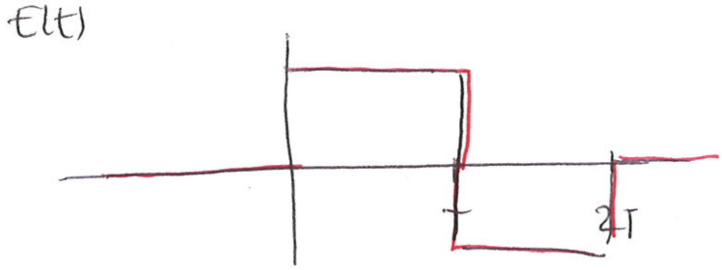
PROBLEMA 1

- $H(t) = \begin{cases} 1 & \text{si } X(t) \geq 0 \\ -1 & \text{si } X(t) < 0 \end{cases}$; $X(t) = A \cdot E(t)$; A unif en $[-3,1]$

$E(t) = \begin{cases} 1 & \text{si } t \in [0,T] \\ -1 & \text{si } t \in [T,2T) \\ 0 & \text{resto} \end{cases}$



- A unif en $[-3,1)$ $\Rightarrow f_a(x) = \begin{cases} 1/4 & \text{si } x \in [-3,1] \\ 0 & \text{resto} \end{cases}$

a) pmf de orden 1.

Tres casos posibles

(C1) $t \notin [0,2T) \Rightarrow E(t) = 0 \Rightarrow X(t) = 0\ \forall t \Rightarrow H(t) = 1\ \forall t$

$P[H(t) = 1] = 1$
$P[H(t) = -1] = 0$

(C2) $t \in [0,T] \Rightarrow E(t) = 1 \Rightarrow X(t) = A$

$P[H(t) = 1] = P[X(t) \geq 0] = P[A \geq 0] \overset{\text{A unif en } [-3,1]}{=} \int_0^1 \frac{1}{4}\, da = \frac{1}{4}$

$P[H(t)] = -1 = P[X(t) < 0] = P[A < 0] = \int_{-3}^{0} \frac{1}{4}\, da = 3/4$

$\Rightarrow$ CASO 2: $P[H(t) \overset{?}{=} 1] = 1/4$
$P[H(t) = -1] = 3/4$

CASO 3 $\quad t \in [T, 2T] \Rightarrow E(t) = -1 \Rightarrow X(t) = -A$

$$p[H(t)=1] = p[X(t) \geq 0] = p[-A \geq 0] = p[A \leq 0] = \int_{-3}^{0} \frac{1}{4} da = 3/4$$

$$p[H(t) = -1] = p[X(t) < 0] = p[-A < 0] = p[A > 0] = \int_{0}^{1} \frac{1}{4} da = 1/4$$

$\Rightarrow$ CASO 3 : $\quad p[H(t)=1] = \frac{3}{4}$

$p[H(t)=-1] = \frac{1}{4}.$

(b) Media de H(t)

si $t \notin [0, 2T] \Rightarrow E[H(t)] = 1 \cdot 1 = 1$

si $t \in [0, T] \Rightarrow E[H(t)] = 1 \cdot \left(\frac{1}{4}\right) + (-1)\left(\frac{3}{4}\right) = -1/2$

si $t \in [T, 2T] \Rightarrow E[H(t)] = 1\left(\frac{3}{4}\right) + (-1)\left(\frac{1}{4}\right) = \frac{1}{2}$

$$m_H(t) = \begin{cases} -1/2 & \text{si } t \in [0, T] \\ +1/2 & \text{si } t \in [T, 2T] \\ 1 & \text{resto.} \end{cases}$$

(c) Funcion pmf de orden 2

$p(H(t_1), H(t_2))$; Distingo 8 casos posibles de t_1 y t_2

CASO 1 $\quad t_1, t_2 \notin [0, 2T] \Rightarrow H(t_1) = H(t_2) = 1 \quad (X(t) = 0 \ \forall t)$

$p[1,1] = 1$

p(resto de comb tienen prob. 0.

