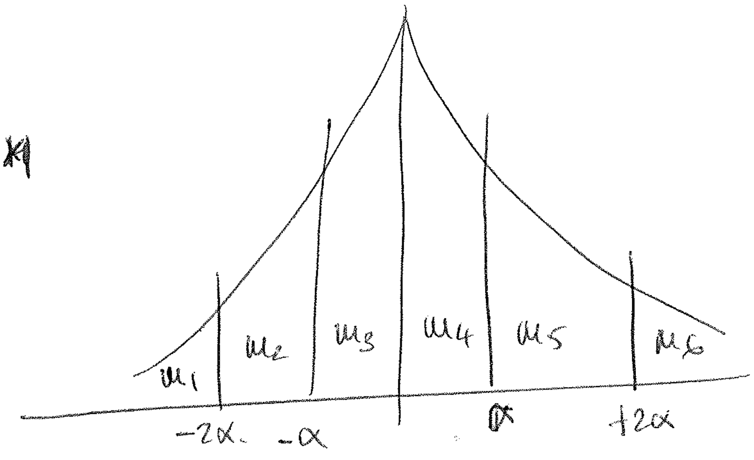
PROBLEMA 1

A

$r = 1000$ m/seg

$$f_X(x) = \text{laplaciano} = \frac{\alpha}{2} e^{-\alpha|x|}$$



$f_X(x)$ simétrico $\Rightarrow$

$$p(m_1) = p(m_6) \;;\; p(m_2) = p(m_5) ;\; p(m_3) = p(m_4)$$

$$p(m_1) = p(m_6) = \int_{-\infty}^{-2\alpha} \frac{\alpha}{2} e^{+\alpha x} dx = \frac{\alpha}{2\alpha} e^{\alpha x}\Big]_{-\infty}^{-2\alpha} = \frac{1}{2}\left[e^{-2\alpha^2} - e^{-\infty}\right] = \frac{e^{-2\alpha^2}}{2}$$

$$p(m_2) = p(m_5) = \int_{-2\alpha}^{-\alpha} \frac{\alpha}{2} e^{+\alpha x} dx = \frac{\alpha}{2\alpha} e^{\alpha x}\Big]_{-2\alpha}^{-\alpha} = \frac{1}{2}\left(e^{-\alpha^2} - e^{-2\alpha^2}\right)$$

$$p(m_3) = p(m_4) = \int_{-\alpha}^{0} \frac{\alpha}{2} e^{+\alpha x} dx = \frac{\alpha}{2\alpha} e^{\alpha x}\Big]_{-\alpha}^{0} = \frac{1}{2}\left(1 - e^{-\alpha^2}\right)$$

$$H(S_q) = \sum_{i=1}^{6} p(m_i) \log \frac{1}{p(m_i)} =$$

$$= e^{-2\alpha^2} \log \frac{2}{e^{-2\alpha^2}} + \left(e^{-\alpha^2} - e^{-2\alpha^2}\right) \log \frac{2}{\left(e^{-\alpha^2} - e^{-2\alpha^2}\right)} +$$

$$+ \left(1 - e^{-\alpha^2}\right) \log \frac{2}{\left(1 - e^{-\alpha^2}\right)} \left(\frac{\text{bit}}{\text{símbolo}}\right)$$

tasa de información: $R_q = r \cdot H(S_q) = 1000 \frac{\text{símbolos}}{\text{seg}} \cdot H(S_q) \frac{\text{bits}}{\text{símb}}$

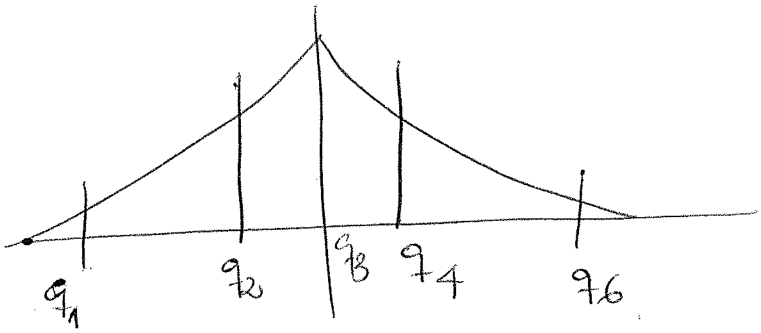
PROBLEMA 1, APARTADO B

Para que $H(S_q)$ sea max $\Rightarrow p(m_i) = 1/6 \ \forall i$

como Laplac. es simétrica

$q_1 = -q_6$

$q_2 = -q_4$



$$p(m_1) = p(m_6) = 1/6 \Rightarrow \int_{-\infty}^{q_1} \frac{\alpha}{2} e^{+\alpha x} dx = 1/6 \Rightarrow \frac{1}{2}\left(e^{\alpha q_1} - 0\right) = 1/6$$

