Research Article

MODELING RESPONSE TIMES FOR TWO-CHOICE DECISIONS

Roger Ratcliff and Jeffrey N. Rouder

Northwestern University

Abstract—*The diffusion model for two-choice real-time decisions is applied to four psychophysical tasks. The model reveals how stimulus information guides decisions and shows how the information is processed through time to yield sometimes correct and sometimes incorrect decisions. Rapid two-choice decisions yield multiple empirical measures: response times for correct and error responses, the probabilities of correct and error responses, and a variety of interactions between accuracy and response time that depend on instructions and task difficulty. The diffusion model can explain all these aspects of the data for the four experiments we present. The model correctly accounts for error response times, something previous models have failed to do. Variability within the decision process explains how errors are made, and variability across trials correctly predicts when errors are faster than correct responses and when they are slower.*

Making decisions is a ubiquitous part of everyday life. In psychology, besides being an object of study in its own right, decision making plays a central role in the tasks used to study basic cognitive functions such as memory, perception, and language comprehension. Frequently, the decisions required in these tasks are rapid two-choice decisions, decisions that are based on information that can be described as varying along a single dimension. Two key features of these decisions are that they occur over time—decisions are never reached instantaneously—and that they are error prone. In this article, we present a model to explain this class of decision processes. The goal is to understand what information drives the decision and how the decision process evolves over time to reach correct and incorrect decisions. The problem is difficult because potential models are constrained to explain multiple empirical measures that interact in complex ways. The measures include mean response times for correct and error responses, the shapes of the distributions of the response times, and the probabilities of correct and error responses. The relation between response time and accuracy is not fixed; it varies according to whether speed or accuracy of performance is emphasized and according to whether one or the other of the responses is more probable or weighted more heavily. In addition, the relation between probability of an error and error response time is not fixed but varies across levels of overall accuracy. Because of these complexities, no previous model has been completely successful. Often, models have dealt with only one measure—accuracy but not response time, or response time but not accuracy. Models that have dealt with response time have usually tried to explain only mean response times for correct responses, not the shapes of response time distributions or response times for errors. Modeling speed–accuracy relationships has usually not been attempted.

In this article, we show how the diffusion model (Ratcliff, 1978, 1981, 1985, 1988; Ratcliff, Van Zandt, & McKoon, 1998) can explain all of these aspects of the data for two-choice perceptual decisions. For the first time, the model provides an integrated account of both the information that drives decisions and how that information is processed over time to produce correct and error responses. The main domain of application is to tasks on which response time is typically under a second. The model may apply when response time is greater than 1 s, but at much longer times, decisions are probably based on multiple decision attempts in which the first decision attempted was sometimes not made or made with too little confidence for the response to be based on that decision (with perhaps different information or different response criteria used in each successive attempt).

A major weakness of all of the models for reaction time is the failure to account for error reaction times. Luce (1986) reviewed data and theory for error reaction times and concluded that there are few systematic studies of error reaction times that can be used to produce comprehensive empirical generalizations, nor is there a comprehensive theoretical account of error reaction times. Empirically, the relationship between correct and error reaction times varies: Sometimes errors are faster than correct responses (mainly when the task is easy and speed is emphasized); sometimes errors are slower than correct responses (mainly when the task is hard and accuracy is emphasized; see Luce, 1986; Swensson, 1972). Ratcliff et al. (1998, see also Smith & Vickers, 1988) presented data showing individual subjects had a crossover, with error responses faster than correct responses at high accuracy, and error responses slower than correct responses at low accuracy. This pattern is very difficult for models to produce; models predict slow errors or fast errors (e.g., Link & Heath, 1975), but most cannot predict both or predict crossovers.

In this article, we show that the diffusion model can explain the relationship between correct and error responses across a range of experimental paradigms while at the same time fitting all the other response time and response probability aspects of the data. The key to the model's success is variability in the decision process: We show this in experiments with perceptual stimuli, but the model is more general than this application; it can potentially have equal success for the two-choice cognitive tasks to which it has been applied previously. These tasks include short- and long-term recognition memory tasks, same/different letter-string matching, lexical decision tasks, numerosity judgments, and visual-scanning tasks (Ratcliff, 1978, 1981; Ratcliff et al., 1998; Strayer & Kramer, 1994).

DIFFUSION MODEL

The diffusion model is a member of the class of sequential-sampling models (accumulator models—Smith & Vickers, 1988; Vickers, 1979; recruitment models—LaBerge, 1962; the runs model—Audley & Pike, 1965). More specifically, the diffusion model is a member of the random-walk class (Feller, 1968; Laming, 1968; Link & Heath, 1975; Stone, 1960). The diffusion model differs from other random-walk models in its assumption that the information that drives a decision process is accumulated continuously over time instead of in discrete steps. Models similar to the diffusion model presented here have been applied to simple reaction time (Smith, 1995) and to decision making (Busemeyer & Townsend, 1993). There is much

Address correspondence to Roger Ratcliff, Psychology Department, Northwestern University, Evanston, IL 60208.

commonality among random-walk and diffusion models, and similarities significantly outweigh differences.

In the diffusion model, the accumulation of information that drives a decision begins from a starting point and continues until the total amount of accumulated information reaches either a positive response boundary or a negative response boundary. The response time for a decision is the time required to reach a decision boundary plus a constant encoding and response-execution time. The rate at which the process approaches a boundary, that is, the mean amount of information accumulated per unit of time, is called the drift rate, v.

The accumulation of information is not constant over time, but instead varies. The variability is assumed to be normally distributed with standard deviation s, a parameter of the model. As a result of this variability, the accumulation process can end up at the wrong boundary. Figure 1a shows the diffusion process with the negative response boundary set at zero and the positive response boundary set at a (solid boundary lines in the figure), with the boundaries equal in distance from z, the starting point ($z = a/2$). The figure shows the paths taken over time by three decision processes, each with the same drift rate (using a discrete approximation; drift rate $v = 0.2$, $a = 0.1$, $z = 0.05$, and $s = 0.1$). Because of the variability in accumulation of information, the decision outcomes for these processes are quite different: One process reaches the positive boundary relatively quickly, a second reaches the negative boundary in error, and the third takes a relatively long time to reach the positive boundary.

On average, a stimulus with a large positive drift rate will approach the positive boundary relatively quickly, and so the probability is relatively low that variability will cause the process to reach the negative boundary by mistake. But a stimulus with an intermediate drift rate will, on average, take longer to reach the correct boundary, and the probability of reaching the wrong boundary in error is larger. In this way, differences in drift rates account for differences between "easy" stimuli and "difficult" ones: For "easy" stimuli, drift rate has an extreme value and responses are fast and accurate on average, whereas for "difficult" stimuli, drift rate is intermediate in value and responses are slower and less accurate on average.

