1.- $a(t) = \text{sen}(2\pi 4000 t + \theta)$

$T_s = 2'5 \cdot 10^{-4} sg = 1/4000 \, sg$ $\quad n = 0 \cdots 31$

$s(n) = \text{sen}(2\pi 4000\, nT + \theta) = \text{sen}(2\pi \frac{\cancel{4000}}{\cancel{4000}} n + \theta) = \text{sen}(2\pi n + \theta) = \underline{\text{sen}(\theta)}$

$s(n) = \text{sen}(\theta)$

i) $s(n) = \text{sen}(\theta) \sum_{k=0}^{31} \delta(n-k)$

ii) $F_s = \frac{1}{T_s} = 4000 \, Hz$
F_{max} de $a(t) = 4000 \, Hz$ $\Big\} \Rightarrow$ no se verifica el teorema de muestreo

iii) $S(\omega) = \sum_{n=-\infty}^{+\infty} s(n) e^{-j\omega n} = \text{sen}(\theta) \sum_{n=-\infty}^{+\infty} e^{-j\omega n} =$ evaluado en $n = 0 \cdots 31$

$= \text{sen}\theta \underbrace{\sum_{n=0}^{31}}_{\text{suma finita}} e^{-j\omega n} = \text{sen}\theta \frac{1 - e^{-j\omega 32}}{1 - e^{-j\omega}}$

θ se comporta como un factor de escala

iv) DFT de 32 puntos

$S(k) = \sum_{n=0}^{32} s(n) e^{-j\frac{2\pi k n}{32}} = \text{sen}\theta \sum_{n=0}^{32} e^{-j\frac{2\pi k n}{32}} =$

$= \text{sen}\theta \frac{1 - e^{-j\frac{2\pi k \cancel{32}}{\cancel{32}}}}{1 - e^{-j\frac{2\pi k}{32}}}$ $\quad k = 0 \cdots 31$

$$S(k) = \begin{cases} 32\,\text{sen}\theta & k = 0 \\ 0 & k \neq 0 \end{cases}$$

2.- $y(n) = y(n-1) + \dfrac{x(n) + x(n-1)}{2}$

i) función de transferencia

$$y(n) - y(n-1) = \frac{1}{2}\left[x(n) + x(n-1)\right]$$

$$Y(z)\left[1 - z^{-1}\right] = \frac{1}{2} X(z)\left[1 + z^{-1}\right]$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{1}{2}\,\frac{1+z^{-1}}{1-z^{-1}}$$

ii) salida del sistema cuando la entrada $x(n) = \operatorname{sen}(n\pi)$ un tono puro de frecuencia $\omega = \pi$.

$$\left|H(\omega)\right|_{\omega=\pi} = \frac{1}{2}\,\frac{1+e^{-j\pi}}{1-e^{-j\pi}} = \frac{1}{2}\,\frac{1+\cos\pi}{1-\cos\pi} = 0$$

$z = e^{j\omega}$

La salida para ese tono es cero

iii) $y(n) = \delta(n) + \delta(n-1)$

$x(n)$?

$$Y(z) = 1 + z^{-1}$$

$$Y(z) = H(z)X(z) \Rightarrow X(z) = \frac{Y(z)}{H(z)} = \frac{1+z^{-1}}{\frac{1}{2}\,\frac{1+z^{-1}}{1-z^{-1}}} = 2\left[1 - z^{-1}\right] \Rightarrow$$

$$x(n) = 2\delta(n) - 2\delta(n-1)$$

(3) $y(n) = x(t) \cdot p(t)$

voy a asumir muestreo con impulsos [($\Delta = 1$ en la figura del ejercicio)

$$Y(\omega) = X(\omega) * P(\omega) \qquad \omega_0 = \frac{2\pi}{T}$$

Periodo de muestreo T,

característica de $p(t)$ = tren de impulsos: los impulsos pares tienen amplitud 1 y los impulsos impares 1/2

si todos los impulsos tuviesen amplitud 1 $\quad P(t) = \sum_{n=-\infty}^{+\infty} \delta(t - nT)$

$$P(\omega) = \frac{2\pi}{T} \sum_{n=-\infty}^{+\infty} \delta(\omega - n\omega_0)$$

