Metody Monte Carlo

Laboratorium 2

Zadanie 2

Kod (C++)

```
#include <iostream>
#include <random>
#include <map>
#include <vector>
#include <gsl/gsl_rng.h>

std::map<unsigned long int, unsigned long int> generator(const gsl_rng_type
*type, int k, int n)
{
  gsl_rng_env_setup();

  const gsl_rng_type *T = type;
  gsl_rng *r = gsl_rng_alloc(T);

  std::map<unsigned long int, unsigned long int> samples;

  for (int i = 0; i < k; i++)
  {
    samples.insert(std::pair<unsigned long int, unsigned long int>(i, 0));
  }

  for (int i = 0; i < n; i++)
  {
    unsigned long int s = gsl_rng_uniform(r) * k;
    ++samples[int(s)];
  }

  gsl_rng_free(r);

  return samples;
}

int main()
{
  std::vector<int> keys;
  keys.push_back(11);
  keys.push_back(51);
  keys.push_back(101);

  std::vector<double> critical_values_0_05;
  critical_values_0_05.push_back(18.307);  // for df = 10
  critical_values_0_05.push_back(67.505);  // for df = 50
  critical_values_0_05.push_back(124.342); // for df = 100

  int n = 100000;

  // Good generator case
  for (int i = 0; i < 3; i++)
  {
    int k = keys[i];
    double average_value = n / k;
    std::map<unsigned long int, unsigned long int> samples =
generator(gsl_rng_ranlux, k, n);
```

```
    unsigned long int sum = 0;

    for (int i = 0; i < k; i++)
    {
      sum += samples[i] * samples[i];
    }

    double result = ((double)k * (double)sum) / (double)n - (double)n;

    std::cout << std::endl;
    std::cout << "Result from GOOD rng generator for " << n << " total probes
and " << k << " intervals is " << result << std::endl;

    if (result > critical_values_0_05[i])
    {
      std::cout << "Result " << result << " is higher than critical value of "
<< critical_values_0_05[i] << std::endl;
    }
    else
    {
      std::cout << "Result " << result << " is lower than critical value of " <<
critical_values_0_05[i] << std::endl;
    }
  }

  // Bad generator case
  for (int i = 0; i < 3; i++)
  {
    int k = keys[i];
    double average_value = n / k;
    std::map<unsigned long int, unsigned long int> samples =
generator(gsl_rng_rand, k, n);
    unsigned long int sum = 0;

    for (int i = 0; i < k; i++)
    {
      sum += samples[i] * samples[i];
    }

    double result = ((double)k * (double)sum) / (double)n - (double)n;

    std::cout << std::endl;
    std::cout << "Result from BAD rng generator for " << n << " total probes and
" << k << " intervals is " << result << std::endl;

    if (result > critical_values_0_05[i])
    {
      std::cout << "Result " << result << " is higher than critical value of "
<< critical_values_0_05[i] << std::endl;
    }
    else
    {
      std::cout << "Result " << result << " is lower than critical value of " <<
critical_values_0_05[i] << std::endl;
    }
  }
  return 0;
}
```

Wyniki

Wynik programu dla n = 1000 (tysiąc)

```
Result from GOOD rng generator for 1000 total probes and 11 intervals is 20.58
Result 20.58 is higher than critical value of 18.307

Result from GOOD rng generator for 1000 total probes and 51 intervals is 67.226
Result 67.226 is lower than critical value of 67.505

Result from GOOD rng generator for 1000 total probes and 101 intervals is
142.714
Result 142.714 is higher than critical value of 124.342

Result from BAD rng generator for 1000 total probes and 11 intervals is 8.942
Result 8.942 is lower than critical value of 18.307

Result from BAD rng generator for 1000 total probes and 51 intervals is 63.044
Result 63.044 is lower than critical value of 67.505

Result from BAD rng generator for 1000 total probes and 101 intervals is 114.232
Result 114.232 is lower than critical value of 124.342
```

Wynik programu dla n = 10000 (dziesięć tysięcy)

```
Result from GOOD rng generator for 10000 total probes and 11 intervals is
11.6676
Result 11.6676 is lower than critical value of 18.307

Result from GOOD rng generator for 10000 total probes and 51 intervals is
58.5464
Result 58.5464 is lower than critical value of 67.505

Result from GOOD rng generator for 10000 total probes and 101 intervals is
120.624
Result 120.624 is lower than critical value of 124.342

Result from BAD rng generator for 10000 total probes and 11 intervals is 10.6798
Result 10.6798 is lower than critical value of 18.307

Result from BAD rng generator for 10000 total probes and 51 intervals is 56.588
Result 56.588 is lower than critical value of 67.505

Result from BAD rng generator for 10000 total probes and 101 intervals is
98.7274
Result 98.7274 is lower than critical value of 124.342
```

