PROBLEMA 1

$$Y_n = X_n + g(n);$$ X_n p.a. iid $E[X_n]=0$

$g(n)$ función determinista $\Rightarrow E[g(n)]=g(n)$

a) $E[Y_n] = E[X_n+g(n)] = E[X_n] + E[g(n)] = g(n)$

$VAR[Y_n] = VAR[X_n+g(n)] = VAR[X_n]$

b) $C_{Y_n}(n_1,n_2) = R_Y(n_1,n_2) - E[Y_{n_1}]E[Y_{n_2}]$

$= R_Y(n_1,n_2) - g(n_1)g(n_2)$

$R_Y(n_1,n_2) = E[Y_{n_1}Y_{n_2}] = E[(X_{n_1}+g(n_1))(X_{n_2}+g(n_2))]$

$= E[X_{n_1}X_{n_2} + g(n_1)\cancelto{0}{E[X_{n_2}]} + g(n_2)\cancelto{0}{E[X_{n_1}]} + g(n_1)g(n_2)]$

$= E[X_{n_1}X_{n_2}] + g(n_1)\cdot g(n_2) \underset{\text{iid}}{=} \underbrace{E[X_{n_1}]E[X_{n_2}]}_{R_X(n_1,n_2)} + g(n_1)g(n_2)$

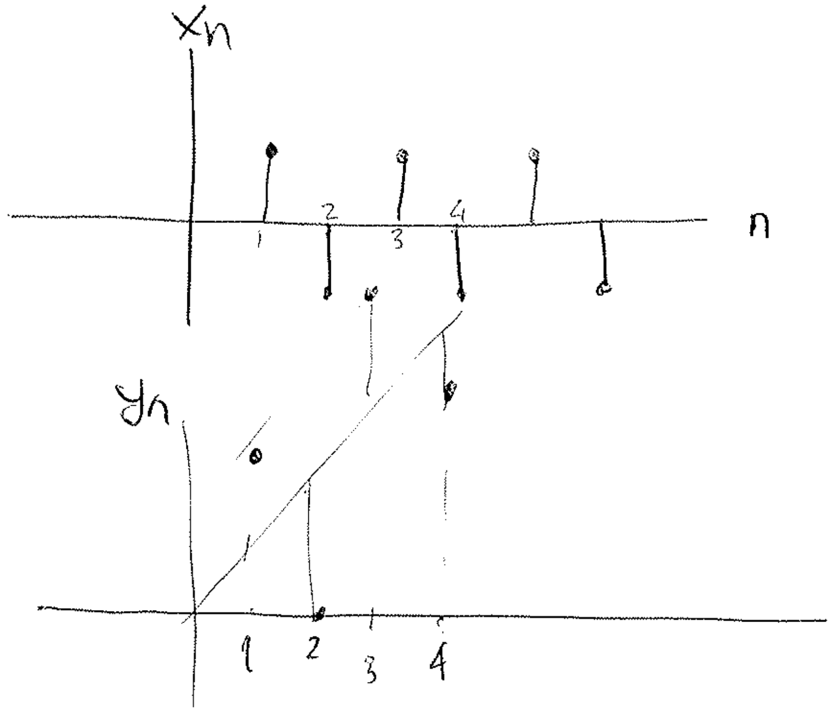
$\Rightarrow C_Y(n_1,n_2) = R_Y(n_1,n_2) - g(n_1)g(n_2) =$

$= R_X(n_1,n_2) + g(n_1)g(n_2) - g(n_1)g(n_2)$

$= R_X(n_1,n_2) = E[X_1]E[X_2] = 0$

iid con $E[X_n]=0$

c) Por ejemplo si X_n ha tomado los valores $\{1, -1, 1, -1, 1, -1\}$



$x_1 = 1 \rightarrow y_1 = 2$

$x_2 = -1 \quad y_2 = 0$

$x_3 = 1 \quad y_3 = 4$

$x_4 = -1 \quad y_4 = 3$

$x_5 = 1 \quad y_5 = 6$

otras realizaciones de X_n originan otras realiz de Y_n

PROBLEMA 2

$F_1 = \{1,2,3\}$ $\quad P_1(x) = \frac{1}{3} \; \forall x$

$F_2 = \{2,4,6\}$ $\quad P_2(x) = \frac{1}{3} \; \forall x$ $\qquad$ F_1 y F_2 equiprob. e indepts

$F =$ suma (F_1, F_2)

F_1	F_2	F	
1	2	3	
1	4	5	
1	6	7	
2	2	4	
2	4	6	
2	6	8	
3	2	5	
3	4	7	
3	6	9	

$\Rightarrow F = \{3, 4, 5, 6, 7, 8, 9\}$

$p(3) = p(4) = p(6) = p(8) = p(9) = \frac{1}{9}$

$p(5) = p(7) = 2/9$

a) $H(F) = \sum_{x \in F} p(x) \log_2 \frac{1}{p(x)} = 5\left(\frac{1}{9}\log_2 9\right) + 2\left(\frac{2}{9}\log_2 \frac{9}{2}\right)$

