

These are just some ideas for a good title...

“Bayesian SDT modeling”

“Bayesian Cognitive modeling applied to a
SDT task: the Mirror Effect meets
individual cognitive modeling,
contaminant analysis and step point
change”

Disclaimer:

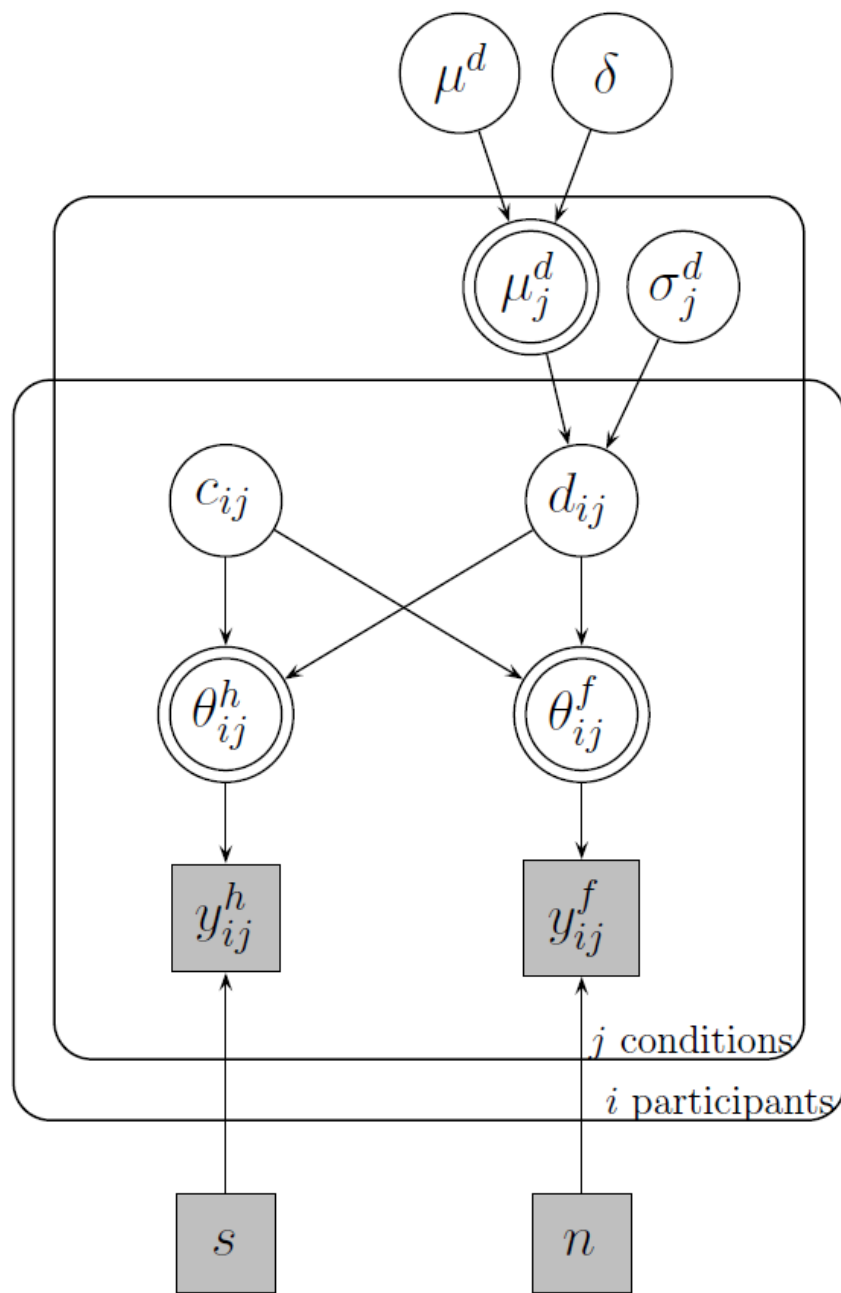
This presentation was made as a tool to “put things together” and begin to fix/set the “story” or the “main course” that I’m planning to cover with my SDT study/paper/presentation.

It’s meant to present, step by step what’s planned to be done and the main findings/plots/estimations to report as a result of each step.

Also, this could be consider a draw for any Talk or presentation derived from this research project.

1. Making sure $d'(A) > d'(B)$

In order to make our results comparable to what has been reported on the literature of the Mirror Effect withing Recognition Memory, we have to be able to guarantee that our two clases of stimuli are indeed different in terms of how “difficult” they are.



$$\mu^d \sim \text{Gaussian}(0, 1)$$

$$\delta \sim \text{Gaussian}(0, 1)$$

$$\mu_A^d \leftarrow \mu + \frac{\delta}{2}$$

$$\mu_B^d \leftarrow \mu - \frac{\delta}{2}$$

$$\sigma_j^d \sim \text{Uniform}(0, 5)$$

$$d_{ij} \sim \text{Gaussian}(\mu_j^d, \sigma_j^d)$$

$$c_{ij} \sim \text{Gaussian}(0, 1)$$

$$\theta_{ij}^h \leftarrow \phi\left(\frac{1}{2}d_{ij} - c_{ij}\right)$$

$$\theta_{ij}^f \leftarrow \phi\left(-\frac{1}{2}d_{ij} - c_{ij}\right)$$

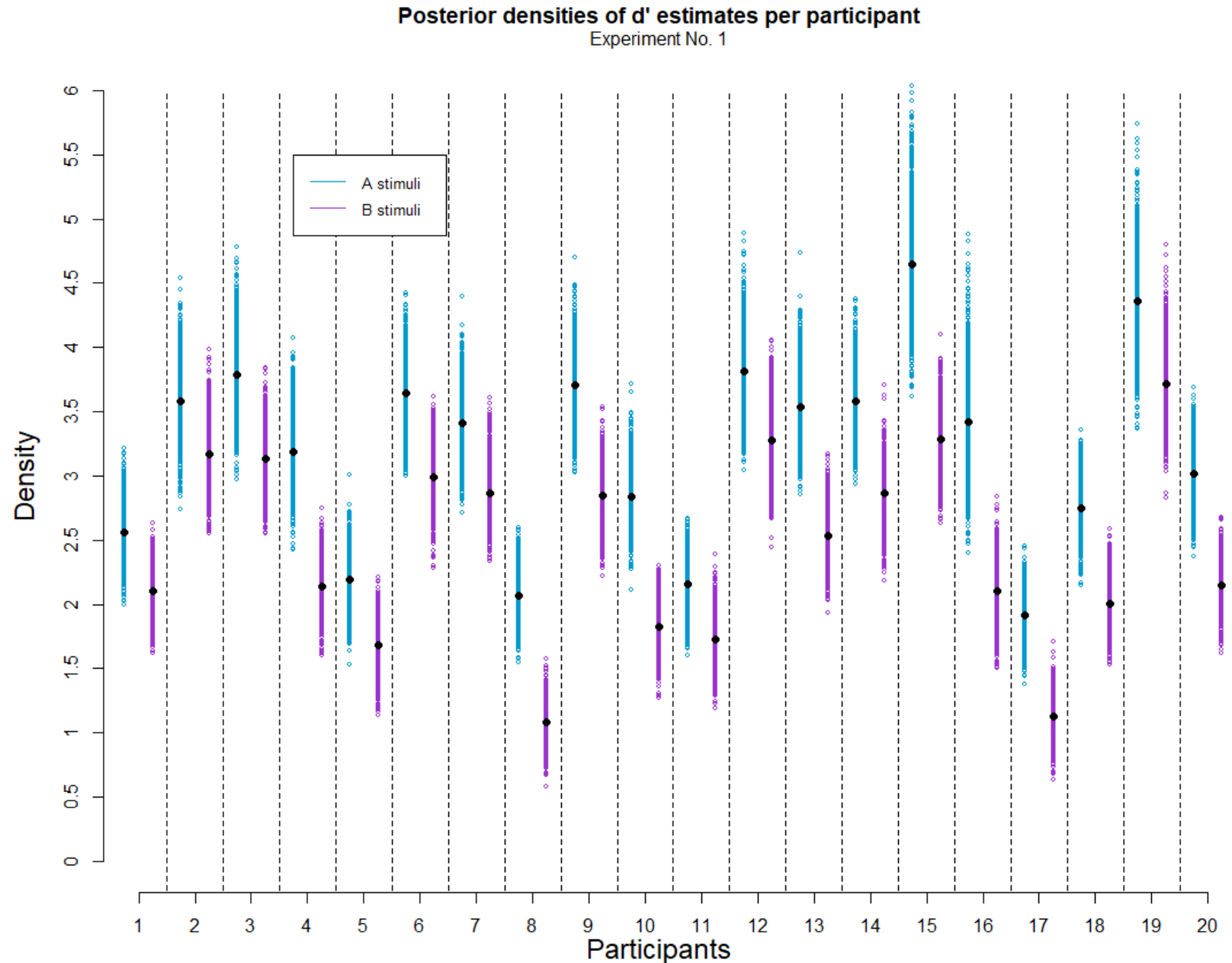
$$y_{ij}^h \sim \text{Binomial}(\theta_{ij}^h, s)$$

$$y_{ij}^f \sim \text{Binomial}(\theta_{ij}^f, n)$$

Plot 1

What's important about this plot?

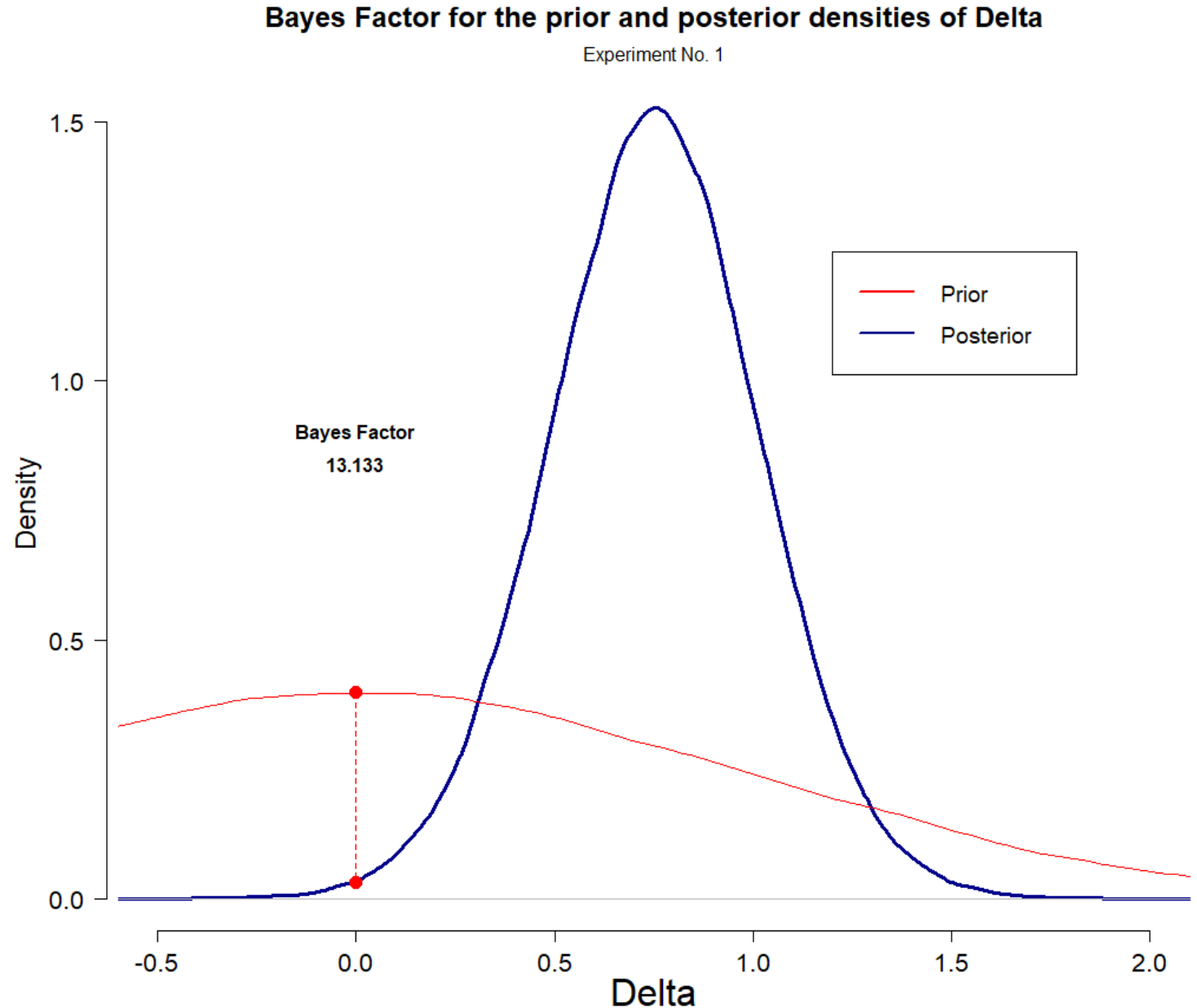
For all participants, d' estimates are greater for A class stimuli than for B class.



Plot 2

What's important about this plot?

