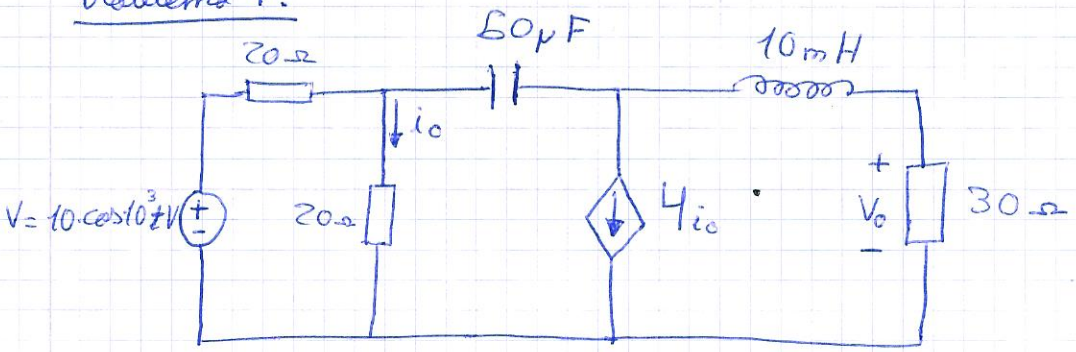
Problemas tema 4

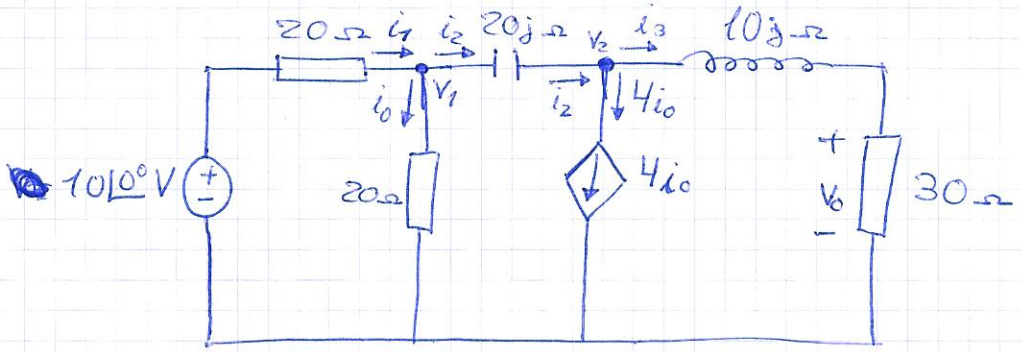
Problema 1:



$$V = 10\angle 0^\circ\, V$$

$$X_c = \frac{1}{-j\cdot\omega\cdot C} = \frac{1}{-(10^3\cdot 50\cdot 10^{-6})j} = +20j\ \Omega$$

$$X_L = j\cdot\omega\cdot L = 10^3\cdot 10\cdot 10^{-3}\, j = 10j\ \Omega$$



Nodo 1 ⇒ $i_1 = i_2 + i_o$

$$\frac{10 - V_1}{20} = \frac{V_1 - V_2}{20j} + \frac{V_1}{20}$$

$$\frac{20j\,(10 - V_1)}{20} = \frac{20j\,(V_1 - V_2)}{20j} + \frac{20j\cdot V_1}{20}$$

$$10j - V_1 j = \frac{V_1 - V_2}{j} + V_1 j$$

$$10 - V_1 = -j(V_1 - V_2) + V_1$$

$$10 - V_1 = -V_1 j + V_2 j + V_1$$

$$2V_1 - V_1 j + V_2 j = 10$$

$$\boxed{(2 - j)\cdot V_1 + (j)\, V_2 = 10}$$

Nodo 2 $\Rightarrow$ $i_2 = i_3 + 4i_o$

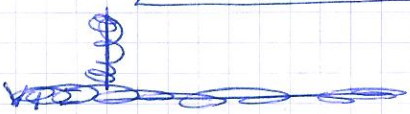
$$\frac{V_1 - V_2}{20j} = \frac{V_2}{30+10j} + \frac{4V_1}{20}$$

$$\frac{10\cdot(V_1 - V_2)}{20j} = \frac{10V_2}{30+10j} + \frac{10\cdot 4V_1}{20}$$

$$-0,5V_1 j + 0,6V_2 j = 0,3V_2 - 0,1V_2 j + 2V_1$$

$$2V_1 + 0,5V_1 j + 0,3V_2 - 0,6V_2 j = 0$$

$$\boxed{(2+0,5j)V_1 + (0,3-0,6j)V_2 = 0}$$



$$\left.\begin{aligned}(2-j)V_1 + (j)V_2 &= 10\\ (2+0,5j)V_1 + (0,3-0,6j)V_2 &= 0\end{aligned}\right\}$$

