Testing attribute-based models in Intertemporal Choice

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One of the main questions in the intertemporal choice literature is how do we evaluate alternatives that differ in outcome and delay, that is, how do we integrate or compare information to choose between an alternative that has an small reward but it is immediately delivered or an alternative that has a larger reward but its delivery is delayed in time. There are two great family models that describe these kinds of choices that have a different integration rule, one is the alternative-based and ~~the other is~~ the attribute-based family. Interval effects is an empirical phenomenon that distinguishes between both families. The objective of the present work is to study interval effects and compare, with Bayesian tools, models that can describe this phenomenon. Results showed that there were subjects that followed the rule, while others didn't. The Direct Differences was the simplest and best model describing the present data set, among the attribute-based models.

Daily, organisms face situations in which they have to choose among outcomes that involve tradeoffs among consequences occurring at different points in time, which are named Intertemporal Choices (Frederick, Loewenstein & O'Donoghue, 2002). There are a lot of examples of these choices, one is when a person has to choose between saving money for retirement or to spend it immediately. Another example would be when a person has to choose between eating a hamburger or ~~to~~ continue the diet. In both examples, one ~~option~~ will fulfil one's needs instantly, while the other option will get you more satisfaction but in the future. In these kinds of choices, it is being observed that people are eager to choose the situation where the reward is immediate but small than to wait for the larger reward. In these examples, it happens that they are used to spend money or eat the hamburger immediately than to save money for retirement or to continue with the diet to look better in the future.

To study these situations, intertemporal choice researchers use a task where people have to choose between a smaller sooner reward (SSR) or a larger later reward (LLR). It is being proposed that people might be choosing the smaller sooner reward over the larger later one, due to the last one loses its effectiveness of reinforcement as it is delayed from the moment of choice (Ainlie, 1974; Mazur, 1984; Rachlin & Green, 1972). Specifically, the change in the value of a reward as a function to its temporal proximity is termed delay discounting (Green, Fry & Myerson, 1994; Rachlin & Green 1972).

Value-based decision making is a process that involves decomposing the alternatives into a set of attributes. It is based on these attributes that an alternative is evaluated and subsequently chosen or rejected (Bhatia, 2013; Bhatia & Stewart, 2018; Rangel, Camerer, & Montague, 2008). This process can be described by choice models which ~~have~~ two elements[1]. The first one is an integration rule that associates or calculates the attributes of the available alternatives into values, also ~~known as~~ the core element of the model. For example, the most used models in intertemporal choice have the integration rule of first calculating the attributes of the same alternative into one value, like the utility of alternatives. The second element is a decision rule that determines which of the alternatives is going to be chosen given the values assigned by the integration rule. For example, choosing the alternative that has more value ~~would be one rule~~.

The objective of this work is to model and evaluate the integration rule contrasting two family models, ~~one is~~ the *alternative-based models* and the ~~other is the~~ *attribute-based models*. These models have different integration rules that describe if people evaluate options independently assigning a ~~value for~~ each alternative or comparing the attributes within alternatives (Scholten, Read & Sanborn, 2014).

To describe the difference between both families, take the example of ~~a choice~~ between a smaller sooner reward (A= $160 in 1 month) and a larger later reward (B= $300 in 4 months). On one hand, in the alternative-based models, the choice is made assigning a subjective value to each alternative A and B, independently. Each value is discounted as a function of its delay, in other words, the outcome is ponderated by the delay, and once the subjective values are calculated, people choose the alternative that has the highest value (Green & Myerson, 2004; Rachlin, Raineri & Cross, 1991). For example, assuming a standard discount, the alternative A would value 139 dollars and the alternative B would value

[1] See Dai & Busemeyer (2014), to explore other elements of choice models.

187 dollars. Because 187 > 139, the alternative to choose is B.

On the other hand, in attribute-based models, options are directly compared among their attributes, and the option favoured by these comparisons is chosen. In other words, the difference between winning now or winning later will be compared against winning ~~more or less~~ (Scholten et al., 2014). Attribute-based functions weight the time advantage vs the outcome advantage. Following ~~the example of the alternatives A and B presented before~~, the difference between outcomes would be 140 dollars and ~~for delays would be~~ 3 months. In this case, it is assumed that there is more ~~advantage~~ waiting ~~for~~ less, than winning more, therefore these differences favour the choice for alternative A.

The difference between the family models lies over the assumption of *additivity in intervals*, alternative-based models assume it while attribute-based models don't. This assumption refers to the total discounting over an interval should not depend on whether and how the interval is subdivided. For example, the discount of 1 year should not depend on whether the year is divided into 12 months ~~(January December)~~ or left undivided (Scholten & Read, 2010). Attribute-based models don't follow additivity in intervals, which allows them to describe some empirical phenomena that alternative based models can't. These phenomena are known as *interval effects*, specifically as subadditivity and superadditivity that are more or less discount in intervals when the periods are divided or undivided (McAlvanah, 2010; Scholten et al., 2014; Scholten & Read, 2006).

First, we will describe the representative models of each family and then the additivity assumption ~~will be presented in detail~~.

Alternative-based models

In this family models, the integration rule ~~is~~ assigning a subjective value to each alternative, then a decision rule could be to choose the alternative with the highest value. Each model in this family describes in a different way the change in the subjective value thought time. The function shape describing the change in subjective value over time is critical to describe several aspects of intertemporal choice (Green, Myerson & McFadden, 1997). These models are the exponential, hyperbolic and hyperboloid, and all of them have dominated the literature in intertemporal choice (Urmisky y Zauberman, 2014).

Consider two alternatives, a smaller (x_s) sooner (t_s) reward, and a larger (x_l) later (t_l) reward. According to the Estandar Economic Theory, the discounting utility of an alternative is given by the exponential function (Samuelson, 1937) assign to each alternative:

$$V_{SS} = (x_s \cdot e^{-k \cdot t_s}) \quad \& \quad V_{LL} = (x_l \cdot e^{-k \cdot t_l}) \tag{1}$$

In this equation, V_{LL} is the subjective value of the larger latter reward and V_{SS} is the subjective value of the smaller sooner reward. k parameter determines the rate at which value decreases with delay: a larger k is associated with steeper discounting, ~~and a smaller k is associated with shallower discounting~~ of the future reward's value. Once the subjective values of each alternative are obtained one rule would be to choose the one that has more value. However, if we have several repetitions of the same question a decision rule ~~could be the one~~ proposed by Luce (1959) that describes the choice as a probabilistic, not ~~an~~ algebraic, phenomenon:

$$P(LL) = \frac{V_{LL}}{V_{LL} + V_{SS}} \tag{2}$$

Exponential discounting predicts that individuals most have consistent preferences thought time, assuming a constant discount rate (Green & Myerson, 1996; Green, Myerson & MacFadden, 1997). However, it is been observed that people are not consistent with their temporal preferences, therefore the hyperbolic model was proposed as an alternative to describe temporal discounting.

~~Taking into consideration~~ the same elements as in Equation 1, the hyperbolic model is ~~the following~~ (Mazur, 1984):

$$V_{LL} = \left(\frac{x_l}{1 + k \cdot t_l}\right) \quad \& \quad V_{SS} = \left(\frac{x_s}{1 + k \cdot t_s}\right) \tag{3}$$

where the probability of choosing the larger later reward could be assing with equation 2. The hyperbolic discounting model implies that the discount rate diminishes with time, while the exponential discount remains constant through time. ~~In other words,~~ the hyperbolic function predicts that the rate of temporal discounting ~~is steeper~~ at the beginning ~~of the period,~~ but is shallower with later delays. ~~In contrast with the exponential model where the discount rate is constant thus time independent~~ (Green & Myerson, 1996). The difference in the rates allows ~~that~~ the hyperbolic model can describe *preference reversal* that is when subjects prefer larger delayed good over a smaller, less delayed one, but as the passage of time makes the smaller one imminent, they change their preferences and choose the smaller less delayed good (Killen, 2009; Green, Fristoe & Myerson, 1994).

