EZ Bayesian Hierarchical Drift Diffusion Model

Based on Joachim's python code

Adriana F. Chavez

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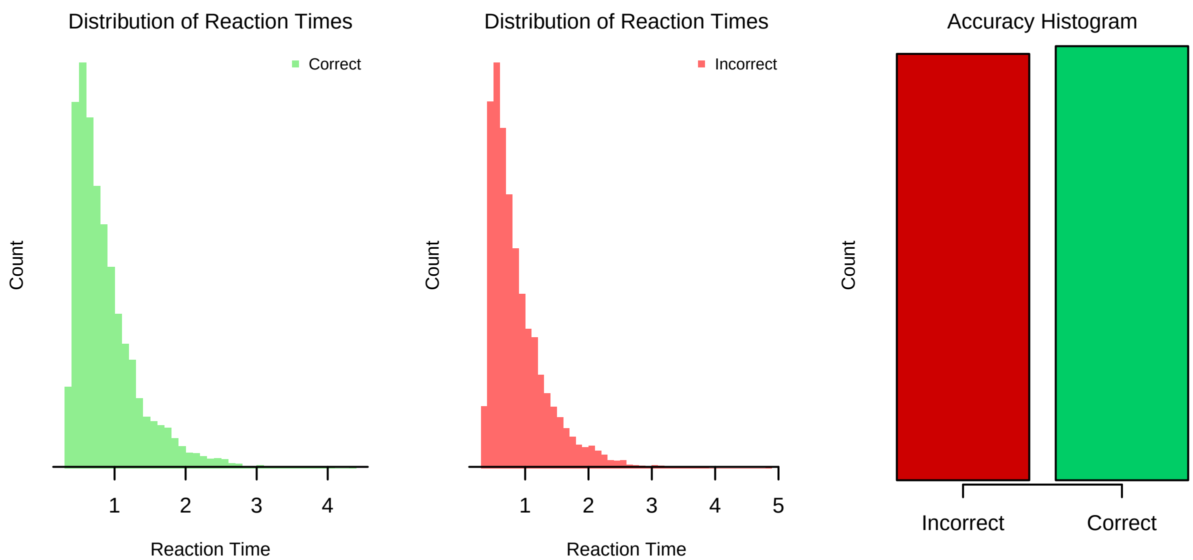
Basic functions to generate DDM data

```
# Part 1: Simulate single trial outcome
simulate_ddm <- function(a, v, dt, max_steps){
     x <- 0
     random_dev <- rnorm(max_steps)
     # Scale step changes by dt
     noise <- random_dev * sqrt(dt)
     drift <- v * dt

     for(i in 2:max_steps){
        this_step = drift + noise[i]
        x = x + this_step
        if(abs(x)>=(a/2)){  break  }
     }
     output <- list("RT" = (i+1)*dt, "C"  = x)
     return(output)
}

# Part 2: Simulate over 'n' trials
wdmrnd <- function(a,v,t,n){
    dt = 0.001
    max_steps = 10 / dt
    rt = rep(NA,n)
    accuracy = rep(NA,n)

    for(i in 1:n){
        X <- simulate_ddm(a, v, dt, max_steps)
        rt[i] <- X$RT
        if(X$C>0){  accuracy[i] <- 1
          }else{    accuracy[i] <- 0  }
    }

    rt = rt + t
    output <- data.frame("RT" = rt, "Accuracy" = accuracy)
    return(output)
}
```

Example data

```
a = 1.50
v = 0.00
t = 0.30
n = 10000

data <- wdmrnd(a, v, t, n)
rt <- data$RT
accuracy <- data$Accuracy
```



Define Simulation Study environment and variables

```
sample_parameters <- function(settings){
  prior <- settings$prior
  nP <- settings$nP
  bound_mean = rnorm(1,prior$bound_mean_mean,prior$bound_mean_sdev)
  drift_mean = rnorm(1,prior$drift_mean_mean,prior$drift_mean_sdev)
  nondt_mean = rnorm(1,prior$nondt_mean_mean,prior$nondt_mean_sdev)
  bound_sdev = runif(1,prior$bound_sdev_lower,prior$bound_sdev_upper)
  drift_sdev = runif(1,prior$drift_sdev_lower,prior$drift_sdev_upper)
  nondt_sdev = runif(1,prior$nondt_sdev_lower,prior$nondt_sdev_upper)
  bound = rnorm(nP,bound_mean, bound_sdev)
  drift = rnorm(nP,drift_mean, drift_sdev)
  nondt = rnorm(nP,nondt_mean, nondt_sdev)

  parameter_set <- list("bound_mean" = bound_mean, "drift_mean" = drift_mean,
```

```
                        "nondt_mean" = nondt_mean, "bound_sdev" = bound_sdev,
                        "drift_sdev" = drift_sdev, "nondt_sdev" = nondt_sdev,
                        "bound" = bound,   "drift" = drift,   "nondt" = nondt)

  return(parameter_set)
}

sample_data <- function(settings, parameter_set){
      for(i in 1:settings$nP){
          data <- wdmrnd(a = parameter_set$bound[i],
                         v = parameter_set$drift[i],
                         t = parameter_set$nondt[i],
                         n = settings$nT[i])
      }
}
```

```
prior <- default_priors()
```

```
## ========== EZBHDDM Priors: ===============
## Bound Mean Mean:    1.5
## Bound Mean Std Dev: 0.2
## Drift Mean Mean:    0
## Drift Mean Std Dev: 0.5
## Non-decision Time Mean Mean: 0.3
## Non-decision Time Mean Std:  0.06
## Bound Std Dev Shape: 0.1
## Bound Std Dev Scale: 0.2
## Drift Std Dev Shape: 0.2
## Drift Std Dev Scale: 0.4
## Non-decision Time Shape: 0.01
## Non-decision Time Scale: 0.05
```

```
class Hddm_Design:
    def __init__(self, participants, trials, prior):
        self.n_Participants    = int(participants)
        self.n_TrialsPerPerson = int(trials)
        self.prior             = prior
        self.parameter_set     = None
        self.data              = None
        self.estimate          = None

    def sample_parameters(self):
        self.parameter_set = Hddm_Parameter_Set()
        self.parameter_set.bound_mean = parameterself.parameter_set.drift_mean = np.random.normal(self.p
