Convex Optimization

Lab 4: Linear Programming (2)

Simplex Method

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Outline

Linear Programming: Simplex Method (1)

- Given $C_b = [0 \quad 0 \quad 0], C_n = [7 \quad 4]$

$$B = \begin{bmatrix} x_3 & x_4 & x_5 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}_{m \times m} \qquad N = \begin{bmatrix} x_1 & x_2 \\ 2 & 1 \\ 1 & 1 \\ 1 & 0 \end{bmatrix}_{m \times p} \qquad b = \begin{bmatrix} 20 \\ 18 \\ 8 \end{bmatrix}_{m \times 1}$$

1. Find out $k = \text{argmin}_{j \in R}\{C_b \cdot B^{-1} \cdot p_j - c_j\}$
2. $\overline{b} = B^{-1}b$, $y_k = B^{-1}p_k$
3. Find out $r = \text{argmin}_i\{\frac{\overline{b}_1}{y_{1k}}, \cdots, \frac{\overline{b}_i}{y_{ik}}, \cdots, \frac{\overline{b}_m}{y_{mk}}\}$
4. $B(:r) \rightleftharpoons N(:, k)$
5. Swap corresponding columns in C_b and C_n

Task: implement Simplex (1)

- Given $C_b = [0 \quad 0 \quad 0], C_n = [7 \quad 4]$

$$B = \begin{bmatrix} x_3 & x_4 & x_5 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}_{m \times m} \qquad N = \begin{bmatrix} x_1 & x_2 \\ 2 & 1 \\ 1 & 1 \\ 1 & 0 \end{bmatrix}_{m \times p} \qquad b = \begin{bmatrix} 20 \\ 18 \\ 8 \end{bmatrix}_{m \times 1}$$

```
1 function [fval, Xb]=simplex(A, b, C)
2     %design a loop to run the Simplex procedure
3 end
```

Linear Programming: Task: implement Simplex (2)

1. Find out $k = \text{argmin}_{j \in R}\{C_b \cdot B^{-1} \cdot p_j - c_j\}$
2. $\overline{b} = B^{-1}b$, $y_k = B^{-1}p_k$
3. Find out $r = \text{argmin}_i\{\frac{\overline{b}_1}{y_{1k}}, \cdots, \frac{\overline{b}_i}{y_{ik}}, \cdots, \frac{\overline{b}_m}{y_{mk}}\}$
4. $B(:r) \rightleftharpoons N(:,k)$
5. Swap corresponding columns in C_b and C_n

```
function [fval, Xb]=simplex(A, b, C)
    %design a loop to run the Simplex procedure
    %define matrix B
    %define matrix N
    %define Cb, Cn
    % Repeat
    %    1. Find out xk
    %    2. Find out r
    %    3. Swap-in xk and Swap-out X_Br
    %    4. Swap ck into Cb, swap-out Cr to Cn
end
```

Outline

Implement Two-phase Simplex

1. Check If $\mathbf{0}$ is the feasible solution
2. If true, call One-phase Simplex
3. Otherwise, construct auxilaury problem
4. Pre-processing, call One-phase Simplex
5. construct the original problem
6. Pre-processing, call one-phase Simplex