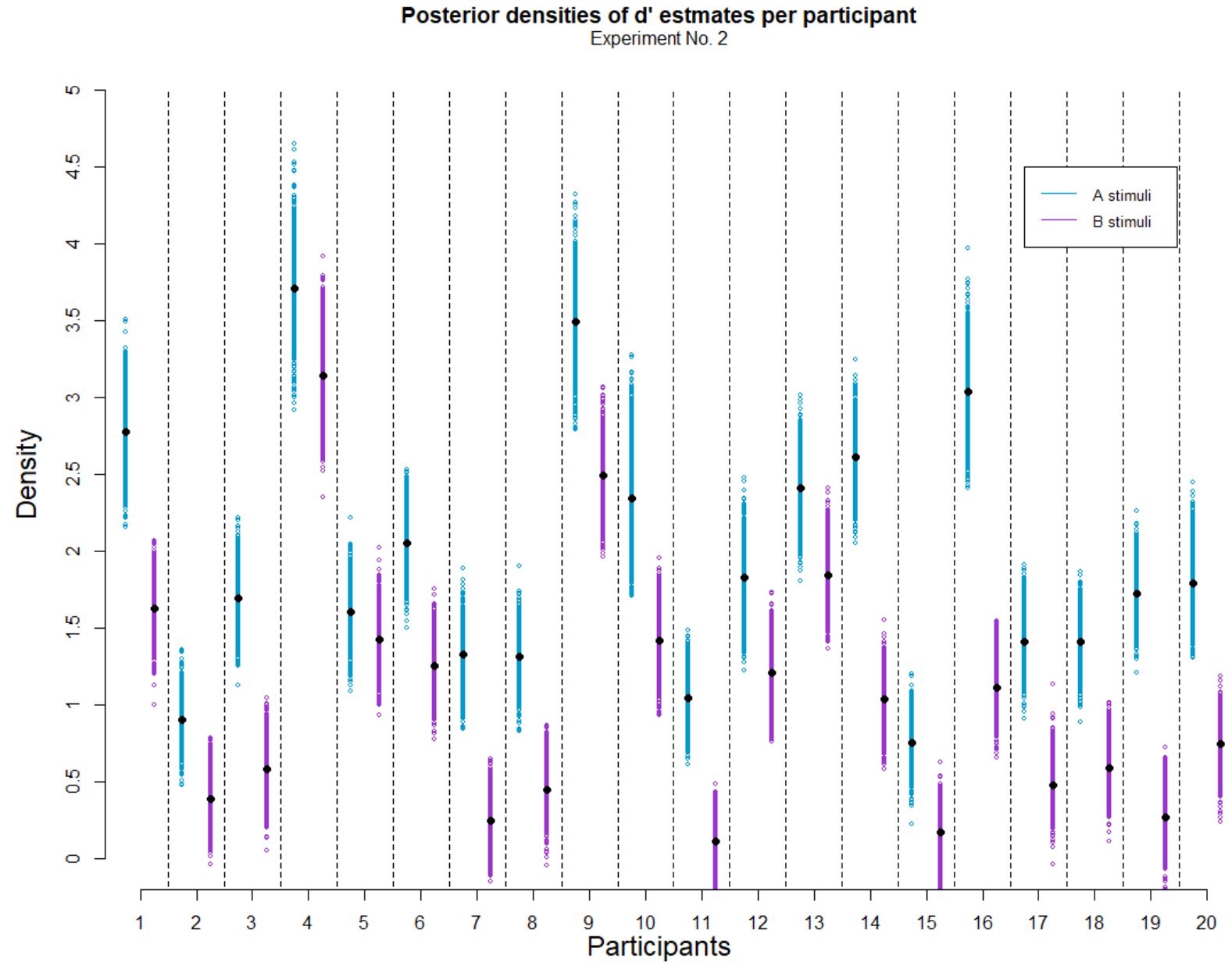
According to the Bayes Factor, the posterior estimates suggest that it's 13 times less likely that the difference between d' across each class of stimuli is 0, than/from what had been stated on the prior.



Plot 1

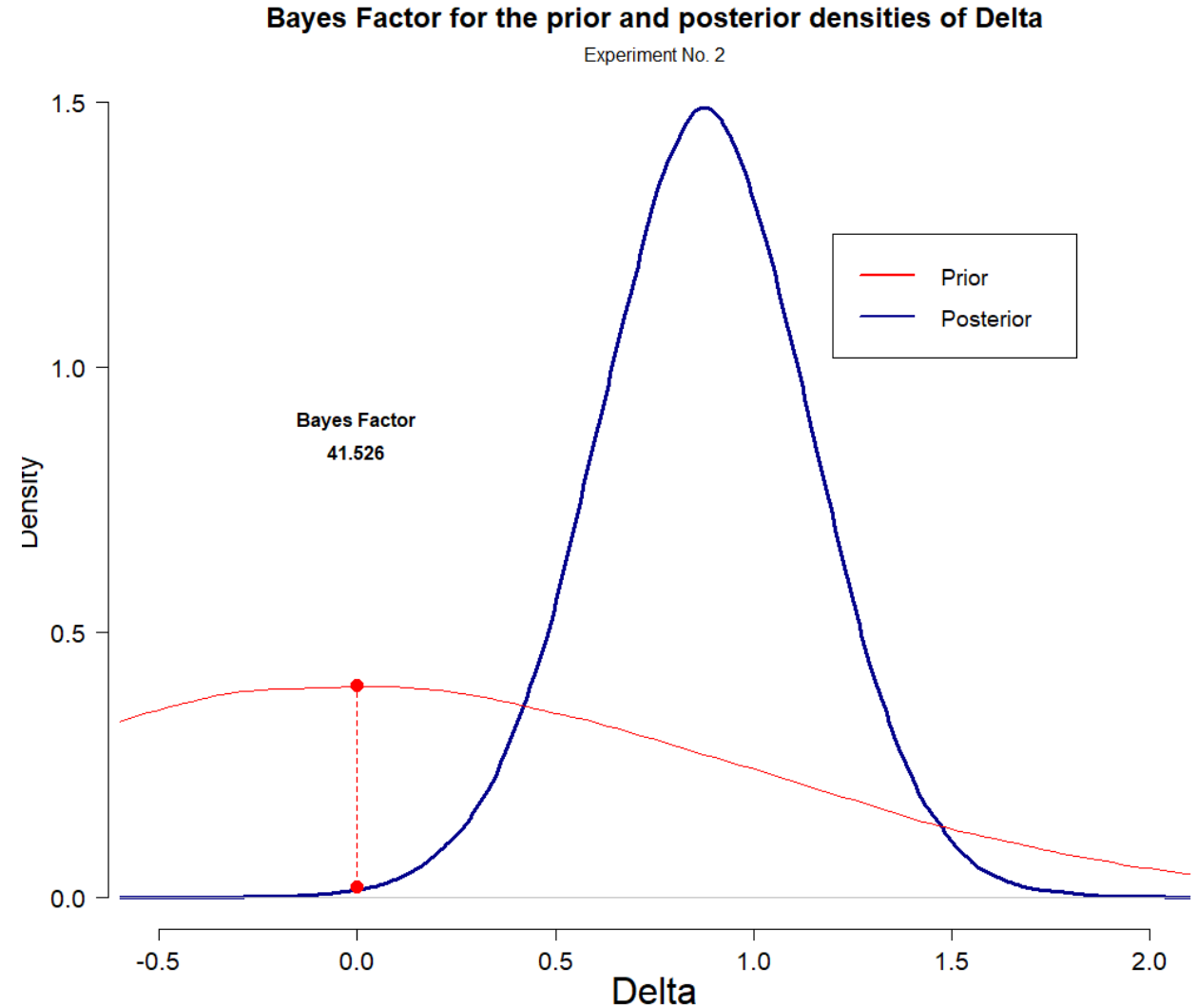
Same things happen in Experiment 2

For all participants, d' estimates are greater for A class stimuli than for B class.



Plot 2

According to the Bayes Factor, the posterior estimates suggest that it's 41.5 times less likely that the difference between d' across each class of stimuli is 0, than/from what had been stated on the prior.



2. Contaminant P

PENDING

In order to make sure that what has been reported on the literature concerning Recognition Memory, we have + that our two classes of stimuli are indeed in terms of how “difficult” they are.

2.1 A “simple” contract

PENDING

based on a

2.1 A “cognitive
contaminant” of the

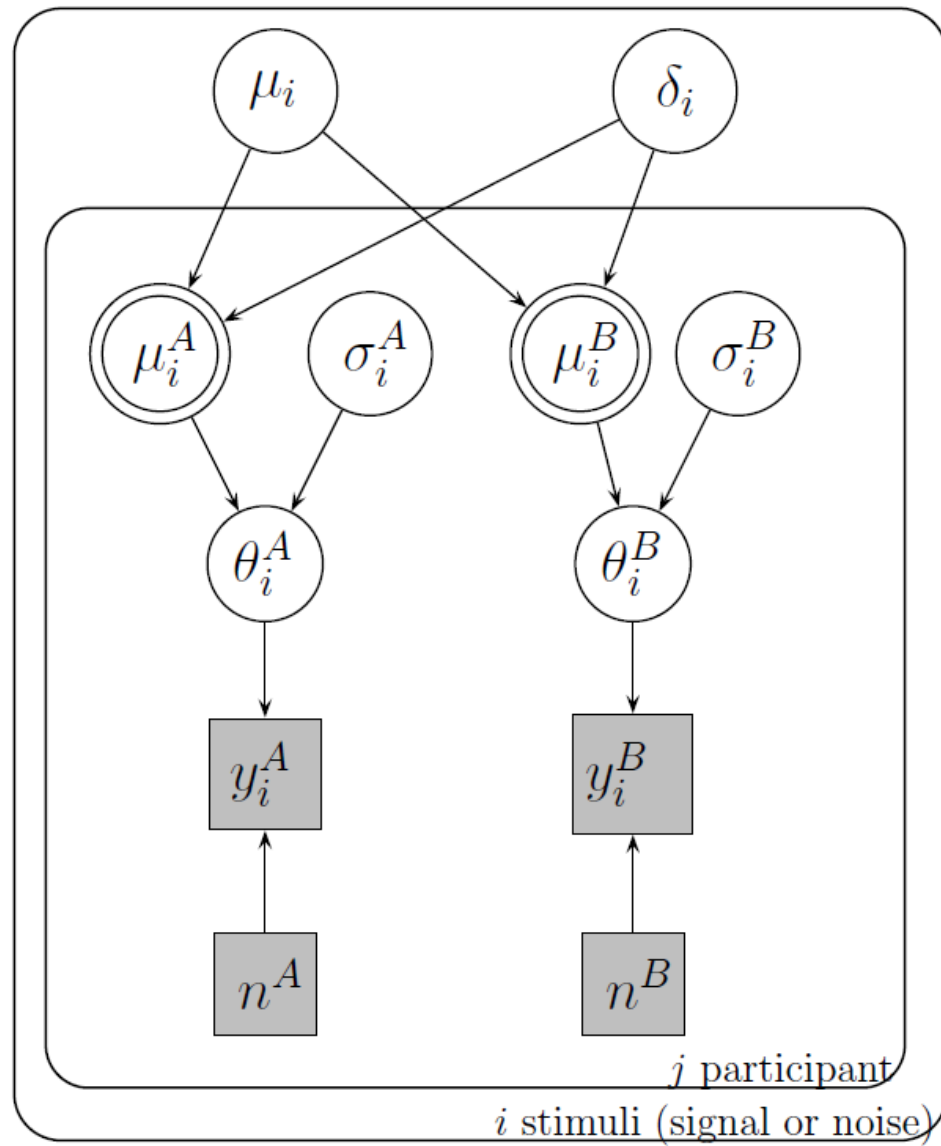
PENDING

3. Looking for the Mirror Effect

Comparing the Hit and F.A. rates across classes of stimuli to see if we can find the same pattern as the one reported in the literatura under the name of “the Mirror Effect”