        self.parameter_set.nondt_mean = np.random.normal(self.prior.nondt_mean_mean, self.prior.nondt_me
        self.parameter_set.bound_sdev = np.random.uniform(self.prior.bound_sdev_lower, self.prior.bound_
        self.parameter_set.drift_sdev = np.random.uniform(self.prior.drift_sdev_lower, self.prior.drift_
        self.parameter_set.nondt_sdev = np.random.uniform(self.prior.nondt_sdev_lower, self.prior.nondt_
        self.parameter_set.bound      = np.random.normal(self.parameter_set.bound_mean, self.parameter_s
        self.parameter_set.drift      = np.random.normal(self.parameter_set.drift_mean, self.parameter_s
        self.parameter_set.nondt      = np.random.normal(self.parameter_set.nondt_mean, self.parameter_s

        return self
```

```
def sample_data(self):
    if not self.parameter_set:
        self.sample_parameters()
    self.data = Hddm_Data().sample(self)
    return self

def estimate_parameters(self):
    # This is the key bit
    code = f"""
    model {{
        # Priors for the hierarchical diffusion model parameters
        bound_mean ~ dnorm({self.prior.bound_mean_mean},  {self.prior.bound_mean_sdev**-2}) T( 0.10
        drift_mean ~ dnorm({self.prior.drift_mean_mean},  {self.prior.drift_mean_sdev**-2}) T(-3.00
        nondt_mean ~ dnorm({self.prior.nondt_mean_mean},  {self.prior.nondt_mean_sdev**-2}) T( 0.05
        bound_sdev ~ dunif({self.prior.bound_sdev_lower}, {self.prior.bound_sdev_upper})
        drift_sdev ~ dunif({self.prior.drift_sdev_lower}, {self.prior.drift_sdev_upper})
        nondt_sdev ~ dunif({self.prior.nondt_sdev_lower}, {self.prior.nondt_sdev_upper})

        for (p in 1:nParticipants) {{
            bound[p] ~ dnorm(bound_mean, pow(bound_sdev, -2)) T( 0.10, 3.00)
            drift[p] ~ dnorm(drift_mean, pow(drift_sdev, -2)) T(-3.00, 3.00)
            nondt[p] ~ dnorm(nondt_mean, pow(nondt_sdev, -2)) T( 0.05, 1.00)

            # Forward equations from EZ Diffusion
            ey[p]  = exp(-bound[p] * drift[p])
            Pc[p]  = 1 / (1 + ey[p])
            PRT[p] = 2 * pow(drift[p], 3) / bound[p] * pow(ey[p] + 1, 2) / (2 * -bound[p] * drift[p
            MDT[p] = (bound[p] / (2 * drift[p])) * (1 - ey[p]) / (1 + ey[p])
            MRT[p] = MDT[p] + nondt[p]

            # Loss functions using MRT, PRT, and Pc
            meanRT[p]  ~ dnorm(MRT[p], PRT[p] * correct[p])
            varRT[p]   ~ dnorm(1/PRT[p], 0.5 * correct[p] * PRT[p] * PRT[p])
            correct[p] ~ dbin(Pc[p], nTrialsPerPerson)
        }}
    }}
    """

    data, valid_indices = self.data.to_jags()

    n_Participants_Left = data['nParticipants']

    # Initial values
    init = { "drift" : np.random.normal(0, 0.1, n_Participants_Left) }

    try:
        model = pyjags.Model(
            progress_bar = False,
            code    = code,
            data    = data,
            init    = init,
            adapt   = 100,
            chains  = 4,
```

```
                threads = 4)
        except Exception as e:
            #error_message = str(e)
            #print(type(error_message))
            #print(error_message)
            #self.data.summary()
            #print(self.data.to_jags())
            #print(self.parameter_set)
            print('e', end='')
            return

        samples = model.sample(400,
                               vars = ['bound_mean', 'drift_mean', 'nondt_mean',
                                       'bound_sdev', 'drift_sdev', 'nondt_sdev',
                                       'bound',      'drift',      'nondt'])

