PROBLEMA 1º

$z(t) = X\cos(2\pi f_0 t) + Y \operatorname{sen}(2\pi f_0 t)$

X: v.a. unif en $(-1,1)$

Y: v.a. unif en $(0,1)$

X, Y independientes.

(a) $m_z(t) = E[z(t)] = E[X\cos(2\pi f_0 t) + Y\operatorname{sen}(2\pi f_0 t)] =$

$E[X\cos(2\pi f_0 t)] + E[Y\operatorname{sen}(2\pi f_0 t)] = \cos(2\pi f_0 t)E[X] +$

$\operatorname{sen}(2\pi f_0 t)\cdot E[Y]$.

$$E[X] = \int_{-1}^{1} x\cdot\frac{1}{2}\,dx = \left.\frac{x^2}{2}\right]_{-1}^{1} = \frac{1}{2} - \frac{1}{2} = 0$$

$$E[Y] = \int_0^1 y\,dy = \left.\frac{y^2}{2}\right]_0^1 = \frac{1}{2}$$

$$\boxed{m_z(t) = \frac{\operatorname{sen}(2\pi f_0 t)}{2}}$$

(b) $R_z(t, t+\tau) = E[z(t)\cdot z(t+\tau)] = E[(X\cos(2\pi f_0 t) + Y\operatorname{sen}(2\pi f_0 t))\cdot$

$(X\cos(2\pi f_0(t+\tau)) + Y\operatorname{sen}(2\pi f_0(t+\tau)))] =$

$E[X\cos(2\pi f_0 t)\cdot X\cos(2\pi f_0(t+\tau)) + X\cdot Y\cos(2\pi f_0 t)\operatorname{sen}(2\pi f_0 t+\tau) +$

$Y\operatorname{sen}(2\pi f_0 t)X\cos(2\pi f_0(t+\tau)) + Y^2\operatorname{sen}(2\pi f_0 t)\operatorname{sen}(2\pi f_0(t+\tau))] =$

$E[X^2\cos(2\pi f_0 t)\cos(2\pi f_0(t+\tau))] + \cancelto{0}{E[X]E[Y]}\cos(2\pi f_0 t)\,sen(2\pi f_0 t+\tau) +$

$\cancelto{0}{E[Y]E[X]}\,sen(2\pi f_0 t)\cos(2\pi f_0(t+\tau)) + E[Y^2]\,sen(2\pi f_0 t)\,sen(2\pi f_0(t+\tau))$

$= E[X^2]\cos(2\pi f_0 t)\cos(2\pi f_0(t+\tau)) + E[Y^2]\,sen(2\pi f_0 t)\,sen(2\pi f_0(t+\tau))$

$$E[X^2] = \int_{-1}^{1} x^2\,\frac{1}{2}\,dx = \frac{x^3}{3\cdot 2}\Big]_{-1}^{1} = \frac{1}{6} + \frac{1}{6} = \frac{1}{3}$$

$$E[Y^2] = \int_{0}^{1} y^2\,dy = \frac{y^3}{3}\Big]_{0}^{1} = \frac{1}{3}$$

$$\Rightarrow R_Z(t,t+\tau) = \frac{1}{3}\left[\cos(\overset{A}{2\pi f_0 t})\cos(\overset{B}{2\pi f_0(t+\tau)}) + sen(\overset{A}{2\pi f_0 t})\,sen\,\overset{B}{2\pi f_0(t+\tau)}\right]$$

$$\cancel{\frac{1}{2}\cos(2\pi f_0 t + 2\pi f_0(t+\tau))}$$

$\cos A\cos B = \frac{1}{2}\cancel{\cos(A+B)} + \frac{1}{2}\cos(A-B)$

$+\quad sen\,A\,sen\,B = \frac{1}{2}\cos(A-B) - \cancel{\cos(A+B)}$

$\cos(A-B) = \cos(2\pi \cancel{f_0 t} - \cancel{2\pi f_0 t} - 2\pi f_0\tau)$

$= \cos(-2\pi f_0\tau)$

$$\Rightarrow \boxed{R_Z(t,t+\tau) = \frac{\cos 2\pi f_0\tau}{3}}$$

; no es WSS pq la media no es cte.

c) Potencia media del proceso:

$$\cancel{P_x} = \cancel{P_z} \; P_z = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} \underbrace{E[x_z(t)]}_{R_z(\tau=0) = \frac{\cos(0)}{3} = \frac{1}{3}} dt =$$

$$P_z = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} \frac{1}{3}\, dt = \left.\frac{T}{3}\right]_{-T}^{T} = \frac{T}{3} + \frac{T}{3} \cdot \frac{1}{2T} = \frac{2T}{2T\cdot 3} = \frac{1}{3}$$

$$\boxed{P_z = \frac{1}{3}}$$

NOTA:

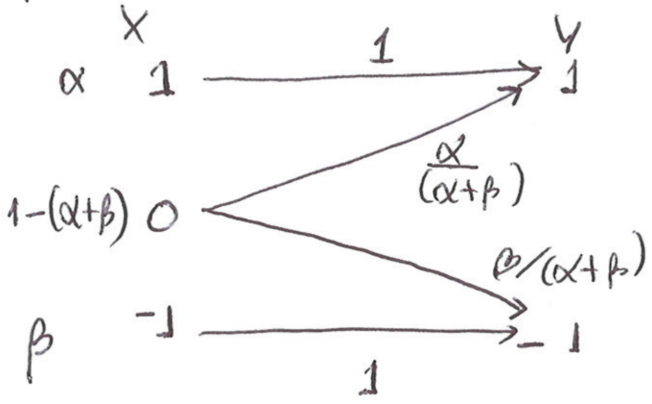
si el proceso es WSS ; $P_x = A\{E[X^2]\} = A\{R_x(0)\}$