Another model that has been used to describe temporal discounting is the hyperboloid function (Green, Fry & Myerson, 1994):

$$V_{LL} = \left(\frac{x_L}{(1 + k \cdot t_L)^{\tau}}\right) \quad \& \quad V_{SS} = \left(\frac{x_S}{(1 + k \cdot t_S)^{\tau}}\right) \tag{4}$$

This function has an additional free parameter τ that represents the nonlinear scaling of amount and time, with less than 1 value, the discounting curve decrements smoothly with larger delays. Notice that ~~in the special case~~ when s equals 1.0, Equation 4 reduces to Equation 3. Numerous studies have compared these three functions to discounting data, and almost invariably, the hyperboloid always provides the better fit, often even after controlling the fact that it has an additional free parameter (Green, Myerson & Vanderveldt, 2014; McKerchar et al., 2009; Myerson & Green, 1995).

Attribute based-models

In the attribute-based family, the integration rule takes the differences between attributes, in other words, the delays and the amount ~~of the outcomes will be compared in-between alternatives~~, each alternative ~~will favour~~ one alternative ~~more than the other which will determine the chosen alternative.~~ There are several models in this family, the in present work we are going to describe three: the Trade-off model (Scholten et al., 2014), the ITCH model (Ericson et al., 2015) and the Direct Differences model (Dai & Busemeyer, 2014; Gonzalez-Vallejo, 2002).

~~Considering again two alternatives, a smaller (x_s) sooner (t_s) reward and a larger (x_l) later (t_l) one,~~ this model holds that intertemporal choice is governed by an attribute-based integration rule, in which the outcome advantage of one option is weighted against the time advantage of the other, ~~and the option with the greatest advantage is chosen~~ (Scholten et al., 2014):

$$P(LL) = \frac{((v(x_l) - v(x_s))^{\frac{1}{\varepsilon}}}{(v(x_l) - v(x_s))^{\frac{1}{\varepsilon}} + (Q(w(t_l) - w(t_s)))^{\frac{1}{\varepsilon}}} \qquad (5)$$

where

$$Q(w(t_s), w(t_l)) = \frac{\kappa}{\alpha} log\left(1 + \alpha\left(\frac{w(t_l) - w(t_s)}{\vartheta}\right)^{\vartheta}\right)$$

~~and $v(x) = \frac{1}{\gamma}log(1 + \gamma x)$ and $w(t) = \frac{1}{\tau}log(1 + \tau t)$, change the subjective value of amounts and delays; x is for amount and t is for delays. The shape of $v(x)$ and $w(t)$, is concave when τ or γ have high values and lineal when they are close to zero, meaning that both parameters reduce the actual value of time and outcome. The tradeoff function, $Q(w(t_s), w(t_l))$, is an S-shaped function over intervals, which accommodates a progression from superadditive (ϑ) to subadditive (α) discounting over intervals of increasing length. All these functions get in $P(LL)$ that assigns the probability of choosing the larger later reward given the differences.~~

Another model that uses the difference between attributes is the ITCH (Intertemporal Choice Heuristic), developed by Ericson, White, Laibson and Cohen (2015):

$$P(LL) = \qquad (6)$$

$$L\left(\beta_0 + \beta_{x_A}(x_l - x_s) + \beta_{x_R}\frac{x_l - x_s}{x*} + \beta_{t_A}(t_l - t_s) + \beta_{t_R}\frac{t_l - t_s}{t*}\right)$$

where β_0 is the intercept, $_R$ means relative and $_A$ absolute, and $(x*, t*)$ represents a reference point that is the arithmetic mean of the two alternatives along each dimension $x* = \frac{x_s + x_l}{2}$, $t* = \frac{t_s + t_l}{2}$. L is the cumulative distribution function of a logistic distribution with ~~a~~ mean ~~of~~ 0 and ~~a~~ variance of 1. Thus, each term of the model represents either an absolute or proportional arithmetic operation that compares the options along a particular dimension (outcome value and time). Each term is multiplied by a parameter~~,~~ β, that represent the weight given to each ~~heuristic~~ when deciding between the two alternatives. The weighted sum of the outcomes predicted by each heuristic ~~then~~ determines the probability of choosing the larger later reward.

The following ~~presented~~ model is a combination of two equations, the integration rule from Dai and Busemeyer (2014) and the decision rule from Gonzalez-Vallejo (2002):

$$P(LL) = \Phi\left(\frac{d - \delta}{\sigma}\right)$$

where

$$d = w(x_l - x_s) - (1 - w)(t_l - t_s) \qquad (7)$$

and d is the difference in-between alternatives, ~~the parameter~~ w can be interpreted as the amount of attention allocated to the money attribute and $1 - w$ ~~the corresponding amount~~ to the delay attribute. The original equation proposed ~~to describe intertemporal choices from~~ Cheng & Gonzalez-Vallejo (2016) considers the same decision rule but the integration rule, with two-dimensional choice options, is the following:

$$d = \left(\frac{max\{|x_l|, |x_s|\} - min\{|x_l|, |x_s|\}}{max\{|x_l|, |x_s|\}}\right) - \left(\frac{max\{|t_l|, |t_s|\} - min\{|t_l|, |t_s|\}}{max\{|t_l|, |t_s|\}}\right) \qquad (8)$$

The Direct Differences Model differs from the Proportional Differences model in the integration rule, the first one considers direct differences of the attributes, while the second one, considers relative differences. The decision rule, given by the probability of choosing the larger later reward, represents the overall advantage ~~of~~ the LL option over the SS option, and Φ is the cumulative distribution function of a standard normal distribution. The decision threshold, δ, is a free parameter that depicts the relative importance of a ~~person to attribute differences; it also represents~~ the degree to which a decision-maker ~~differentially~~ weighs the attributes ~~in the trade-off.~~ It is seen that $\delta < 0$ is the propensity to select one attribute, and that for $\delta > 0$, this propensity decrease~~s.~~ Finally, σ is a measure of the variability in utility difference.

Additivity

The most important assumption that distinguishes ~~the~~ alternative based models from ~~the~~ attribute-based models, is that the ~~first one assumes additivity, while the second doesn't.~~ *Additivity in intervals* ~~entitles~~ that the total discounting over an interval should not depend on whether and how the interval is subdivided (Cheng & Gonzalez Vallejo, 2016; Read, 2001; Scholten, & Read, 2010). To show additivity in intervals we present figure 1. in which we have 4 bars, each one represent~~s~~ a question, the leftmost end indicates the smaller reward, and the rightmost end the larger reward. The delays and outcomes are indicated in the superior axis and the letters A, B, C and D identify each alternative. Question 1, for example, offers A=*$ 5150 pesos in one week* against B=$5300 pesos in two weeks. This graphic represents two procedures: a segmented one and non-segmented one; questions 1, 2 and 3~~,~~ are the segmented procedure where ~~the~~ alternatives are divided into subintervals, while question 4 is the non-segmented procedure with an undivided alternative that covers ~~the~~ subintervals. Following the additivity assumption means that the two procedures most ~~have~~ the same discount, in other words, the discount is additive when both procedures produce the same subjective value (Scholten & Read, 2010). Contrary, the discount is not additive when the subjective values of both procedures evoke different results. As a matter of fact, attribute-based models assume no-additivity, proposing that the subjective values are not just in function of its delay, but also depend on the interval between the delays of each alternative (Read, 2001).

