@Norm2d

Implement the MATLAB class `@Norm2d` as a class folder

Properties

Mean

A 2×1 vector $M = \begin{pmatrix} \mu_1 \\ \mu_2 \end{pmatrix}$

Covariance

A 2×2 positive definite matrix $\Sigma = \begin{pmatrix} \sigma_1^2 & c_{12} \\ c_{12} & \sigma_2^2 \end{pmatrix}$

Precision

A 2×2 matrix T that is the inverse of the covariance matrix: $T = \Sigma^{-1}$

Correlation

A scalar ρ that is $c_{12}/(\sigma_1\sigma_2)$

Methods

Constructor `Norm2d(Mu, Sigma)`

Make sure that the user can make a Norm2d object without providing parameters. Also make sure T is always correct

Getters and setters

Make sure I can get and set Mean and Covariance, and that I can get Precision and Correlation, and that these are always consistent

Ordinary methods

1. Probability density function `pdf(X,Mu,Sigma)`

The bivariate normal probability density function is given by

$$p(x_1, x_2|M, \Sigma) = \frac{1}{2\pi\sigma_1\sigma_2\sqrt{1-\rho^2}} \exp\left(-\frac{1}{2}\frac{z}{1-\rho^2}\right),$$

where

$$z = \left(\frac{x_1-\mu_1}{\sigma_1}\right)^2 - 2\rho\left(\frac{x_1-\mu_1}{\sigma_1}\right)\left(\frac{x_2-\mu_2}{\sigma_2}\right) + \left(\frac{x_2-\mu_2}{\sigma_2}\right)^2,$$

and

$$\rho = cor(x_1, x_2) = \frac{c_{12}}{\sigma_1 \sigma_2}$$

is the correlation of x_1 and x_2 and c_{12} is the covariance.

Make sure I can input a $2 \times n$ matrix for X and obtain a $1 \times n$ output

2. Log-PDF `logpdf(X,Mu,Sigma)`

The log-pdf will be useful in the future. You'll want to implement it *efficiently*. Don't just compute the pdf and take the logarithm – first you should mathematically simplify this:

$$l(x_1, x_2 | M, \Sigma) = \log(p(x_1, x_2 | M, \Sigma))$$

and implement the result

3. Cumulative distribution function `cdf(X,Mu,Sigma)`

Use the `mvncdf()` function from the Statistics toolbox

4. Log-CDF `logcdf(X,Mu,Sigma)`

Use the `mvncdf()` function from the Statistics toolbox

5. Random number generation `rng(Mu,Sigma,size)`

A sample from a bivariate normal distribution can be simulated by first simulating a point from the marginal distribution of one of the random variables and then simulating from the second random variable conditioned on the first:

$$x_1 \sim N(\mu_1, \sigma_1)$$
$$x_2 \sim N\left(\mu_2 + \sigma_2 \rho \left(\frac{x_1 - \mu_1}{\sigma_1}\right), \sigma_2^2 \sqrt{1 - \rho^2}\right)$$

Make sure this function returns a matrix of size `size`

6. Deviance `deviance(Data,Mu,Sigma)`

The data set D is a $2 \times n$ matrix:

$$D = \begin{pmatrix} d_{11} & d_{12} \\ d_{21} & d_{22} \\ \vdots & \vdots \\ d_{n,1} & d_{n,2} \end{pmatrix}$$

The deviance is the summed logpdf of all points in a data set, times -2:

$$dev(D|M, \Sigma) = -2 \sum_{i=1}^{n} l(d_{i,1}, d_{i,2}|M, \Sigma)$$

Make sure this returns a scalar