        # Annoying management of sample object...  First move individual parameters to their own fields
        for i in np.arange(0, n_Participants_Left):
            samples.update({'bound_'+str(valid_indices[i]): samples['bound'][i,:,:],
                            'drift_'+str(valid_indices[i]): samples['drift'][i,:,:],
                            'nondt_'+str(valid_indices[i]): samples['nondt'][i,:,:], })

        # ... remove the old unwieldy matrices
        for s in ["bound", "drift", "nondt"]:
            samples.pop(s)

        # Start a new dict with estimates only
        estimate = { "bound": [np.nan] * self.n_Participants,
                     "drift": [np.nan] * self.n_Participants,
                     "nondt": [np.nan] * self.n_Participants
                   }

        for varname in ['bound_mean', 'drift_mean', 'nondt_mean',
                        'bound_sdev', 'drift_sdev', 'nondt_sdev']:
            estimate.update({varname: np.mean(samples[varname])})

        # ... make new, wieldy matrices
        for i in valid_indices:
            estimate['bound'][i] = np.mean(samples['bound_'+str(i)])
            estimate['drift'][i] = np.mean(samples['drift_'+str(i)])
            estimate['nondt'][i] = np.mean(samples['nondt_'+str(i)])

        # Copy estimate to design object
        self.estimate = Hddm_Parameter_Set()
        self.estimate.bound_mean = estimate['bound_mean']
        self.estimate.drift_mean = estimate['drift_mean']
        self.estimate.nondt_mean = estimate['nondt_mean']
        self.estimate.bound_sdev = estimate['bound_sdev']
        self.estimate.drift_sdev = estimate['drift_sdev']
        self.estimate.nondt_sdev = estimate['nondt_sdev']
        self.estimate.bound      = estimate['bound']
        self.estimate.drift      = estimate['drift']
        self.estimate.nondt      = estimate['nondt']
```

```
    def __str__(self):
        output = [
            "Hddm_Design Parameters:",
            f"Number of Participants: {self.n_Participants}",
            f"Trials Per Person:      {self.n_TrialsPerPerson}",
            f"Prior:                  {self.prior}",
            f"Parameter Set:          {self.parameter_set}",
            f"Data:                   {self.data}"
        ]
        return '\n'.join(output)
```

```
class Hddm_Data():
    def __init__(self, person = None, rt = None, accuracy = None, n_TrialsPerPerson = None):
        self.person            = person
        self.rt                = rt
        self.accuracy          = accuracy
        self.n_TrialsPerPerson = n_TrialsPerPerson

    @staticmethod
    def sample(design):
        T = design.n_TrialsPerPerson
        P = design.n_Participants
        parameters = design.parameter_set

        person_list   = []
        rt_list       = []
        accuracy_list = []

        for p in range(P):
            accuracy = 0
            while np.sum(accuracy) == 0:
                rt, accuracy = wdmrnd(parameters.bound[p], parameters.drift[p], parameters.nondt[p], T)
            person_list.extend([p] * T)  # Repeat the participant ID for T trials
            rt_list.extend(rt)
            accuracy_list.extend(accuracy)

        # Convert lists to NumPy arrays for consistency and potential performance benefits
        person   = np.array(person_list)
        rt       = np.array(rt_list)
        accuracy = np.array(accuracy_list)

        return Hddm_Data(person, rt, accuracy, T)

    def summary(self):
        if self.person is None or self.rt is None or self.accuracy is None:
            print("Data not available.")