Not only does the accumulation of information vary within the course of a decision, but the drift rate for the same, nominally equivalent, stimulus also varies across trials. In a memory task, for example, the same word *dog* might be remembered better on one trial than another, or a subject might better attend to it as a stimulus on one trial than another. The assumption about variability in drift was made because it seemed necessary to deal with variability in encoding in memory. Later, it turned out to be necessary to account for response signal functions asymptoting as a function of response signal lag (Ratcliff, 1978). Specifically, drift rate is assumed to be normally distributed across trials with standard deviation η, a parameter of the model. As we explain later, the assumption of variability in drift rate across trials of the same stimulus causes the diffusion model to predict slower response times for incorrect responses than for correct responses.

Speed-accuracy trade-offs are modeled by the boundary positions. When accuracy is emphasized, the boundaries are set far from the starting point; response times are slow and accuracy is high, as shown by the solid boundary lines in Figure 1a. When speed is emphasized, the boundaries are moved closer to the starting point, as illustrated by the dotted boundary lines in Figure 1a. Response times are shorter, and processes that would have hit the correct boundary are now more likely to hit the wrong boundary by mistake (the left-most T in Fig. 1a), leading to lowered accuracy.

The distributions of response times in two-choice tasks are positively skewed. The geometry of the diffusion process predicts this shape. Figure 1b illustrates drift rates and response time distributions for two decisions that have the same variability in accumulation of information (the same s) but are different in difficulty (different drift rates). For each of the two drift rates represented by the two distributions, the figure plots the average path to reach the positive boundary for the fastest and the slowest responses (the random lines in Fig. 1a are replaced by straight lines). The difference between the drift rates for the fastest responses, shown by the left-most X, is equal to the difference between the drift rates for the slowest responses, shown by the right-most X. These equal differences in drift rate translate into

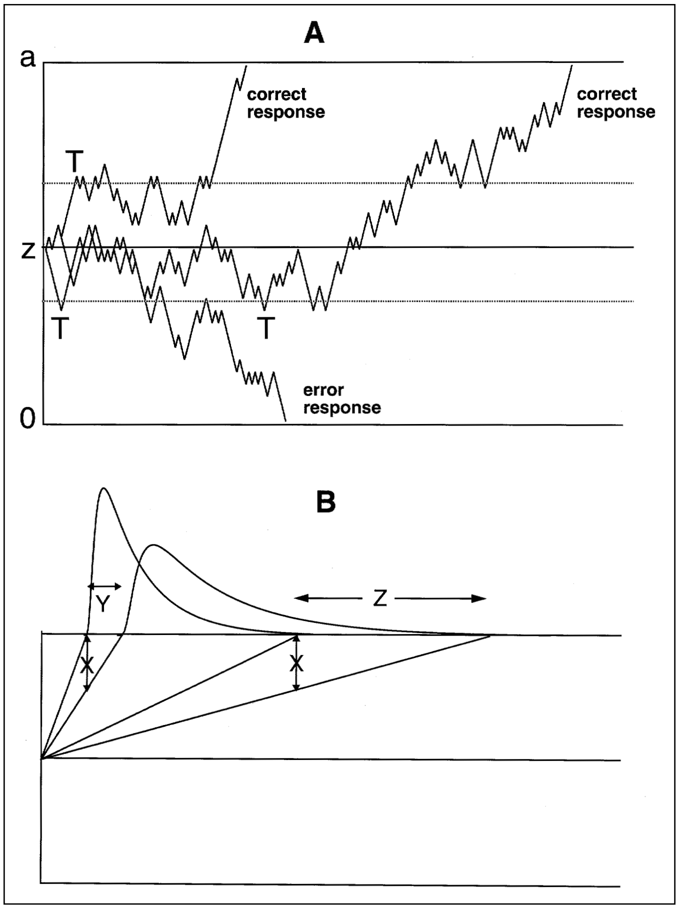

Fig. 1. Illustration of the diffusion model. The sample paths in (a) are derived from a random walk designed to mimic the diffusion process (the continuous version of the random walk). The bottom boundary is set to zero, the starting point of the walk to z, and the upper boundary to a. If the boundaries were moved in to the dotted lines, the processes would terminate at the points T. The straight diagonal lines in (b) represent average paths for two conditions in which the fastest responses differ in mean drift by X, and the slowest responses differ in mean drift by X. The two curves at the upper decision boundary show illustrative distributions of reaction times for these two conditions. The distributions show that the same difference in mean drift leads to smaller differences between the shortest response times (Y) than between the longest response times (Z), illustrating the skewing of the response time distribution that is usually obtained empirically when conditions vary in difficulty.

unequal differences in response time (*Y* and *Z* in Fig. 1b) that give positive skew.

The diffusion model, as described so far, has been shown by previous research to accurately predict most—but not quite all—accuracy and response time measures for decision processes in many two-choice tasks. For example, for recognition memory, the model has been shown to provide an explanation, almost a complete explanation, of how the familiarity of a stimulus drives decision processes through time to produce correct and incorrect decisions (Ratcliff, 1978).

What was missing in previous applications of the diffusion model, as just described, was an accurate account of the relationship between response times for correct decisions and incorrect decisions. When, for the first time, we had sufficient computer power to fully explore the parameter spaces of the model, we discovered that variability in two parameters provides all that is necessary to account for the relative speed of correct versus error responses. One of these parameters is the across-trial variability in drift rate for nominally equivalent stimuli (parameter η) that has always been part of the diffusion model, and the other is variability in the starting point (parameter s_z). Starting-point variability has not been explicitly implemented before in the diffusion model (cf. Ratcliff, 1981), but has been used previously in other random-walk models (Laming, 1968; Rouder, 1996) to account for short error response times.

How variability across trials in drift rate and starting point interact with the diffusion process to explain the relationship between error and correct response times is illustrated in Figure 2. Variability across trials means that, for any stimuli with mean drift rate *v*, mean response time and accuracy are a function of the averages across trials of all the varying starting-point values and all the varying drift-rate values. We illustrate the effects of this averaging in Figure 2 by averaging not over the whole distributions of possible starting points and drift rates, but instead over only two values of each. Figure 2a shows what happens when two values of drift rate are averaged, each, for the purposes of this example, assumed to have zero between-trial variability. The mean response time for correct (positive) responses for the process with drift rate v_1 is 400 ms, and the mean response time for the process with drift rate v_2 is 600 ms; the mean probabilities of correct responses are .95 and .80, respectively. With the starting point equidistant from the two boundaries ($z = a/2$), correct and error responses have equal mean response times (see Laming, 1968; Stone, 1960). Averaging response times from the v_1 and v_2 processes for correct responses and for error responses, weighting by probability of termination at the appropriate boundaries, produces a mean error response time (560 ms) slower than the mean correct response time (491 ms). Figure 2b shows what happens when two values of the starting point (z_1 and $a - z_1$) are averaged (for the same drift rate, *v*). The weighted mean response time for errors is faster than the weighted mean response time for correct responses. When variability in drift rate and variability in starting point are combined, error responses can be slower than correct responses at intermediate levels of accuracy but faster than correct responses at extreme levels of accuracy. This pattern is shown by some of the subjects in the experiments described later.

