PROBLEMA 1

$X(n)$, $Y(n)$ indeptes en n y discretos

$m_x = 1/\sqrt{2}$ $\quad R_x(k) = (1/2)^k$

$m_y = 2$ $\quad R_y(k) = (1/3)^k$

(a) $Z(n) = X(n) + Y(n)$; energía promedio $= R_Z(0) = E[Z(n)^2]$

$$E[Z(n)^2] = E[X(n)^2 + Y(n)^2] + 2E[X(n)]\overset{X,Y \text{ indeptes}}{\cdot} E[Y(n)] =$$

$$= E[X(n)^2] + E[Y(n)^2] + 2 \cdot m_x \cdot m_y =$$

$$R_x(0) + R_y(0) + 2m_x \cdot m_y = (1/2)^0 + (1/3)^0 + 2 \cdot \left(\frac{2}{\sqrt{2}}\right)$$

$$= \underline{2 + 2\sqrt{2}} = \text{energía promedio}$$

(b) $$C_Z(n, n+k) = \underbrace{E[Z(n)Z(n+k)]}_{R_Z(n,n+k)} - m_Z(n) \cdot m_Z(n+k)$$

$$* R_Z(n,n+k) = E[Z(n)Z(n+k)] = E[(X(n)+Y(n))(X(n+k)+Y(n+k))] =$$

$$= E[X(n)X(n+k)] + E[X(n) \cdot Y(n+k)] + E[Y(n) \cdot X(n+k)] + E[Y(n)Y(n+k)]$$

$$= R_x(k) + R_y(k) + 2m_x \cdot m_y = (1/2)^k + (1/3)^k + 2\sqrt{2}$$

$$* \ m_Z(n) \cdot m_Z(n+k) = \left(\frac{1}{\sqrt{2}} + 2\right)^2 = 4 + \frac{1}{2} + \frac{2 \cdot 2}{\sqrt{2}} = 4'5 + 2\sqrt{2}$$

$$\Rightarrow C_Z(n,n+k) = R_Z(n,n+k) - m_Z(n) m_Z(n+k) =$$

$$= (1/2)^k + (1/3)^k + 2\sqrt{2} - 4'5 - 2\sqrt{2}$$

$$= (1/2)^k + (1/3)^k - 4'5$$

(c) Muestreo de orden 1 de $X(n)$, $f_X(x) = \lambda e^{-\lambda x}$ $0 < x < \infty$

la condición de que $\int_0^\infty \lambda e^{-\lambda x} dx = 1$ se cumple

para cualquier valor de λ $\Rightarrow$ no lo puedo usar para fijar la λ.

- usando que $E[X(n)] = 1/\sqrt{2}$:

$$E[X(n)] = \int x \lambda e^{-\lambda x} dx \underset{\substack{\uparrow \\ \text{tablas de integrales}}}{=} \lambda \cdot \frac{1}{\lambda^2} = \frac{1}{\lambda} = \frac{1}{\sqrt{2}} \,,\ \lambda > 0$$

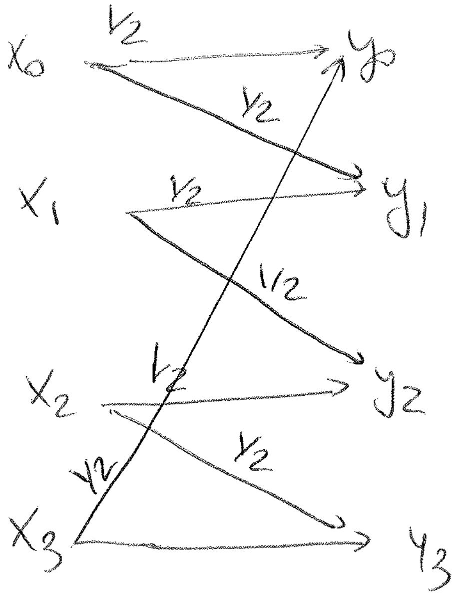
$$\Rightarrow \underline{\lambda = \sqrt{2}}$$