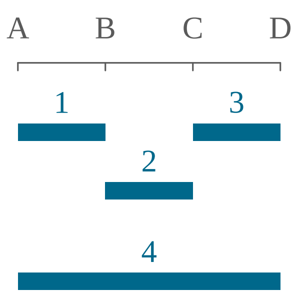


Figure 1. Each bar represents a question, the leftmost end indicates the smaller reward and the rightmost end the larger reward. Questions 1, 2 and 3 represent the segmented procedure; question 4 represents the complete procedure.

Following the example, one measure used to represent additivity is the Discount fraction (Cheng & Gonzalez-Vallejo, 2016) of an interval:

$$F[t_A, t_B] = \frac{1 + \kappa * t_A}{1 + \kappa * t_B} \tag{9}$$

that means the proportion of the money that is subjectively left after this interval. For this case, is about the period between alternatives A and B (question 1). Therefore, the models that satisfy additivity in intervals will have to satisfy the following equivalence:

$$F[t_A, t_D] = F[t_A, t_B] * F[t_B, t_C] * F[t_C, t_D] \tag{10}$$

Where the discount fraction of a complete interval (question 4), most be equivalent to the discount fractions of the subintervals combined (questions 1, 2, and 3). If the discount is equivalent between the discount fraction of the complete intervals and the multiplication of the subinterval fractions, then people should choose the same alternative in both intervals and subintervals. In other words, if they chose the LLR in the subintervals, they most choose it also in the complete interval, and in the same way with the SSR, they have to choose consistently in subintervals and the complete intervals.

However, some studies have found choice inconsistencies over this assumption and found *Interval~~s~~ effects* (McAlvanah, 2010; Scholten & Read, 2006; 2010; Read, 2001), specifically they are:

Superadditivity: more discount over complete intervals than over subintervals, and empirically mean~~s~~ choosing the smaller sooner reward in full intervals (question 4), and the larger-later in subintervals (questions 1, 2 and 3).

Subadditivity: more discount over subintervals than over full intervals, and empirically mean~~s~~ choosing the smaller sooner reward (questions 1, 2 and 3) in subintervals, and the larger later one in intervals (question 4).

Choice Variability

In the studies that have tested the attribute-based models, researchers not necessarily have taken into consideration the stochastical nature of choice, observed in the individual preferences when the same alternatives are presented several times. The experimental protocol of repetition allows untying the behavior variability from the structural inconsistency of preferences. As a matter of fact, it is being found that if people take into consideration choice variability, the subjects present preferences that do not violate the additivity assumption (Cavagnaro & Davis-Stober, 2014; Dai, 2016; Gonzalez-Vallejo, 2002; Regenwetter, Dana & Davis-Stober, 2011; Regenwetter & Davis-Stober, 2012). It is important to mention that these authors evaluated the inconsistency of preferences thought the transitivity axiom (for alternatives A, B and C, the preference of A over B and B over C implies the preference of A over C) with stochastical versions of it. The violation of this axiom (intransitivity) (Tversky, 1969) would be similar to the violations from the additivity assumption but with risky rewards (Scholten et al., 2014).

Additionally, in the studies that found interval effects, they analyzed the data assuming that there is just one parameter describing the behavior from all subjects. However, it is known that results analyzed in a pooled way can substantially differ from the results evaluated at an individual level (Estes, 1956, Wagenmakers, Lee, Lodewycks & Iverson, 2008). For example, the Condorcet paradox shows that individual preferences enter in conflict with majority preferences, especially when the preferences differ from one individual to another. Actually, it is being found that if data is analyzed individually, few people present inconsistencies (Dai, 2016; Dai & Busemeyer, 2014; Regenwetter, Dana & Davis-Stober, 2010). This shows the importance of observing individual data and analyze how they differ from the population, especially intending to found conclusions about the empirical phenomena and the process behind it and not regarding the way the data was analyzed.

The alternative-based family has dominated the intertemporal choice literature. However, this family can't describe interval effects, while the attribute-based family can. Given both families models and the different approximations to the problem of preference inconsistency (i.e. intransitivity), it has been observed that interval effects aren't replicated if you consider choice variability and individual data analysis. Due to different results found regarding this topic, it was decided to do an experiment that would replicate interval effects taking into account choice variability and individual data analysis. Additionally, we studied and compared different models, with Bayesian tools, to find the one that better describes the data.

Method

Participants. 25 School of Psychology students. For their participation, they entered a raffle where they could win a card of Netflix, iTunes or Spotify, according to their preference.

Procedure. The procedure was made in one session that lasted about 35 minutes. Each individual presented the experimental task in a closed cubicle, without noise and in a desk computer. All subjects read and sign an informant consent. The task was developed in PsychoPy v1.83.04 (Pierce, 2007). The positions of the smaller and larger options were randomized across trials (see Appendix A for instructions).

Experimental Desing. The task was based on the Scholten et al. (2014) ~~2nd study.~~ The design consisted of 12 fixed alternatives that have within them linear increments; the increment was \$150 Mexican pesos for outcomes and one week for time. The smallest outcome was 5150 and the largest was 6500, the shortest interval was one week and the largest was 10 weeks. The combination of pairs alternatives created 22 questions which were classified in 4 sets: 1) Small intervals, 2) Medium intervals, 3) Long intervals/large outcomes, and 4) Long intervals/Small outcomes (see Appendix B). To consider choice variability, each question was presented 10 times, in such manner, there were 220 trials. This method allows to evaluate intervals and subintervals, for example, questions 1, 2, 3, 7, 8 and 9 have the smallest intervals and are subintervals of the question 18, that has the largest interval.

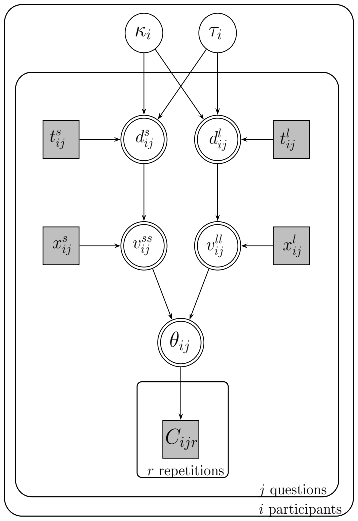
Bayesian Cognitive Modeling

The evaluated models were 1) hyperboloid, 2) ITCH, 3) Trade-off, 4) Proportional Differences and 5) Direct Differences, the last one is a version of the fourth model. There are several advantages, as well as, examples of intertemporal choice models evaluated by Bayesian Statistics (see Chavez et al, 2017; Lee, 2018; Nilsson, Rieskamp & Wagenmakers, 2011; Vincent, 2016; Wagenmakers et al, 2016; 2008). Among the advantages is that allows to describe uncertainty over the possible parameter values thought posterior densities, besides of allowing to estimate parameters at an individual and group levels, without changing the assumptions of the mathematical model.

The ~~used~~ notation for describing Bayesian analysis was adopted from Lee & Wagenmakers (2014). In this representation, shaded nodes correspond ~~to~~ observed variables, whereas unshaded nodes stand for latent variables. Double-bordered nodes are ~~deterministic,~~ while single-bordered nodes are stochastic. Circles represent continuous variables, and squares portray discrete variables.