$$V_1 = \frac{\begin{vmatrix} 10 & j \\ 0 & 0,3-0,6j \end{vmatrix}}{\begin{vmatrix} 2-j & j \\ 2+0,5j & 0,3-0,6j \end{vmatrix}} = 1,574\angle -24'15^\circ \text{ V}$$

$$V_2 = \frac{\begin{vmatrix} 2-j & 10 \\ 2+0,5j & 0 \end{vmatrix}}{\begin{vmatrix} 2-j & j \\ 2+0,5j & 0,3-0,6j \end{vmatrix}} = 6,457\angle 88'69^\circ \text{ V}$$

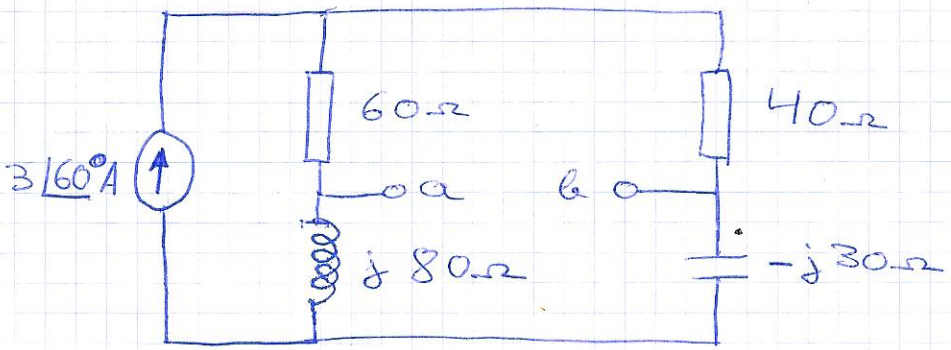
$$i_3 = \frac{V_2}{30+10j} = \frac{6,457\angle 88'69^\circ}{31,62\angle 18,43^\circ} = 0,204\angle 70,26^\circ \text{ A}$$

$V_o = i_3 \cdot R$

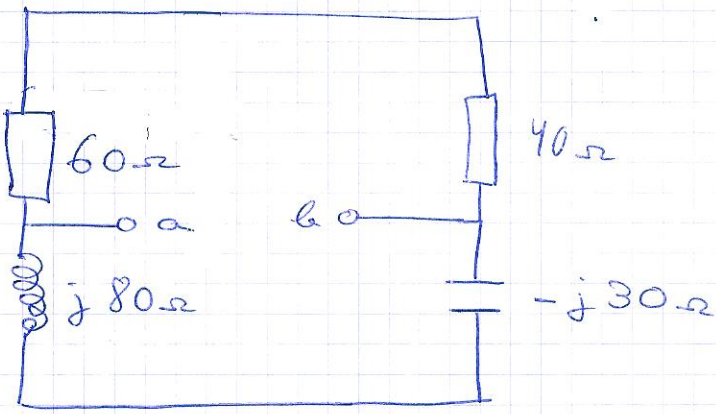
$V_o = 0,204\angle 70,26^\circ \cdot 30 = 6,12\angle 70,26^\circ \text{ V}$

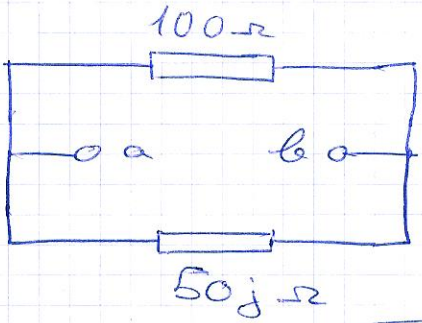
(2) $\boxed{V_o = 6,12 \cdot \cos(1000t + 70,26^\circ)\text{ V}}$

Problema 2:



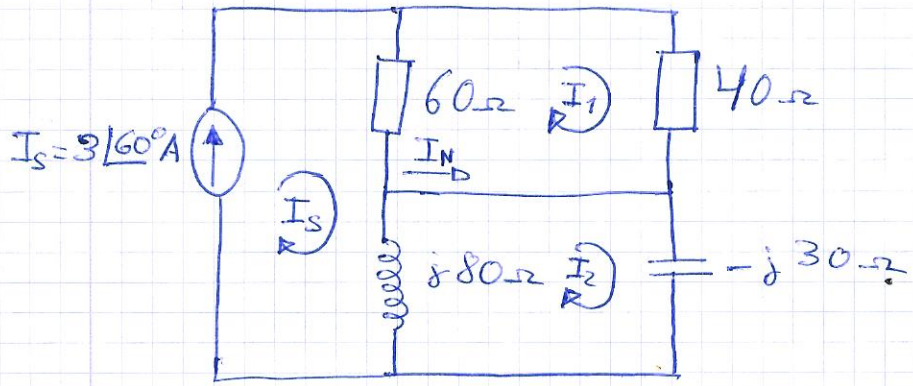
Z_N:





$$Z_N = \frac{100 \cdot 50j}{100 + 50j} = 20 + 40j\ \Omega$$

$$\boxed{Z_N = 20 + 40j\ \Omega = 44{,}72 \angle 63{,}43^\circ\ \Omega}$$



$I_S = 3\angle 60°\ A = 1,5 + 2,6j\ A$

Malla 1 $\Rightarrow$ $(40+60)\cdot I_1 - 60\cdot I_S = 0$

$100\, I_1 - 60\cdot(1,5+2,6j) = 0$

$100\, I_1 - 90 - 156j = 0$

$I_1 = \dfrac{90+156j}{100}$

$I_1 = 0,9 + 1,56j\ A = 1,804\angle 60,02°\ A$

Malla 2 $\Rightarrow$ $(80j - 30j)\cdot I_2 - 80j\cdot I_S = 0$

$50j\, I_2 - 80j\cdot(1,5+2,6j) = 0$

$50j\, I_2 - 120j + 208 = 0$

$I_2 = \dfrac{-208+120j}{50j}$

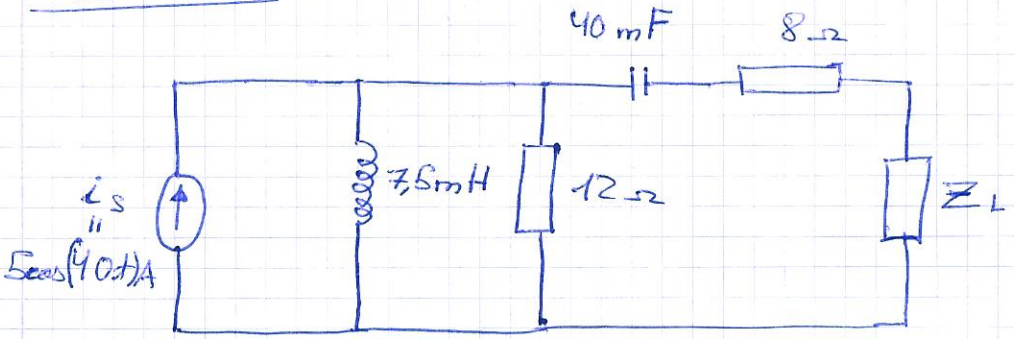
$I_2 = 2,4 + 4,16j\ A = 4,80\angle 60,02°\ A$

$I_N = I_2 - I_1$

~~$I_N = 1,5 + 2,6j\ A$~~ ~~$= 2,999\angle 60,02°\ A$~~

$\boxed{I_N = 1,5 + 2,6j\ A = 2,999\angle 60,02°\ A}$

Problema 3:

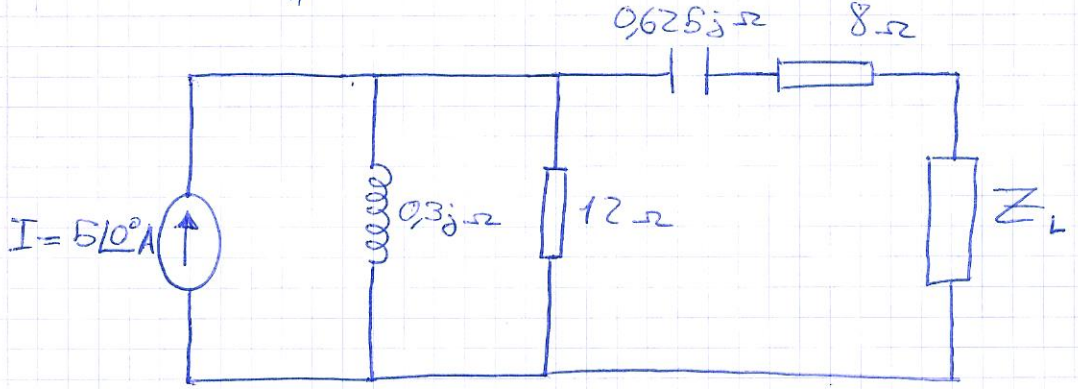


$\omega = 40\ rad/s \qquad I = 5\angle 0°\ A$

$$X_L = j \cdot \omega \cdot L = 40 \cdot (7{,}5 \cdot 10^{-3})\, j = 0{,}3\, j\ \Omega$$