$$e^{\alpha q_1} = 1/3 \Rightarrow \alpha q_1 = \ln(1/3) \Rightarrow \boxed{q_1 = \ln(1/3)/\alpha}$$

$$p(m_2) = p(m_5) = \int_{q_1}^{q_2} \frac{\alpha}{2} e^{+\alpha x} dx = 1/6 \Rightarrow \frac{1}{2}\left[e^{\alpha x}\right]_{q_1}^{q_2} = \frac{1}{2}\left(e^{\alpha q_2} - \underbrace{e^{\alpha q_1}}_{= 1/3}\right) = 1/6$$

$$\Rightarrow e^{\alpha q_2} - \frac{1}{3} = 1/3 \Rightarrow e^{\alpha q_2} = 2/3$$

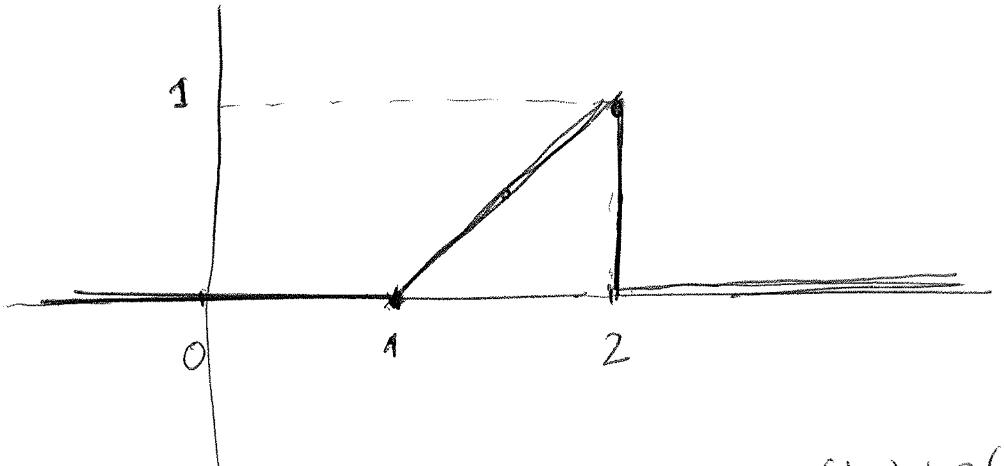
$$\boxed{q_2 = \ln(2/3)/\alpha}$$

$$p(m_3) = p(m_4) = \int_{q_2}^{q_3} \frac{\alpha}{2} e^{+\alpha x} dx = 1/6 \Rightarrow \frac{1}{2}\left[e^{\alpha q_3} - \underbrace{e^{\alpha q_2}}_{2/3}\right) = 1/6 \Rightarrow$$

$$\Rightarrow e^{\alpha q_3} - 2/3 = 1/3 \Rightarrow e^{\alpha q_3} = 3/3 = 1 \Rightarrow \boxed{q_3 = 0}$$

PROBLEMA 2 $\quad X(t) = A \cdot g(t)$; A = v.a. discreta $\begin{cases} P[A=1] = 1/2 \\ P[A=2] = 1/4 \\ P[A=3] = 1/4 \end{cases}$

$$g(t) = \begin{cases} 0 & \text{si } t < 1 \\ t-1 & \text{si } 1 \leq t \leq 2 \\ 0 & \text{si } t > 2 \end{cases}$$



(A) $E[X(t)] = E[A] \cdot g(t)$; $E[A] = 1(1/2) + 2(1/4) + 3(1/4) = 7/4$

$$m_X(t) = \begin{cases} 0 & \text{si } t < 1 \\ \frac{7}{4}(t-1) & \text{si } 1 \leq t \leq 2 \\ 0 & \text{si } t > 2 \end{cases}$$