$$FA(A) < FA(B) < H(B) < H(A)$$

3.1 Comparing binomial response rates



$$\mu_h \sim \text{Uniform}(0.5, 1)$$

$$\mu_{fa} \sim \text{Uniform}(0, 0.5)$$

$$\delta \sim \text{Uniform}(0, 0.5)$$

$$\mu_h^A \leftarrow \mu_h + \frac{\delta}{2}$$

$$\mu_h^B \leftarrow \mu_h - \frac{\delta}{2}$$

$$\mu_{fa}^A \leftarrow \mu_{fa} - \frac{\delta}{2}$$

$$\mu_{fa}^B \leftarrow \mu_{fa} + \frac{\delta}{2}$$

$$\sigma_i^A, \sigma_i^B \sim \text{Uniform}(0, 1)$$

$$\theta_i^A \sim \text{Gaussian}(\mu_i^A, \sigma_i^A)$$

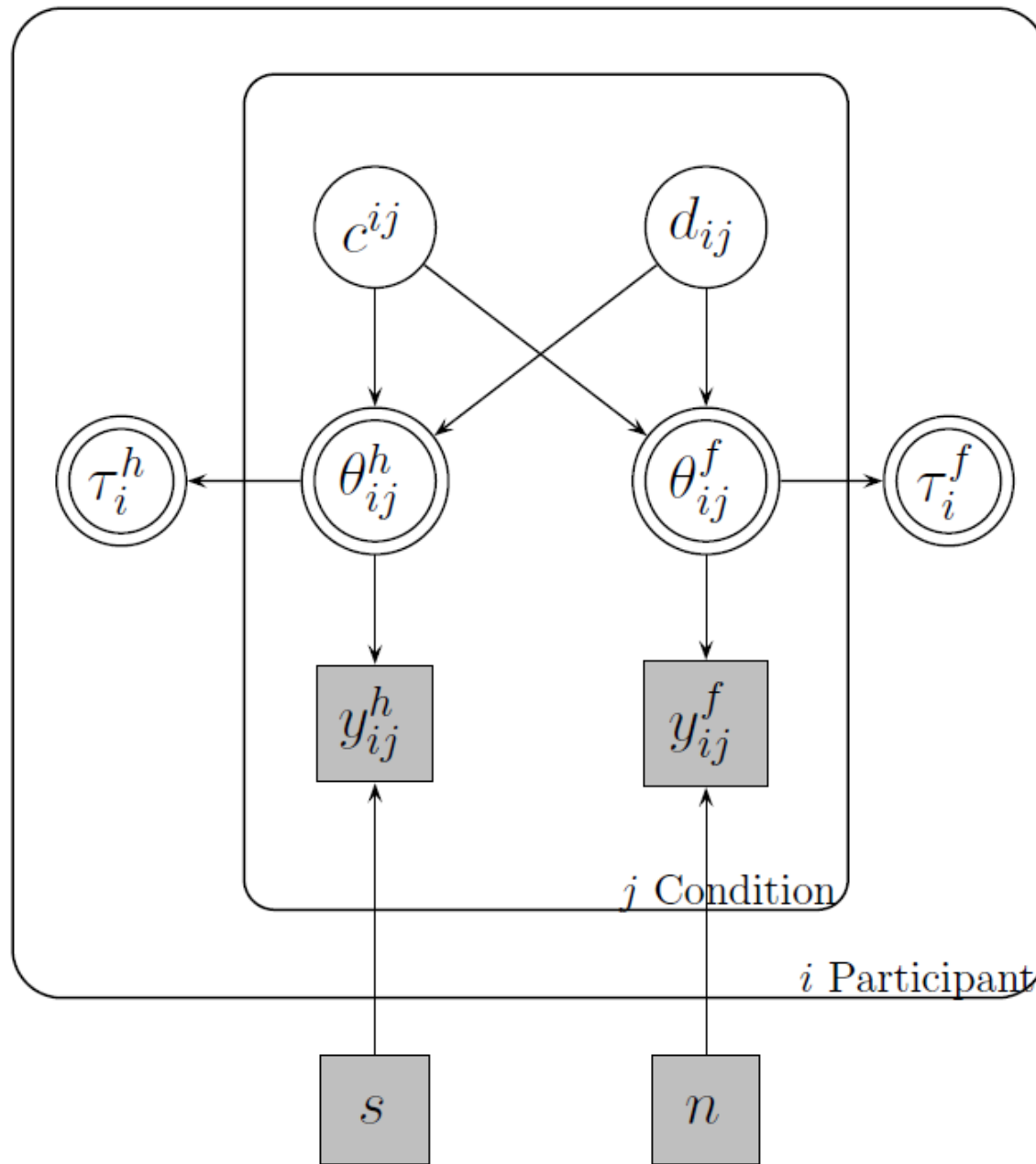
$$\theta_i^B \sim \text{Gaussian}(\mu_i^B, \sigma_i^B)$$

$$y_i^A \sim \text{Binomial}(\theta_i^A, n^A)$$

$$y_i^B \sim \text{Binomial}(\theta_i^B, n^B)$$

3.2 Comparing Hit rates and F.A. rates in the context of a Bayesian **cognitive** model

Using a Bayesian cognitive SDT model to look through the differences observed between the Hit and F.A. rates across classes of stimuli



$$y_{ij}^h \sim \text{Binomial}(\theta_{ij}^h, s)$$

$$y_{ij}^f \sim \text{Binomial}(\theta_{ij}^f, s)$$

$$\theta_{ij}^h \leftarrow \phi\left(\frac{1}{2}d_{ij} - c_{ij}\right)$$

$$\theta_{ij}^f \leftarrow \phi\left(-\frac{1}{2}d_{ij} - c_{ij}\right)$$

$$d_{ij} \sim \text{Gaussian}(0, 1)$$

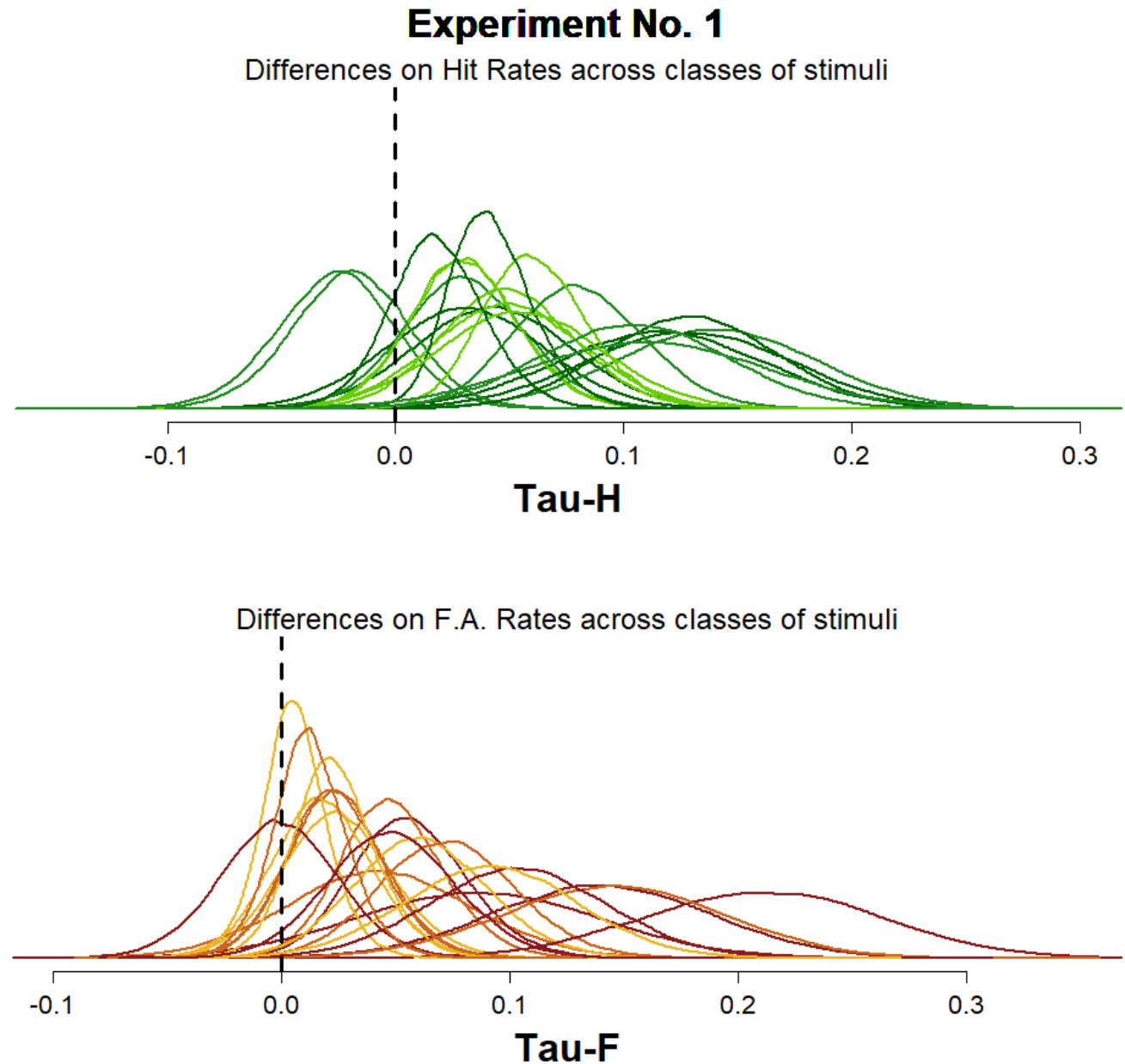
$$c_{ij} \sim \text{Gaussian}(0, 1)$$

$$\tau_i^h \leftarrow \theta_{iA}^h - \theta_{iB}^h$$

$$\tau_i^f \leftarrow \theta_{iB}^f - \theta_{iA}^f$$

Plot 1

Based on the individual Tau-H and Tau-F posterior distributions being shown, it's clear that there's a lot of variability (even if “at first glance” one might be tempted to say that the majority of the posterior density lies to the right side of the “0 differences between classes” point).

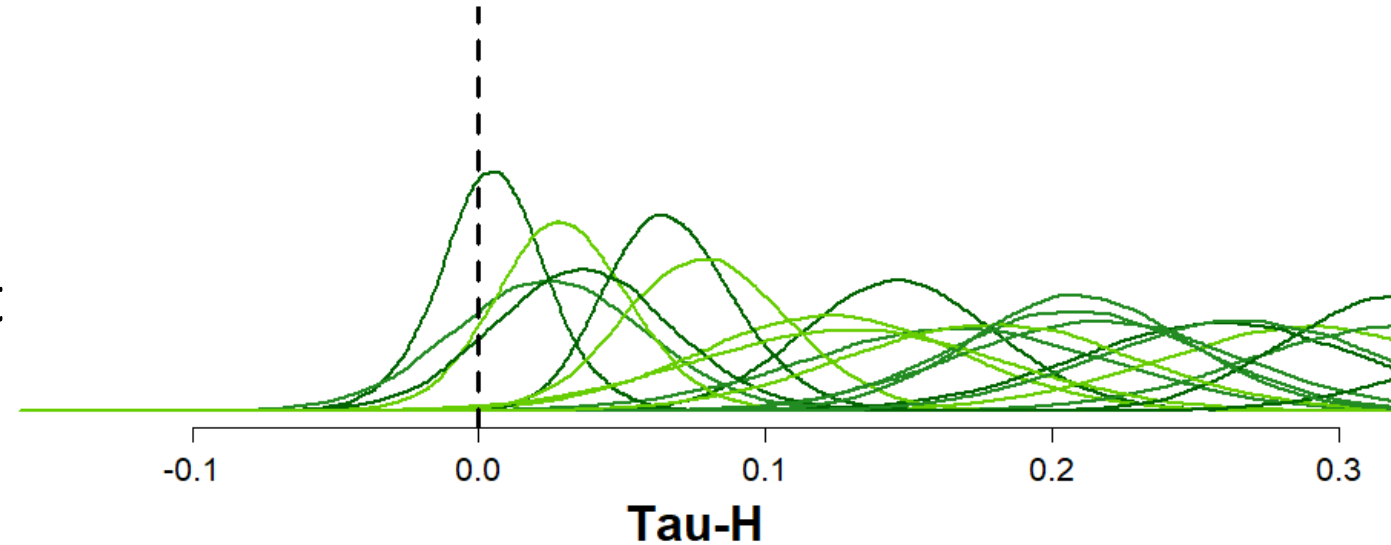


Plot 1

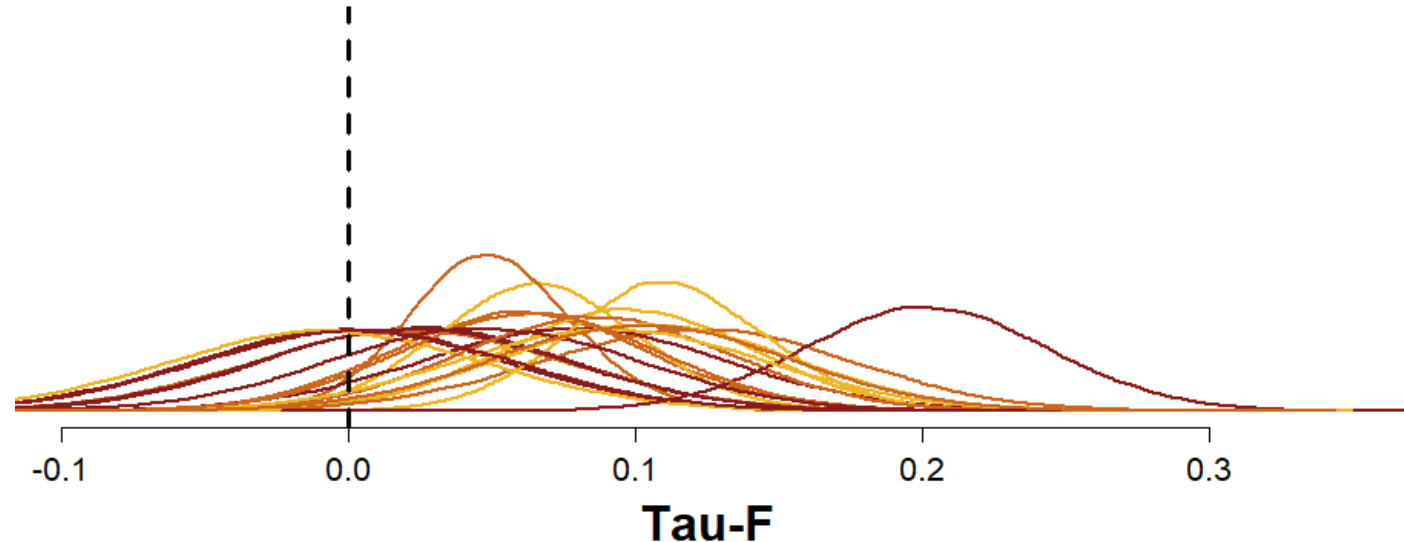
For **Experiment 2**, you can see that the estimates for the differences between the Hit Rates has clearly increased, at the same time that the estimates made for the differences between the FA rates seem to be a Little bit more clustered around 0.

