Convex Optimization

Lab 3: Linear Programming (1)

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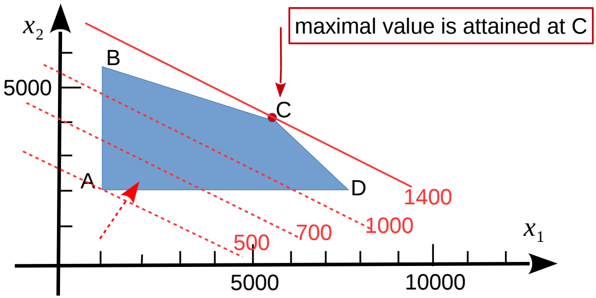
Outline

Linear Programming: solve the problem with graph

$$\text{maximize } 0.1 * x_1 + 0.2 * x_2$$

$$\text{subject to} \begin{cases} x_1 + x_2 \leq 10000 \\ x_1 \geqslant 1000 \\ x_2 \geqslant 2000 \\ 10 * x_1 + 30 * x_2 \leq 180000 \end{cases} \quad (1)$$



Linear Programming: the standard form

$$\text{minimize } f(x), x \in R^n$$
$$\text{s.t.} \quad \begin{aligned} A \cdot x &\preceq b \\ A_e \cdot x &= b_e \\ lb \preceq x &\preceq ub \end{aligned} \tag{2}$$

- 'Maximize problem' can be converted to 'minimize problem'
- $Ax \preceq b$ covers all inequations
- $A_e x = b_e$ covers all equalitions
- lb and ub are the lower and upper bounds for x respectively
- Observations:
  1. The target is a linear function
  2. All conditions are linear
  3. The region scoped by all conditions is **convex**
  4. Target function must be **convex** too!!

Linear Programming: solve it by Matlab command (1)

$$\text{minimize } f(x), x \in R^n$$
$$\text{s.t.} \quad \begin{aligned} A \cdot x &\preceq b \\ A_e \cdot x &= b_e \\ lb \preceq x &\preceq ub \end{aligned} \tag{2}$$

- [**x**, **fval**]=linprog(f, A, b, A_e, b_e, lb, ub);

$$max. \quad 0.1*x_1+0.2*x_2 \quad \Longrightarrow \quad min. \quad -1*(0.1*x_1+0.2*x_2)$$

$$\begin{bmatrix} x_1+x_2 \leqslant 10000 \\ 10*x_1+30*x_2 \leqslant 180000 \end{bmatrix} \quad \Longrightarrow \quad \underset{\mathbf{A}}{\begin{bmatrix} 1 & 1 \\ 10 & 30 \end{bmatrix}} \text{ x } \underset{\boldsymbol{x}}{\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}} \leqslant \underset{\mathbf{b}}{\begin{bmatrix} 10000 \\ 180000 \end{bmatrix}}$$

$$\begin{bmatrix} 1000 \leqslant x_1 \\ 2000 \leqslant x_2 \end{bmatrix} \quad \Longrightarrow \quad \underset{\mathbf{lb}}{\begin{bmatrix} 1000 \\ 2000 \end{bmatrix}} \leqslant \underset{\boldsymbol{x}}{\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}}$$

Linear Programming: solve it by Matlab command (2)

- [**x**, **fval**]=linprog(f, A, b, A_e, b_e, lb, ub);

$$\textit{max.}\ \ 0.1*x_1+0.2*x_2 \quad \Longrightarrow \quad \textit{min.}\ \ -1*(0.1*x_1+0.2*x_2)$$

$$\begin{bmatrix} x_1+x_2\leqslant 10000 \\ 10*x_1+30*x_2\leqslant 180000 \end{bmatrix} \quad \Longrightarrow \quad \overset{\textbf{A}}{\begin{bmatrix} 1 & 1 \\ 10 & 30 \end{bmatrix}} \text{ x } \overset{\boldsymbol{x}}{\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}} \leqslant \overset{\textbf{b}}{\begin{bmatrix} 10000 \\ 180000 \end{bmatrix}}$$

$$\begin{bmatrix} 1000\leqslant x_1 \\ 2000\leqslant x_2 \end{bmatrix} \quad \Longrightarrow \quad \underset{\textbf{lb}}{\begin{bmatrix} 1000 \\ 2000 \end{bmatrix}} \leqslant \underset{\boldsymbol{x}}{\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}}$$

- [**x**, **fval**]=linprog(f, A, b, [], [], lb, []);