(B) pmf de orden 1

- si $t_1 \notin [1,2]$ $\quad g(t) = 0 \Rightarrow p[X=0] = 1$
  $p = 0$ para el resto de valores

- si $t_1 \in (1,2) \Rightarrow g(t) = t-1$.

dado t_1 $\quad \left.\begin{array}{l} p(X(t_1) = 1(t_1-1)) = 1/2 \\ p(X(t_1) = 2(t_1-1)) = 1/4 \\ P(X(t_1) = 3(t_1-1)) = 1/4 \end{array}\right]$ los casos posibles para $t_1 \in (1,2)$

(c) Dados t_1 y t_2 instantes de muestreo,

genero $\overline{X} = (x(t_1), x(t_2))$ y tengo cuatro escenarios posibles

C1 • si $t_1, t_2 \notin [1,2]$ $\Rightarrow$ $\cancel{X(t}$ $p(0,0) = 1$

$p(X_1, X_2) = 0$ por el resto del valores.

C2 • si $t_1 \notin [1,2]$
$t_2 \in [1,2]$ $\Rightarrow$ tengo la variable bidimensional $(0, X_2)$ con prob:

$p(0, t_2 - 1)) = 1/2$
$p(0, 2(t_2-1) = 1/4$
$p(0, 3(t_2-1) = 1/4$

resto de combinac. de valores con prob $p(X_1, X_2) = 0$

C3 • si $t_1, t_2 \in [1,2]$

tengo la variable bidimensional (X_1, X_2) con prob.

$p[((t_1 - 1, t_2 - 1)] = \frac{1}{2} \cdot 1/2 = \frac{1}{4}$
$p[(t_1 - 1, 2(t_2-1))] = 1/2 \cdot \frac{1}{4} = 1/8$
$p[(t_1 - 1, 3(t_2-1))] = 1/2 \cdot 1/4 = 1/8$
$p[2(t_1-1), (t_2-1)] = 1/4 \cdot 1/2 = 1/8$
$p[2(t_1-1), 2(t_2-1)] = 1/4 \cdot 1/4 = 1/16$
$p[2(t_1-1), 3(t_2-1)] = \frac{1}{4} \cdot 1/4 = 1/16$
$p[3(t_1-1), (t_2-1)] = 1/4 \cdot 1/2 = 1/8$
$p[3(t_1-1), 2(t_2-1)] = 1/4 \cdot 1/4 = 1/16$
$p[3(t_1-1), 3(t_2-1)] = 1/4 \cdot 1/4 = 1/16$

C4: • si $t_1 \in [1,2]$, $t_2 \notin [1,2]$ → tengo la variable bidimensional $[X_1, 0]$

$P[t_1-1, 0] = 1/2$

$P[2(t_1-1), 0] = 1/4$

$P[3(t_1-1), 0] = 1/4$

(D) Determinar la autocorrelación del proceso

$R_{X_1X_2}(t_1,t_2) = E[X(t_1) \cdot X(t_2)] = E[A g(t_1) \cdot A g(t_2)]$

$= E[A^2 \cdot g(t_1) g(t_2)] = E[A^2] \cdot g(t_1) \cdot g(t_2)$

$E[A^2] = (1^2) \cdot \frac{1}{2} + 2^2 \cdot \frac{1}{4} + 3^2 \cdot \frac{1}{4} = \frac{1}{2} + \frac{4}{4} + \frac{9}{4} = \frac{2+4+9}{4} = \frac{15}{4}$

distingo 3 casos posibles según los valores de t_1 y t_2

C1: • si t_1 y $t_2 \notin [1,2]$ ⇒ $g(t_1) \cdot g(t_2) = 0$ → $R_X(t_1,t_2) = 0$

C2 • si $t_1 \in [1,2]$ y $t_2 \notin [1,2]$ ó $t_1 \notin [1,2]$ y $t_2 \in [1,2]$

⇒ $g(t_1) \cdot g(t_2) = 0$ ⇒ $R_X(t_1,t_2) = 0$

C3 • si $t_1, t_2 \in [1,2]$ ⇒ $g(t_1) \cdot g(t_2) = (t_1-1)(t_2-1)$

⇒ $R_X(t_1,t_2) = \frac{15}{4}(t_1-1)(t_2-1)$.