$H(F) = 2'7255$ bits/símbolo

b) $I(F, F_1) = H(F) - H(F|F_1)$

$$\boxed{H(F|F_1) = \sum_{x \in F_1} P_1(x)\, H(F|x_{F_1})}$$ donde

$$\boxed{H(F|x \in F_1) = \sum_{x' \in F} P_F(x'_F | x \in F_1) \log_2 \frac{1}{P_F(x'_F | x \in F_1)}}$$

$$H(F|x_{F_1} = 1) = \underbrace{p(x'_F = 3 | x_{F_1} = 1)}_{1/3} \log_2 \frac{1}{p(x'_F = 3 | x_{F_1} = 1)} +$$

$$\underbrace{p(x'_F = 4 | x_{F_1} = 1)}_{0} \log_2 \frac{1}{p(x'_F = 4 | x_{F_1} = 1)} + \underbrace{p(x'_F = 5 | x_{F_1} = 1)}_{1/3} \log_2 \frac{1}{p(x'_F = 5 | x_{F_1} = 1)}$$

$$+ \underbrace{p(x'_F = 6 | x_{F_1} = 1)}_{0} \log_2 \frac{1}{p(x'_F = 6 | x_{F_1} = 1)} + \underbrace{p(x'_F = 7 | x_{F_1} = 1)}_{1/3} \log_2 \frac{1}{p(x'_F = 7 | x_{F_1} = 1)} +$$

$$+ \underbrace{p(x'_F = 8 | x_{F_1} = 1)}_{0} \log_2 \frac{1}{p(x'_F = 8 | x_{F_1} = 1)} + \underbrace{p(x'_F = 9 | x_{F_1} = 1)}_{0} \log_2 \frac{1}{p(x'_F = 9 | x_{F_1} = 1)}$$

$$\Rightarrow H(F | x_{F_1} = 1) = \log_2 3$$

del mismo modo, $H(F | x_{F_1} = 2) = \log_2 3$ y $H(F | x_{F_1} = 3) = \log_2 3$

$$\Rightarrow H(F|F_1) = p(x_{F_1} = 1) \cdot H(F | x_{F_1} = 1) +$$

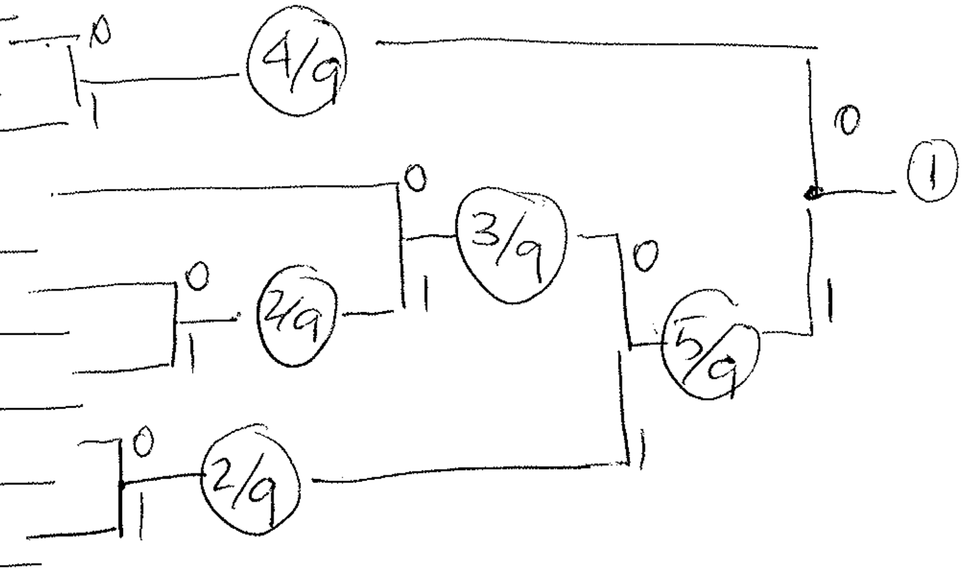
$$p(x_{F_1} = 2) \cdot H(F | x_{F_1} = 2) + p(x_{F_1} = 3) \cdot H(F | x_{F_1} = 3)$$

$$= \log_2 3 = 1'5850 \text{ bits/seg}$$

$I(F, F_1) = H(F) - H(F|F_1) = 2'7255 - 15850 = 1'1405$ bits/seg

c) Código Huffman: (uno de los posibles implementaciones)

Símbolo	Probabilidad.
5	2/9
7	2/9
3	1/9
4	1/9
6	1/9
8	1/9
9	1/9



5 — 00
7 — 01
3 — 100
4 — 1010
6 — 1011
8 — 110
9 — 111

$$\bar{N} = 2\left(\frac{2}{9}\right) + 2\left(\frac{2}{9}\right) + 3\left(\frac{1}{9}\right) + 3\left(\frac{1}{9}\right) + 3\left(\frac{1}{9}\right) + 4\left(\frac{1}{9}\right) + 4\left(\frac{1}{9}\right)$$

$$= \frac{25}{9} \text{ bits/símbolo}$$

d)

$H(F|F_1)$ = Cantidad de incertidumbre sobre la salida de F, que queda después de conocer la salida de F_1

$\boxed{H(F|F_1) = H(F|F_2)}$ (Podemos calcular $H(F|F_2)$ o razonarlo)

↓

Como son iguales,
me dará lo mismo conocer la salida de F_1 o la de F_2