In most quantitative models, there is no variability in the values of parameters; they are fixed to simplify applications of the models. Although variability in parameter values across trials would be expected, it has been left out of models because it was thought not to change a model's predictions (and in most cases, it will not change predictions). However, it is by incorporating parameter variability explicitly into fits of the diffusion model to the data from the experiments we describe next that the model provides a complete explanation of decision processes (see also Van Zandt & Ratcliff, 1995).

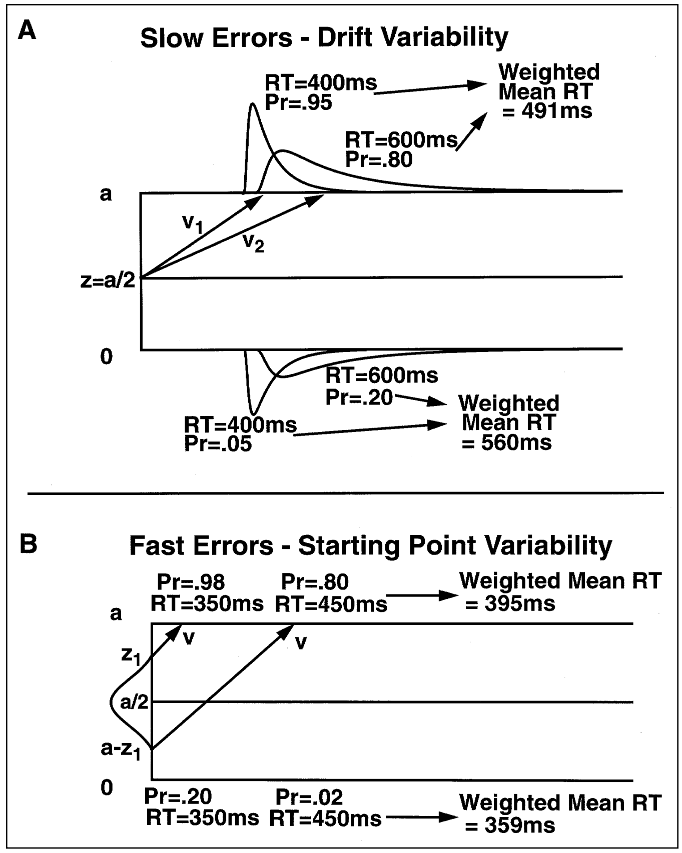

Fig. 2. Illustration of how parameter variability in the diffusion model leads to fast and slow error responses. In (a), two processes have drift rates v_1 and v_2, and the starting point, *z*, is halfway between the two boundaries. The diagonal lines ending in arrows represent average paths, and the curves at the decision boundaries show distributions of response times (RTs) for the two processes. Correct and error responses have equal RTs (400 ms and 600 ms, respectively). The average of these RTs (exemplifying variability in drift across trials) weighted by probability of response (Pr) leads to slow error responses relative to correct responses. In (b), the effect of variability in starting point is illustrated. Each of the two average paths begins from an extreme of the distribution of starting points centered at *z*/2. Processes starting at z_1 hit the correct boundary with high accuracy and short RT, and errors are slow. Processes starting at $a - z_1$ hit the correct boundary with lower accuracy and longer RT, and errors are fast. The weighted average gives fast errors.

EXPERIMENTS 1, 2, AND 3

In each experiment, the subjects were asked to discriminate perceptual stimuli as belonging to one of two response categories. In Experiment 1, subjects were asked to decide whether the overall brightness of pixel arrays displayed on a computer monitor was "high" or "low" (Fig. 3a). The brightness of a display was controlled by the proportion of the pixels that were white. For each trial, the proportion of white pixels was chosen from one of two distributions, a high distribution or a low distribution, each with fixed mean and standard deviation (Fig. 3b). Feedback was given after each trial to tell the subject whether his or her decision had correctly indicated the distribution from which the stimulus had been chosen. Other than this feedback, a subject had no information about the distributions. Because the distributions overlapped substantially, a subject could not be highly accurate. A display with 50% white pixels, for example, might have come from the high distribution on one trial and the low distribution on another. Experiment 2 was similar except that the discrimination was between red and green stimuli. In Experiment 3, subjects were asked to decide whether two stimuli had the same or different brightness. The diffusion model was also applied to the data from an experiment in which subjects were asked to decide whether auditory tones were "high" or "low" (Espinoza-Varas & Watson, 1994).

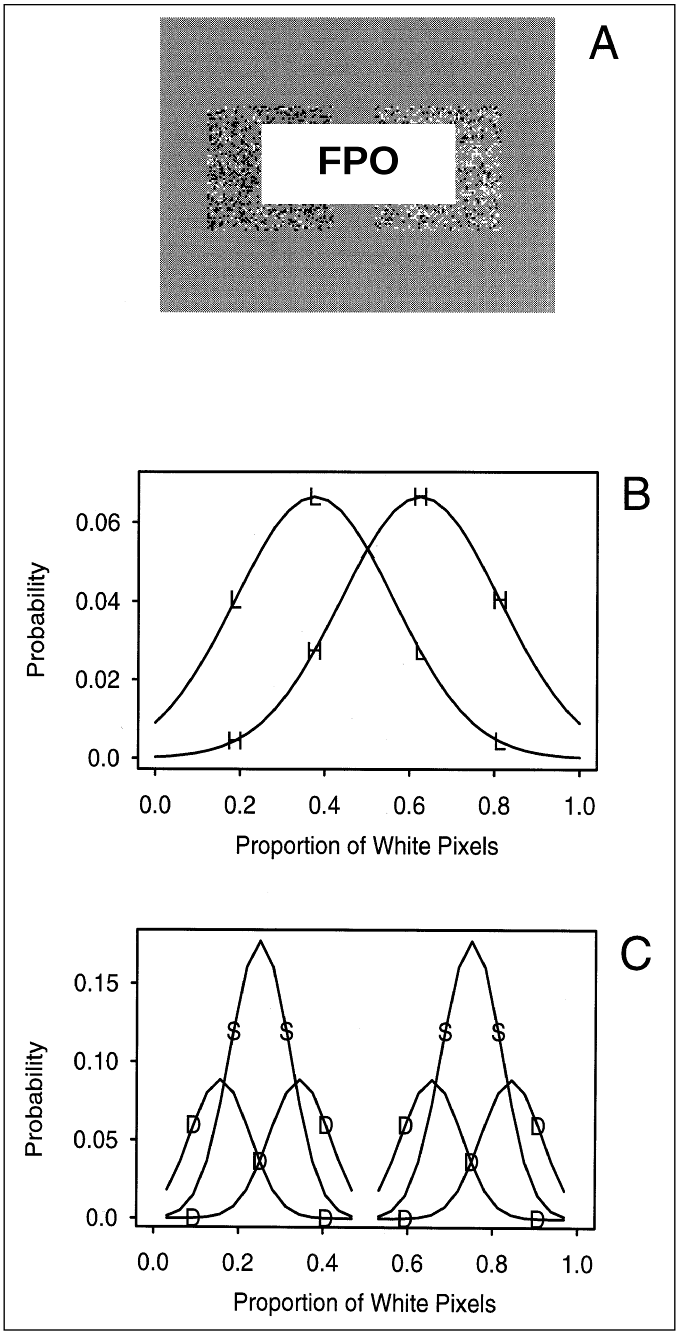

Fig. 3. Sample stimuli for Experiment 1 (a), distributions used in stimulus selection for Experiments 1 and 2 (b), and stimulus selection probabilities for Experiment 3 (c). H and L refer to high versus low brightness (green vs. red dots in Experiment 2), S refers to the *same* distribution (used when the two stimuli were to be selected from the same distribution), and D refers to the *different* distributions (one stimulus was selected from the *different* distribution to one side of the *same* distribution, and the other stimulus was selected from the *different* distribution to the other side of the *same* distribution).