CASO 2 $\quad t_1 \notin [0,2T]$ $\quad \Rightarrow X(t_1) = 1$ siempre
$t_2 \in [0,T]$ $\quad \Rightarrow$

los posibles valores son $(1,1)$, $(1,-1)$ $\Rightarrow \begin{cases} P(1,1) = 1/4 \\ P(1,-1) = 3/4 \end{cases}$

resto de comb. tienen prob $= 0$

CASO 3 $\quad t_1 \notin [0,2T]$
$t_2 \in [T,2T)$ $\quad \Rightarrow$ $P(1,1) = 3/4$
$P(1,-1) = 1/4$
resto de combinac. tiene prob $= 0$

CASO 4 $\quad t_1 \in [0,T]$
$t_2 \in [0,T)$ $\quad \Rightarrow$ $E(t_1) = E(t_2) = 1$
$X(t_1) = A = X(t_2)$

$P(1,1) = 1/4$
$P(-1,-1) = 3/4$
resto de combinac. tienen probabilidad 0.

CASO 5: $\quad t_1 \in [T,2T)$ $\quad \Rightarrow E(t_1) = E(t_2) = -1$
$t_2 \in [T,2T]$ $\quad X(t_1) = -A = X(t_2)$

$P(1,1) = 3/4$
$P(-1,-1) = 1/4$
resto de combinac. tiene prob $= 0$

CASO 6 $t_1 \in [0,T] \rightarrow x(t_1) = A$

$t_2 \in [T,2T] \rightarrow x(t_2) = -A$

$$P[H(t_1)=1,\ H(t_2)=-1] = P[(A \geq 0) \cap (-A < 0)] = P[A \geq 0] = 1/4$$

$$P[H(t_1)=1,\ H(t_2)=1] = P[(A \geq 0) \cap (-A \geq 0)] = 0$$

$$P[H(t_1)=-1,\ H(t_2)=-1] = P[(A<0) \cap P(A \geq 0)] = 0$$

$$P[H(t_1)=-1,\ H(t_2)=1] = P[(A<0) \cap P(-A \geq 0)] = 3/4$$

$$\Rightarrow \begin{cases} P(1,-1) = 1/4 \\ P(-1,1) = 3/4 \end{cases}$$

resto de combinac tiene prob $= 0$

CASO 7 $t_1 \in [T,2T]$, $t_2 \notin [0,2T]$ siguiendo el mismo razonamiento

$$\begin{cases} P(1,1) = \frac{3}{4} \\ P(-1,1) = \frac{1}{4} \end{cases}$$

resto de combinac tiene prob $= 0$

caso 8 $t_1 \in [0,T]$, $t_2 \notin [0,2T]$

$$P(1,1) = 1/4$$

$$P(-1,1) = 3/4$$

(d) Autocorrelación de $H(t)$:

Distinguiendo de nuevo los 8 casos:

CASO 1 $\quad t_1, t_2 \notin [0, 2T]$

$H(t_1) = H(t_2) = 1 \quad \Rightarrow \quad R_H(t_1, t_2) = E[H(t_1)H(t_2)] = 1 \cdot 1 [p(1,1)] = 1$

C2 $\quad t_1 \notin [0, 2T] \rightarrow H(t_1) = 1 \ \forall t$

$t_2 \in [0, T] \rightarrow H(t_2) = \begin{cases} 1 & \text{con prob} = 1/4 \\ -1 & \text{con prob} = 3/4 \end{cases}$

$R_H(t_1, t_2) = E[H(t_1)H(t_2)] = 1 \cdot 1 \cdot p(1,1) + 1(-1)\, p(1,-1)$

$= \frac{1}{4} - \frac{3}{4} = -\frac{1}{2}$

CASO 3 $\quad t_1 \notin [0, 2T] \quad \rightarrow p(1,1) = 3/4$

$t_2 \in [T, 2T] \quad p(1,-1) = 1/4$

resto de comb con prob $= 0$

$R_H(t_1, t_2) = E[H(t_1)H(t_2)] = 1 \cdot 1 \left(\frac{3}{4}\right) + (1)(-1)\left(\frac{1}{4}\right) = \frac{3}{4} - \frac{1}{4} = \frac{1}{2}$