All models share ~~the following structure:~~ They have three rectangles that enclose independent replications of 1) the number of participants, i; 2) the number of questions, j; 3) the number of repetitions for the same questions, r. Individual responses, C_{ijr}, were modelled with a Bernoulli process with parameter θ_{ij}, that represents the probability of choosing the larger reward. Node x_{ij}^{s} is the small outcome, and x_{ij}^{l} is the larger outcome. Node t_{ij}^{s} represents the sooner delay and t_{ij}^{l} is the later delay.

Hyperboloid Model. Figure 2 is the Bayesian graphical model of the hyperboloid function (equation 3) were the probability of choosing the larger reward, θ_{ij}, is obtained with the decision rule originally proposed by Luce (1959), however, this version adds a noise parameter, ϵ, that allows stochastic error (Andersen et al, 2010); in the present work this parameter was not estimated individually. The discounted values, v_{ij}^{ll} and v_{ij}^{ss}, are obtained with the multiplication of the discount factor and the outcome. The discount factor, d_{ij}^{sl}, has the hyperboloid structure with two parameters: κ_i and τ_i, estimated individually and with a prior of a normal distribution with a mean of zero and a standard deviation of one, truncated for positive values. It is important to mention that the discount factor of this model is easily replaced by functions such as the exponential and the hyperbolic, keeping the other elements of the model without modification. In fact, these last models were also evaluated, however, the adequacy of these was really low and very similar between them. Because of this, just the hyperboloid function was used to be compared with the other models.



$$\tau_i, \kappa_i \sim \mathrm{dnorm}(0,1)_{T(0,)}$$

$$d_{ij}^{[s,l]} \leftarrow \frac{1}{\left(1+\kappa_i * t_{ij}^{s,l}\right)^{\tau_i}}$$

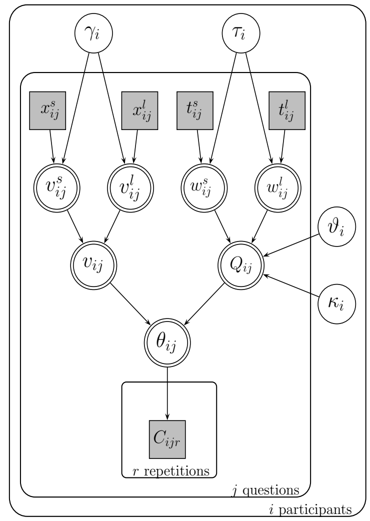
$$v_{ij}^{ll} \leftarrow d_{ij}^{l} * x_{ij}^{l}$$

$$v_{ij}^{ss} \leftarrow d_{ij}^{s} * x_{ij}^{s}$$

$$\theta_{ij} \leftarrow \frac{\left(v_{ij}^{ll}\right)^{1/\varepsilon}}{\left(v_{ij}^{ll}\right)^{1/\varepsilon}+\left(v_{ij}^{ss}\right)^{1/\varepsilon}}$$

$$C_{ijr} \sim \mathrm{Bernoulli}(\theta_{ij})$$

Figure 2. Bayesian Cognitive Modeling of the Hiperboloid Model



$$\vartheta_i \sim \mathrm{Lognorm}(1,1)$$

$$\tau_i, \gamma_i, \kappa_i \sim \mathrm{Lognorm}(0,1)$$

$$v_{ij}^{s,l} \leftarrow \tfrac{1}{\gamma_i}\log(1+\gamma i * x_{ij}^{s,l})$$

$$w_{ij}^{s,l} \leftarrow \tfrac{1}{\tau_i}\log(1+\tau i * t_{ij}^{s,l})$$

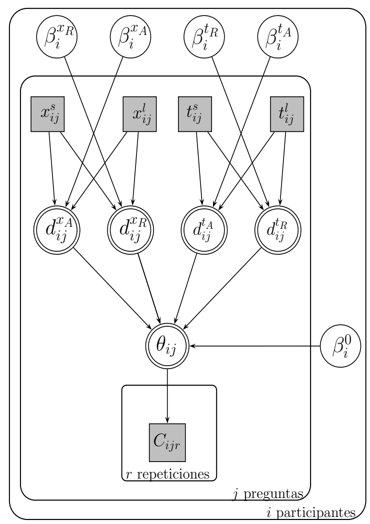
$$Q_{ij} \leftarrow \kappa_i \log\left(1+\left(\frac{w_{ij}^{l}-w_{ij}^{s}}{\vartheta_i}\right)^{\vartheta_i}\right)$$

$$v_{ij} \leftarrow v_{ij}^{l} - v_{ij}^{s}$$

$$\theta_{ij} \leftarrow \frac{v_{ij}^{1/\varepsilon}}{v_{ij}^{1/\varepsilon}+Q_{ij}^{1/\varepsilon}}$$

$$C_{ijr} \sim \mathrm{Bernoulli}(\theta_{ij})$$

Figure 3. Bayesian Cognitive Modeling of the Hiperboloid Model



$$\beta_i^0, \beta_i^{xR}, \beta_i^{xA}, \beta_i^{tR}, \beta_i^{tA} \sim \mathrm{dnorm}(0,1)$$

$$d_{ij}^{xA} \leftarrow \beta_i^{xA} * (x_j^l - x_j^s)$$

$$d_{ij}^{xR} \leftarrow \beta_i^{xA} * \frac{(x_j^l - x_j^s)}{0.5*(x_j^l + x_j^s)}$$

$$d_{ij}^{tA} \leftarrow \beta_i^{tA} * (t_j^l - t_j^s)$$

$$d_{ij}^{tR} \leftarrow \beta_i^{tA} * \frac{(t_j^l - t_j^s)}{0.5*(t_j^l + t_j^s)}$$

$$y_{ij} \leftarrow \beta_i^0 + d_{ij}^{xA} + d_{ij}^{xR} + d_{ij}^{tA} + d_{ij}^{tR}$$

$$\theta_{ij} \leftarrow \Phi(y_{ij})$$

$$C_{ijr} \sim \mathrm{Bernoulli}(\theta_{ij})$$

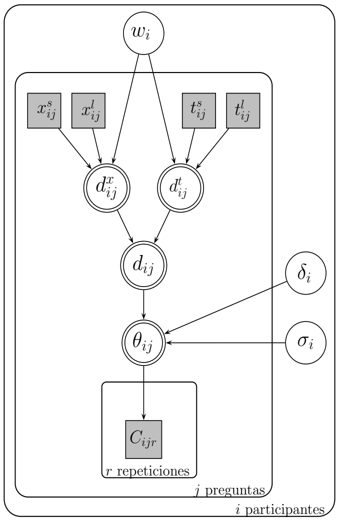
Figure 4. Bayesian Cognitive Modeling of the ITCH model

Trade-off Model. The Trade-off model (figure 3 and equation 5) has the same decision rule as the hyperboloid model which assigns θ_{ij} probability. The Q_{ij} is the Trade-off function contains κ_i and ϑ_i that go over differences between the weighted delays. The weights' delays are assigned with $w_{ij}^{[s,l]}$ and have τ parameter that diminishes the weight of time. The same mathematical structure diminishes the subjective value to the outcomes but with γ_i parameter. α parameter was not considered in the present analysis because our data didn't show this pattern and because it increased variability in the estimation when it was added. All free parameters were estimated at an individual level, except ϵ. Lognormal prior distributions were set with mean zero and standard deviation of one, except for ϑ with a mean of one. This model was based on the statistical work of Scholten et al, (2014) were they made a model comparison with Bayesian analysis. However, they evaluated the models assuming that all participants were described by one parameter but we assumed that each participant has a different parameter distribution.

