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Tarea Sistema de Control por realimentación de estados.

$$G(s) = \frac{20(s+5)}{s(s+1)(s+4)}$$

$$U(s) \longrightarrow \boxed{\frac{1}{s^3+5s^2+4s}} \xrightarrow{X_1(s)} \boxed{\cancel{0}s^2+20s+100} \longrightarrow Y(s)$$

$$\frac{X_1(s)}{R_1(s)} = \frac{1}{s^3+5s^2+4s}$$

$$(s^3+5s^2+4s)X_1(s) = U(s)$$

$$\dddot{x}_1 + 5\ddot{x}_1 + 4\dot{x}_1 = u$$

$$x_1 = x_1$$
$$x_2 = \dot{x}_1$$
$$x_3 = \dot{x}_1 = \ddot{x}_1$$
$$\dot{x}_3 \ldots \dddot{x}_1$$
$$\dot{x}_3 = -5x_3 - 4x_2 + u$$

$$y(s) = (b_2 s^2 + b_1 s + b_0)X_1(s)$$
$$= (0s^2 + 20s + 100)X_1(s)$$
$$= (20s + 100)X_1(s)$$

↓ aplicando $\mathcal{L}^{-1}$

$$= 20\underbrace{\dot{x}_1}_{x_2} + 100x_1 \longrightarrow \boxed{y = 20x_2 + 100x_1}$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -4 & -5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$y = \begin{bmatrix} 100 & 20 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

$$\%\,OS = e^{-(\zeta\pi/\sqrt{1-\zeta^2})} \times 100$$

$$\ln(0{,}095) = \ln\left(e^{-(\zeta\pi/\sqrt{1-\zeta^2})}\right)$$

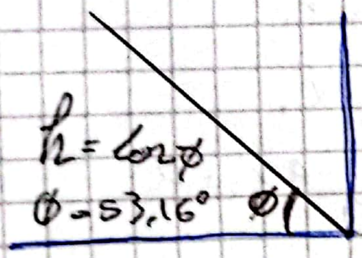
$$-\left(2{,}3539\left(\sqrt{1-\zeta^2}\right)\right)^2 = (-\zeta\pi)^2$$

$$5.5407\,(1-\zeta^2) = \zeta^2\pi^2$$

$$5{,}5407 - 5.5407\,\zeta^2 = \zeta^2\pi^2$$

$$5{,}5407 = \zeta^2(\pi^2 + 5{,}5407)$$

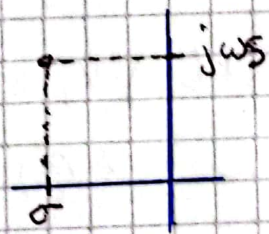
$$\zeta^2 = \frac{5{,}5407}{\pi^2 + 5{,}5407} \longrightarrow \zeta = 0{,}5996$$



$s = \sigma + j\omega_d$ $\quad$ acos(0,5996)