CASO 4: $\quad t_1 \in [0, T] \quad \rightarrow p(1,1) = 1/4$

$t_2 \in [0, T] \quad p(-1,-1) = 3/4$

resto prob $= 0$

$R_H(t_1, t_2) = E[H(t_1)H(t_2)] = (1)(1)\left(\frac{1}{4}\right) + (-1)(-1)\left(\frac{3}{4}\right) = 1$

__CASO 5__: $t_1 \in [T, 2T]$
$t_2 \in [T, 2T)$

$$\rightarrow \begin{cases} p(1,1) = \frac{3}{4} \\ p(-1,-1) = 1/4 \\ \text{resto con prob} = 0 \end{cases}$$

~~[illegible]~~

$$R_H(t_1, t_2) = E[H(t_1) H(t_2)] = (1)(1)\left(\frac{3}{4}\right) + (-1)(-1)\left(\frac{1}{4}\right) = 1$$

__CASO 6__ $t_1 \in [0, T]$
$t_2 \in [T, 2T]$

$$\rightarrow \begin{cases} p(1,-1) = 1/4 \\ p(-1,1) = 3/4 \\ \text{resto con prob} = 0 \end{cases}$$

$$R_H(t_1, t_2) = E[H(t_1) \cdot H(t_2)] = (1)(-1)\left(\frac{1}{4}\right) + (-1)(1)\left(\frac{3}{4}\right) = -1$$

__CASO 7__ $t_1 \in [T, 2T]$
$t_2 \notin [0, 2T]$

$$\rightarrow \begin{cases} p(1,1) = \frac{3}{4} \\ p(-1,1) = 1/4 \\ \text{resto de comb. con prob} = 0 \end{cases}$$

$$R_H(t_1, t_2) = (1)(1)(3/4) + (-1)(1)(1/4) = \frac{1}{2}$$

__CASO 8__
$t_1 \in [0, T]$
$t_2 \notin [0, 2T)$

$$\rightarrow \begin{cases} p(1,1) = 1/4 \\ p(-1,1) = 3/4 \end{cases}$$

$$R_H(t_1, t_2) = E[H(t_1) H(t_2)] = (1)(1)\left(\frac{1}{4}\right) + (-1)(1)\left(\frac{3}{4}\right) = -\frac{1}{2}$$

PROBLEMA 2

$X = \{N, S, E\}$, Fuente de Markov de orden 1.

$$\begin{pmatrix} 0'4 & 0'1 & 0'5 \\ 0'2 & 0'5 & 0'3 \\ 0'25 & 0'6 & 0'15 \end{pmatrix} = \begin{pmatrix} P(N|N) & P(S|N) & P(E|N) \\ P(N|S) & P(S|S) & P(E|S) \\ P(N|E) & P(S|E) & P(E|E) \end{pmatrix}$$

a) Determinar la entropía de la fuente $H(X)$

~~PASE~~ $P(SSEE) = P(S) \cdot P(S|S) \cdot P(E|S) \cdot P(E|E) = 0'0135$

$$= P(S) \cdot 0'5 \cdot 0'3 \cdot 0'15 = 0'0135$$

$$\Rightarrow P(S) = \frac{0'0135}{0'0225} = 0'6$$

$$P(ENS) = P(E)\, P(N|E)\, P(S|N) = P(E) \cdot 0'25 \cdot 0'1 = 0'0075$$

$$\Rightarrow P(E) = \frac{0'0075}{0'025} = 0'3$$

$$P(N) = 1 - P(S) - P(E) = 1 - 0'9 = 0'1$$

$$H(X) = P(N) \cdot \log_2 \frac{1}{P(N)} + P(E) \log_2 \frac{1}{P(E)} + P(S) \log_2 \frac{1}{P(S)}$$

$$= 0'1 \log_2(10) + 0'3 \log_2(3'33) + 0'6 \log_2 1'66 = 1'2916 \text{ bits/símb}$$

(b) Entropía condicional $H(X|S)$

$$H(X|S) = p(N|S)\log_2 \frac{1}{p(N|S)} + p(S|S)\log_2 \frac{1}{p(S|S)} +$$

$$p(E|S)\log_2 \frac{1}{p(E|S)} = 0'2\log_2 \frac{1}{0'2} + 0'5\log_2 \frac{1}{0'5} +$$