ITCH. In this model (figure 4 and equation 6), the probability of choosing the larger later reward, θ_{ij}, is defined by Φ that is the cumulative distribution function of a standard normal distribution [2]. The probit function adds the intercept term, β_1, and four differences multiplied by a different β that represents the weight given to each difference. The differences can be either an absolute d^A or proportional d^R and are assigned to outcome and delay, separately. All parameters have a normal prior distribution, with mean 0 and standard deviation of 1.

Direct Differences. The figure 5 represents Direct Difference model from equation 7). The probability of choosing the larger later reward is designated also by the cumulative normal distribution evaluated at d. The parameter δ_i is the decision threshold that indexes the relative importance of time and outcome in the choices. σ_i is the stander deviation and represents internal variability. The model weights the differences thought w that can be interpreted as the amount of attention allocated to each attribute. w_i is ponderated by outcome differences and $1 - w_i$ ponderated by delay differences. The prior distributions are from a normal distribution with mean 0 and standard deviation of 1; however, σ is trun-

[2] The original model uses the cumulative distribution of a logistic distribution with mean of 0 and variance of 1, however, in the present work we used the probit model instead. Several authors have compared logit vs the probit models and have concluded that in practice both provide similar fits (Agresti, 2007; Chambers & Cox, 1967; Hahn & Soyer, 2005).



$$\sigma_i \sim \text{dnorm}(0,1)_{T(0,)}$$

$$\delta_i, w_i \sim \text{dnorm}(0,1)$$

$$d^x_{ij} \leftarrow w_i * (x^l_j - x^s_j)$$

$$d^t_{ij} \leftarrow (1-w_i) * (t^l_j - t^s_j)$$

$$dij \leftarrow d^x_{ij} - d^t_{ij}$$

$$\theta_{ij} \leftarrow \Phi\left(\frac{d_{ij}-\delta_i}{\sigma i}\right)$$

$$C_{ijr} \sim \text{Bernoulli}(\theta_{ij})$$

Figure 5. Bayesian Cognitive Modeling of the Direct Differences model

cated to positive values. The Proportional differences model was evaluated with the same structure as the direct differences model, however, the assignation of d_{ij} node was with equation 8, thus parameter w wasn't estimated.

Results

Figure 6 shows six graphics, the top left ~~is the actual data with the 10 repetitions for each question~~, where blue is the choice for the LLR and red for the SSR. Participants are ordered by the LL choices they made. Questions are ordered by the size of the interval, indicating the set at the top of the graphic. For example, participants 6, 2 and 7, that are found in the inferior part of the graphic, always chose the smaller sooner reward in the small, medium and large intervals. Similarly, participants 4, 17 and 9, that are found in the superior part of the graphic, chose the larger later reward in all interval sizes. There are some participants that chose the larger later reward in the three sets of large outcomes but in the small outcomes set, they chose otherwise; for example, participants 5 and 8. These mentioned patterns follow the additivity assumption, due to they chose in the same way independently of the interval size. It is important to notice that the set of small outcomes, almost all participants chose the smaller sooner reward and there wasn't a clear effect of the interval size for small outcomes.

Conversely, participants that are found in the middle part of the graphic, present at least one data pattern that follows the attribute-based models, meaning that they chose depending on the interval size. For example, participant 24 choose the larger later reward in the small intervals but the smaller sooner reward in the medium and large intervals, pattern known as superadditivity. Similarly, other participants presented patterns where, as the size of the interval increased, their choices changed from choosing the larger later reward to chose the smaller sooner one. Because of these individual differences the necessity of finding a model that can describe patterns that are consistent with the presence and absence of interval effects.

The same figure 6 shows the predictions for each model and the top right graphic is the prediction for the hyperboloid model. On it, we can find more yellow around in the small and medium interval sizes, which means that the model is predicting the choice for the smaller sooner reward when the actual datum was for the larger later reward. For large intervals, there is a presence of yellow and green; and for small outcomes, there is more presence of green, which means that the model predicted the choice for the larger later reward when the actual data was for the smaller sooner reward. This model has the lowest percentage of prediction (63%), in comparison with the other models. The misprediction of this model is related to the fact that it follows the additivity assumption.

The PD model (bottom left graphic) has the second-worst prediction (69%). It can be observed that it has a misprediction pattern in the small and medium intervals where there is a presence between yellow and blue; and in the large intervals and small outcomes, the patter goes between red and green. Even this is an attribute-based model, its performance is not as good as the other models.

The ITCH and DD models have a very similar prediction in the percentage (81%) and the mispredictions. Both models have more presence of green in the medium and large sets, low presence of yellow in the small intervals. In small outcomes, participants that choose the smaller sooner reward all time, there is a presence of yellow. It seems that the DD model is presenting a little bit more of misprediction than the ITCH model. However, none of them has a pattern in the choices prediction.

The Trade-off model has the highest prediction percentage (83%). Small intervals have a very low presence of yellow, but in the medium and large intervals, the presence of yellow, as well as green, is found. For the small outcomes set, there seems to be more presence of misprediction, even when all participants choices were always for the smaller sooner reward.

The Trade-off, ITCH and DD models have similar performance, thus in the following part, we are going to make a parameter analysis of each model and observe which one has a better description of the present data set.

Parameter evaluation

To evaluate the model that better describes the data, a parameter analysis will be presented. The figure 7 (showed in the Appendix) are the posterior distributions of the parameters from the Trade-off model for each participant. The line