En el caso del ejercicio

$$P(t) = \sum_{n=-\infty}^{+\infty} \left[\delta(t - 2nT) + \frac{1}{2}\delta(t - (2n+1)T) \right] = \qquad T' = 2T, \quad \omega_0' = \omega_0/2$$

$$\sum_{n=-\infty}^{+\infty} \delta(t - 2nT) + \frac{1}{2} \sum_{n=-\infty}^{+\infty} \delta(t - T - 2nT) = P_1(t) + \frac{1}{2} P_2(t)$$

$$P_1(\omega) = 2\pi \sum_{n=-\infty}^{+\infty} \frac{1}{2T} \delta(\omega - n\,\omega_0/2)$$

$$P_2(\omega) = 2\pi\, e^{-j\omega T_0} \sum_{n=-\infty}^{+\infty} \frac{1}{2T} \delta(\omega - n\cdot\omega_0/2)$$

por la propiedad de desplazamiento temporal de la T Fourier el espectro de magnitud no se altera, solamente el espectro de fase sufre un cambio de ωT

$$P(\omega) = \left[\frac{\cancel{2}\pi}{\cancel{2}T} \sum_{n=-\infty}^{+\infty} \delta(\omega - n\,\omega_0/2) + \frac{1}{2} \frac{\cancel{2}\pi}{\cancel{2}T} e^{-j\omega T_0} \sum_{n=-\infty}^{+\infty} \delta(\omega - n\,\omega_0/2) \right] =$$

$$= \frac{\pi}{T} \sum_{n=-\infty}^{+\infty} \delta(\omega - n\,\omega_0/2) \left[1 + \frac{1}{2} e^{-j\omega T} \right]$$

$$Y(\omega) = X(\omega) * \frac{\pi}{T} \sum_{n=-\infty}^{+\infty} \delta\left(\omega - n\frac{\omega_0}{2}\right) \left[1 + \frac{1}{2} e^{-j\omega T} \right] =$$

$$\frac{\omega_0}{2} \left[X(\omega) * \delta_{\omega_0/2}(\omega) + \frac{1}{2} X(\omega) * \delta_{\frac{\omega_0}{2}}(\omega)\, e^{-j\omega T} \right]$$

$$Y(\omega) = \frac{\omega_0}{2} \sum_{n=-\infty}^{+\infty} X\left[\omega - n\frac{\omega_0}{2}\right] + \frac{1}{2}\frac{\omega_0}{2} \left[X(\omega) * \delta_{\frac{\omega_0}{2}}(\omega)\, e^{-j\omega T} \right] \qquad (*)$$

Estudiemos $P(\omega)$

recordemos $\omega_0 = \frac{2\pi}{T}$

$\frac{\omega_0}{2} = \pi/T$

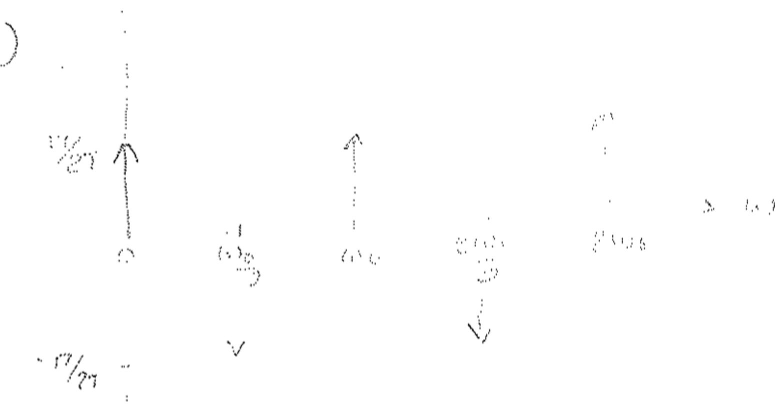
$$P(\omega) = \frac{\pi}{T} \sum_{n=-\infty}^{+\infty} \delta\left(\omega - n\frac{\omega_0}{2}\right)\left[1 + \frac{1}{2}e^{-j\omega T}\right]$$

este término introduce un cambio de fase

a)

← término $\frac{\pi}{T} \sum_{n=-\infty}^{+\infty} \delta\left(\omega - n\frac{\omega_0}{2}\right)$

b)



← Término $\frac{\pi}{2T} \sum_{n=-\infty}^{+\infty} \delta\left(\omega - n\frac{\omega_0}{2}\right)e^{-j\omega T}$