Method

Subjects

The subjects, paid $6 per experimental session, were recruited by advertisements from the population of undergraduates at Northwestern University. Three subjects participated in Experiment 1 for ten 35-min sessions, and 3 other subjects participated in both Experiments 2 and 3, four sessions of 35 min for each experiment. In each experiment, subjects participated in one 35-min practice session before the experimental sessions.

Stimuli

The stimulus display for Experiment 1 was a square that was 64 pixels on each side and subtended 3.8° of visual angle on a PC-VGA monitor. Figure 3a shows examples of two stimuli; the left example is low in brightness; the right is high. In each square, 3,072 randomly chosen pixels were neutral gray, like the background, and the remaining 1,024 pixels were either black or white; the proportion of white to black pixels provided the brightness manipulation. There were 33 equally spaced proportions from zero (all 1,024 pixels were black) to 1 (all 1,024 pixels were white). The two distributions from which the bright and dark stimuli were chosen were centered at .375 (low brightness) and .625 (high brightness), and they each had a standard deviation of .1875.

For Experiment 2, all 4,096 pixels in the square were either red or green, and the proportion of red to green pixels varied from .375 to .625 in 33 equally spaced proportions. The means for the distributions from which the red and green stimuli were chosen were .469 and .531, respectively, and the standard deviation of each distribution was .047.

In Experiment 3, subjects were asked to decide whether two stimuli came from the same or different distributions of brightness. The display was two squares (one above the other) like those used in Experiment 1. The stimuli varied across the whole brightness dimension; for example, two stimuli from the same distribution could be both high in brightness or both low in brightness, and two stimuli from different brightness distributions could both be from the brighter end of the scale or the lower end. To construct the stimuli for each trial, first a proportion of white pixels was selected from 16 possible values, equally spaced on a uniform distribution of values between .25 and .75. For a trial for which the stimuli were to come from the same distribution of brightness, this selected value was the mean of the distribution, and its standard deviation was .07. For a trial for which the stimuli were to come from different distributions, the means of the two distributions were offset ± .046 from the selected value; their standard deviations were .07. Figure 3c shows the *same* and *different* distributions for two examples: with the selected proportion .25 and with the selected proportion .75.

Procedure

A subject's task was to decide, on each trial, from which distribution, high or low brightness in Experiment 1, red or green in Experiment 2, or same or different brightness in Experiment 3, the observed stimulus (stimuli) had been sampled. Subjects made their decision by pressing one of two response keys. On each trial, a 500-ms foreperiod, during which the display consisted solely of neutral gray, was followed by presentation of the stimulus; presentation was terminated by the subject's response. In Experiment 1, speed-versus-accuracy instructions were manipulated. For some blocks of trials, subjects were instructed to respond as quickly as possible, and a "too slow" message followed every response longer than 550 ms. For other blocks of trials, subjects were instructed to be as accurate as possible, and a "bad error" message followed incorrect responses to stimuli from the extreme ends of the distributions. Experiment 1 had ten 35-min sessions, and Experiments 2 and 3 had four sessions. In Experiment 1, subjects switched from emphasis on speed to emphasis on accuracy every 204 trials. Each session consisted of eight blocks of 102 trials per block, for a total of 8,160 trials per subject. In Experiments 2 and 3, there was no speed-accuracy manipulation. Each session consisted of eight blocks of 102 trials, for a total of 3,264 trials per subject in each experiment. For all trials in each experiment, subjects were instructed to maintain a high level of accuracy while responding quickly, and an "error" message indicated incorrect