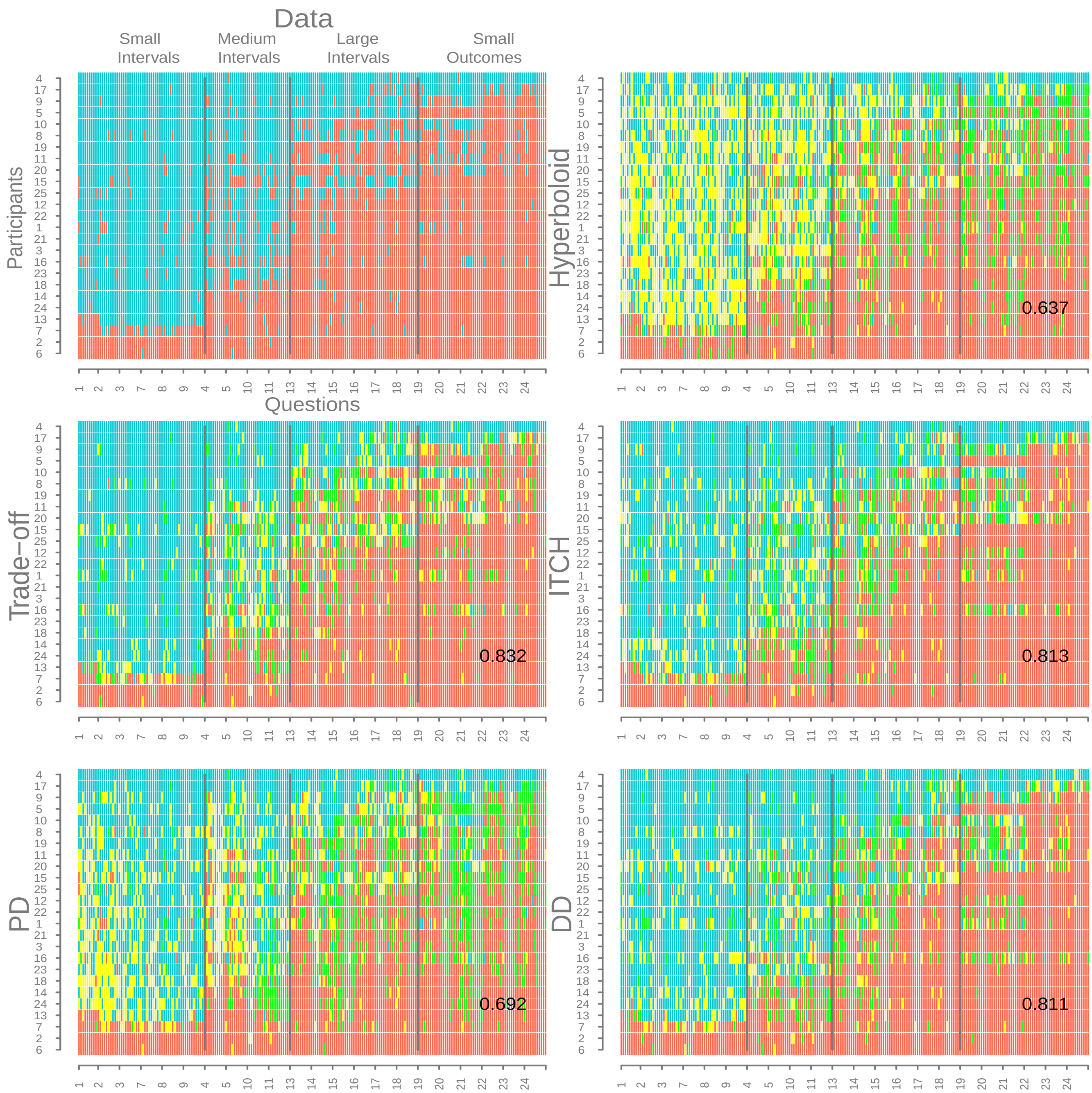


Figure 6. Data and model predictions. Each graphic represents the choices, including the 10 repetitions, made by each participant. The X-axis is the questions ordered by the size of the interval and indicated by a grey line (the four sets presented before). The Y-axis is the participants ordered by the number of choices made for the LLR. In the top left graphic, red is the choice for the SSR and blue for the LLR. The other graphics are the predictions of the evaluated models indicated in the Y-axis; green is the model prediction for the LLR when the actual choice was for the SSR and yellow is the prediction for the SSR when the choice was for the LLR, red is the correct prediction for SSR and blue for the LLR. The percentage of good prediction is shown at the bottom right of each graphic.

indicates the interval of the posterior and the point is the mean, in this case, they are ordered by ϑ parameter. τ parameter has a narrow range for all posterior, except for participants 2, 6, and 9 that have a wide posterior distribution.

ϑ parameter, the one that describes superadditivity, is well defined in a certain interval for all participants. γ and κ parameters reach values higher than 80 in their posterior distributions for some participants. Besides, participants that have higher values of ϑ, found in the bottom part of the graphics, have wider a posterior distribution. This could mean that the model might be over-parametrized; the posterior distributions that have high and wide values indicate that there are a lot of possibilities, from the model, that are generating the data-set, which is not good for model estimation. Additionally, there is not a clear group formed in-between participants' parameters. The simulator created for this model, couldn't recreate similar data patterns, with the estimated posterior means, as the real data found.

Figure 8 shows the interval posterior distributions of the ITCH model for each participant, ordered by the β_x^A parameter. There are not clear clusters formed except for a relation between β_t^A and β_x^A. When participants have a posterior with a narrow interval and values close to zero in β_x^A, they have narrow intervals and close to zero for β_t^A; also, participants have a wide posterior in both parameters. All participants in the parameter β_t^R, have a narrow posterior distribution and with negative values. Parameter β_x^R is presenting a problem because the means of each participant are very similar. Additionally, the posterior intervals are the same as those established in the prior distribution. This could mean that the relative outcome differences are not giving enough information to update the information of this parameter. This result was analyzed in detail and the simulations showed that the relative βs need great changes in the parameter values to generate different data patterns. Additionally, the clusters didn't match subjects choices and it was difficult to observe different data patterns with the simulator. Thus, this model might be over-parametrized and the relative differences are not giving to much information.

Figure 9 are the posterior distribution intervals for the Direct Differences model; ordered by δ parameter. In this graphic, there is a clear cluster formed by parameters σ and δ. When the values are less than -2.5 in δ, they all have more variability and are larger than 1 in σ. When they are higher than -2.5 in δ, there is a clear group in σ with posteriors less than 1 and with narrow posteriors; for these parameter values, there is a correlation between both parameters. Finally, there is a group for participants 7, 6, and 2, which behave in a similar way between σ and δ. For w parameter, all posterior intervals are narrow except for 6 and 2 participants. As a matter of fact, all clusters are very similar to the observed data and the parameter simulations created similar data to the one actually found.

Results showed that the model that better describes the present data is the Direct Differences model because it has a better parameter interpretation taking into consideration the present data set. Compared to the ITCH and Trade-off models, the interpretation was difficult to generate. Besides, with the model simulator, the Direct Differences model can replicate this data and the ITCH and Trade-off model can't.

Discussion

The objective of the present work was to study and compare the hyperbolic, Trade-off, ITCH, Proportional and Direct Differences model; the last four models can describe interval effects.

The analyzed data had a presence and absence of interval effects. In other words, participants change their choices depending on the interval size, while others didn't. Most participants that presented interval effects, presented the superadditivity pattern in which they chose the larger later reward in the smallest intervals, but the smaller sooner in the largest intervals. A small number of participants were consistent with the additivity pattern in which they chose either the larger latter or smaller sooner rewards independently of the interval sizes.

There is more choice variability in the intervals of medium size than in small and large intervals. These results seem to be consistent with some research that found that when the features of the alternatives are similar, then is harder to make a decision, and if the features are very different is easier to make a decision (Referencia). The repetition protocol of the same question allowed to separate choice variability from the structural inconsistency of preferences.

The hyperboloid model, given that, is the model that better describes temporal discounting data among the alternative based family, is the one that works the worst comparing attribute based-models. These results are not unexpected because several articles founded similar results (Chen & Gonzalez-Vallejo, 2016; Ericson et al., 2015; Dai & Busemeyer, 2014; Scholten et al., 2014). This model predicted that it was equally probable to choose any of both alternatives (SSR and LLR), even when actual choices were, for example, for the larger-later reward in all repetitions. The reason for this misprediction is the additivity assumption that the model accomplishes.

Despite the fact that the Trade-off model was proposed to describe interval effects, it holds some issues. First, the mathematical structure is complex and it contains a lot of parameters that seem to be explaining the same part of the decision process (over-parametrization); going against the parsimony principle which describes that a model should be simple and avoid unnecessary assumptions. Second, there was a failure related to large and small outcomes set that have the same interval length. The model evaluation showed that if a participant starts to choose the larger later reward in medium and large intervals, that should also happen in intervals of the same length but in small outcomes. Because of this, the same predictions will be done in intervals that have the same length, without care of the outcome size. However, as we observed in the present data set, the interval size didn't matter in small outcomes and participants always chose the smaller sooner reward.

The ITCH model has an integration rule also based on attributes, but it takes into consideration the relative and absolute differences. In the present data, it was found that the parameters that refer to the relative differences have difficulties to update its posterior distribution regarding the given information. This could mean that is an over-parametrized model where the proportional differences are not giving more information than absolute differences. This was confirmed with the Proportional differences model performance, which considers just relative differences but presented the lowest prediction from the attribute-based model.