Linear Programming: solve it by Matlab command (3)

$$\text{minimize } -0.1*x_1 - 0.2*x_2$$
$$\text{subject to } \begin{cases} x_1 + x_2 \leq 10000 \\ 10*x_1 + 30*x_2 \leq 180000 \\ x_1 \geqslant 1000 \\ x_2 \geqslant 2000 \end{cases} \tag{3}$$

1. $f = -0.1*x_1 - 0.2*x_2$
2. A=[1 1;10 30];
   b=[10000;180000]
3. lb=[1000;2000]; ub=[]

- Steps:
  1. syms x1 x2;
  2. c=[-0.1,-0.2];
  3. A=[1 1;10 30];
  4. b=[10000;180000]
  5. Ae=[]; be=[];
  6. lb=[1000;2000]; ub=[];
  7. x=linprog(c, A, b,Ae, be, lb, ub);
- Output: x=[6000; 4000]

Outline

1. Linear Programming: solve by matlab

2. Linear Programming: the problems

Linear Programming: diet problem (1)

Diet problem: suppose there are three foods available, corn, milk, and bread, and there are restrictions on the number of calories (between 2000 and 2250) and the amount of Vitamin A (between 5000 and 50,000). The following table lists, for each food, the cost per serving, the amount of Vitamin A per serving, and the number of calories per serving. The maximum servings for each food should be no higher than 10.

Table: Food, Costs, V-A, and Calories

Food	Cost/serving	V-A	Cal.
Corn	\$0.18	107	72
Milk	\$0.23	500	121
Bread	\$0.05	0	65

- Target: minimize the costs while satisfying the restrictions on calories and Vitamin A

Production Plan

Ms. Li produces widgets. To make 100 left-handed widgets she uses 1 pound of metal and 5 pounds of fiberglass. To make 100 right-handed widgets she uses 2 pounds of metal and 3 pounds of fiberglass. Each week Ms. Li has 65 pounds of metal and 150 pounds of fiberglass delivered. She makes a profit of $2.50 per right-handed widget and $2.00 per left-handed widget. How many widgets of each type should Ms. Li produce to maximize profit?

Sovle LP problem by SpreadSheet (1)

- Given following LP problem

$$\text{Max. } 3x_1 + 5x_2$$
$$\text{s.t.} \begin{cases} 3x_1 + x_2 \leq 6 \\ x_1 + x_2 \leq 4 \\ x_1 + 2x_2 \leq 6 \\ x_1, x_2 \geq 0 \end{cases} \tag{4}$$

Table: Tableau

	z	x_1	x_2	s_1	s_2	s_3	
	1	-3	-5	0	0	0	0
s_1	1	3	1	1	0	0	6
s_2	1	1	1	0	1	0	4
s_3	1	1	2	0	0	1	6

- Solve the problem by Tableau operations with Spreadsheet
- Verify it with "linprog" in Matlab

Sovle LP problem by SpreadSheet (2)

- Given following LP problem

$$\text{Max.}\ 12x_1 + 8x_2 + 10x_3$$
$$\text{s.t.} \begin{cases} 3x_1 + 2x_2 + x_3 \leq 120 \\ 5x_1 + 4x_2 + 3x_3 \leq 300 \\ x_1 + x_2 \leq 50 \\ x_1, x_2, x_3 \geq 0 \end{cases} \tag{5}$$

Table: Tableau

	z	x_1	x_2	x_3	s_1	s_2	s_3	
	1	-12	-8	-10	0	0	0	0
s_1	1	3	2	1	1	0	0	120
s_2	1	5	4	3	0	1	0	300
s_3	1	1	1	0	0	0	1	50

- Solve the problem by Tableau operations with Spreadsheet
- Verify it with "linprog" in Matlab

Sovle LP problem by SpreadSheet (3)

- Given following LP problem

$$
\begin{aligned}
&\text{Max. } 4x_1 + 6x_2 + 3x_3 + x_4 \\
&\text{s.t.} \begin{cases} 1.5x_1 + 2x_2 + 4x_3 + 3x_4 \leq 550 \\ 4x_1 + x_2 + 2x_3 + x_4 \leq 700 \\ 2x_1 + 3x_2 + x_3 + 2x_4 \leq 200 \\ x_1, x_2, x_3 \geq 0 \end{cases}
\end{aligned} \tag{6}
$$

Table: Tableau

	z	x_1	x_2	x_3	x_4	s_1	s_2	s_3	
	1	-4	-6	-3	-1	0	0	0	0
s_1	1	1.5	2	4	3	1	0	0	550
s_2	1	4	1	2	1	0	1	0	700
s_3	1	2	3	1	2	0	0	1	200

- Solve the problem by Tableau operations with Spreadsheet
- Verify it with “linprog” in Matlab