1. Solve the s-wave Schrodinger equation for the ground state and the first excited state of the hydrogen atom:

$$\frac{d^2y}{dx^2} = A(r)u(r), A(r) = \frac{2m}{\hbar^2}[V(r) - E] \tag{1}$$

where,

$$V(r) = -\frac{e^2}{r} \tag{2}$$

Here, m is the reduced mass of the electron. Obtain the energy eigenvalues and plot the corresponding wavefunction. Remember that the ground state energy of the hydrogen atom is $\approx -13.6eV$. Take $e = 3.795(eVÅ)^{1/2}$, $\hbar c = 1973(eVÅ)$ and $m = 0.511 \times 10^6 eV/c^2$

2. Solve the s-wave radial Schrodinger equation for an atom:

$$\frac{d^2y}{dx^2} = A(r)u(r), A(r) = \frac{2m}{\hbar^2}[V(r) - E] \tag{3}$$

where, m is the reduced mass of the system(which can be chosen to be the mass of an electron), for the screened coulomb potential

$$V(r) = -\frac{e^2}{r}e^{-}r/a \tag{4}$$

Find the energy (in eV) of the ground state of the atom to an accuracy of three significant digits. Also, plot the corresponding wavefunction. Take $e = 3.795(eVÅ)^{1/2}, m = 0.511 \times 10^6 eV/c^2$, and $a = 3Å, 5Å, 7Å$. In these units $\hbar c = 1973(eVÅ)$. The ground state energy is expected to be above -12 eV in all three cases.

3. Solve the s-wave radial Schrodinger equation for an atom:

$$\frac{d^2y}{dx^2} = A(r)u(r), A(r) = \frac{2m}{\hbar^2}[V(r) - E] \tag{5}$$

For the anharmonic oscillator potential

$$V(r) = \frac{1}{2}kr^2 + \frac{1}{3}br^3 \tag{6}$$

for the ground state energy (in MeV) of particle to an accuracy of three significant digits. Also, plot the corresponding wave function. Choose
$m = 940MeV/c^2, k = 100MeVfm^{-2}, b = 0, 10, 30MeVfm^{-3}$ in these units,
$c\hbar = 197.3MeVfm$. The ground state energy is expected to lie between 90 and 110 MeV for all three cases.

4. Solve the s-wave radial Schrodinger equation for the vibrations of hydrogen molecule:

$$\frac{d^2y}{dx^2} = A(r)u(r), A(r) = \frac{2\mu}{\hbar^2}[V(r) - E] \tag{7}$$

Where μ is the reduced mass of the two-atom system for Morse potential

$$V(r) = D(r^{-2\alpha r\prime} - r^{\alpha r\prime}), r\prime = \frac{r - r_0}{r} \tag{8}$$

Find the lowest vibrational energy (in MeV) of the molecule to an accuracy of three significant digits. Also plot the corresponding wave function.
Take: $m = 940 \times 10^6 eV/c^2, D = 0.755501eV, \alpha = 1.44, r_0 = 0.131349\AA$.