Experiment No. 2

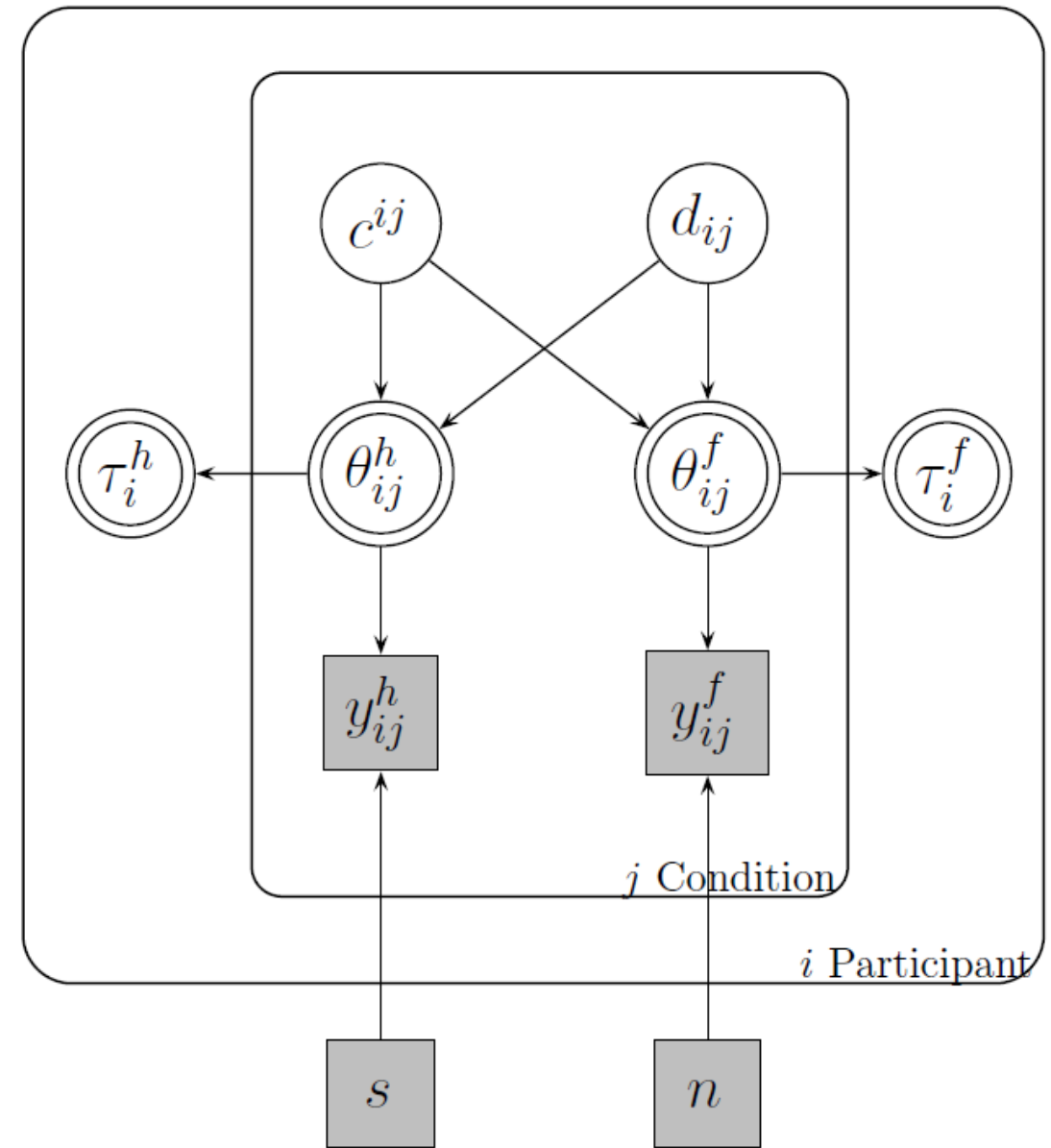
Differences on Hit Rates across classes of stimuli



Differences on F.A. Rates across classes of stimuli



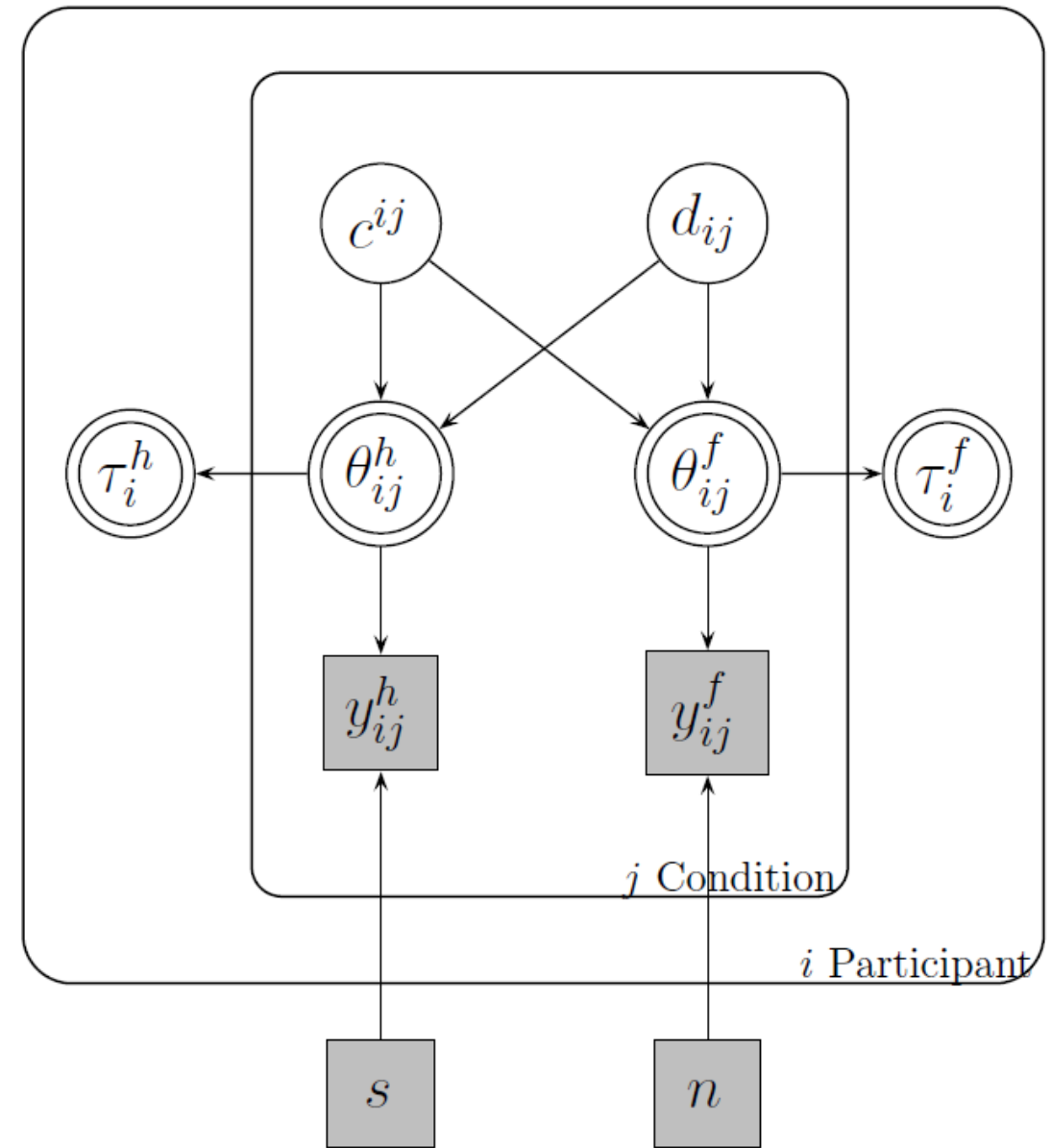
Given that this particular model was constructed so that both Taus are deterministically estimated, based on the individual estimations made for θ_{ij}^H and θ_{ij}^F , conducting a Bayes Factor or any other density ratio measure might not seem like a straight-forward thing to do.

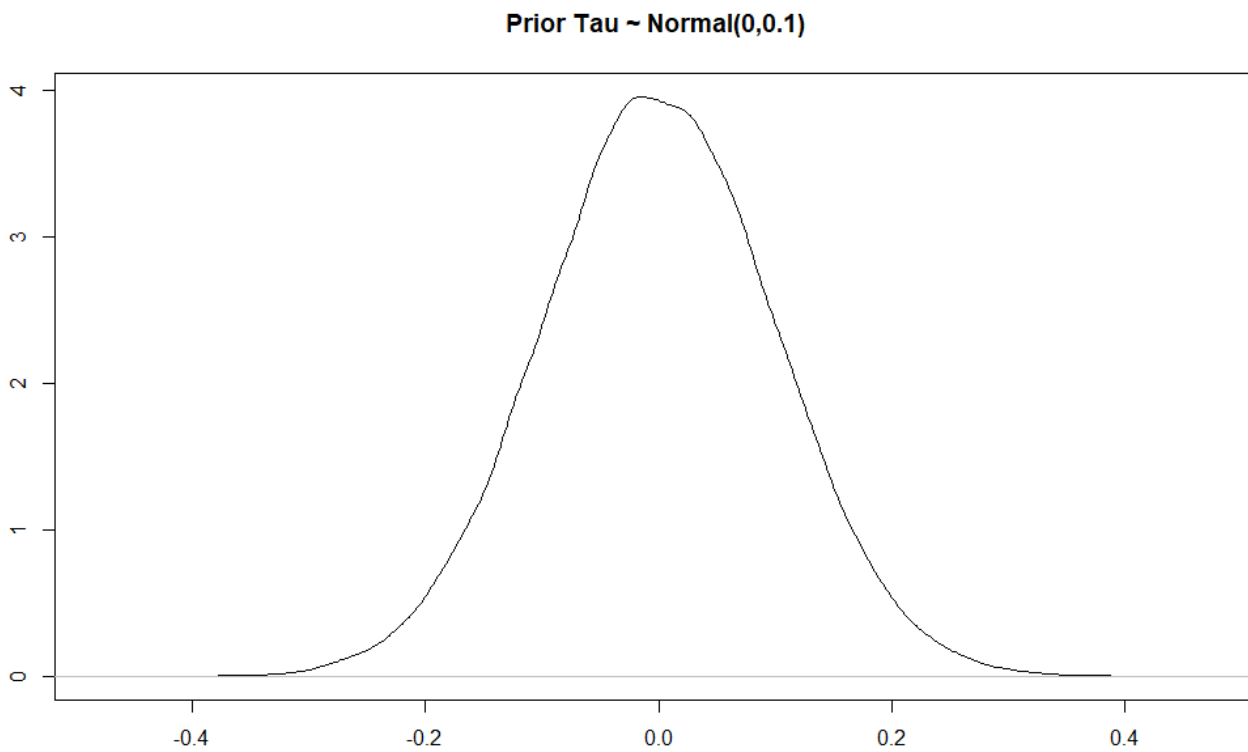


Given that this particular model was constructed so that both Taus are deterministically estimated, based on the individual estimations made for θ_{ij}^H and θ_{ij}^F , conducting a Bayes Factor or any other density ratio measure might not seem like a straight-forward thing to do.

