice experiments, there was a series of yes-no experiments under the same experimental conditions, except that only four light intensity increments were used. These were the same as the four greatest intensities used in the forced-choice experiments, reduced by adding a .1 fixed filter. In the first four of these sessions, two values of a priori probabilities, $P(SN)$ equal to .8 and .4, were used. The observers were informed of the value of $P(SN)$ before each experimental session. No values or costs were incorporated in these four sessions, which were thus excluded from the analysis as practice sessions

The test experiments consisted of 12 sessions in each of which all of the information necessary for the calculation of a β (the best best possible decision level) was furnished the observers. While they did not know the formal calculation of β, that they knew the direction of cutoff change indicated by a change in any of these factors was suggested by the fact that the obtained values of $P_N(A)$ varied approximately with changes in the information given them The values and costs were made real to the observers, for they were actually paid in cash. It was possible for them to earn as much as two dollars extra in a single experimental session as a result of this payment.

The first four sessions each carried the same value of β as $P(SN) = .8$ and the same payment was maintained. A high value of $P_N(A)$, or false-alarm rate, resulted. In the next four sessions with $P(SN)$ held at .8, $K_{N \cdot A}$ and $V_{N \cdot CA}$ were gradually increased from session to session (not within sessions) until $P_N(A)$ dropped to a low value. Then $P(SN)$ was dropped to .4, and K_{NA} and $V_{N \cdot CA}$ were reduced so that for the thirteenth session $P_N(A)$ stayed low. The last three sessions successively involved increases in $V_{S \cdot NA}$ and $K_{SN \cdot CA}$, again forcing $P_N(A)$ toward a higher value.

RESULTS

Figures 9 and 10 show scatter diagrams of $P_{SN}(A)$ vs. $P_N(A)$ for a particular intensity of signal and for a single observer. These scatter diagrams can be used to estimate d'. In Fig. 9 the estimate of d' is .7. In Fig. 10, the estimate of d' is 1.3. Each d' estimated in this way is based on 560 observations. A procedure similar to this was used for the d's for each of four signals for each of the four observers.

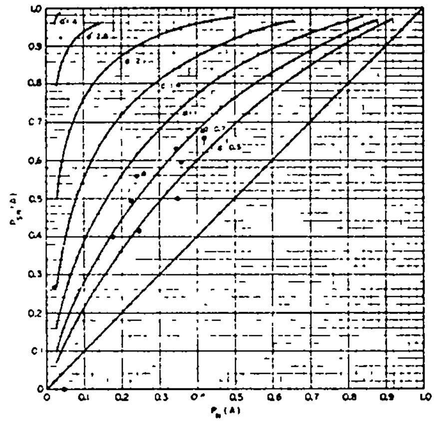

FIG. 9. A scatter diagram of $P_{SN}(A)$ vs. $P_N(A)$

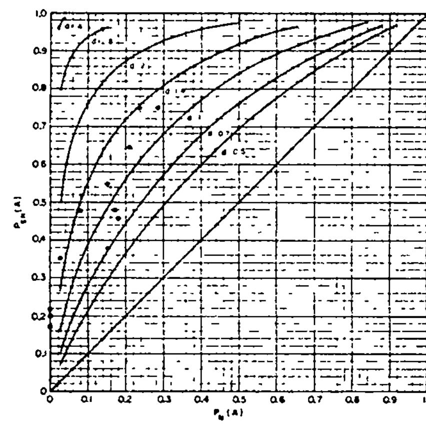

FIG 10. A scatter diagram of $P_{SN}(A)$ vs. $P_N(A)$

In the forced-choice experiment the estimates of d' are made by entering our forced-choice curve (Fig. 8), using the observed percentage correct as an estimate of $P(C)$. Figure 11 shows log d' as a function of log signal in-