- usando que $E[X(n)^2] = R_X(0) = \left(\frac{1}{2}\right)^0 = 1$:

$$R_X(0) = \int_0^\infty x^2 \lambda e^{-\lambda x} dx = \cancel{\lambda} \cdot \frac{2!}{\lambda^{\cancel{3}2}} = 1$$

$$\frac{2}{\lambda^2} = 1 \Rightarrow \underline{\lambda = \sqrt{2}}$$

PROBLEMA 2



$P(x_0) = 0'2 \quad P(x_1) = 0'4 \quad P(x_2) = 0'3 \quad P(x_3) = 0'1$

$P(y_0|x_0) = 1/2 \quad P(y_1|x_0) = 1/2$

$P(y_1|x_1) = 1/2 \quad P(y_2|x_1) = 1/2$

$P(y_3|x_2) = 1/2 \quad P(y_2|x_2) = 1/2$

$P(y_3|x_3) = 1/2 \quad P(y_0|x_3) = 1/2$

$p(x_0,y_0) = p(y_0|x_0)\, p(x_0) = 0'2 \cdot 0'5 = 0'1$

$P(x_0,y_1) = P(y_1|x_0)\, p(x_0) = 1/2 \cdot 0'2 = 0'1$

$P(x_1,y_1) = P(y_1|x_1)\, p(x_1) = 1/2 \cdot 0'4 = 0'2$

$p(x_1,y_2) = p(y_2|x_1)\, p(x_1) = 1/2 \cdot 0'4 = 0'2$

$P(x_2,y_2) = p(y_2|x_2)\, p(x_2) = 1/2 \cdot 0'3 = 0'15$

$P(x_2,y_3) = p(y_3|x_2)\, p(x_2) = 1/2 \cdot 0'3 = 0'15$

$p(x_3,y_3) = P(y_3|x_3)\, p(x_3) = 1/2 \cdot 0'1 = 0'05$

$P(x_3,y_0) = P(y_0|x_3)\, p(x_3) = 1/2 \cdot 0'1 = 0'05$

$P(y_0) = P(x_0,y_0) + p(x_3,y_0) = 0'05 + 0'1 = 0'15$

$P(y_1) = p(x_0,y_1) + P(x_1,y_1) = 0'1 + 0'2 = 0'3$

$p(y_2) = p(x_1,y_2) + p(x_2,y_2) = 0'2 + 0'15 = 0'35$

$p(y_3) = P(x_2,y_3) + p(x_3,y_3) = 0'15 + 0'05 = 0'2$

Ⓐ Información mutua del canal

$$I(X,Y) = H(Y) - H(Y|X)$$

$$H(Y) = \sum_{y_j} p(y_j) \log \frac{1}{p(y_j)} = 0'15 \log \frac{1}{0'15} + 0'3 \log \frac{1}{0'3} + 0'35 \log \frac{1}{0'35}$$

$$+ 0'2 \log \frac{1}{0'2} = 1'9261 \text{ bits}$$

$$H(Y|X) = \sum_{x_i} \sum_{y_j} p(x_i, y_j) \cdot \log \frac{1}{p(y_j|x_i)} = \sum_{x_i} \sum_{y_j} p(x_i) \cdot p(y_j|x_i) \log \frac{1}{p(y_j|x_i)}$$

$$= \sum_{\forall x_i} p(x_i) \cdot \frac{1}{2} \log 2 = 1 \text{ bit}$$

$$\Rightarrow I(X,Y) = H(Y) - H(Y|X) = 1'9261 \text{ bits} - 1 \text{ bit} = 0'9261 \text{ bits}$$

Ⓑ Capacidad del canal:

$$C = \max_{\{p(x_i)\}} (I(X,Y)) = \max_{\{p(x_i)\}} [H(Y) - H(Y|X)] =$$

$$\max \left[H(Y) - \underbrace{\sum_{x_i} p(x_i) \cdot \left[\frac{1}{2} \cdot \log 2\right] \cdot 2}_{=1} \right]$$

sea cual sea la distribución de $p(x_i)$

$$\Rightarrow C = \max_{\{p(x_i)\}} [H(Y) - 1] = \max_{\{p(x_i)\}} [H(Y)] - 1 = 2 - 1 = 1 \text{ bit/ } \text{símbolo}$$

Supongo símbolos equiprobables

para $H(Y)_{max} = \frac{4}{4} \log_2 4 = 2 \text{ bits}$

Hará falta comprobar que existe una distribución de $p(x_i)$ con la que se consigue $p(y_i) = 1/4 \ \forall i$

Habrá que resolver el sistema

$$\frac{1}{4} = p(y_0) = p(y_0|x_0) \cdot p(x_0) + p(y_0|x_3)\, p(x_3)$$
$$\frac{1}{4} = p(y_1) = p(y_1|x_0)\, p(x_0) + p(y_1|x_1) \cdot p(x_1)$$
$$\frac{1}{4} = p(y_2) = p(y_2|x_1)\, p(x_1) + p(y_2|x_2)\, p(x_2)$$
$$\frac{1}{4} = p(y_3) = p(y_3|x_3)\, p(x_3) + p(y_3|x_2)\, p(x_2)$$

→ resolviendo encontramos que $p(x_i) = 1/4 \ \forall i$ satisface las ecuaciones.