It is under this disclaimer that we conducted a Bayes Factor for every individual Tau taking as a reference the following, not stated in the model, “prior distribution”:

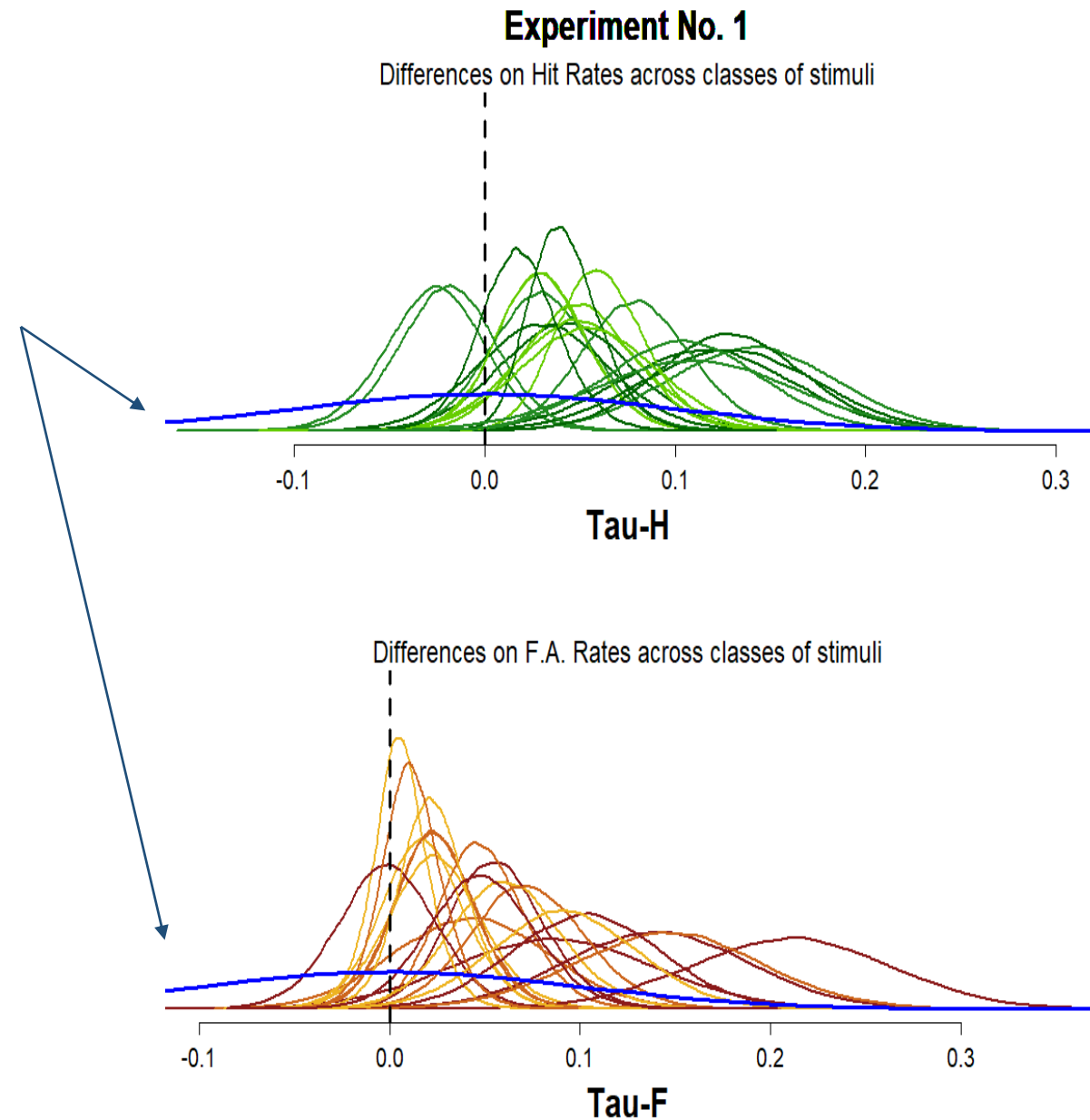
$$\text{prior}(\text{Tau}_i^j \sim \text{Normal}(0,0.2))$$





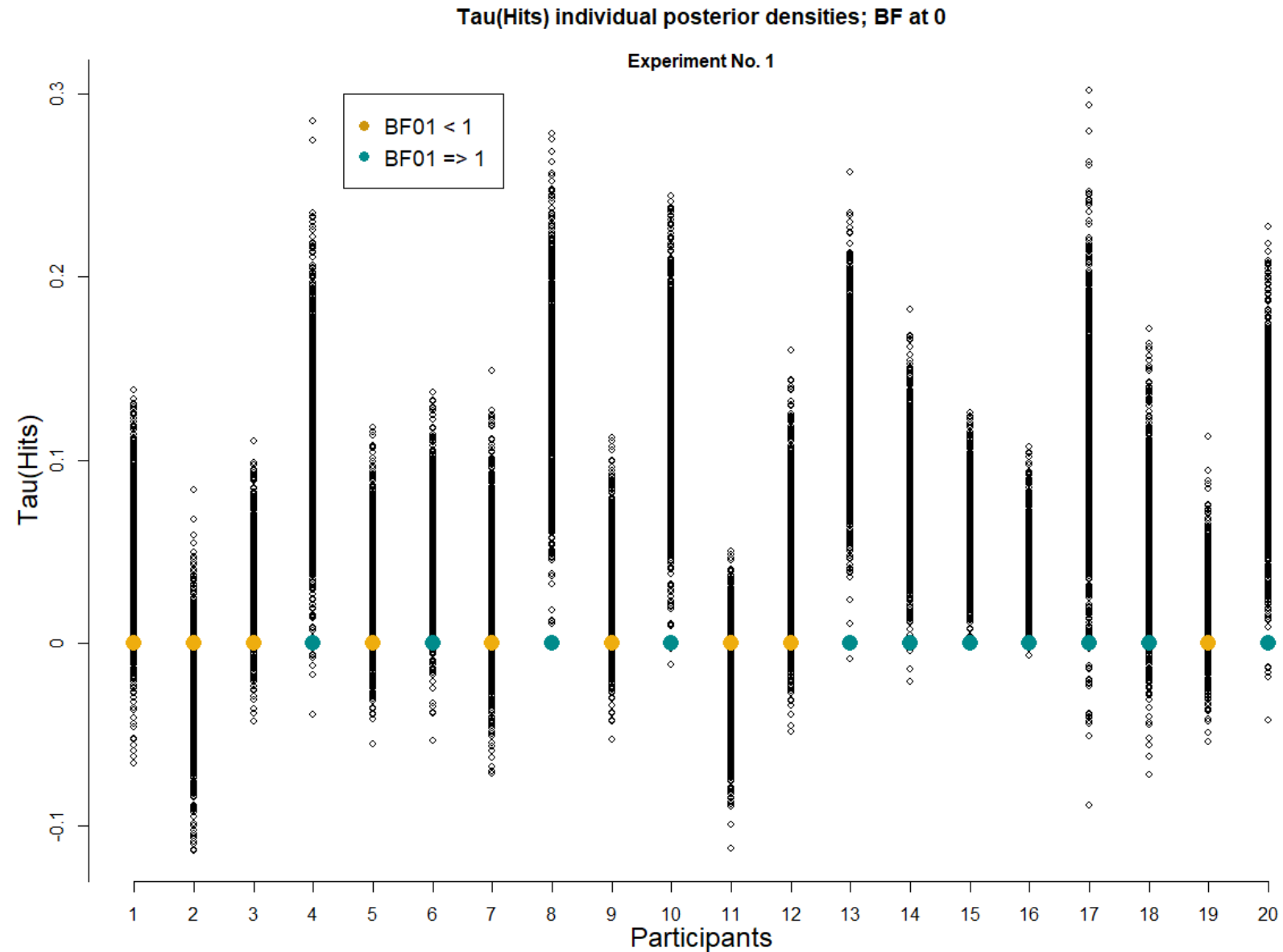
It is under this disclaimer that we conducted a Bayes Factor for every individual Tau taking as a reference the following, not stated in the model, “prior distribution”:

$$\text{prior}(\text{Tau}_i^j \sim \text{Normal}(0, 0.1))$$



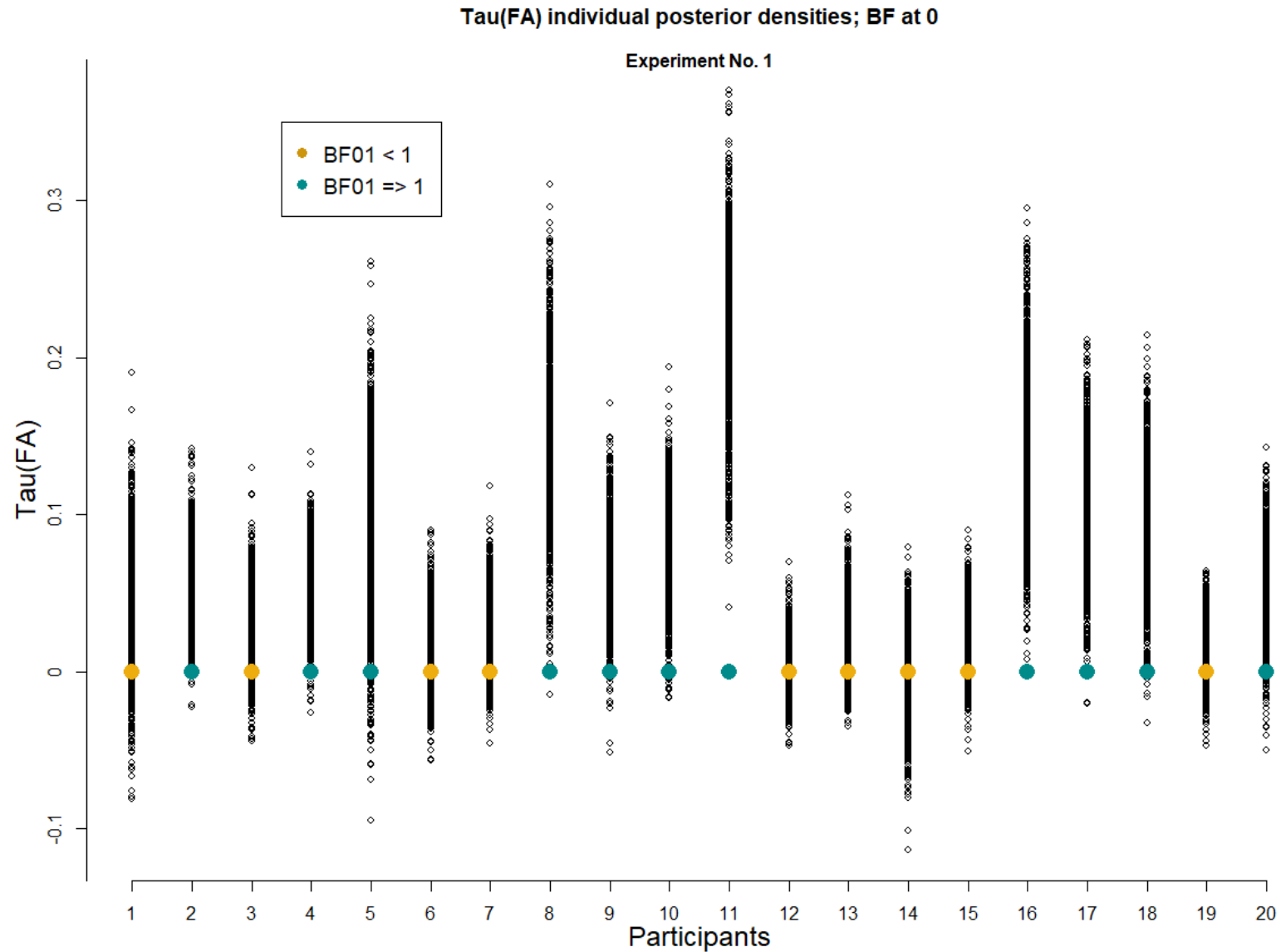
Plot 1

We can see that for **Experiment No. 1**, there are 9 participants (Golden color) who show a **greater posterior density at the 0 difference point between the Hit rates** for each class of stimuli, compared to the artificial prior distribution proposed for reference.



Plot 1

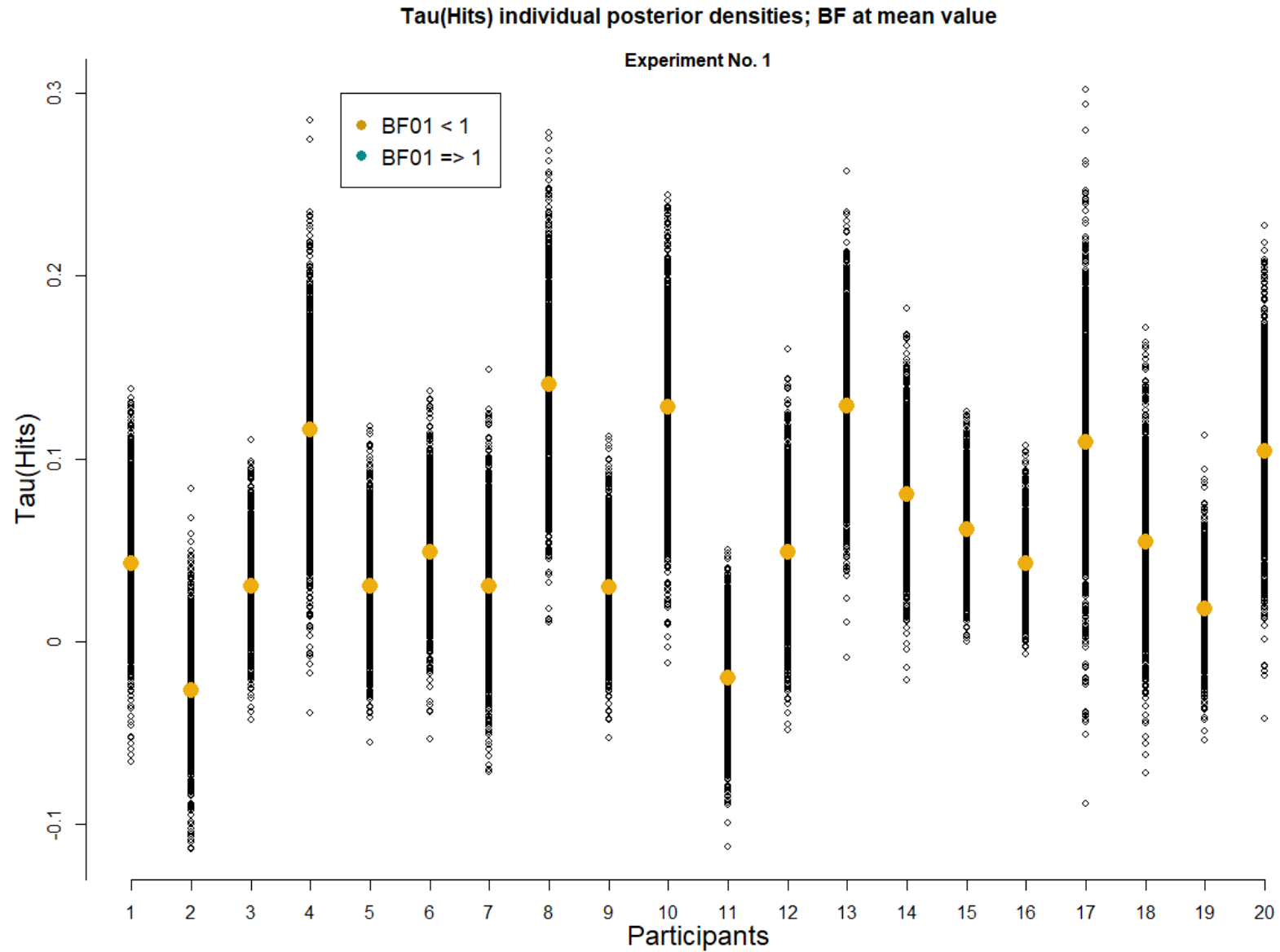
We can see that for **Experiment No. 1**, there are 9 participants (Golden color) who show a **greater posterior density at the 0 difference point between the False Alarms rates** for each class of stimuli, compared to the artificial prior distribution proposed for reference.