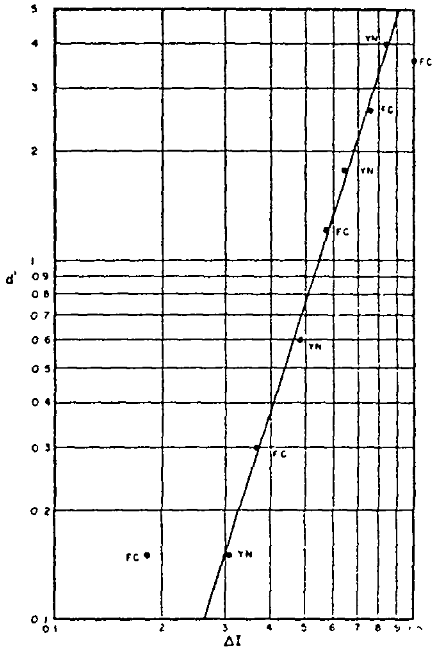

FIG. 11. Log d' vs. Log ΔI for Observer 1

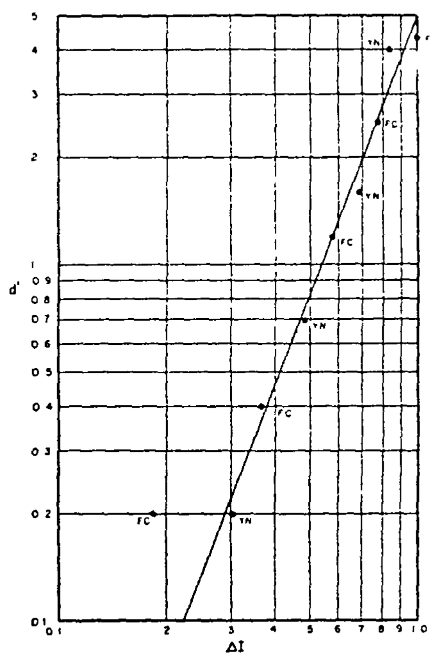

FIG. 12. Log d' vs. Log ΔI for Observer 2

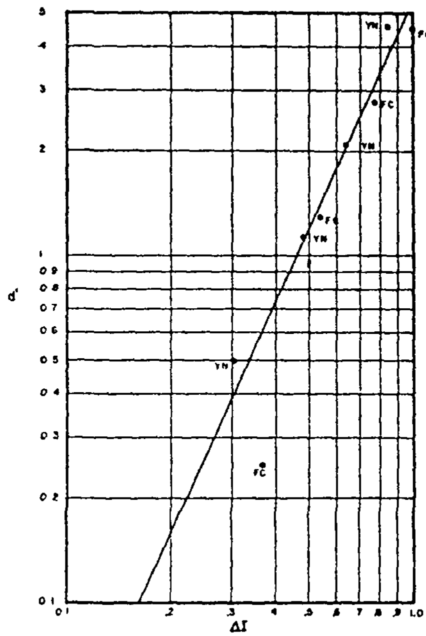

FIG. 13. Log d' vs Log ΔI for Observer 3

tensity for the first observer, the estimates of d' being from both forced-choice and yes-no experiments. In general the agreement is good. The deviation of the forced-choice point at the top can be explained on the basis of inadequate experimental data for the determination of the high probability involved. The deviation of the low point is unexplained. Figure 12 is the same plot for the second observer, showing about the same picture. Figure 13 is for the third observer, showing not quite as good a fit, but nevertheless satisfactory for psychological experiments. For this observer, the lowest point for forced choice is off the graph to the right of the line.

Figures 14, 15, and 16 show the predictions for forced-choice data (when yes-no data are used to estimate d') for the three observers. Note that the lowest point is on the curve in both of the first two cases,