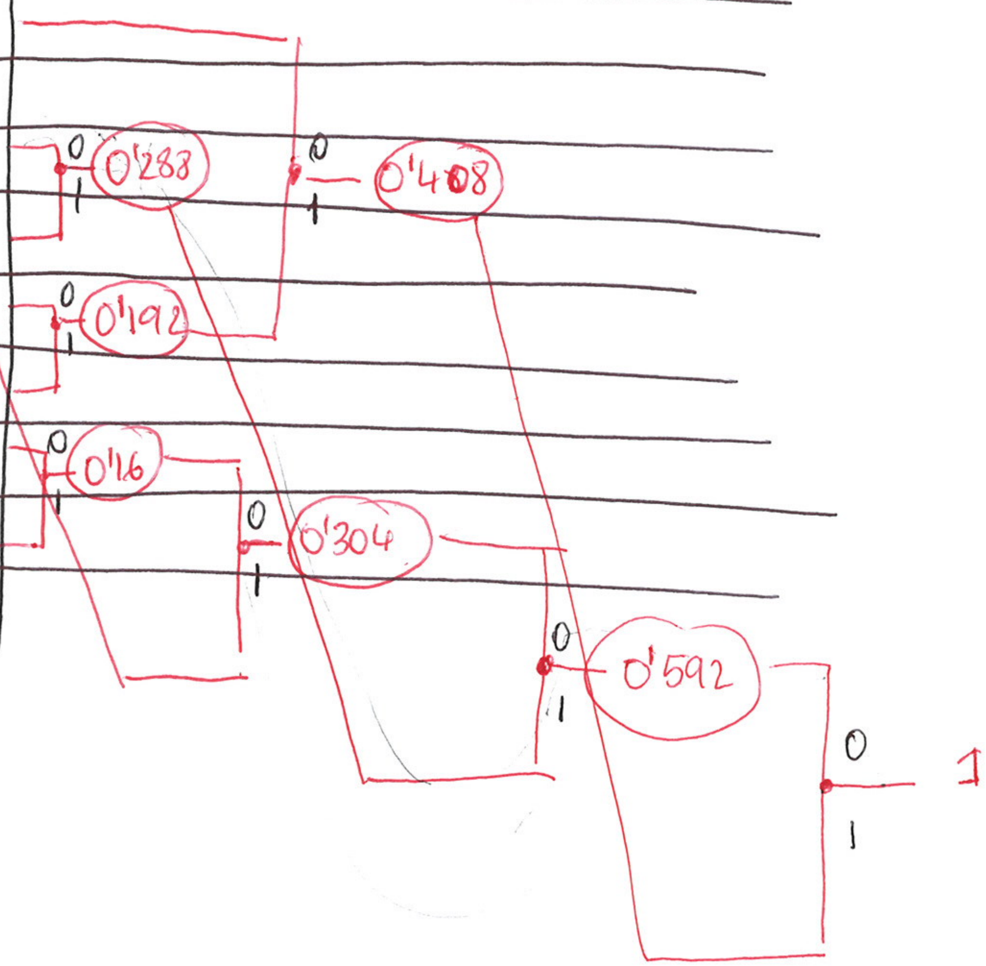
$$+ 0'3\log_2 \frac{1}{0'3} = 1'4855 \text{ bits/símbolo}$$

(c) si $P(N) = 0'4$ $P(S) = 0'6$ y $P(E) = 0$
código Huffman para una extensión de orden 3 de la fuente

Los símbolos de orden 3 que pueden darse son los que no contienen el símbolo E. Suponiendo que son independientes entre sí.

$P(NNN) = 0'064$
$P(NNS) = 0'096$
$P(NSN) = 0'096$
$P(SNN) = 0'096$
$P(NSS) = 0'144$
$P(SNS) = 0'144$
$P(SSN) = 0'144$
$P(SSS) = 0'216$

Simbolo	Probabilidad
SSS	0'216
NSS	0'144
SNS	0'144
SSN	0'144
NNS	0'096
NSN	0'096
SNN	0'096
NNN	0'064



SSS 10

NSS 001

SNS 010

SSN 011

NNS 110

NSN 111

SNN 0000

NNN 0001