Plot 2

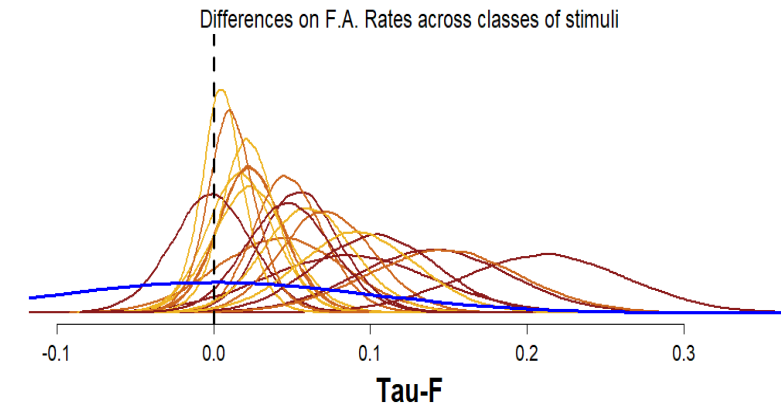
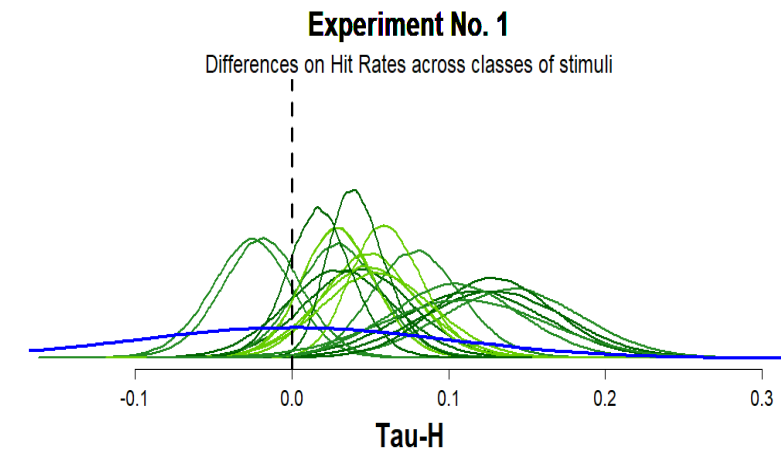
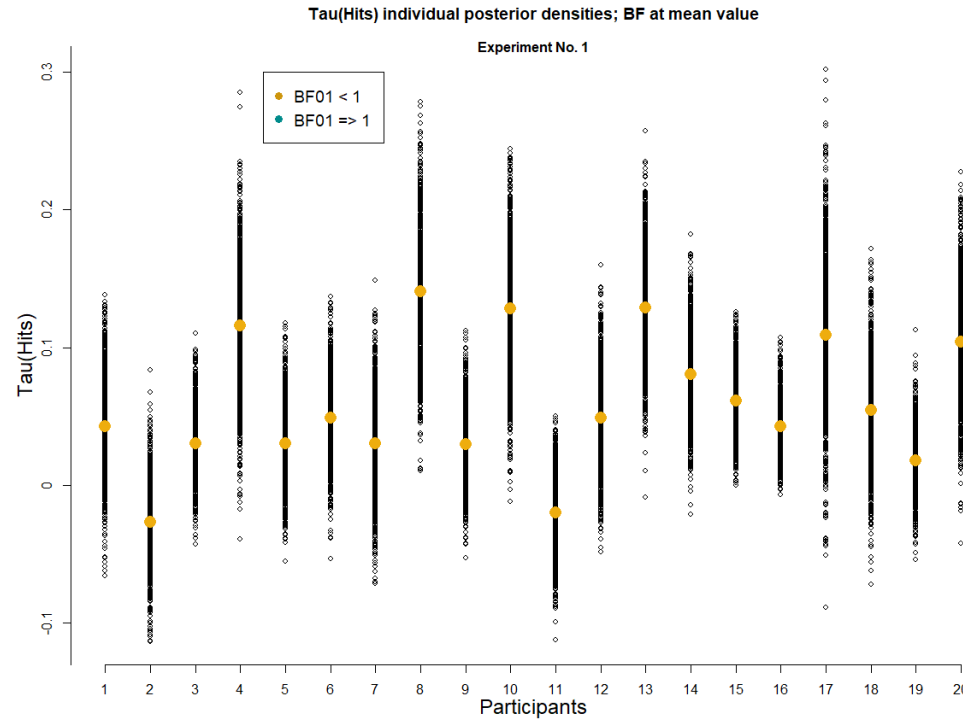
We can see that for **Experiment No. 1**, all participants had a greater posterior density at their mean value than the prior at this very same point.

I have doubts on whether or not this makes sense, please see the next slide for the details...



Plot 2

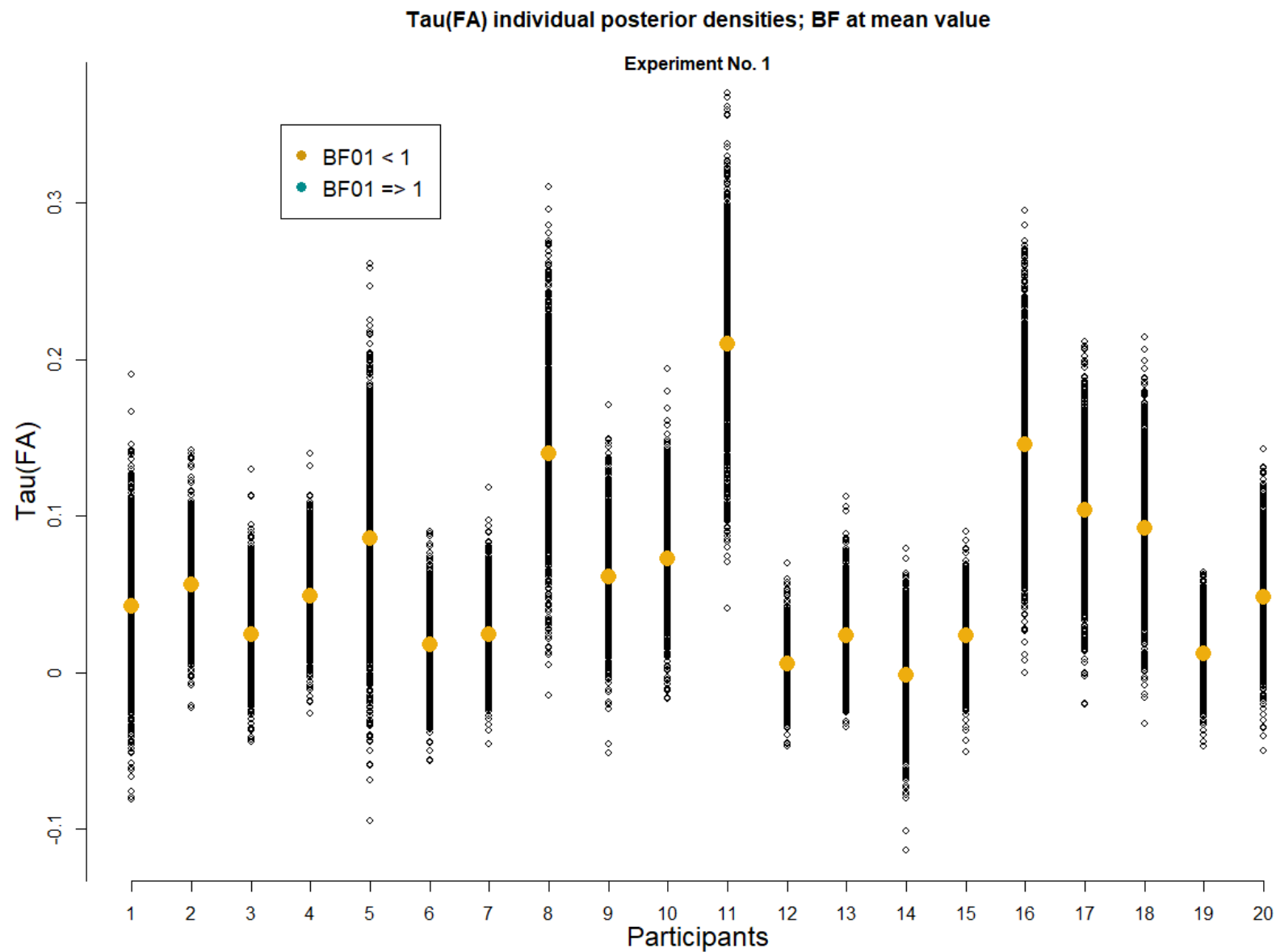
We can see that for **Experiment No. 1**, all participants had a greater posterior density at their mean value than the prior at this very same point.



Does this comparison even make sense? I mean, it's pretty clear that no matter what the posterior distribution looks like... its mean is always going to have a greater density than the artificial prior

Plot 2

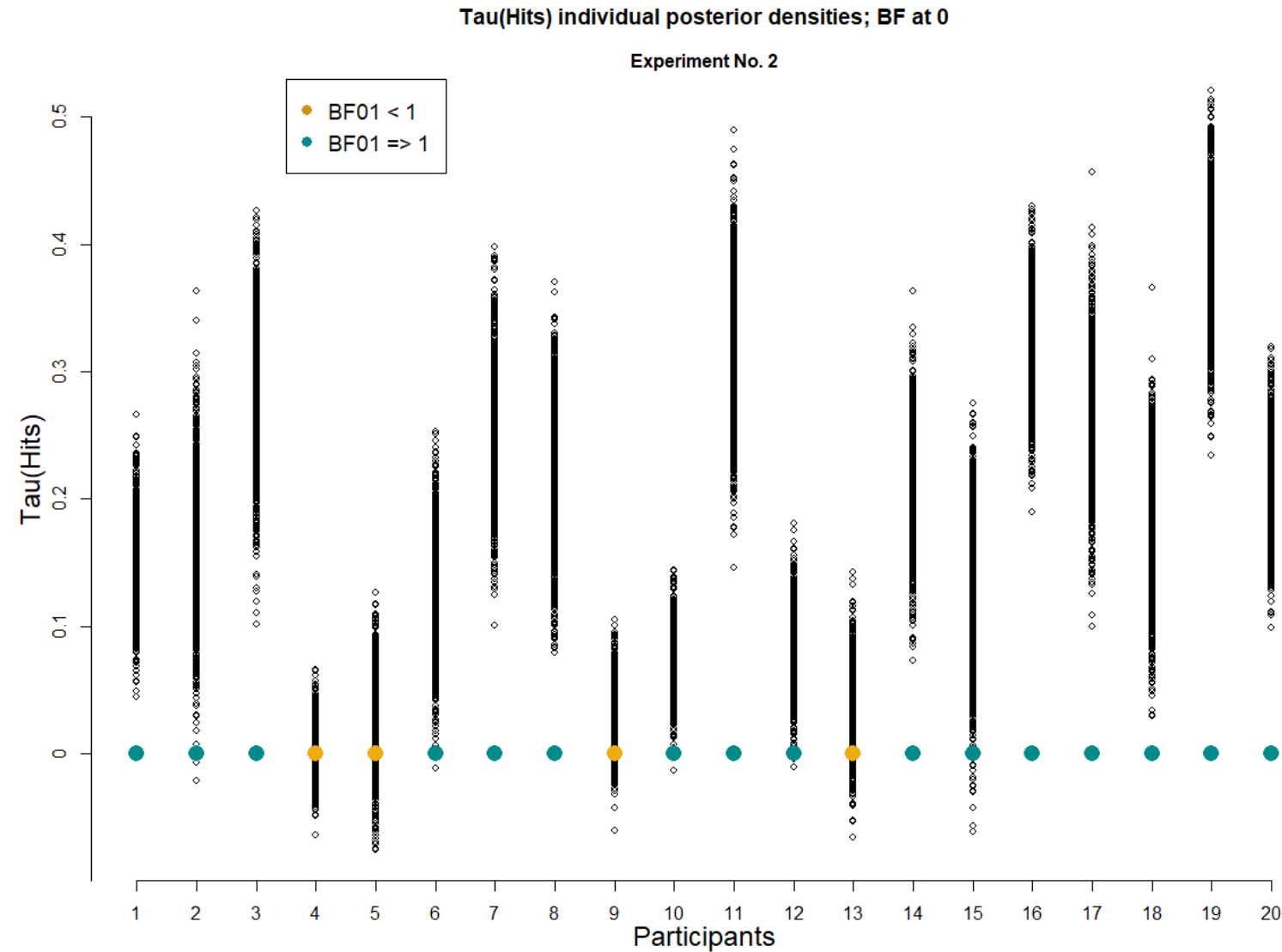
Same thing happens for the differences between the False Alarm rates from each class of stimuli and its Bayes Factor.