The Direct Differences model is the simplest one from the attribute-based family because it doesn't have a complex mathematical structure. This model is interesting because it has the decision rule of attributes and it explains the same percentage of prediction as to the other models, but with fewer parameters. Additionally, the model assumes a stochastic process that describes the perceived differences in-between attributes and the choice probability increases monotonically as a function of the direct differences. For example, if the attention is focused to time, then people will choose the immediate reward, but if the attention is focused on the outcome, then people will choose the larger reward even the delay. Another important finding with the DD model is that there was a correlation for some participants, while for others there wasn't. This could be related to the fact that there are two parameters that describe the given attention to one attribute than to the other, however, w ponderate then directly, while δ is substracting the ponderated differences. σ parameter describes choice variability, which increases depending on which were participants' choices.

The parameter estimation of this model contained less variability in their posterior distributions because the range of its distributions was bounded in a certain part of the parameter. This allowed confirming the clusters where the range of the posterior distributions, in fact, changes subjects behavior, which is good for a model.

Conclusions

In conclusion, the attribute based-rule seems to be more adequate to describe the presence and absence of interval effects. The model that better works with the present database is the Direct Differences model. However, is important to keep studying this model to determine systematically the choice factors that affect the parameters of this model.

Very complicated models are difficult to interpret because even they give a good percentage of prediction, the theoretical interpretation is difficult to produce. This produces uncertainty about the functioning of the model, and how it is describing the data.

The models that were studied in the present work were very different, even those that share the same integration rule. It was difficult to compare them taking into consideration a single measure that examines its performance. However, the analysis of the parameters allowed to observe the theoretical and internal performance of each model, and then give an idea of which was the model that worked better to describe the present data based.

Given that alternative based models can't describe the present data set, these have been very useful in describing risky behaviours, such as addictions and impulsivity. Thus, one of the following steps of the attribute-based models is to study if they can describe in the same the mentioned behaviours.

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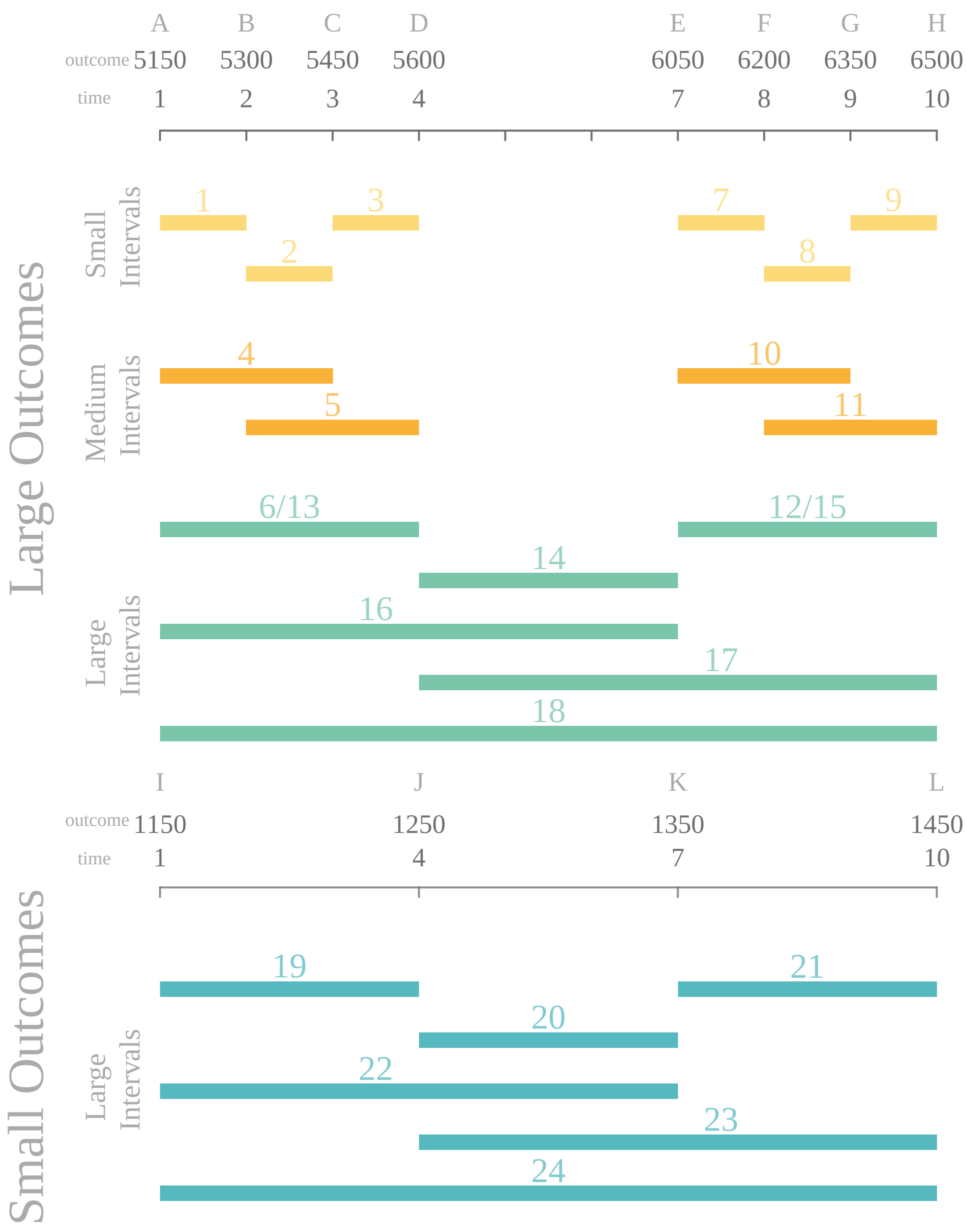
Appendix A

Intructions for the time task:

Showing up next there will be a series of hypothetical pairs of alternatives, which you should choose one of them according to your preference. Each one of the alternatives is different in outcome of money and time of deliver, for example: Which alternative do you prefer? A = 300 pesos in 6 weeks or B = 400 pesos in 6 weeks. If you choose ~~the letter A entails~~ that you would receive 300 pesos ~~within~~ 5 weeks ~~counting~~ from now, if you choose the letter B you would receive 400 pesos within 6 weeks counting from now.

The instructions for selecting the alternatives were the following:

To choose between alternatives you most conduct the mouse towards the letter of your preference and it will change to an orange color. ~~Once that the mouse is inside the letter~~ you most click in order to choose the alternative of your preference. After you have chosen the alternative given the click, the screen will show you which was the alternative that you chose. In order to continue with the next questions, you will have to click in the center of the screen. In the next question you will have to chose between other pair of alternatives with different outcomes of money and delays ~~and you will have to choose again.~~ There is no wrong or right answers, we are just interested in which option you would prefer. Each one of the questions is important, choose carefully. If you are ready, click ~~to begin with the experiment~~.