$E[x^2(t)] = \overline{X^2}$

Pero sólo si el proceso es WSS

③

PROBLEMA 2



$X = \{1, 0, -1\}$

$$\begin{pmatrix} P(Y=1|X=1) & P(Y=0|X=1) \\ 1 & 0 \\ P(Y=1|X=0) & P(Y=-1|X=0) \\ \frac{\alpha}{\alpha+\beta} & \frac{\beta}{\alpha+\beta} \\ P(Y=-1|X=1) & P(Y=-1|X=-1) \\ 0 & 1 \end{pmatrix}$$

$P[X=1] = \alpha$ $\quad 0 < \alpha+\beta \leq 1$

$P[X=0] = 1-\alpha-\beta$

$P[X=-1]\ \beta$

(a) entropía de los símbolos de salida

$$H(Y) = P(Y=1)\log\frac{1}{P(Y=1)} + P(y=-1)\log\frac{1}{P(y=-1)}$$

$$P(y=1) = P(y=1|X=1) \cdot P(X=1) + P(y=1|X=0)\,P(X=0) + \cancel{P(y=1|X=-1)\,P(X=-1)}$$

0

$$\frac{(1-\alpha-\beta)\alpha}{\alpha+\beta} = \frac{\alpha-\alpha^2-\beta\alpha}{\alpha+\beta}$$

$$= 1 \cdot \alpha + [1-(\alpha+\beta)] \cdot \frac{\alpha}{(\alpha+\beta)} = \alpha + \cancel{\alpha\alpha\beta}$$

$$= \frac{\alpha(\alpha+\beta) + \alpha - \alpha^2 - \beta\alpha}{\alpha+\beta} = \frac{\cancel{\alpha^2} + \beta + \alpha \cancel{- \alpha^2} - \beta\alpha}{\alpha+\beta}$$

B

$$P[Y=1] = P[Y=1 \mid X=1]\, P[X=1] + P[Y=1 \mid X=0]\, P[X=0]$$

$$= 1 \cdot \alpha + \frac{\alpha}{\alpha+\beta}\left[1-\alpha-\beta\right] = \alpha + \frac{\alpha - \alpha^2 - \alpha\beta}{\alpha+\beta} =$$

$$= \frac{\alpha^2 + \beta\alpha + \alpha - \alpha^2 - \beta\alpha}{\alpha+\beta} = \frac{\alpha}{\alpha+\beta}$$

$$P[Y=-1] = P[Y=-1 \mid X=-1]\, P[X=-1] + P[Y=-1 \mid X=0]\, P[X=0]$$

$$= \beta + \frac{\beta}{\beta+\alpha}\left[1-\alpha-\beta\right] = \frac{\cancel{\beta^2} + \cancel{\alpha\beta} + \beta - \cancel{\beta\alpha} - \cancel{\beta^2}}{\alpha+\beta}$$

$$\Rightarrow P[Y=-1] = \frac{\beta}{\alpha+\beta}$$

$$\boxed{H[Y] = \frac{\alpha}{\alpha+\beta}\log_2\frac{\alpha+\beta}{\alpha} + \frac{\beta}{\alpha+\beta}\log_2\frac{\alpha+\beta}{\beta}}$$

(b) $I(X;Y) = H(Y) - H(Y|X)$

$$H(Y|X) = P[X=1]\cdot H(Y|X=1) + P[X=-1]\cdot H(Y|X=-1) +$$
$$P[X=0]\cdot H(Y|X=0)$$

$$H(Y|X=1) = \sum_j p(y_j|X=1)\log\frac{1}{p(y_j|X=1)} =$$

$$\underbrace{p(y=1|X=1)}_{1}\log\frac{1}{p(y=1|X=1)} + \underbrace{p(y=-1|X=1)}_{0}\log\frac{1}{p(y=-1|X=1)}$$

$$\Rightarrow H(Y|X=1) = 0$$
$H(Y|X=-1) = 0$ (por las mismas razones)

$$H(Y|X=0) = p(y=1|X=0)\log\frac{1}{p(y=1|X=0)} +$$
$$p(y=-1|X=0)\log\frac{1}{p(y=-1|X=0)} =$$
$$= \frac{\alpha}{\alpha+\beta}\log\frac{\alpha+\beta}{\alpha} + \frac{\beta}{\alpha+\beta}\log\frac{\alpha+\beta}{\beta}$$

$$\Rightarrow H(Y|X) = p[X=0]\cdot H[Y|X=0] =$$
$$(1-(\alpha+\beta))\cdot\left[\frac{\alpha}{\alpha+\beta}\log\frac{\alpha+\beta}{\alpha} + \frac{\beta}{\alpha+\beta}\log\frac{\alpha+\beta}{\beta}\right]$$

$$I(X;Y) = H(Y) - H(Y|X) = \dots$$

$$= \alpha \log \frac{\alpha+\beta}{\alpha} + \beta \log \frac{\alpha+\beta}{\beta}.$$

(c) C si $\alpha = \beta$.

$$I(X;Y)\Big|_{\alpha=\beta} = 2\alpha \log_2 \frac{2\alpha}{\alpha} = 2\alpha \,, \quad 0 < \alpha+\beta \leq 1$$

$$0 < 2\alpha \leq 1$$

$$\Rightarrow \alpha \leq \frac{1}{2}$$

$C = I_{max} \Rightarrow \alpha = \frac{1}{2}$; es decir que $p(X=0) = 0$

Cuando $\alpha = \frac{1}{2}$ no se producen borrados y

$C = 1$ bit/símbolo