# PROBLEM SET 4: APPLIED MATHEMATICS 216

Due: Monday March 28 at 11:59pmt

**Problems.**

(1) **Unfair dice** In this homework problem, you are going to use 'tensorflow-probability' to deal with an unfair dice, i.e., a dice that has different probability of settling with each of the face 1 to 6 facing up, instead of the $p = 1/6$ equal probability in the case of a fair dice.

You are provided with 5000 data entries of this dice. Each entry is a length-6 vector with one element being 1 and 0 for the rest. For example, $[0, 1, 0, 0, 0, 0]$ means the "2" face of this dice landed facing up. This form is also the data form generated by 'tfp.distribution.Bernoulli' when you feed multiple probability to it.

You are going to estimate the 6 probabilities describing this unfair dice (6 face). $\tilde{p} = [p1, p2, p3, p4, p5, p6]$. Keep in mind that they sum up to 1.

(a) Performing an MLE to estimate $\tilde{p}$

(b) Use MAP to estimate $\tilde{p}$. Select three different prior distribution (if the distribution is parametrized, select three different enough parameters). Using 5000 sample, compare which prior gives the best estimation of why.
[Hint: If the optimization takes too long, try to run a certain amount of steps instead of setting a criteria for the gradient. Check the remaining gradient and determine whether to increase the number of steps]

(c) Using Monte-Carlo sampling, estimate the 6 probabilities of $\tilde{p}$ again.

(2) **Inference a complicated distribution with graphical model** A graphical model provides a visual overview of inderdepedency structures of random variables to compactly represent their joint distribution. In this problem, your task is to discover the model of data generating process for the following graphical model. Before proceeding, see "Graphical model" part of "Going Bayesian" section notebook for review of notations.

(a) Using the illustration of the graphical model in Problem 2 notebook, write down the factored expression of *joint* distribution specified by this graphical model.

(b) Infer the complicated joint distribution! Please follow the step-by-step guidance of the Problem 2 notebook.