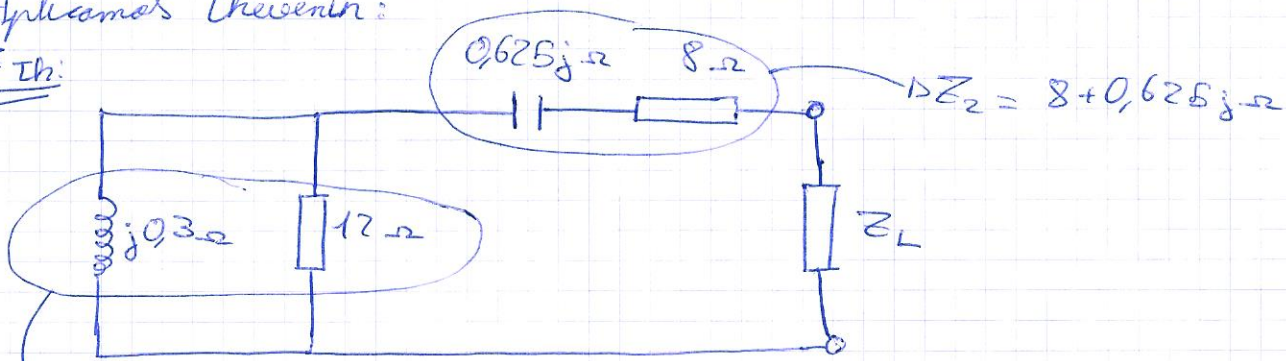
$$X_C = \frac{1}{-j\,\omega \cdot C} = \frac{1}{-40 \cdot (40 \cdot 10^{-3})\, j} = 0{,}625\, j\ \Omega$$

Circuito equivalente:

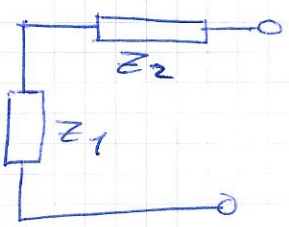


Aplicamos Thevenin:

Z_{Th}:



$$Z_1 = \frac{0{,}3j \cdot 12}{12 + 0{,}3j} = 7{,}5 \cdot 10^{-3} + 0{,}3\, j\ \Omega$$



$$Z_{Th} = Z_L = Z_1 + Z_2$$

$$Z_L = (7{,}5 \cdot 10^{-3} + 0{,}3\, j) + (8 + 0{,}625\, j)$$

$$\boxed{Z_L = 8{,}0075 + 0{,}925\, j\ \Omega}$$

$P_{máx}$:

$V = I \cdot R$

$$P = \frac{|V_{Th}|^2}{8 \cdot Z_L}$$

$V = 5 \cdot 8{,}061 \angle 6{,}59°$

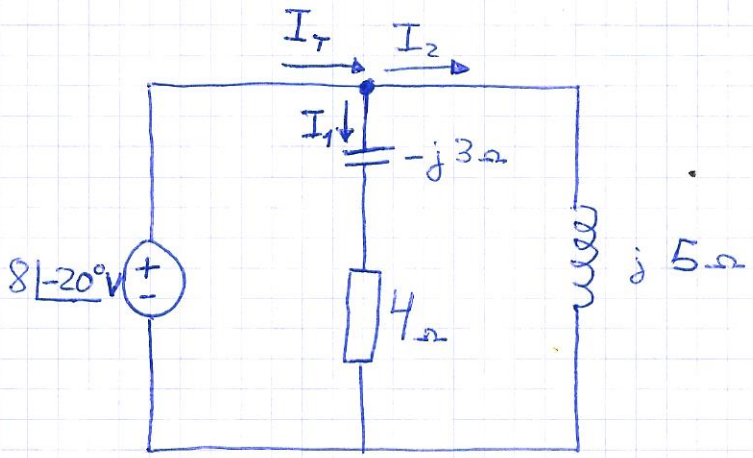
$V = 40{,}30 \angle 6{,}59°\ V$

~~[illegible]~~

$$P = \frac{|40{,}30|^2}{8 \cdot 8{,}061}$$

$\boxed{P = 25{,}18\ W}$

Problema 4:



$$I_1 = \frac{8\angle -20°}{4-3j} = \frac{8\angle -20°}{5\angle -36{,}87°} = 1{,}6\angle 16{,}87°\ A$$

$$I_2 = \frac{8\angle -20°}{5j} = \frac{8\angle -20°}{5\angle 90°} = 1{,}6\angle -110°\ A$$

$$I_T = I_1 + I_2 = 1{,}6\angle 16{,}87° + 1{,}6\angle -110° = 1{,}431\angle -46{,}56°\ A$$