            return

        unique_persons = np.unique(np.array(self.person))
        print("{:<10} {:<20} {:<20} {:<20}".format("Person", "Mean Accuracy", "Mean RT (Correct)", "Var

        for person_id in unique_persons:
            # Filter data for current person
```

```
            person_indices  = np.where(self.person == person_id)
            person_rts      = np.array(self.rt)[person_indices]
            person_accuracy = np.array(self.accuracy)[person_indices]

            # Compute the metrics
            mean_accuracy       = np.mean(person_accuracy)
            correct_rts         = person_rts[person_accuracy == 1]  # only accurate responses
            mean_rt_correct     = np.mean(correct_rts) if len(correct_rts) > 0 else np.nan
            variance_rt_correct = np.var(correct_rts) if len(correct_rts) > 0 else np.nan

            print("{:<10} {:<20.3f} {:<20.3f} {:<20.3f}".format(person_id, mean_accuracy, mean_rt_corre

    def to_jags(self):
        if self.person is None or self.rt is None or self.accuracy is None:
            return None

        unique_persons = np.unique(np.array(self.person)).astype(int)
        nParticipants  = len(unique_persons)

        # Initialize arrays to NaN for storing metrics
        sum_accuracy        = np.zeros(nParticipants, dtype=int)
        mean_rt_correct     = np.full(nParticipants, np.nan)
        variance_rt_correct = np.full(nParticipants, np.nan)

        # Loop over unique persons and compute metrics
        for person_id in unique_persons:
            # Filter data for the current person
            person_indices  = self.person == person_id
            person_rts      = self.rt[person_indices]
            person_accuracy = self.accuracy[person_indices]

            # Update metrics
            sum_accuracy[person_id] = np.sum(person_accuracy)
            correct_rts = person_rts[person_accuracy == 1]  # only accurate responses

            if correct_rts.size > 1:
                mean_rt_correct[person_id]     = np.mean(correct_rts)
                variance_rt_correct[person_id] = np.var(correct_rts)

        # Filter out participants with NaN values in any metric
        valid_indices = ~(
            np.isnan(mean_rt_correct) |
            np.isnan(variance_rt_correct)
        )

        # Extract valid metrics
        sum_accuracy        = sum_accuracy[valid_indices].tolist()
        mean_rt_correct     = mean_rt_correct[valid_indices].tolist()
        variance_rt_correct = variance_rt_correct[valid_indices].tolist()
        nParticipants       = len(sum_accuracy)  # Update nParticipants after filtering

        return {
            "nTrialsPerPerson": int(self.n_TrialsPerPerson),
```

```python
            "nParticipants": nParticipants,
            "meanRT": mean_rt_correct,
            "varRT": variance_rt_correct,
            "correct": sum_accuracy,
        }, unique_persons[valid_indices]

    def __str__(self):
        output = [
            "Hddm_Data Details:",
            f"Person:    {self.person}",
            f"RT:        {self.rt}",
            f"Accuracy:  {self.accuracy}"
        ]
        return '\n'.join(output)
```

```python
class Hddm_Parameter_Set:
    def __init__(self,
                 bound_mean = None, bound_sdev = None, bound = None,
                 drift_mean = None, drift_sdev = None, drift = None,
                 nondt_mean = None, nondt_sdev = None, nondt = None):
        self.bound_mean = bound_mean
        self.bound_sdev = bound_sdev
        self.bound      = bound
        self.drift_mean = drift_mean
        self.drift_sdev = drift_sdev
        self.drift      = drift
        self.nondt_mean = nondt_mean
        self.nondt_sdev = nondt_sdev
        self.nondt      = nondt

    def __sub__(self, other):
        if not isinstance(other, Hddm_Parameter_Set):