$e^{-j\omega T}$ introduce un desfase que es ωT

entonces $\omega = 0$ $e^{-j\omega T} = 1$

$\omega = \frac{\omega_0}{2} \Rightarrow e^{-j\frac{\omega_0}{2}T} = e^{-j\pi} = -1$

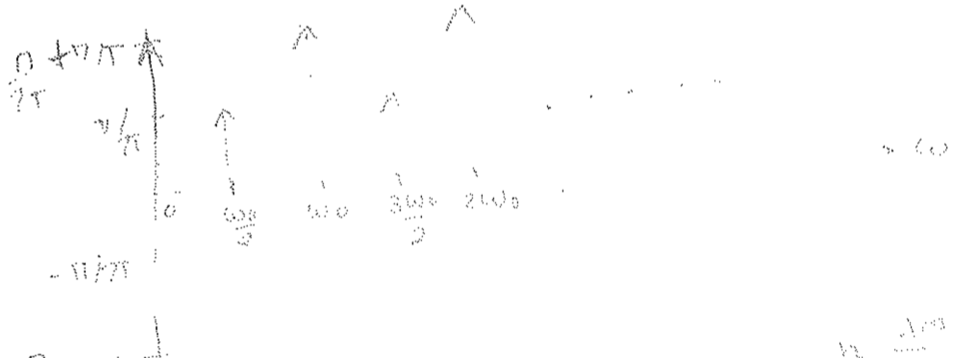
$\omega = \omega_0 \Rightarrow e^{-j2\pi} = 1$

$\omega = \frac{3\omega_0}{2} \Rightarrow e^{-j\frac{3}{2}\frac{2\pi}{T}T} = -1$

$\omega = m\omega_0 \Rightarrow e^{-j\omega T} = 1$
↳ m par

$\omega = n\omega_0$ $e^{-j\omega T} = -1$
↳ impar

$P(\omega)$ será la suma de a) y b)



$$Y(\omega) = \frac{\omega_0}{2} \sum_{n=-\infty}^{+\infty} X\left(\omega - n\frac{\omega_0}{2}\right) + \frac{1}{2}\frac{\omega_0}{2}(-1)^n \sum_{n=-\infty}^{+\infty} X\left(\omega - n\frac{\omega_0}{2}\right)$$

b) La señal sería recuperable dependiendo de si $\frac{f_s}{2} \geq 2 f_{max}$ de $X(\omega)$)

(4)

entrada $x(n) = \left(\frac{1}{2}\right)^n u(n) - \left(\frac{1}{4}\right)^{n-1} u(n-1)$

salida $y(n) = \left(\frac{1}{3}\right)^n u(n)$

a) Respuesta al impulso

Usando la T. Z

$$X(z) = TZ\left[\left(\frac{1}{2}\right)^n u(n)\right] - TZ\left[\left(\frac{1}{4}\right)^{n-1} u(n-1)\right] =$$

$$X(z) = \frac{1}{1-\frac{1}{2}z^{-1}} - \frac{z^{-1}}{1-\frac{1}{4}z^{-1}} = \frac{1-\frac{1}{4}z^{-1} - z^{-1} + \frac{1}{2}z^{-2}}{(1-\frac{1}{2}z^{-1})(1-\frac{1}{4}z^{-1})}$$

$$Y(z) = \frac{1}{1-\frac{1}{3}z^{-1}}$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{\frac{1}{1-\frac{1}{3}z^{-1}}}{\frac{1-\frac{5}{4}z^{-1}+\frac{1}{2}z^{-2}}{(1-\frac{1}{2}z^{-1})(1-\frac{1}{4}z^{-1})}} = \frac{(1-\frac{1}{2}z^{-1})(1-\frac{1}{4}z^{-1})}{(1-\frac{1}{3}z^{-1})(1-\frac{5}{4}z^{-1}+\frac{1}{2}z^{-2})}$$

$$H(z) = \frac{(1-\frac{1}{2}z^{-1})(1-\frac{1}{4}z^{-1})}{(1-\frac{1}{3}z^{-1})(1-\frac{5-j\sqrt{7}}{8}z^{-1})(1-\frac{5+j\sqrt{7}}{8}z^{-1})} =$$