Same question as before, of course...

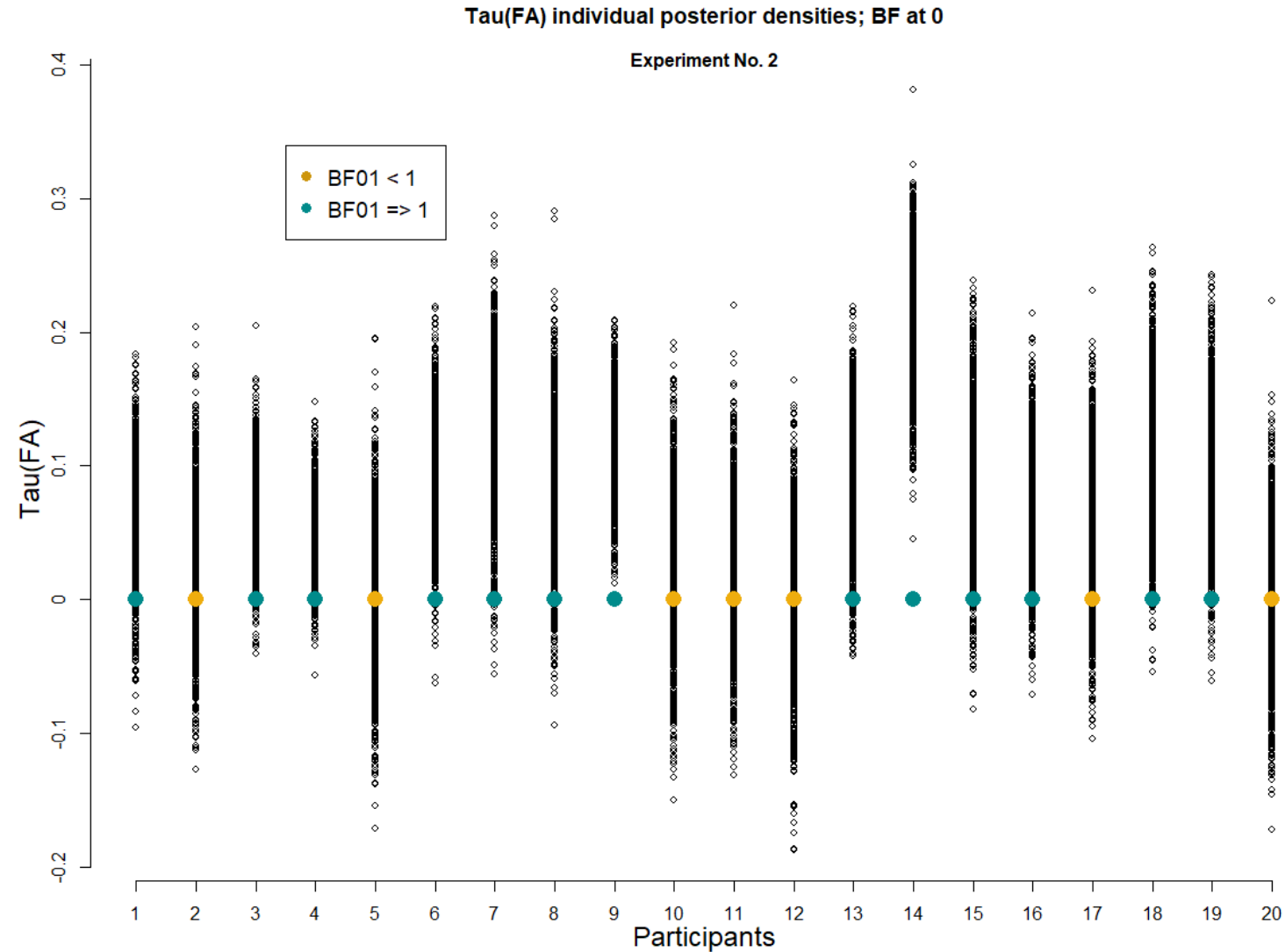
Plot 1

We can see that for **Experiment No. 2**, there are **4 participants** who show a **greater posterior density at the point of “0 differences”** between the Hit rates of each class of stimuli, compared to the artificial prior.



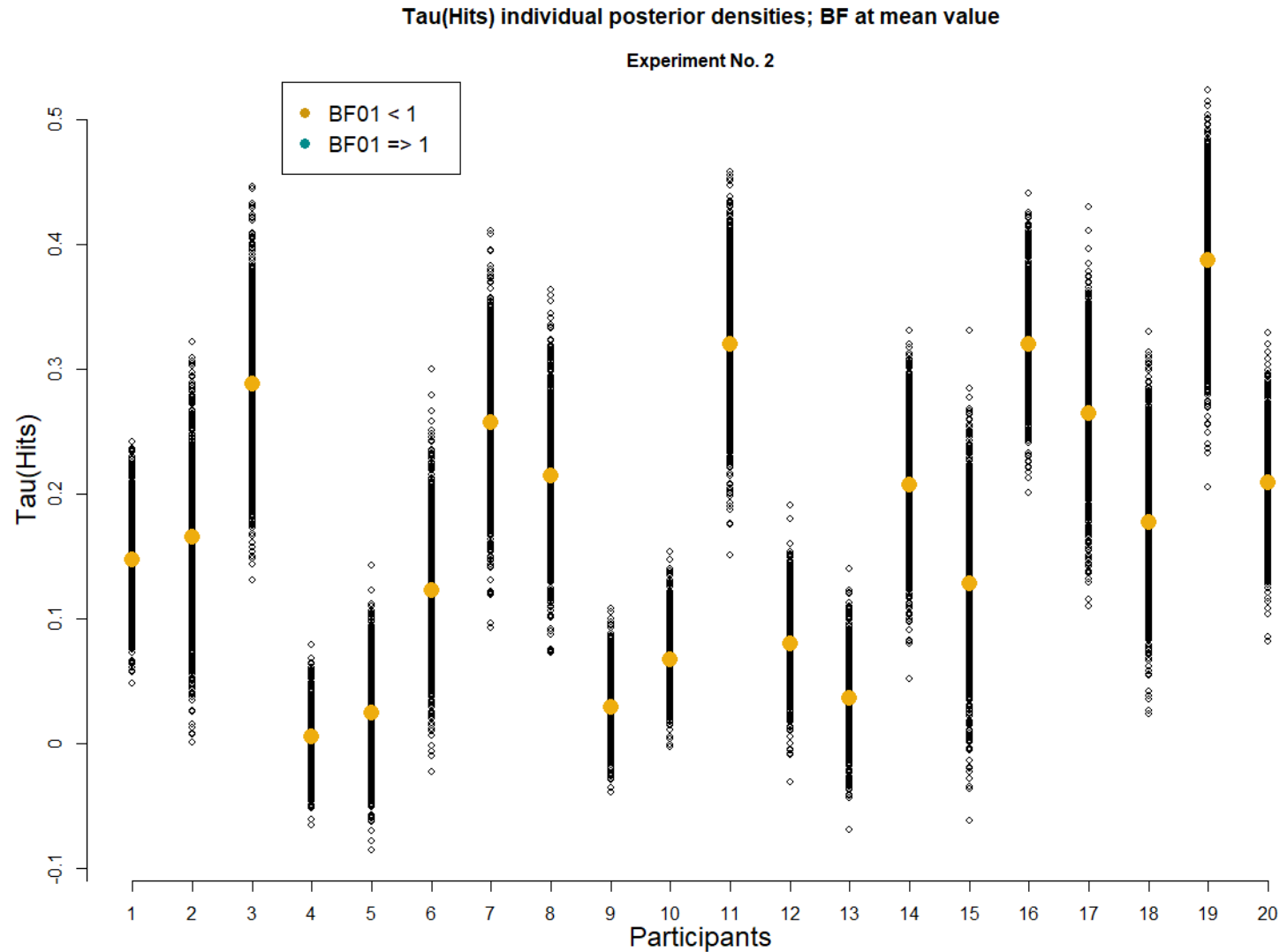
Plot 1

We can see that for **Experiment No. 2**, there are **7 participants** who show a **greater posterior density at the point of “0 differences”** between the False Alarms rates of each class of stimuli, compared to the artificial prior.



Plot 2

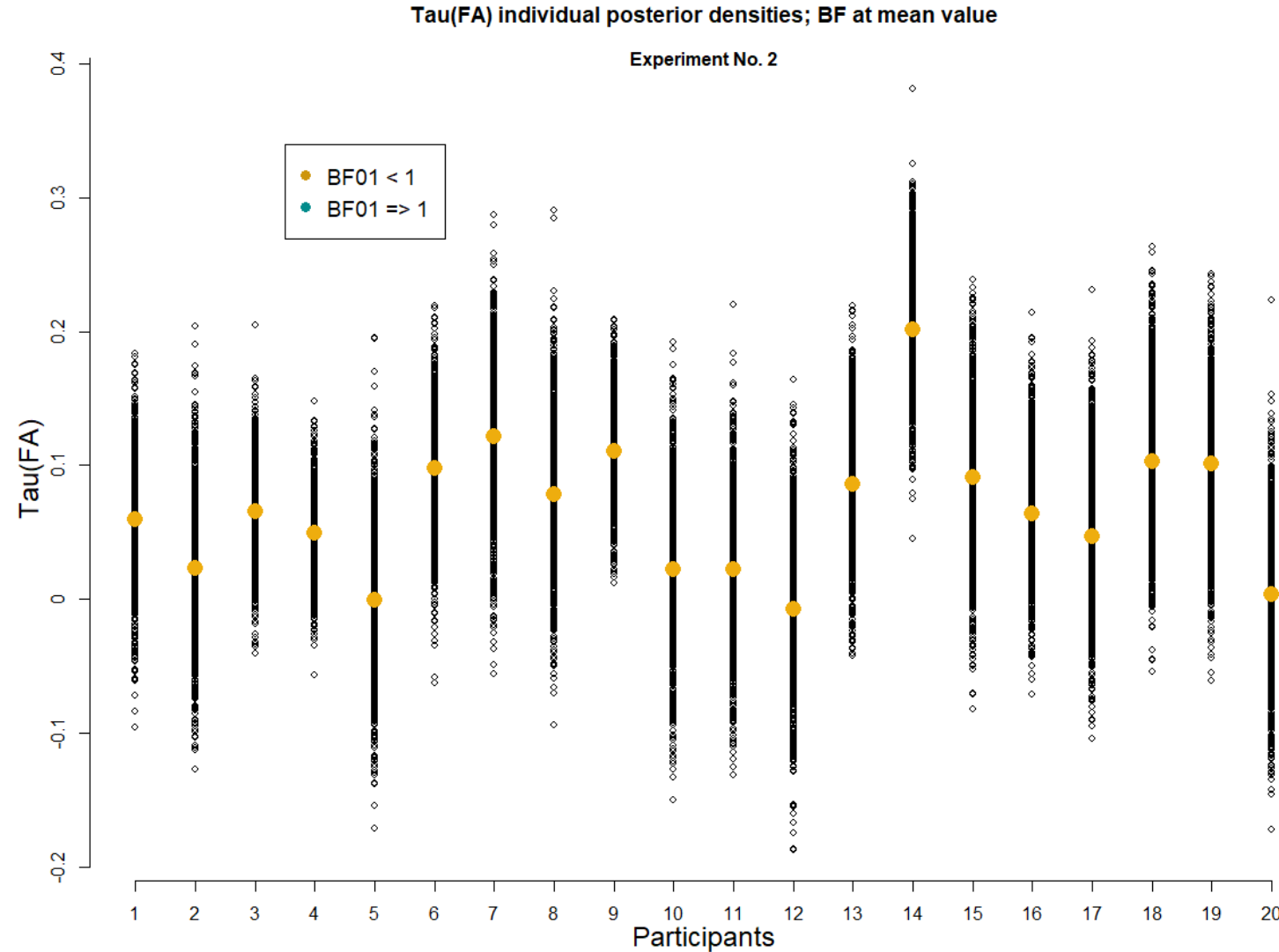
Once again, for the **differences between the Hit rates at Experiment No. 2** when we measure the Bayes Factor between the artificial prior and the posterior distributions at the mean value of the posterior, all participants show a greater posterior density than the prior.



Same question as before, of course...

Plot 2

Once again, for the **differences between the False Alarms rates at Experiment No. 2** when we measure the Bayes Factor between the artificial prior and the posterior distributions at the mean value of the posterior, all participants show a greater posterior density than the prior.



Same question as before, of course...

Some sort of conclusion from the past section

Many of the main findings/empirical phenomena reported within Psychology tend to do it in terms of the “mean performance” of all participants.

It is interesting to note that when we conducted a step-by-step replication of the statistical tests that had been reported within the Recognition Memory literature focused on studying the Mirror Effect (t-test with an arcsine comparison of the response rates), we also find “evidence for the Mirror Effect” in our **merely perceptual** task, (which was itself a very interesting finding in terms of what it suggest about the validity of the models and theories developed to account for the Mirror Effect as a Recognition Memory phenomenom).