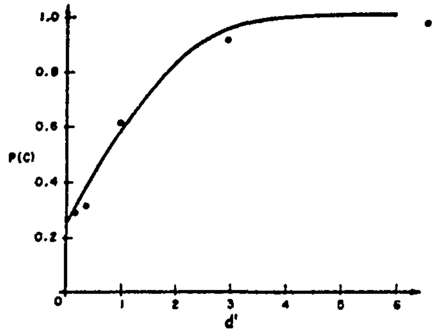

FIG. 14. Prediction of forced-choice data from yes-no data for Observer 1

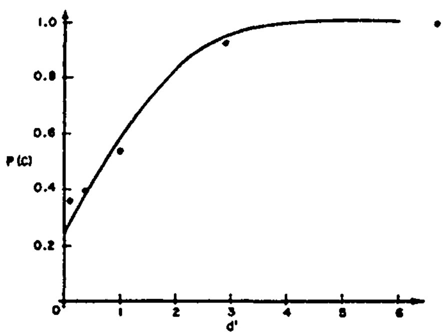

FIG. 15. Prediction of forced-choice data from yes-no data for Observer 2

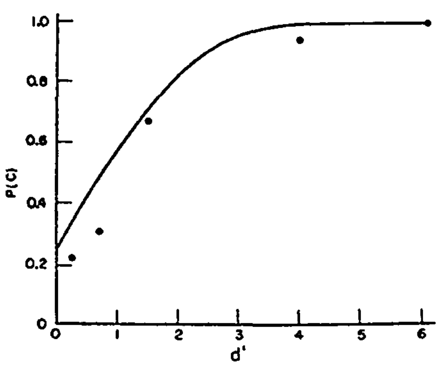

FIG. 16. Prediction of forced-choice data from yes-no data for Observer 3

suggesting that the deviation which appeared on the curves in Fig. 11, 12, and 13 is not significant.

DISCUSSION

The results satisfy the criterion of internal consistency. The theory also turns out to be consistent with the vast amount of data in the literature, for, when the d' vs. ΔI function for any one of the observers is used to predict probability of detection as a function of ΔI in terms of this theory, the result closely approximates the type of curve frequently reported. Shapes of curves thus furnish no basis for selecting between the two theories, and a decision must rest on the other arguments.

According to conventional theory, application of the chance correction should yield corrected values of $P_{SN}(A)$ which are independent of $P_N(A)$, or should yield corrected thresholds in the conventional sense which are independent of $P_N(A)$. Rank-order correlations for the three observers between $P_N(A)$ and corrected thresholds (.30, .71, .67) are highly significant; the combined $p \ll .001$. This is a result consistent with theory presented here.

Another method of comparison is to fit the scatter diagrams (Fig. 9 and 10) by straight lines. According to the independence theory, these straight lines should intercept the point (1.00, 1.00). Sampling error would be expected to send some of the lines to either side of this point. There are 12 of these scatter diagrams, and all 12 of these lines intersect the line $P_{SN}(A) = 1.00$ at values of $P_N(A)$ between 0 and 1.00 in an order which would be predicted if these lines were arcs of the curves $P_{SN}(A)$ vs. $P_N(A)$ as defined by the theory of signal detectability.

Two additional sessions were run in which the observers were permitted three categories of response (yes, no, and doubtful), and were told to be sure of being correct if they responded either yes or no. Again, two a priori probabilities (.8 and .4) were employed, and again $P_N(A)$ was correlated with $P(SN)$. The observers, interviewed after these sessions, reported that their "yes" responses were based on "phenomenal" seeing.

This does not mean that the observers were abnormal because they hallucinated. It suggests, on the other hand, that phenomenal seeing develops through experience, and is subject to change with experience. Psychological as well as physiological factors are involved. Psychological "set" is a function of β, and after experience with a given set one begins to see, or not to see, rather automatically. Change the set, and the level of seeing changes. The experiments reported here were such that the observers learned to adjust rapidly to different sets.

CONCLUSIONS

The following conclusions are advanced: (*a*) The conventional concept of a threshold, or a threshold region, needs re-evaluation in the light of the present theory that the visual detection problem is the problem of detecting signals in noise. (*b*) The hypothesis that false alarms are guesses is rejected on the basis of statistical tests. (*c*) Change in neural activity is a power function of change in light intensity. (*d*) The mathematical model of signal detection is applicable to problems of visual detection. (*e*) The criterion of seeing depends on psychological as well as physiological factors. In the experiments reported here the observers tended to use optimum criteria. (*f*) The experimental data support the assumption of a logical connection between forced-choice and yes-no techniques developed by the theory.

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