$$= \frac{A}{(1-\frac{1}{3}z^{-1})} + \frac{B+jC}{(1-\frac{5-j\sqrt{7}}{8}z^{-1})} + \frac{B-jC}{(1-\frac{5+j\sqrt{7}}{8}z^{-1})} \Rightarrow$$

$$(1-\tfrac{1}{2}z^{-1})(1-\tfrac{1}{4}z^{-1}) = A\left[1-\tfrac{10}{8}z^{-1}+\tfrac{1}{2}z^{-2}\right] + (1-\tfrac{1}{3}z^{-1})\left[2B - \tfrac{1}{8}z^{-1}(10B-2C\sqrt{7})\right]$$

$$1-\frac{3}{4}z^{-1}+\frac{1}{8}z^{-2} = A - \frac{10A}{8}z^{-1} + \frac{A}{2}z^{-2} + 2B - \frac{10B-2C\sqrt{7}}{8}z^{-1} + \frac{10B-2C\sqrt{7}}{24}z^{-2} - \frac{2B}{3}z^{-1}$$

$$\left.\begin{aligned} 1 &= A + 2B \\ -\frac{3}{4} &= -\frac{10}{8}A - \frac{10B-2C\sqrt{7}}{8} - \frac{2}{3}B \\ \frac{1}{8} &= \frac{A}{2} - \frac{10B-2C\sqrt{7}}{24} \end{aligned}\right\} \quad \begin{aligned} A &= 0.27 \\ B &= 0.36 \\ C &= -0'61 \end{aligned}$$

$$H(z) = \frac{0.27}{1-\frac{1}{3}z^{-1}} + \frac{0.36-0'61j}{(1-\frac{5-j\sqrt{7}}{8}z^{-1})} + \frac{0.36+0'6j}{(1-\frac{5+j\sqrt{7}}{8}z^{-1})}$$

$$h(n) = 0.27\left(\frac{1}{3}\right)^n u(n) + \left[0.7\,e^{j\,\mathrm{arctg}\,0'6/0.36}\left(\frac{5-j\sqrt{7}}{8}\right)^n + 0.7\,e^{-j\,\mathrm{arctg}\,0'6/0.36}\left(\frac{5+j\sqrt{7}}{8}\right)^n\right]u(n)$$

$$h(n) = 0'77\left(\frac{1}{3}\right)^n u(n) + 0.7\left[e^{j\,arctg(1'66)}\left((0.7)e^{-j\,arctg(0.52)}\right)^n + (0.7)^n e^{-j\,arctg(1'66)}\left(e^{+j\,arctg(0.52)}\right)^n\right] u(n)$$

$$\boxed{h(n) = 0'77\left(\frac{1}{3}\right)^n u(n) + 0.7\,(0.7)^n\, 2\cos(0.54\,n)\, u(n)}$$

b) Ec. en diferencias:

$$H(z) = \frac{\left(1-\frac{1}{2}z^{-1}\right)\left(1-\frac{1}{4}z^{-1}\right)}{\left(1-\frac{1}{3}z^{-1}\right)\left(1-\frac{5}{4}z^{-1}+\frac{1}{2}z^{-2}\right)} = \frac{Y(z)}{X(z)}$$

$$Y(z)\left[\left(1-\frac{1}{3}z^{-1}\right)\left(1-\frac{5}{4}z^{-1}+\frac{1}{2}z^{-2}\right)\right] = X(z)\left[\left(1-\frac{1}{2}z^{-1}\right)\left(1-\frac{1}{4}z^{-1}\right)\right]$$

$$Y(z)\left[1-\frac{5}{4}z^{-1}+\frac{1}{2}z^{-2}-\frac{1}{3}z^{-1}+\frac{5}{12}z^{-2}-\frac{1}{6}z^{-3}\right] =$$

$$X(z)\left(1-\frac{1}{4}z^{-1}-\frac{1}{2}z^{-1}+\frac{1}{8}z^{-2}\right)$$

$$Y(z)\left[1-\frac{19}{12}z^{-1}+\frac{11}{12}z^{-2}-\frac{1}{6}z^{-3}\right] = X(z)\left[1-\frac{3}{4}z^{-1}+\frac{1}{8}z^{-2}\right)$$

$$\boxed{y(n)-\frac{19}{12}y(n-1)+\frac{11}{12}y(n-2)-\frac{1}{6}y(n-3) = x(n)-\frac{3}{4}x(n-1)+\frac{1}{8}x(n-2)}$$

