

Assignment-5

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1

$$\begin{aligned}f(x) &= -5x^2 + 5x \\f^*(y) &= \sup_{x \in \mathbb{R}} (yx - f(x)) \\&= \sup_{x \in \mathbb{R}} (yx + 5x^2 - 5x) \\&= +\infty, \quad (x \rightarrow +\infty)\end{aligned}$$

Can't plot.

2

$\because a \leq x \leq b$, f is convex

$$\begin{aligned}\therefore f(x) &= f\left(\frac{b-x}{b-a}a + \frac{x-a}{b-a}b\right) \\ &= f\left(\frac{b-x}{b-a}a + \left(1 - \frac{b-x}{b-a}\right)b\right) \\ &\leq \frac{b-x}{b-a}f(a) + \left(1 - \frac{b-x}{b-a}\right)f(b) \\ &= \frac{b-x}{b-a}f(a) + \frac{x-a}{b-a}f(b)\end{aligned}$$

3

$$\text{dom } h = \text{dom } f \cap \text{dom } g$$

$\because \text{dom } f, \text{dom } g$ are convex

$\therefore \text{dom } h$ is convex

$$\forall (x < y) \in \text{dom } h, k \in [0, 1]$$

$$\begin{aligned}h(kx + (1-k)y) &= f(kx + (1-k)y) + g(kx + (1-k)y) \\ &\leq kf(x) + (1-k)f(y) + kg(x) + (1-k)g(y) \\ &= k(f(x) + g(x)) + (1-k)(f(y) + g(y)) \\ &= kh(x) + (1-k)h(y)\end{aligned}$$

Q.E.D.

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