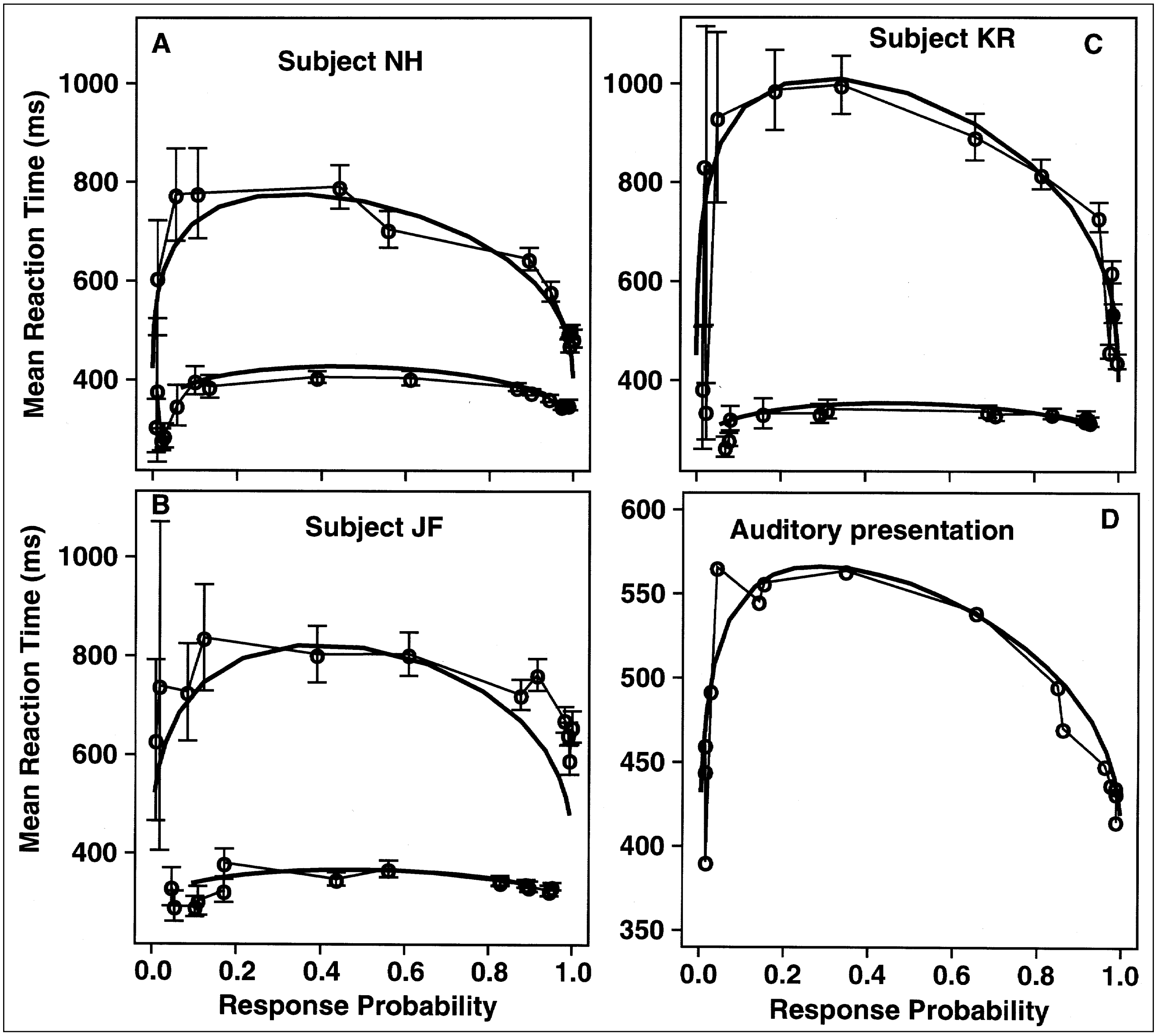

Fig. 4. Latency-probability functions for 3 subjects for the speed (lower curves) and accuracy (upper curves) conditions of Experiment 1 (a, b, and c) and average over subjects for the auditory experiment (d) (Espinoza-Varas & Watson, 1994, Experiment 3). The thick continuous line is the theoretical prediction, and the circles are the data points. Error bars represent 2 standard deviations in mean response time (for d, standard deviations were not available). Correct responses are to the right of the .5 point for response probability, errors are to the left of the .5 point, and each correct response to the right with probability p has a corresponding error response to the left with probability $1 - p$.

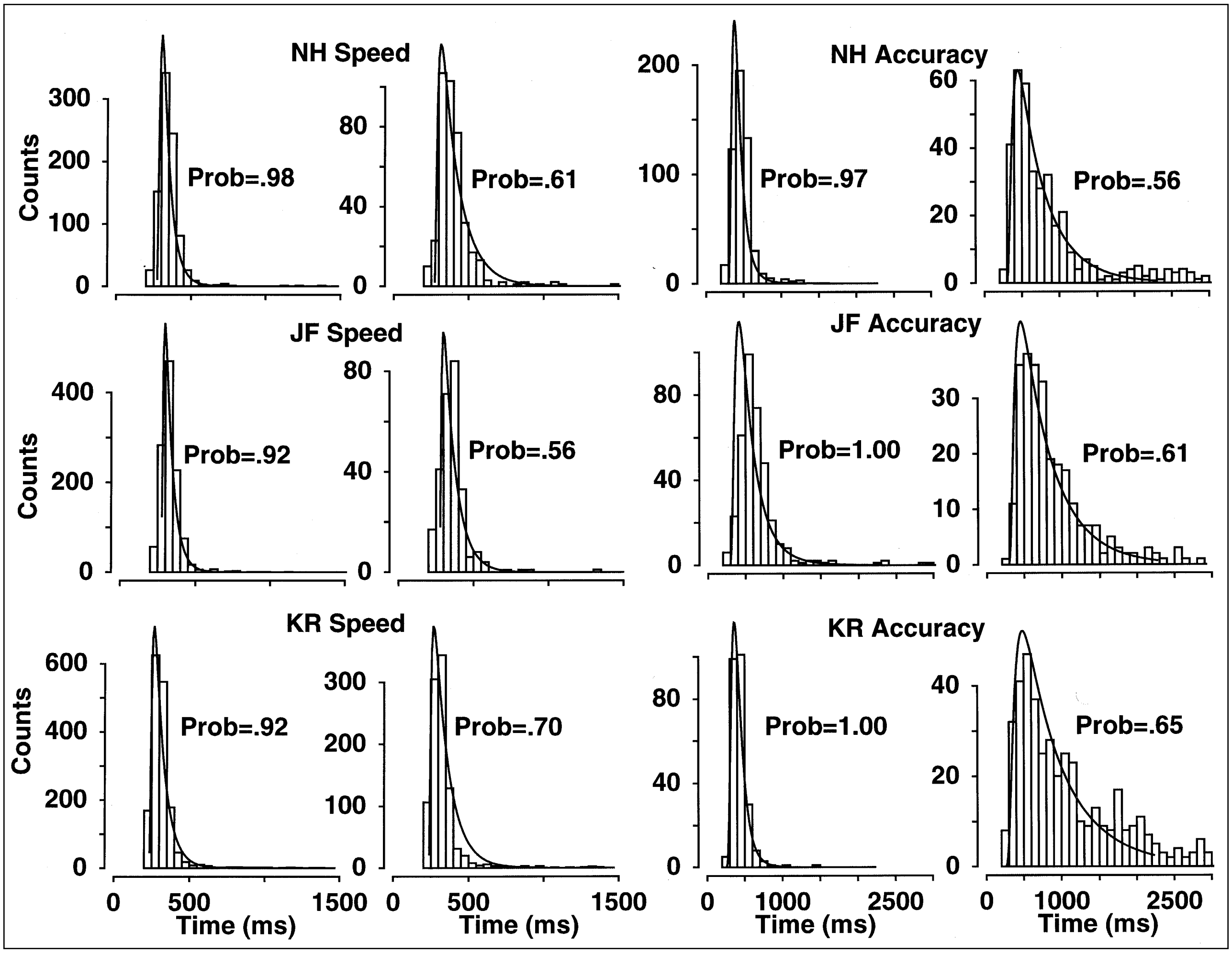

Fig. 5. Response time distributions for 3 subjects from Experiment 1. For each subject, two sample distributions from each of the speed and accuracy conditions are shown, one distribution for responses with relatively high probability ("Prob") and one for responses with intermediate probability. In each panel, the curve represents the theoretical predictions.

responses. Responses were followed by a 300-ms blank interval, and the error message was displayed for 300 ms after the blank interval.

Model Fits and Results

The parameters of the model include the distance between the boundaries (a), the mean distance of the starting point (z) from the bottom boundary, and a parameter T_{er} for encoding and response-execution processes that are not part of the decision process. There are also drift rates, v, one for each stimulus condition (i.e., in Experiment 1, one value of v for each of the 33 possible levels of brightness). There are also two between-trial variability parameters: The starting point and the drift rate are both assumed to vary across trials with normal distributions with standard deviations s_z and η, respectively. Variability in drift rate within a trial is a scaling parameter (i.e., if it were altered, other parameters could be scaled to produce exactly the same fits), and it was fixed at 0.1. Fits of the model were accomplished using the equations in Ratcliff (1978), minimizing the differences between observed and predicted values of accuracy and between observed and predicted values of μ and τ of fits of the ex-Gaussian distribution (Ratcliff & Murdock, 1976) to the empirical response time distributions (see Ratcliff, Van Zandt, & McKoon, in press, for further details; note that we now fit the model using cumulative reaction time distributions, and the results are almost the same as those using the ex-Gaussian).