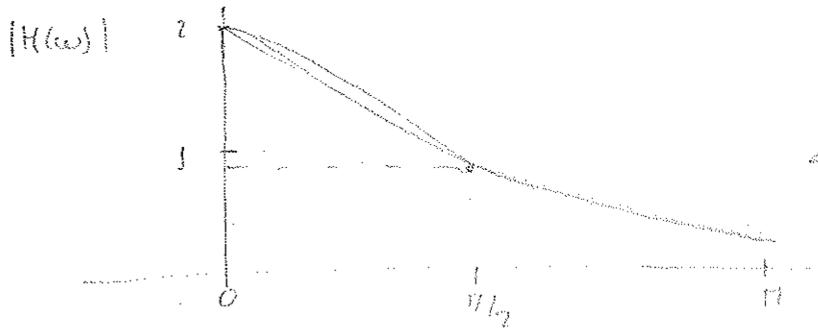
c) $$H(e^{j\omega}) \equiv H(\omega) = \frac{\left(1-\frac{1}{2}e^{-j\omega}\right)\left(1-\frac{1}{4}e^{-j\omega}\right)}{\left(1-\frac{1}{3}e^{-j\omega}\right)\left(1-\frac{5}{4}e^{-j\omega}+\frac{1}{2}e^{-2j\omega}\right)}$$

$$H(\omega=0) = \frac{\left(1-\frac{1}{2}\right)\left(1-\frac{1}{4}\right)}{\left(1-\frac{1}{3}\right)\left(1-\frac{5}{4}+\frac{1}{2}\right)} = 2'25$$

$$H(\omega=\pi/2) = \frac{\left(1-j\frac{1}{2}\right)\left(1-j\frac{1}{4}\right)}{\left(1-j\frac{1}{3}\right)\left(1-\frac{5}{4}j-\frac{1}{2}\right)} = \frac{\left(1-j\,\frac{1}{2}\right)\left(1-\frac{1}{4}j\right)}{\left(1-\frac{1}{3}j\right)\left(\frac{1}{2}-\frac{5}{4}j\right)}$$

$$|H(\omega)|_{\omega=\pi/2} = \frac{\left(\sqrt{1+\frac{1}{4}}\right)\left(\sqrt{1+\frac{1}{16}}\right)}{\left(\sqrt{1+\frac{1}{9}}\right)\left(\sqrt{\frac{1}{4}+\frac{25}{16}}\right)} = 0.8$$

$$H(\omega)\Big|_{\omega=\pi} = \frac{\left(1+\frac{1}{2}\right)\left(1+\frac{1}{4}\right)}{\left(1+\frac{1}{3}\right)\left(1+\frac{5}{4}+\frac{1}{2}\right)} = 0.5$$



← Es un filtro paso bajo

(5) $H_1(z) = \dfrac{1-z^{-1}}{1-az^{-1}}$ $\quad a < 1$

a) Para anular $\omega = \pi/4$ y sus armónicos

$\omega = 0$

$\omega = \pm\pi/4$

$\omega = \pm 2\pi/4 = \pm\pi/2$

$\omega = \pm 3\pi/4$

$\omega = \pm 4\pi/4 = \pm\pi$

necesito 8 ceros a las frecuencias $\omega = 0$, $\omega = \pm\pi/4$, $\omega = \pm 3\pi/4$ y $\omega = \pm\pi$

$$H_2(z) = \frac{1-z^{-8}}{1-az^{-8}}.$$

La e. en dif cuya función de transferencia es $H_1(z)$

$$\frac{Y_1(z)}{X_1(z)} = H_1(z) = \frac{1-z^{-1}}{1-az^{-1}}$$

$$Y_1(z)(1-az^{-1}) = X_1(z)(1-z^{-1})$$

$$y_1(n) - a\,y_1(n-1) = x_1(n) - x_1(n-1)$$

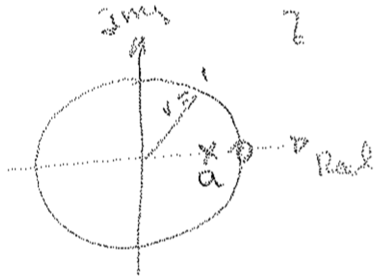
La ec. en diferencias cuya f. de transf. es $H_2(z)$ será