A B C D E F G H
outcome 5150 5300 5450 5600 6050 6200 6350 6500
time 1 2 3 4 7 8 9 10
Large Outcomes
Small Intervals
1 3 7 9
2 8
Medium Intervals
4 10
5 11
Large Intervals
6/13 12/15
14
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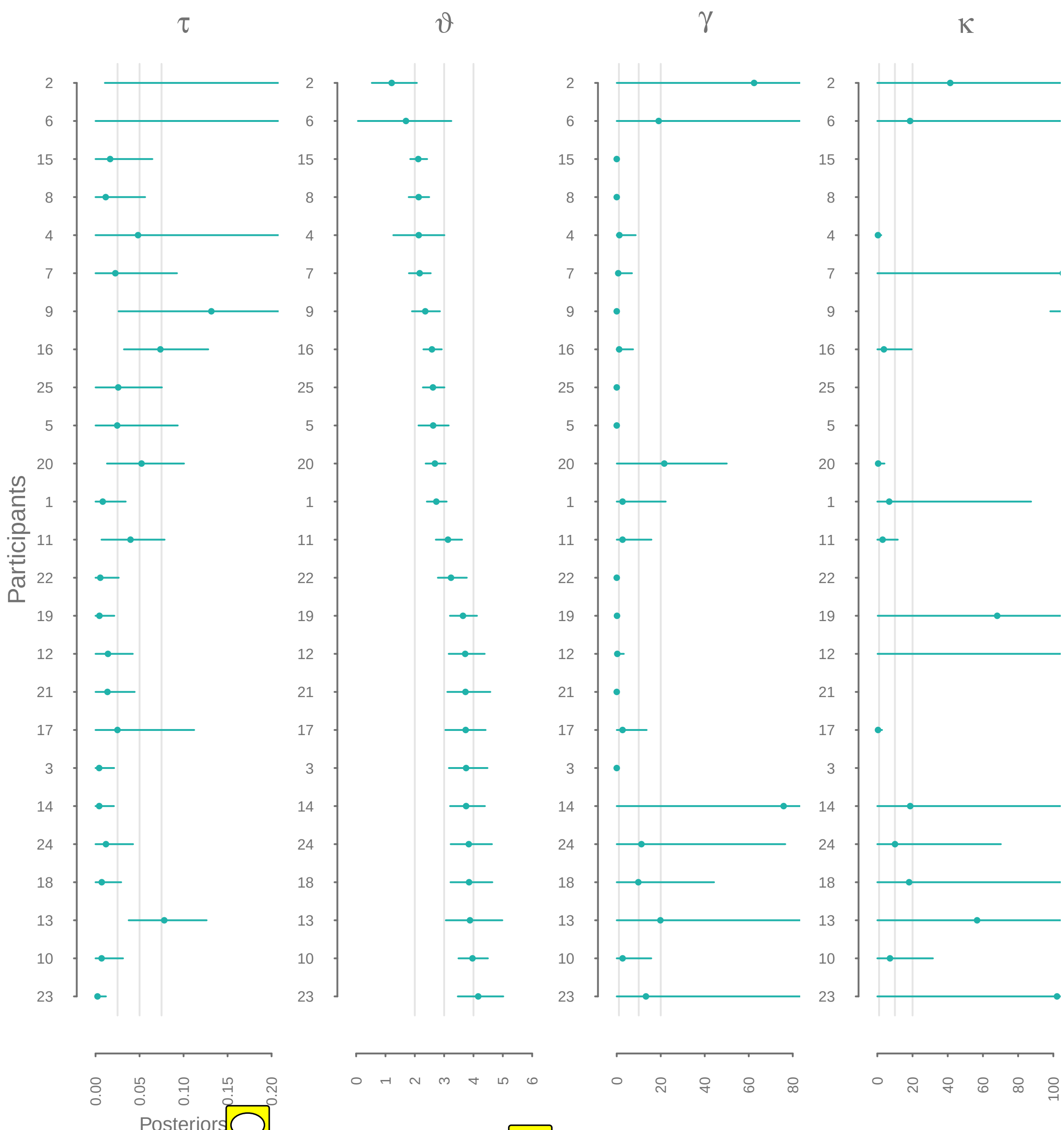

Figure 7. Posterior distribution Intervals for each participant the Trade-off model.

Figur Posterior distribution Intervals for each participant for the ITCH model.

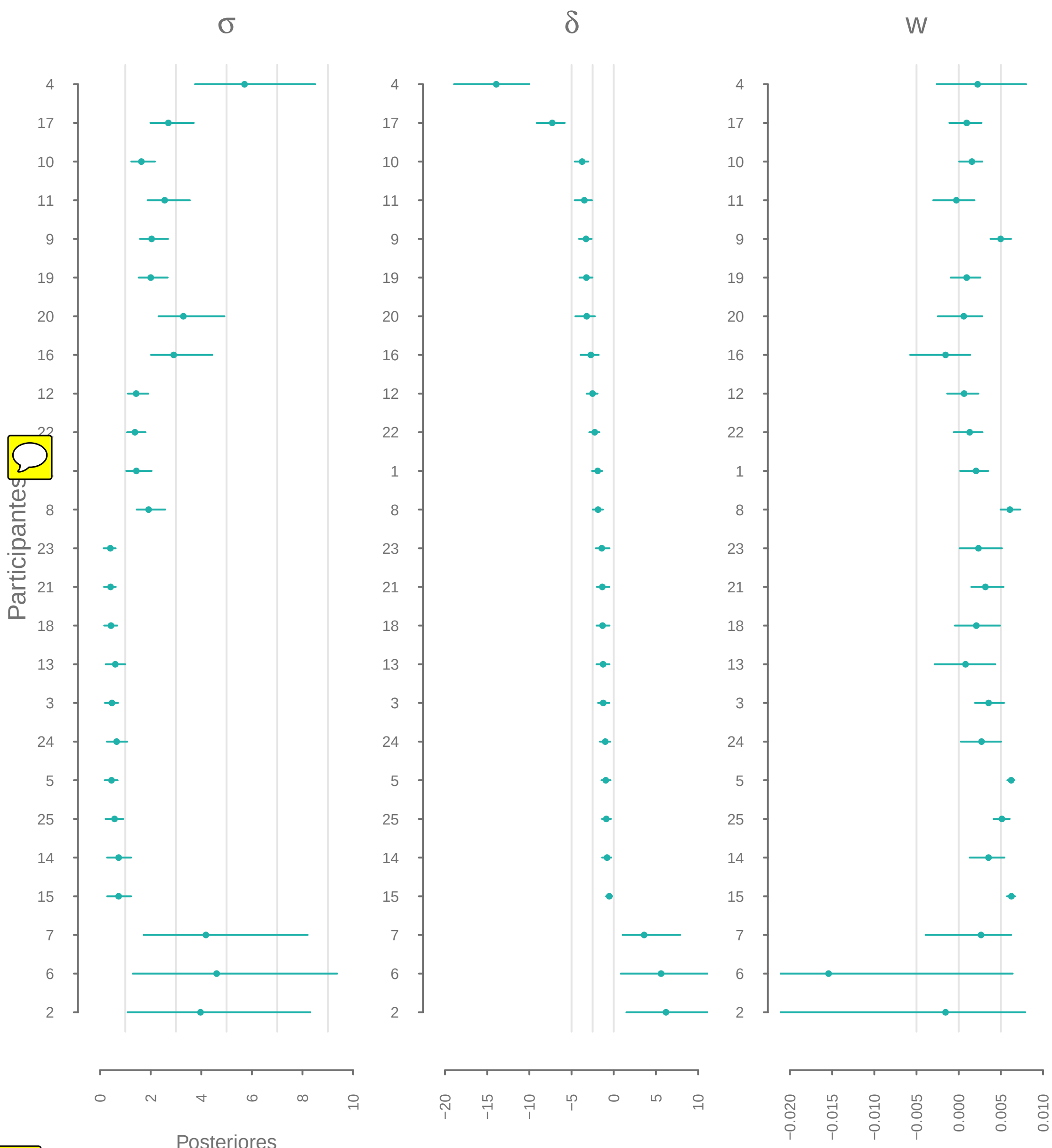

Figure 9. Posterior distribution Intervals for each participant for the Direct Differences model.