The data and fits of the model to the data are displayed in latency-probability functions (Audley & Pike, 1965; Vickers, Caudrey, & Willson, 1971) to show the relationship between response time and accuracy. For each stimulus (or group of similar stimuli), mean response time is plotted against response probability (see, e.g., Fig. 4). Without variability in drift rates or starting point and with boundaries equidistant from the starting point, the diffusion model predicts a symmetric latency-probability function (see also Ratcliff et al., in press). For a correct response with a probability of .8, for example, response time would be the same as for an error response with a probability of .2. (For discrimination tasks, we use the term error as a shorthand for the response that is less likely to be correct.) The addition of variability makes the function asymmetric, representing the relative speeds of correct and error response times.

Figures 4a, 4b, and 4c and Figure 5 show the fits of the diffusion model to the data from Experiment 1, which studied brightness discrimination. The top curves in Figure 4 show data from the accuracy condition, and the bottom curves show data from the speed condition. The curves for "high" and "low" responses were almost identical, so they were averaged together—correct responses with correct responses and error responses with error responses. With speed emphasized, response times were about constant—about 400 ms—across the accuracy range. With accuracy emphasized, response times varied from 500 to 900 ms. Error response times were slower than correct response times in the middle of the accuracy range, and error response times were a little faster than correct response times at the extremes of the accuracy range. For example, for subject K.R., correct response times were about 450 ms for stimuli for which the probability of a correct response was very high (e.g., above .95, the far right points on the latency-probability function), whereas error response times for those same stimuli (response probability less than .05, the far left points) were faster, about 350 to 400 ms. In contrast, for stimuli for which the probability of a correct response was between .5 and .9, correct response times ranged from 750 ms to 850 ms, whereas error response times for those same stimuli (response probability between .5 and .1) were slower, between 900 and 1,000 ms. Figure 5 shows sample empirical response time distributions and fits of the model to them.

The fits of the diffusion model match the empirical latency-probability functions and the response time distributions for Experiment 1 with only drift rate (v) varying across the stimulus conditions. For each subject, T_{er} and η were fixed across all stimulus conditions. The boundary parameter a was also fixed across conditions, but it had two values, one for the speed condition and another for the accuracy condition. The mean value of the starting point z was set to $a/2$, and variability in the starting point was fixed at $0.1z$. The values of the parameters of the model are shown in Table 1. The fits of the model are particularly noteworthy in that they capture both the large differences in speed versus accuracy conditions (several hundred milliseconds in reaction time) and the patterns of correct versus error response times (errors faster than correct responses at extreme accuracy values and errors slower than correct responses at less extreme error values).

Figure 6a shows how the diffusion model reveals the stimulus information that is driving the decision process. For all 3 subjects, the value of drift rate v for each stimulus is a linear transformation of the probability that the stimulus was drawn from the high versus the low distribution. Thus, all 3 subjects based their decisions on probability matching. As a consequence, all the different values of v, one for each stimulus, that were used to fit the model to data can be replaced by only the two parameters needed to linearly transform stimulus probability to drift (see the legend of Fig. 6). (Note that in this paradigm, decision-bound models would produce predictions very similar to those of stimulus probability; e.g., Maddox & Ashby, 1993; Nosofsky & Palmeri, 1997; so we cannot rule out the decision-bound model in favor of models that predict stimulus probability as the function driving drift rate.)

In sum, the model has only five free parameters: two parameters to transform the probability of a stimulus being drawn from the high distribution to drift rate, across-trial variability in drift rate, a boundary parameter, and an encoding and response time parameter. With these five parameters, the model accounts for data with literally hundreds of degrees of freedom—including the relative probabilities and response times of correct and error responses across the whole range of accuracy and the variance and shape of the response time distributions. With the addition of a sixth parameter, a second value of a, the model also explains the data for conditions in which speed versus accuracy is emphasized.

The results of Experiment 2, which studied red/green discrimination, are generally similar to those of Experiment 1: The

Table 1. *Diffusion model parameters for the four experiments*

Subject	Condition	a	z	T_{er}	η	s_z
		Experiment 1: brightness discrimination				
N.H.	Speed	0.079	0.0395 (a/2)	0.260	0.063	0.1z
	Accuracy	0.160	0.0800 (a/2)	0.260	0.063	0.1z
J.F.	Speed	0.080	0.0400 (a/2)	0.274	0.093	0.1z
	Accuracy	0.148	0.0740 (a/2)	0.274	0.093	0.1z
K.R.	Speed	0.073	0.0365 (a/2)	0.228	0.082	0.1z
	Accuracy	0.186	0.0930 (a/2)	0.228	0.082	0.1z
		Experiment 2: red/green discrimination				
J.S.	—	0.120	0.068	0.378	0.120	0.1z
J.B.	—	0.120	0.052	0.369	0.071	0.1z
M.J.	—	0.132	0.061	0.311	0.120	0.1z
		Experiment 3: same/different discrimination				
J.S.	—	0.116	0.046	0.406	0.068	0.1z
J.B.	—	0.133	0.056	0.374	0.088	0.1z
M.J.	—	0.103	0.050	0.350	0.136	0.1z
		Auditory experiment				
Average over subjects	—	0.102	0.051	0.363	0.193	0.012