            return None

        return Hddm_Parameter_Set(
            bound_mean = self.bound_mean - other.bound_mean,
            bound_sdev = self.bound_sdev - other.bound_sdev,
            drift_mean = self.drift_mean - other.drift_mean,
            drift_sdev = self.drift_sdev - other.drift_sdev,
            nondt_mean = self.nondt_mean - other.nondt_mean,
            nondt_sdev = self.nondt_sdev - other.nondt_sdev,
            bound      = self.bound - other.bound if self.bound is not None and other.bound is not None
            drift      = self.drift - other.drift if self.drift is not None and other.drift is not None
            nondt      = self.nondt - other.nondt if self.nondt is not None and other.nondt is not None
        )

    def __str__(self):
        output = [
            "Hddm_Parameter_Set Details:",
            f"Bound Mean:              {self.bound_mean}",
            f"Bound Std Dev:           {self.bound_sdev}",
            f"Drift Mean:              {self.drift_mean}",
            f"Drift Std Dev:           {self.drift_sdev}",
```

```
            f"Non-decision Time Mean:  {self.nondt_mean}",
            f"Non-decision Time Std:   {self.nondt_sdev}",
            f"Bound:                   {self.bound}",
            f"Drift:                   {self.drift}",
            f"Non-decision Time:       {self.nondt}"
        ]
        return '\n'.join(output)
```

Run simulations

Simple example

```
prior = Hddm_Prior()
np.random.seed(seed = 188) # This doesn't work
design = Hddm_Design(participants=20, trials=50, prior=prior)
design.sample_parameters()
design.sample_data()
design.estimate_parameters()
```

```
nSim <- 200
settings <- list("nPart"   = 50,
                 "nTrials" = 150,
                 "prior"   = prior)
prior <- default_priors()

tru = [Hddm_Parameter_Set()] * K
est = [Hddm_Parameter_Set()] * K
err = [Hddm_Parameter_Set()] * K

for(k in 1:nSim){
    set.seed(k)
    cat("Iteration", k+1, "of", nSim)
    design = Hddm_Design(participants=20, trials=50, prior=prior)
    design.sample_parameters()
    design.sample_data()
    #print(design.parameter_set)
    #design.data.summary()
    design.estimate_parameters()
    tru[k] = design.parameter_set
    est[k] = design.estimate
    if design.estimate is not None:
        err[k] = (design.estimate - design.parameter_set)
    else:
        err[k] = None
    if (k+1) % 100 == 0:
        print(f'. {k+1} of {K}\n', end='')
    else:
        print('.', end='')
}
```

```
def recovery_plot(x, y, parameterName, ttl):
    fontsize = 10

    plt.figure(figsize=(2, 2))

    plt.scatter(x, y, color='b', s=3)
    plt.grid()
    plt.gca().set_aspect('equal')

    xax = np.linspace(min(x), max(x), 100)

    plt.plot(xax, xax, '--')

    plt.xlabel('Simulated value', fontsize=10)
    plt.title('Group mean ' + parameterName, fontsize=10)

    output_path = "ezrecovery_" + parameterName + ".pdf"
    plt.savefig(output_path, format='pdf', bbox_inches='tight')

    plt.show()
```

```
x = [np.nan] * K
y = [np.nan] * K
for k in range(K):
    if err[k] is not None:
       x[k] = tru[k].nondt_mean
       y[k] = est[k].nondt_mean

recovery_plot(x, y, 'nondt', 'Group mean nondt')

x = [np.nan] * K
y = [np.nan] * K
for k in range(K):
    if err[k] is not None:
       x[k] = tru[k].drift_mean
       y[k] = est[k].drift_mean

recovery_plot(x, y, 'drift', 'Group mean drift')

x = [np.nan] * K
y = [np.nan] * K
for k in range(K):
    if err[k] is not None:
       x[k] = tru[k].bound_mean
       y[k] = est[k].bound_mean

recovery_plot(x, y, 'bound', 'Group mean bound')

x = np.empty(0)
y = np.empty(0)
for k in range(K):
    if err[k] is not None:
       x = np.append(x, tru[k].drift)
```

```
        y = np.append(y, est[k].drift)

recovery_plot(x, y, 'drift', 'Individual drift rates')
```