Wynik programu dla n = 100000 (sto tysięcy)

```
Result from GOOD rng generator for 100000 total probes and 11 intervals is
3.7698
Result 3.7698 is lower than critical value of 18.307

Result from GOOD rng generator for 100000 total probes and 51 intervals is
39.5957
Result 39.5957 is lower than critical value of 67.505
```

```
Result from GOOD rng generator for 100000 total probes and 101 intervals is
89.7254
Result 89.7254 is lower than critical value of 124.342

Result from BAD rng generator for 100000 total probes and 11 intervals is 6.8201
Result 6.8201 is lower than critical value of 18.307

Result from BAD rng generator for 100000 total probes and 51 intervals is
53.4493
Result 53.4493 is lower than critical value of 67.505

Result from BAD rng generator for 100000 total probes and 101 intervals is
118.543
Result 118.543 is lower than critical value of 124.342
```

Wynik programu dla n = 1000000 (milion)

```
Result from GOOD rng generator for 1000000 total probes and 11 intervals is
6.53801
Result 6.53801 is lower than critical value of 18.307

Result from GOOD rng generator for 1000000 total probes and 51 intervals is
44.1295
Result 44.1295 is lower than critical value of 67.505

Result from GOOD rng generator for 1000000 total probes and 101 intervals is
95.0461
Result 95.0461 is lower than critical value of 124.342

Result from BAD rng generator for 1000000 total probes and 11 intervals is
6.39226
Result 6.39226 is lower than critical value of 18.307

Result from BAD rng generator for 1000000 total probes and 51 intervals is
57.7413
Result 57.7413 is lower than critical value of 67.505

Result from BAD rng generator for 1000000 total probes and 101 intervals is
121.386
Result 121.386 is lower than critical value of 124.342
```

Jak możemy zauważyć, im więcej iteracji w badaniu (liczba n) tym dystrybucja elementów jest bardziej zróżnicowana. Dla przypadku n = 1000 (tysiąc) zachodzi przekroczenie rozbieżności 5%, i co ciekawe jest to dla "dobrego" generatora (Ranlux), który został polecony w zadaniu:

"*wartości testu chi-kwadrat dla dwóch generatorów – „dobrego" (np. Mersenne Twister czy* ***Ranlux****) i „złego" (np. randu z biblioteki GSL)*"

Generator "zły", czyli rand z biblioteki GSL, wykazuje lepsze właściwości dla próby jedynie 1000 iteracji.

Sytuacja zmienia się dla iteracji większych od 10^4. Tutaj możemy zaobserwować iż generator "dobry" (Ranlux) wykazuje wynik znacznie mniejszy od wartości krytycznej, w porównaniu do generatora złego (rand). Można tym samym uznać, że przy założeniu liczby iteracji na poziomie przynajmniej 10^5 dobry generator liczb pseudolosowych wykazuje niższą entropię.

Zadanie 3
Kod (Python)

Program wykonuje pętle iteracji, w liczbie zdefiniowanej parametrem **iterations**. Algorytm został umieszczony w metodzie **generate_linear_results**. Wykonuje on zadaną liczbę iteracji i zwraca trzy pożądane parametry w postaci listy:

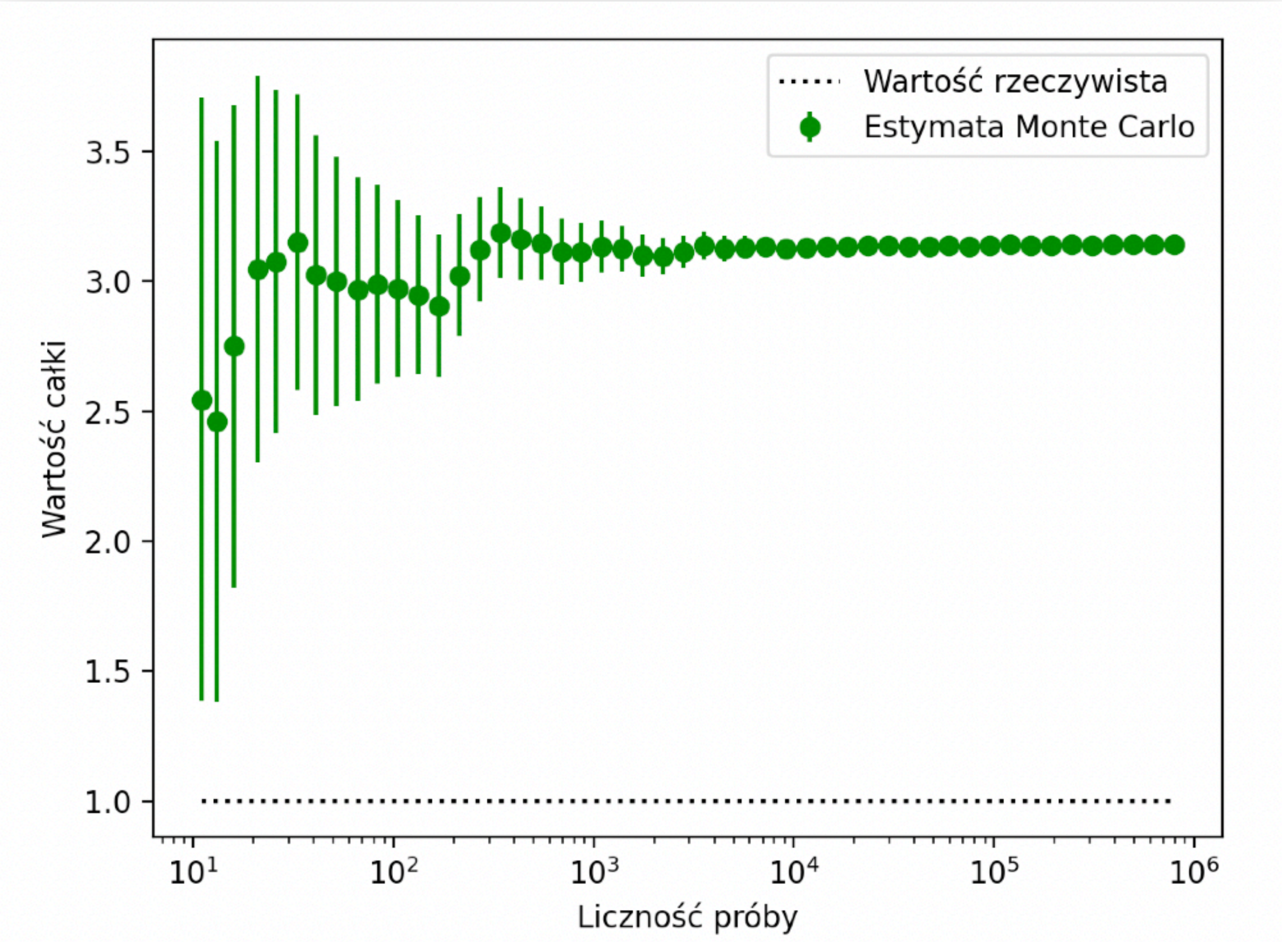
1. **total_number_of_tries** - Liczność próby
2. **pi_estimations** - estymata wyniku
3. **standard_deviations** - odchylenie standardowe

Wyniki te zostały zapisane z krokiem logarytmicznym, do pliku tekstowego zad3.txt i później użyte w generowaniu wykresu.

```python
from random import uniform
from math import sqrt
from numpy import logspace


def generate_linear_results(number_of_tries):
  points_inside_radix = 0
  total_numer_of_tries = []
  pi_estimations = []
  standard_deviations = []
  V = 4  # because we are operating in 2x2 area

  for n in range(number_of_tries):
    # Get random number form (-1, 1)
    x = uniform(-1, 1)
    y = uniform(-1, 1)

    # Calculate distance from center (0, 0)
    distance = x * x + y * y

    if distance <= 1:
      points_inside_radix += 1

    N = n + 1
    pi = 4 * points_inside_radix / N
    M = points_inside_radix
    standard_deviation = V * sqrt((1 / N) * (M / N) * (1 - (M / N)))

    total_numer_of_tries.append(N)
    pi_estimations.append(pi)
    standard_deviations.append(standard_deviation)

  return (total_numer_of_tries, pi_estimations, standard_deviations)


def main():
  iterations = 1000000

  total_numer_of_tries, pi_estimations, standard_deviations =
generate_linear_results(iterations)

  with open("zad3.txt", "w") as file:
    for i in logspace(1, 6, dtype='int'):
      if i >= iterations:
        return
      file.write(f'{total_numer_of_tries[i]} {pi_estimations[i]}
{standard_deviations[i]}\n')


if __name__ == '__main__':
  main()
```

Wyniki

Wykres otrzymano wykorzystując gotowy kod w języku Python, dostępny na stronie przedmiotu. Jest to wykres z zaznaczonymi przedziałami ufności (*errorbars*).



Jak możemy zauważyć im większa liczność próby, tym słupek błędu mniejszy, a tym samym wynik jest dokładniejszy. Dla iteracji rzędu 10^5 możemy przyjąć, iż wynik jest wystarczająco dokładny do podstawowych obliczeń (dokładność rzędu dwóch miejsc po przecinku):

`790605` **`3.1424643153028375`** `0.0018462117036397517`

Jest to ostatni wiersz w pliku zad3.txt

Bibliografia

- Wartości krytyczne dla funkcji chi-kwadrat, gdy znamy stopień swobody oraz procent maksymalnej rozbieżności 5% ze strony: https://www.medcalc.org/manual/chi-square-table.php