$\zeta\omega_n$

$s = \sigma + j\omega_d$



$$t_s = \frac{4}{\sigma}$$

$$0{,}74 = \frac{4}{\sigma}$$

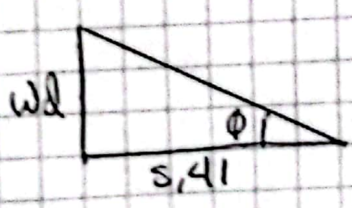
$\sigma = 4/0{,}79$

$\sigma = 5{,}405$

$\sigma = \zeta\omega_n$

$5{,}405 = 0{,}5976\,\omega_n$

$\boxed{\omega_n = 9.02}$ rN/s



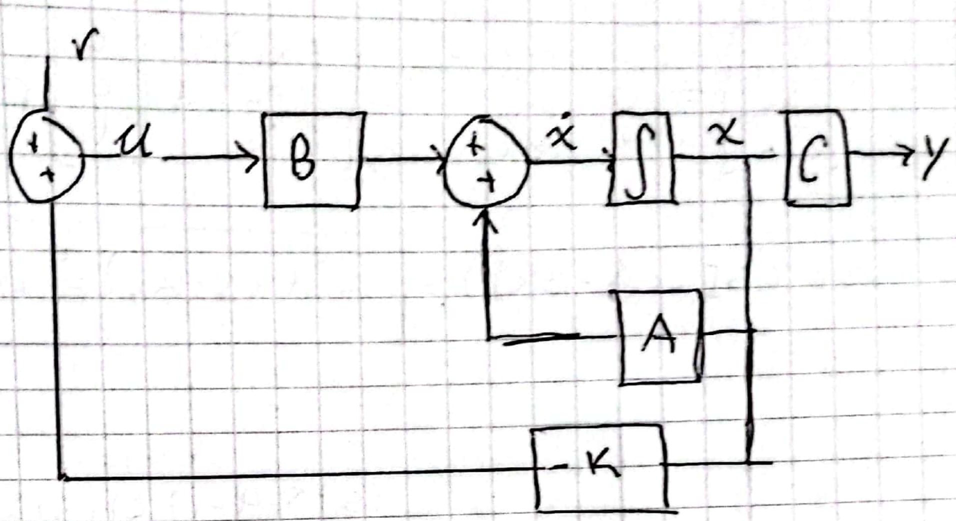
$\tan\phi = \omega_d/5{,}41$

$\tan d(53{,}16)\,5{,}41 = \omega_d$

$\omega_d = 7{,}21$

$$\dot{x} = Ax + Bx$$

$$y = Cx$$



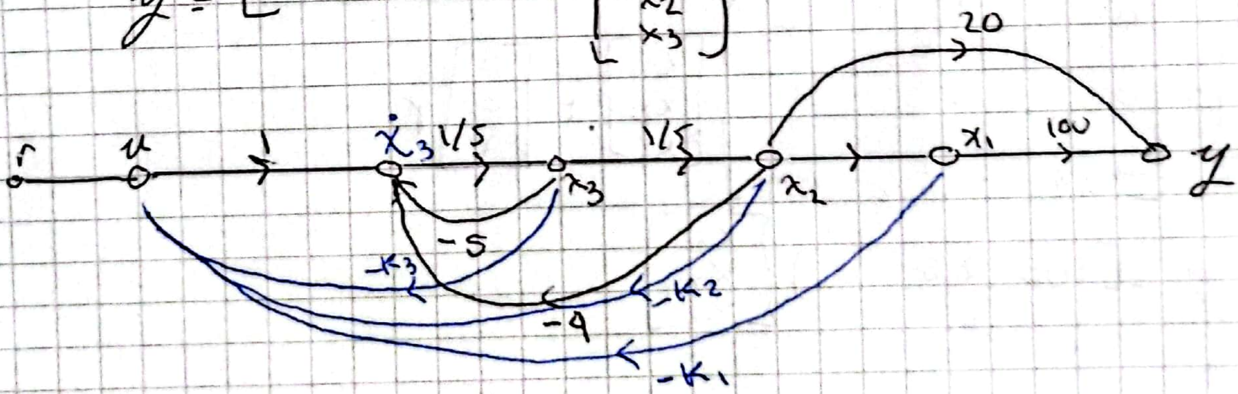
$$\dot{x} = Ax + Bu$$

$$= Ax + B(-Kx + r)$$

$$\boxed{\dot{x} = (A - BK)x + Br}$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -4 & -5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u$$

$$y = \begin{bmatrix} 160 & 20 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$



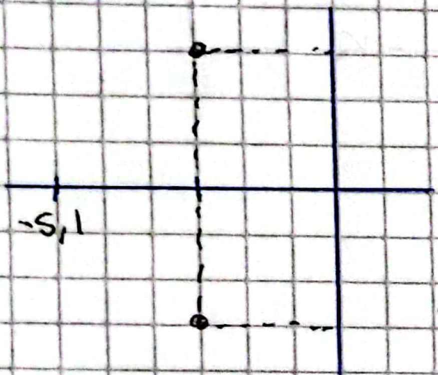
$$\dot{x}_3 = -4x_2 - 5x_3 + u$$

$$= -4x_3 - 5x_3 + [-K_3x_3 - K_2x_2 - K_1x_1]$$

$$= -K_1x_1 - (4 + K_2)x_2 - (5 + K_3)x_3 + r$$

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\\ x_3\end{bmatrix} = \begin{bmatrix}0 & 1 & 0\\ 0 & 0 & 1\\ -K_1 & -(4+K_2) & -(5+K_3)\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix} + \begin{bmatrix}0\\ 0\\ 1\end{bmatrix} r$$

$$\det(sI-(A-BK)) = \boxed{s^3+(5+K_3)s^2+(4+K_2)s+K_1=0}$$



$$(s+5,4-j7,2)(s+5,4+j7,2)(s+5,1)$$

$$s^3+15,9s^2+136,22s+413,83=0$$

$$s^3+(5+K_3)s^2+(4+K_2)s+K_1 = s^3+15,9s^2+136,22s+413,83$$

$5+K_3=15,9$, $K_3=10,9$ | $(4+K_2)s=136,22s$ // $4+K_2=136,22$ // $K_1=413,83$

$K_2=132,22$

$$\begin{bmatrix}\dot{x}_1\\ x_2\\ \dot{x}_3\end{bmatrix} = \begin{bmatrix}0 & 1 & 0\\ 0 & 0 & 1\\ -413,8 & -136,22 & -15,9\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix} + \begin{bmatrix}0\\ 0\\ 1\end{bmatrix} r$$

$$y = \begin{bmatrix}100 & 20 & 0\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix}$$