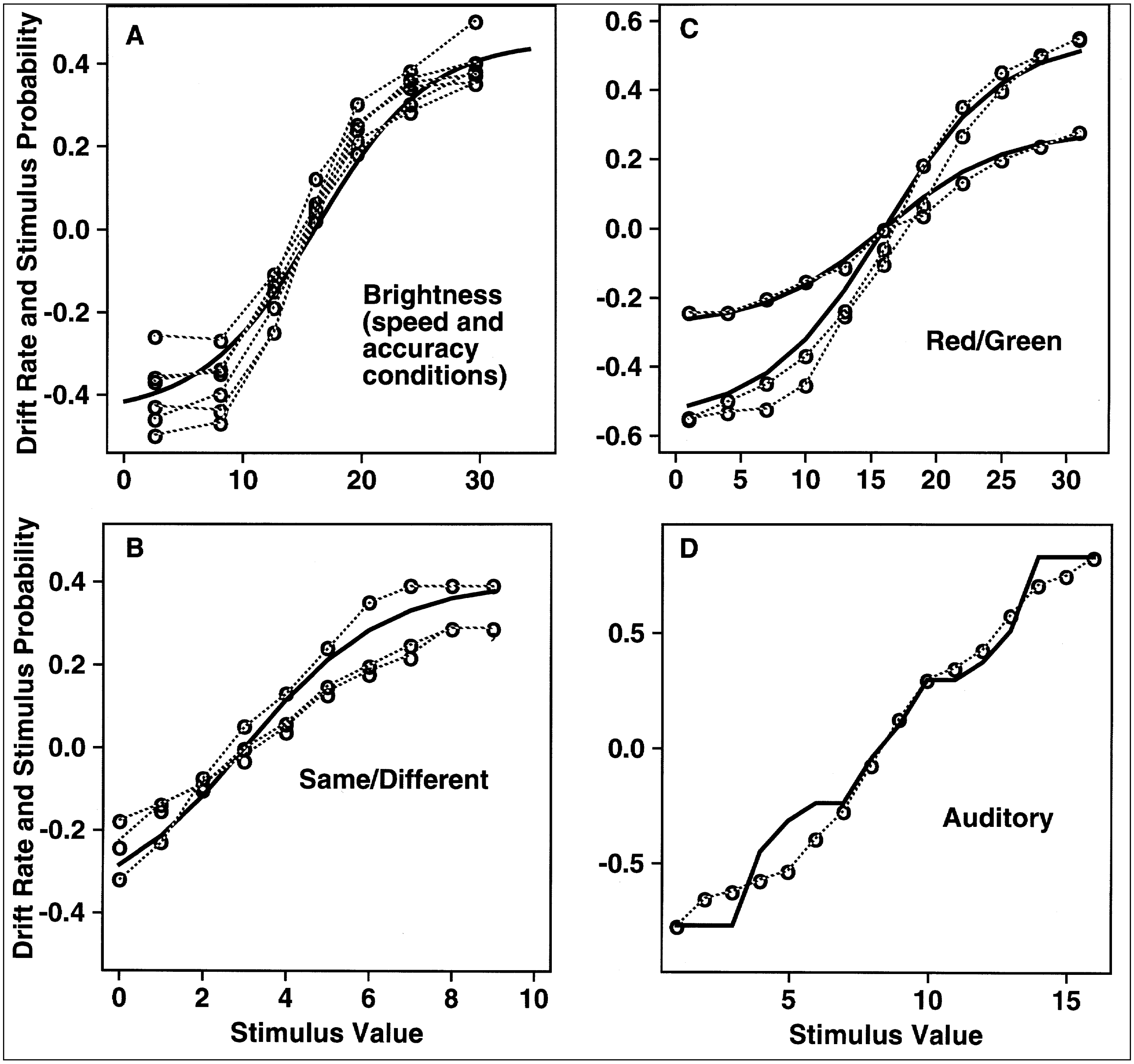

Fig. 6. Subjects' drift rates plotted against stimulus values for each experiment (dotted lines) and the probability with which each stimulus value was selected from the high distribution (Experiment 1), the red distribution (Experiment 2), the *same* distribution (Experiment 3), and the high-tone distribution (auditory experiment). Probabilities (thick lines) are transformed to the same scale as drift rate. For Experiment 1 (a), there are six drift-rate functions (3 subjects and two conditions), and the transformation from probability (p) to drift is $0.9p - 0.45$. For Experiment 3 (b), there are three drift-rate functions (one for each of 3 subjects), and the transformation from probability to drift is the same as for Experiment 1. For Experiment 2 (c), there are three drift-rate functions (one for each of 3 subjects), and two transformations from probability to drift are used: $1.1p - 0.55$ and $0.56p - 0.28$ (this latter transformation is for a subject who had a slight red/green discrimination problem). For the auditory experiment (d), there is one drift-rate function (one condition averaged over subjects), and the transformation from probability to drift is $1.6p - 0.8$.

latency-probability functions (Fig. 7) show the same kinds of asymmetries in correct versus error response times, and the drift rate for each subject matched the probability that a stimulus was drawn from the red distribution (Fig. 6). The red and green latency-probability functions were not mirror images of each other (as were the high and low functions for brightness), so they are plotted separately. The difference in the red/green latency-probability functions for red and green responses is fit in the model by asymmetric response boundaries (i.e., $z \neq a/2$); this adds one parameter to the model (Table 1). Response time distributions are not shown, but the model fit them as well as the distributions for brightness discrimination. The model fit the data with only six free parameters (see Table 1).