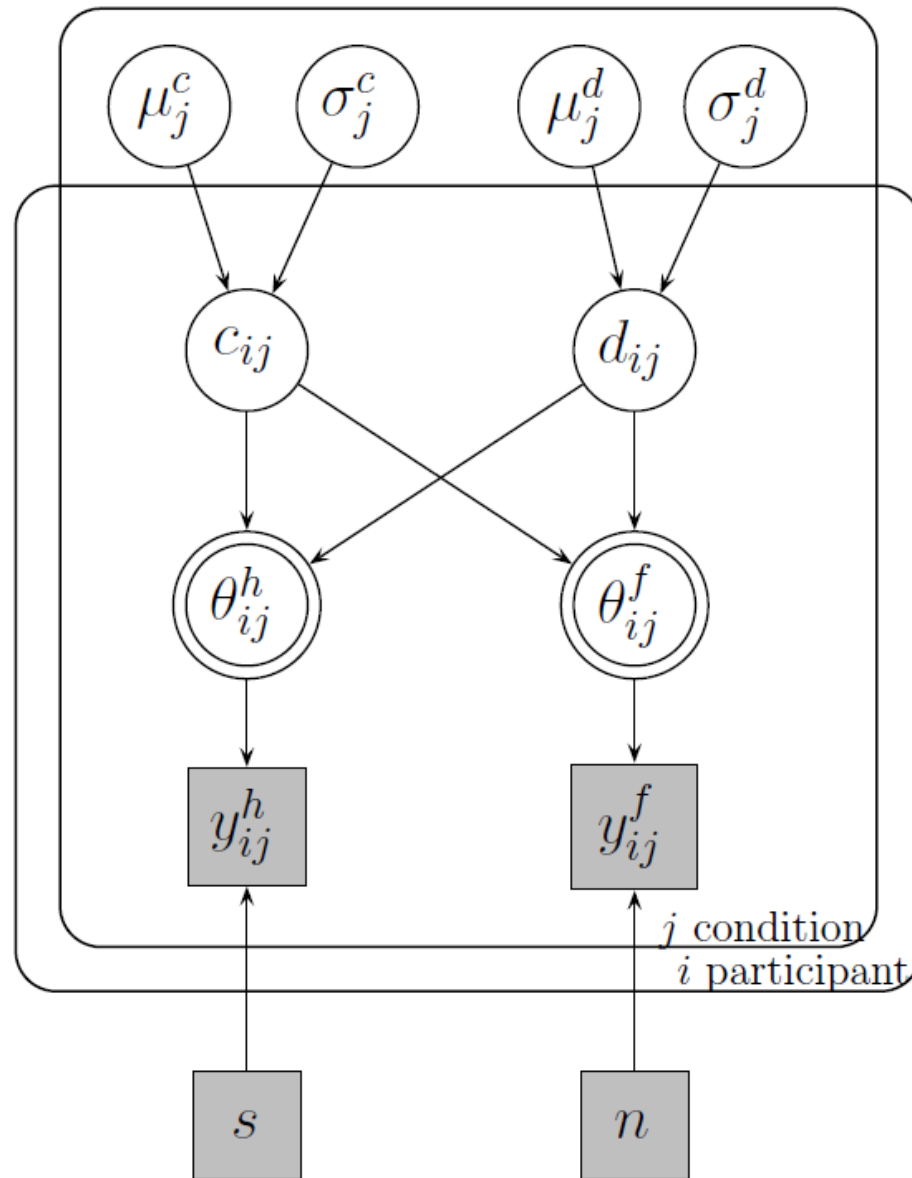
However, the presented variation of a Bayesian SDT cognitive model seems to be suggesting that the Mirror Effect could be a phenomena that holds for the “whole group” analysis, but not the individual level.

The present study could serve as a clear reference of the advantages that Bayesian Cognitive modeling has in terms of the general conclusions that can arise from its application, given the great power it has shown to deal with the individual data.

(I'm trying to stay conservative and not take it too far... but this is the general direction that the conclusions arised from this first part would have)

3. Bayesian hierarchical cognitive modeling of our data

Ok, we already know what's happening in terms of our d' , but, is there any good reason not to look into c estimates as well?



$$y_{ij}^h \sim \text{Binomial}(\theta_{ij}^h, s)$$

$$y_{ij}^f \sim \text{Binomial}(\theta_{ij}^f, n)$$

$$\theta_{ij}^h \leftarrow \phi\left(\frac{1}{2}d_{ij} - c_{ij}\right)$$

$$\theta_{ij}^f \leftarrow \phi\left(-\frac{1}{2}d_{ij} - c_{ij}\right)$$

$$d_{ij} \sim \text{Gaussian}(\mu_j^d, \sigma_j^d)$$

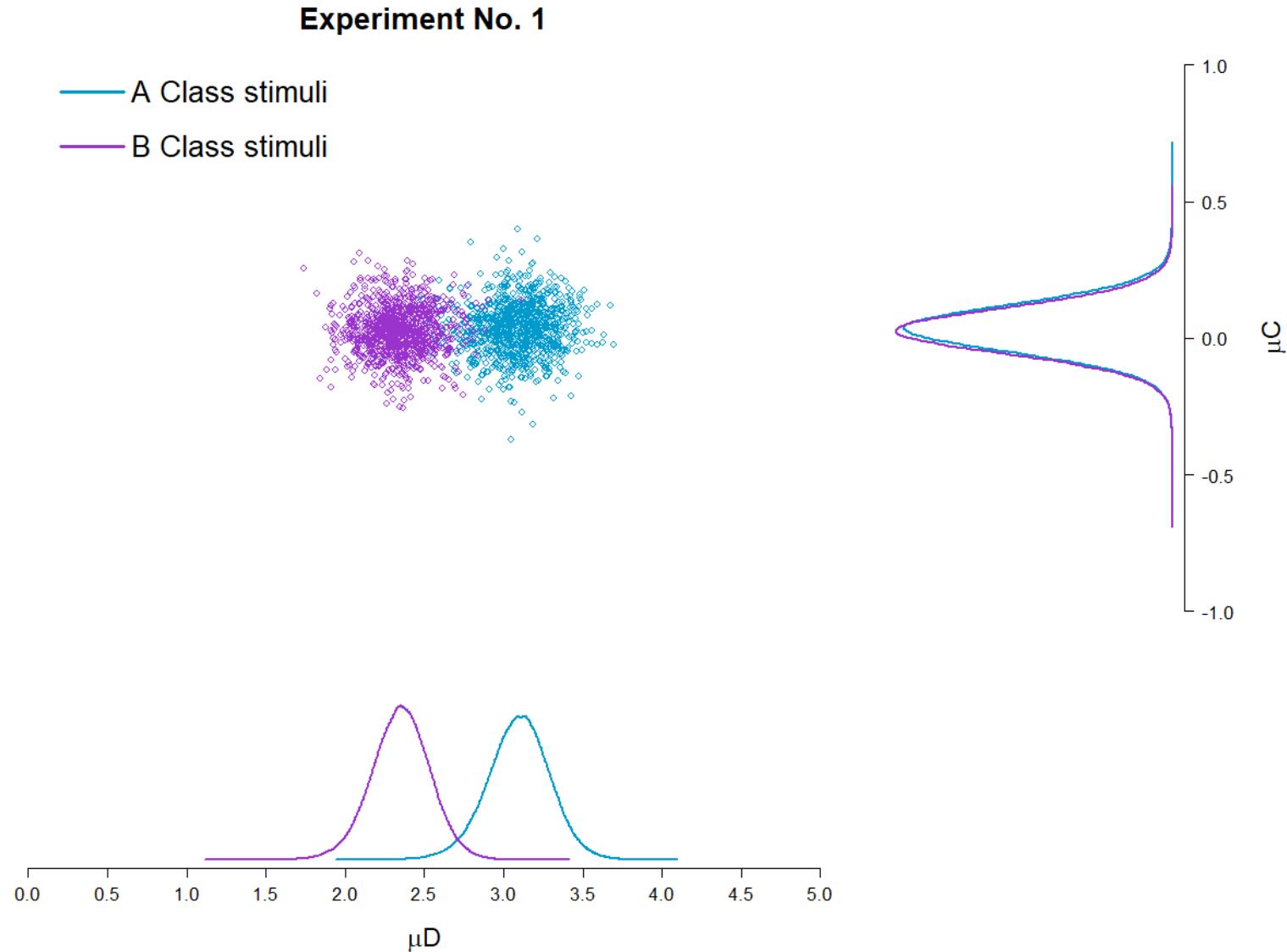
$$c_{ij} \sim \text{Gaussian}(\mu_j^c, \sigma_j^c)$$

$$\mu_j^c, \mu_j^d \sim \text{Gaussian}(0, 2)$$

$$\sigma_j^c, \sigma_j^d \sim \text{Gamma}(1, 1)$$

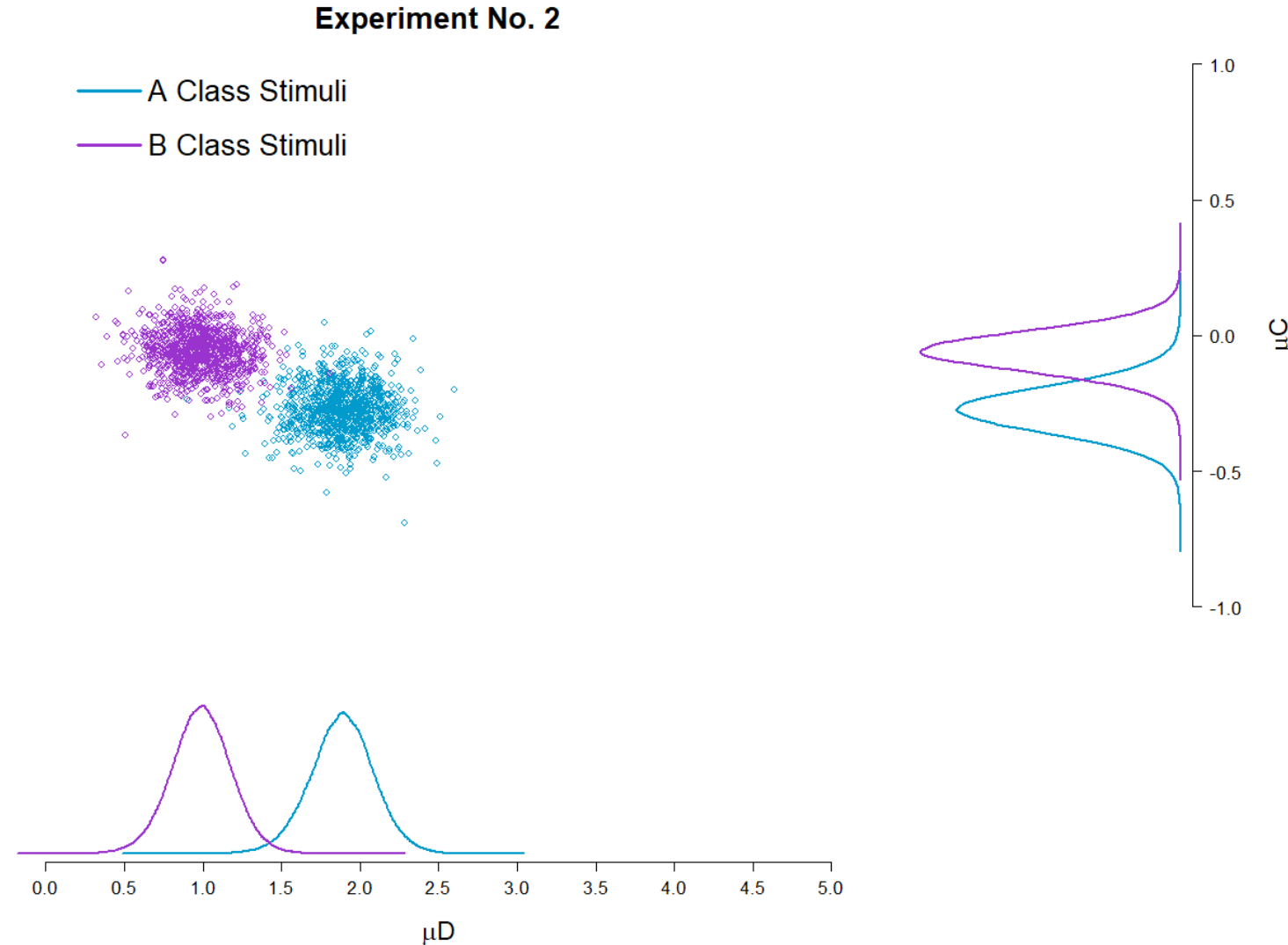
Plot 1

It is interesting to note that **for Experiment No. 1**, yes, some differences are found in terms of the mean d' estimations for each class of stimuli, but this doesn't happen for C, which according to the proposed hierarchical model could be described by the same mean value for both classes of stimuli.



Plot 1

Even more interesting should be the fact that this doesn't hold up **for Experiment No. 2**, where differences between classes of stimuli are observed both for d' and C .



4. Step change model

Do participants B or C across trials?

PENDING

5. Testing an Un Variance

What can confidence rating data?

PENDING

5.1 Are participants
of the Confidential Hearings ?

PENDING