

1. Solve the s-wave Schrodinger equation for the ground state and the first excited state of the hydrogen atom: $\frac{d^2y}{dx^2} = A(r)u(r), A(r) = \frac{2m}{\hbar^2}[V(r)-E]$ where, $V(r) = -\frac{e^2}{r}$. Here, m is the reduced mass of the electron. Obtain the energy eigenvalues and plot the corresponding wavefunction. Remember that the ground state energy of the hydrogen atom is ≈ -13.6 eV. Take $e = 3.795(\text{eV}\text{\AA})^{1/2}$, $\hbar c = 1973(\text{eV}\text{\AA})$ and $m = 0.511 \times 10^6 \text{ eV}/c^2$.
2. Solve the s-wave radial Schrodinger equation for an atom: $\frac{d^2y}{dx^2} = A(r)u(r), A(r) = \frac{2m}{\hbar^2}[V(r)-E]$ where m is the reduced mass of the system (which can be chosen to be the mass of an electron), for the screened coulomb potential $V(r) = -\frac{e^2}{r} e^{-r/a}$. Find the energy (in eV) of the ground state of the atom to an accuracy of three significant digits. Also, plot the corresponding wavefunction. Take $e = 3.795 (\text{eV}\text{\AA})^{1/2}$, $m = 0.511 \times 10^6 \text{ eV}/c^2$, and $a = 3\text{\AA}, 5\text{\AA}, 7\text{\AA}$. In these units $\hbar c = 1973(\text{eV}\text{\AA})$. The ground state energy is expected to be above -12 eV in all three cases.
3. Solve the s-wave radial Schrodinger equation for an atom: $\frac{d^2y}{dx^2} = A(r)u(r), A(r) = \frac{2m}{\hbar^2}[V(r)-E]$

For the anharmonic oscillator potential $V(r) = \frac{1}{2}kr^2 + \frac{1}{3}br^3$

for the ground state energy (in MeV) of particle to an accuracy of three significant digits. Also, plot the corresponding wave function. Choose $m = 940 \text{ MeV}/c^2, k = 100 \text{ MeVfm}^{-2}, b = 0, 10, 30 \text{ MeVfm}^{-3}$ in these units, $\hbar c = 197.3 \text{ MeVfm}$. The ground state energy is expected to lie between 90 and 110 MeV for all three cases.

4. Solve the s-wave radial Schrodinger equation for the vibrations of hydrogen molecule: $\frac{d^2y}{dx^2} = A(r)u(r), A(r) = \frac{2\mu}{\hbar^2}[V(r)-E]$

Where μ is the reduced mass of the two-atom system for Morse potential $V(r) = D(r^{-2\alpha} - r^{-\alpha})^2$, $r^{-\alpha} = \frac{r - r_0}{r}$

Find the lowest vibrational energy (in MeV) of the molecule to an accuracy of three significant digits. Also plot the corresponding wave function.

Take: $m = 940 \times 10^6 \text{ eV}/c^2, D = 0.755501 \text{ eV}, \alpha = 1.44, r_0 = 0.131349 \text{\AA}$