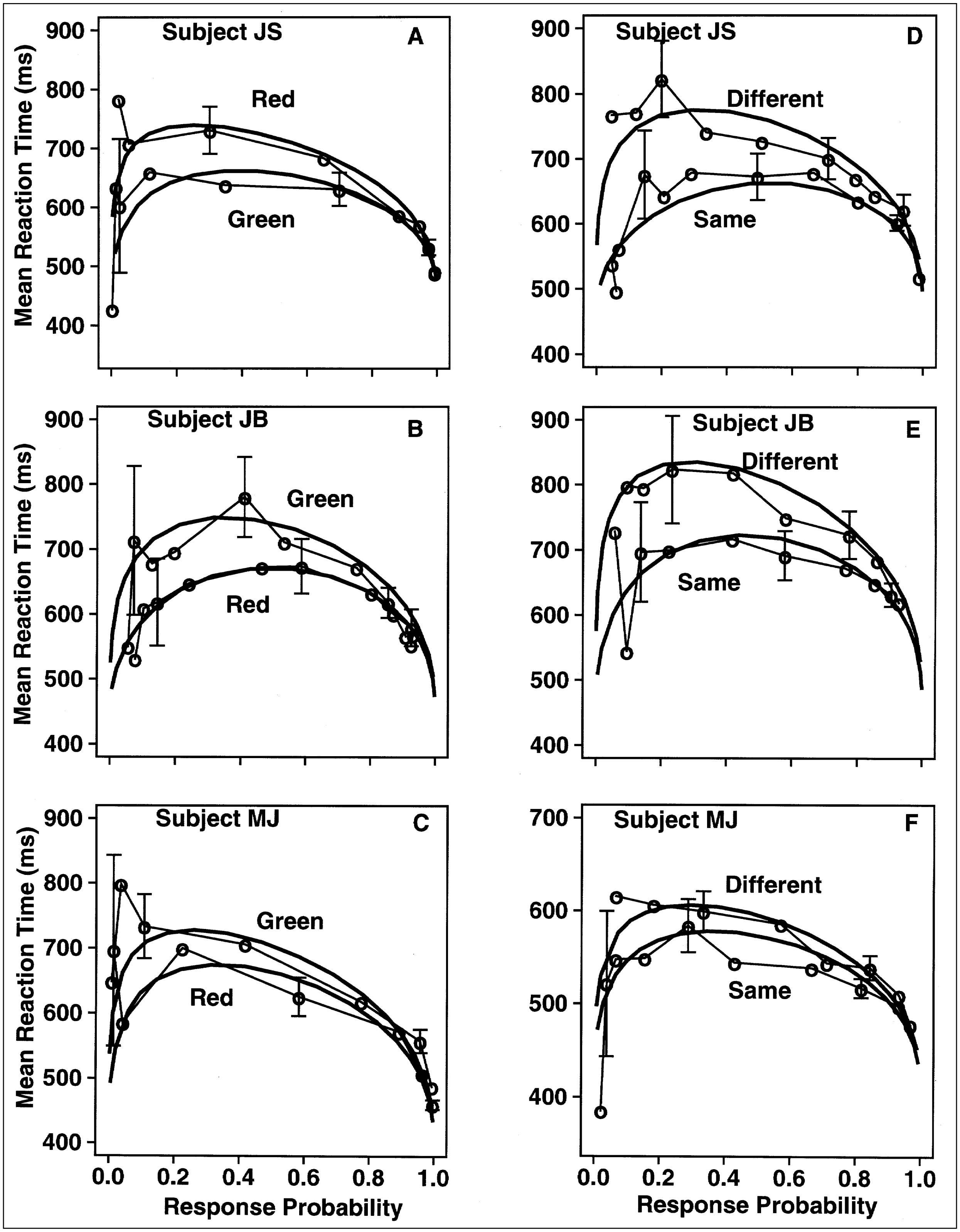

Fig. 7. Latency-probability functions for red/green (a, b, and c) and same/different (d, e, and f) discriminations for 3 subjects in each experiment. In each panel, the thick continuous line represents the theoretical prediction, and the circles represent the data (error bars represent 2 standard deviations). Correct responses are to the right of the .5 point for response probability, errors are to the left of the .5 point, and each correct response to the right with probability p has a corresponding error response to the left with probability $1 - p$.

In Experiment 3, subjects were asked to decide whether two stimuli were the same or different in brightness. This task does not allow a simple stimulus dimension to govern performance; instead, similarity between the two stimuli had to be computed (equivalent to an exclusive-OR computation). Despite the difference in the task, the model fit the latency-probability functions (Fig. 7) and response time distributions, and the drift rate closely corresponded to the probability that a stimulus was drawn from the *same* distribution (Fig. 6), the same correspondence as in the other experiments.

To add generality, we fit the model to data from an auditory discrimination experiment (Espinoza-Varas & Watson, 1994, Experiment 3). In that experiment, the frequency of tones was varied on a log scale from low to high, and stimuli were drawn from one of two distributions (high or low tone) on a log scale in a manner analogous to that of Experiment 1. Figure 4d shows the fit of the model to the latency-probability function (for the neutral, high-discriminability; instructed; and statistical-decision conditions, averaged over subjects). Figure 6 shows the correspondence of drift rate to the probability that the stimulus was drawn from the high distribution, the same relationship as in Experiments 1 through 3.

Parameter values for the fits for Experiment 3 and the auditory experiment are shown in Table 1.

CONCLUSION

The results of the experiments show a remarkable set of fits of a few-parameter model to data with a very large number of degrees of freedom. The model explains how both correct and incorrect decisions are made over time, how their relative speeds change as a function of experimental conditions, and why response time distributions have their characteristic shapes. No other current model can account for this complete pattern of data. The key feature of the model that allows it to deal with the complexities of error response times is the assumption that parameters of the model (drift rate and starting point) are variable from trial to trial. Other random-walk models might achieve the same success with similar assumptions about variability in parameters.

The diffusion model revealed that all the subjects based their decisions on stimulus probability (or some very similar function), and that this was true in both speed and accuracy conditions and for all four of the psychophysical judgments that were investigated. This is a rare example of a model allowing discovery of the stimulus information that drives decision processes (see Estes, 1995, for further discussion of stimulus probability and cognitive models). The diffusion model provides an account of both the stimulus information and the processing that makes use of that information.

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REFERENCES

Audley, R.J., & Pike, A.R. (1965). Some alternative stochastic models of choice. *The British Journal of Mathematical and Statistical Psychology, 18,* 207–225.
Busemeyer, J.R., & Townsend, J.T. (1993). Decision field theory: A dynamic-cognitive approach to decision making in an uncertain environment. *Psychological Review, 100,* 432–459.
Espinoza-Varas, B., & Watson, C. (1994). Effects of decision criterion on latencies of binary decisions. *Perception & Psychophysics, 55,* 190–203.
Estes, W.K. (1995). Response processes in cognitive models. In R.F. Lorch & E.J. O'Brien (Eds.), *Sources of coherence in text comprehension* (pp. 51–71). Hillsdale, NJ: Erlbaum.
Feller, W. (1968). *An introduction to probability theory and its applications.* New York: John Wiley & Sons.
LaBerge, D.A. (1962). A recruitment theory of simple behavior. *Psychometrika, 27,* 375–396.
Laming, D.R.J. (1968). *Information theory of choice reaction time.* New York: Wiley.
Link, S.W., & Heath, R.A. (1975). A sequential theory of psychological discrimination. *Psychometrika, 40,* 77–105.
Luce, R.D. (1986). *Response times.* New York: Oxford University Press.
Maddox, W.T., & Ashby, F.G. (1993). Comparing decision bound and exemplar models of categorization. *Perception & Psychophysics, 53,* 49–70.
Nosofsky, R.M., & Palmeri, T.J. (1997). An exemplar based random walk model of speeded classification. *Psychological Review, 104,* 266–300.
Ratcliff, R. (1978). A theory of memory retrieval. *Psychological Review, 85,* 59–108.
Ratcliff, R. (1981). A theory of order relations in perceptual matching. *Psychological Review, 88,* 552–572.
Ratcliff, R. (1985). Theoretical interpretations of speed and accuracy of positive and negative responses. *Psychological Review, 92,* 212–225.
Ratcliff, R. (1988). Continuous versus discrete information processing: Modeling the accumulation of partial information. *Psychological Review, 95,* 238–255.
Ratcliff, R., & Murdock, B.B., Jr. (1976). Retrieval processes in recognition memory. *Psychological Review, 83,* 190–214.
Ratcliff, R., Van Zandt, T., & McKoon, G. (in press). Comparing connectionist and diffusion models of reaction time. *Psychological Review.*
Rouder, J.N. (1996). Premature sampling in random walks. *Journal of Mathematical Psychology, 40,* 289–296.
Smith, P.L. (1995). Psychophysically principled models of visual simple reaction time. *Psychological Review, 102,* 567–591.
Smith, P.L., & Vickers, D. (1988). The accumulator model of two-choice discrimination. *Journal of Mathematical Psychology, 32,* 135–168.
Stone, M. (1960). Models for choice reaction time. *Psychometrika, 25,* 251–260.
Strayer, D.L., & Kramer, A.F. (1994). Strategies and automaticity: I. Basic findings and conceptual framework. *Journal of Experimental Psychology: Learning, Memory, and Cognition, 20,* 318–341.
Swensson, R.G. (1972). The elusive tradeoff: Speed versus accuracy in visual discrimination tasks. *Perception & Psychophysics, 12,* 16–32.
Van Zandt, T., & Ratcliff, R. (1995). Statistical mimicking of reaction time distributions: Mixtures and parameter variability. *Psychonomic Bulletin & Review, 2,* 20–54.
Vickers, D. (1979). *Decision processes in visual perception.* New York: Academic Press.
Vickers, D., Caudrey, D., & Willson, R.J. (1971). Discriminating between the frequency of occurrence of two alternative events. *Acta Psychologica, 35,* 151–172.

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