COGS 205B: Computational Lab Skills for Cognitive Scientists I

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Part 7: Metropolis-Hastings sampling

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- MH algorithms need some customization and fine-tuning to be most efficient
- One intuition for the MH algorithm is that it is a rejection sampler with an adaptive envelope

Metropolis-Hastings sampler

Given a function $f(\theta) \propto p(\theta|D)$, and a symmetric candidate generating distribution $Q(a|b) = Q(b|a)$, a Metropolis-Hastings sampling algorithm proceeds as follows:

1. Set $i \leftarrow 1$ and choose $R \leftarrow 1000$
2. Choose, arbitrarily, $\theta^{(0)}$
3. Draw a randomly selected θ^c from $Q\left(\theta|\theta^{(i-1)}\right)$
4. Compute the acceptance ratio $\alpha = \frac{f(\theta^c)}{f\left(\theta^{(i-1)}\right)} = \frac{p(\theta^c|D)}{p\left(\theta^{(i-1)}|D\right)}$
5. Draw a randomly selected u from $U(0,1)$. If $\alpha > u$, set $\theta^{(i)} \leftarrow \theta^c$, otherwise set $\theta^{(i)} \leftarrow \theta^{(i-1)}$
6. Set $i \leftarrow i + 1$. If $i \leq R$, return to Step 3, otherwise halt

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- We also repeat the procedure a few times with different values for $\theta^{(0)}$ to ensure that the algorithm converges to the same stationary distribution. Several convergence statistics exist, with Geweke's and Gelman's $\hat{R}$ being the most popular

Sampling in MATLAB

Exercise: Metropolis sampling in MATLAB

Write a new class called `Metropolis()`

The constructor should work by taking an anonymous function as input (call the property `Target` of type `function_handle`), with other inputs such as parameters of the algorithm optional (with defaults)

In addition to the standard methods, `Metropolis()` should have at least a method `DrawSamples(N)` to draw N samples

You should assume that `TargetLogPdf` has exactly one input, but the input may be a vector

Finally, make sure the `disp()` method outputs something informative about the current state of the sampler