En la fuente:

$$S_F = \frac{V \cdot I_T^*}{2} = \frac{8\angle -20° \cdot 1{,}431\angle 46{,}56°}{2} = 5{,}72\angle 26{,}56° = 5{,}12 + 2{,}56j\ V\cdot A$$

En el capacitador:

$$S_C = \frac{V \cdot |I_1|}{2} = \frac{Z \cdot |I_1|^2}{2} = \frac{(-3j)\cdot 1{,}6^2}{2} = -3{,}84j\ V\cdot A$$

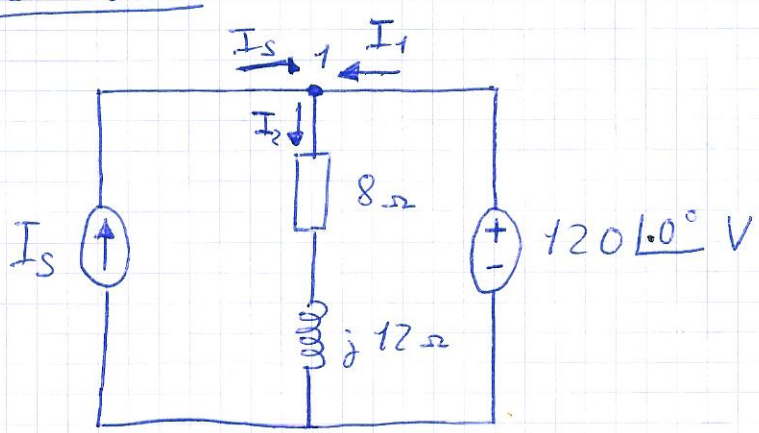
En la resistencia:

$$S_R = \frac{Z_R \cdot |I_1|^2}{2} = \frac{4j \cdot 1{,}6^2}{2} = 5{,}12j\ V\cdot A$$

En la bobina:

$$S_B = \frac{Z_B \cdot |I_2|^2}{2} = \frac{5j \cdot 1{,}6^2}{2} = 6{,}4j\ V\cdot A$$

Problema 5:



Nodo 1 ⇒ $I_S + I_1 = I_2$

$$I_S = I_2 - I_1$$

$$I_S = \frac{120}{8+12j} - I_1$$

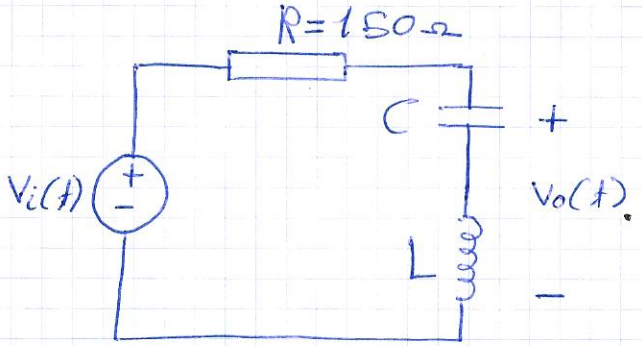
$$I_1^* = \frac{S}{V} = \frac{2500-400j}{120} = 20,83-3,33j = 20,83+3,33j$$

$$I_S = \frac{120}{8+12j} - (20,83+3,33j) = \boxed{-16,71-10,25j\ A}$$

$$\Downarrow$$

$$\boxed{19,18\angle -147,69°\ A}$$

Problema 6:



$f_0 = 200\ Hz$

$$B = w_2 - w_1 = 2\pi \cdot (f_2 - f_1)$$

$$B = 2\pi \cdot 100 = 200\pi\ rad/s$$

$$L = \frac{R}{B}$$

$$\boxed{L = \frac{150}{200\pi} = 0{,}2387\,H}$$

$$w_0 = 2\pi \cdot f_0$$

$$w_0 = 2\pi \cdot 200 = 400\pi$$

$$w_0 = \frac{1}{\sqrt{L \cdot C}}$$

$$400\pi = \frac{1}{\sqrt{0{,}2387 \cdot C}}$$

$$(400\pi)^2 = \frac{1}{0{,}2387 \cdot C}$$

$$C = \frac{1}{0{,}2387 \cdot (400 \cdot \pi)^2}$$

$$\boxed{C = 2{,}653 \cdot 10^{-6}\,F = 2{,}653\,\mu F}$$