$$y_2(n) - a\,y_2(n-8) = x_2(n) - x_2(n-8)$$

b) $H_1(z)$

ceros $1 - z^{-1} = 0$ $\quad z = 1$

polos $1 - az^{-1} = 0$. $\quad z = a$



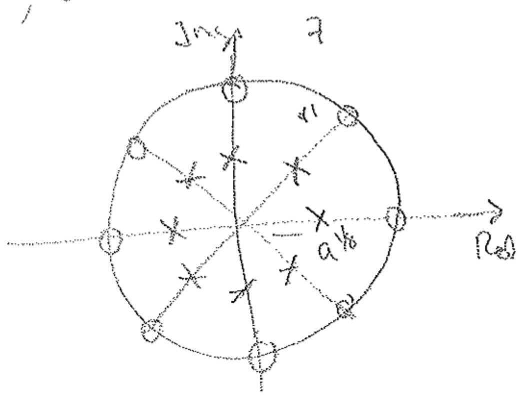
$H_2(z)$

ceros $1 - z^{-8} = 0$. $\quad z_i \Rightarrow 1,\ e^{\pm j\pi/4},\ e^{\pm j\pi/2},\ e^{\pm j3\pi/4},\ -1$

polos $1 - az^{-8} = 0$ $\quad p_i \Rightarrow a^{1/8},\ a^{1/8}e^{\pm j\pi/4},\ a^{1/8}e^{\pm j\pi/2},\ a^{1/8}e^{\pm j3\pi/4},\ -a^{1/8}$

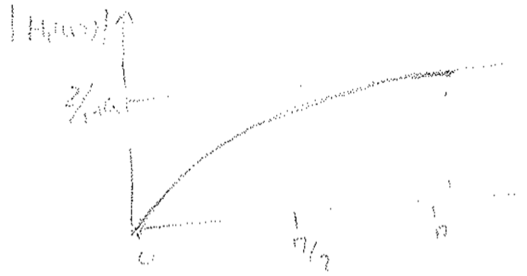
$z_i = (1)^{1/8}\, e^{j(2k\pi/8)}$ $\quad k = 0, 1, \ldots 8$

$p_i = (a)^{1/8}\, e^{j(2\pi k/8)}$ $\quad k = 0, 1, \ldots 8$

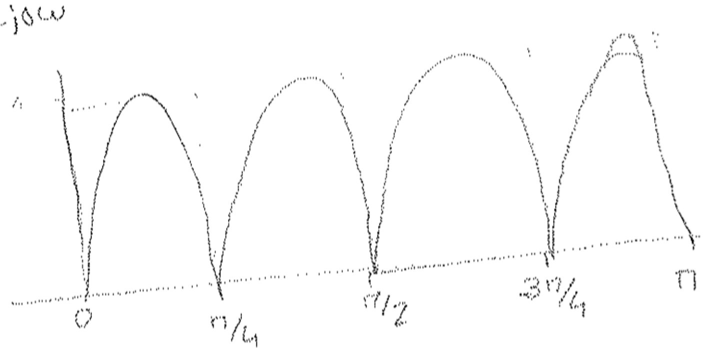


$$H_1(\omega) = \frac{1 - e^{-j\omega}}{1 - a e^{-j\omega}}$$

$$H_1(\omega)\Big|_{\omega=0} = 0 \qquad H_1(\omega)\Big|_{\omega=\pi} = \frac{1+1}{1+a} = \frac{2}{1+a}$$



$$H_2(\omega) = \frac{1 - e^{-j8\omega}}{1 - a e^{-j8\omega}}$$



$e^{-j\omega}$ $\cos\omega - j\,\mathrm{sen}\,\omega$

$1 - j\frac{5}{4} + \frac{1}{2}j$

$\cos \cancel{2}\frac{\pi}{\cancel{2}} - j\,\mathrm{sen}\,\cancel{2}\frac{\pi}{2}$

$\cos(\pi) - j\,\mathrm{sen}\,\pi$

$\cos 2\pi - \cancel{j\,\mathrm{sen}\,2\pi}$

$\frac{5}{4} + \frac{1}{3}$

$\frac{15}{12} + \frac{4}{12}$

$\frac{6}{12} + \frac{5}{12}$