Convex Optimization

Lab 6: Convex Functions

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Autumn Semester 2025

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General steps in Two-phase Simplex

A. Stage-1: Solve the auxiliary problem
   1. Set k=1, Repeat
   2. Find out r by $\text{argmin}_r\{B^{-1}\cdot b./B^{-1}\cdot N(:,k)\}$
   3. Swap-in $N(:,k)$ to B, Swap-out $B(:,r)$ to N_2
   4. $Cn(k) \rightleftharpoons Cb(r)$
   5. Find out k by $\text{argmin}_k\{C_b\cdot B^{-1}N - C_n\}$

B. State-2: Solve the original problem
   1. Set $C_b = [0\ \cdots\ 0]$, $C_n = -C\cdot N\cdot B^{-1}$
   2. $B_2 = N, N_2 = B^{-1}$, $b = B^{-1}\cdot b$
   3. Repeat
   4. Find out k by $\text{argmin}_k\{C_b\cdot B_2^{-1}N_2 - C_n\}$
   5. Find out r by $\text{argmin}_r\{B_2^{-1}\cdot b./B_2^{-1}\cdot N_2(:,k)\}$
   6. Swap-in $N_2(:,k)$ to B_2, Swap-out $B_2(:,r)$ to N_2
   7. $Cn(k) \rightleftharpoons Cb(r)$

Linear Programming: implement Two-phase Simplex (2)

1. Implement function phase1(), return B, N, b
2. Implement function phase2(), return B, N, b

```
function [fval, Xb]=simplexP2(A, b, C)
    %design a loop to run the Simplex procedure
    %define matrix B
    %define matrix N
    %define Cb, Cn
    %k=1
    %[B, N, b] = phase1(B, Cb, N, Cn, b);
    %Cb = [0 0 0];
    %Cn=-C*N*inv(B);
    %[B, N, b] = phase2(B, Cb, N, Cn, b);
    %fval = C*b;
end
```

Linear Programming: implement Two-phase Simplex (3)

1. Implement function phase1(), return B, N, b
2. Implement function phase2(), return B, N, b

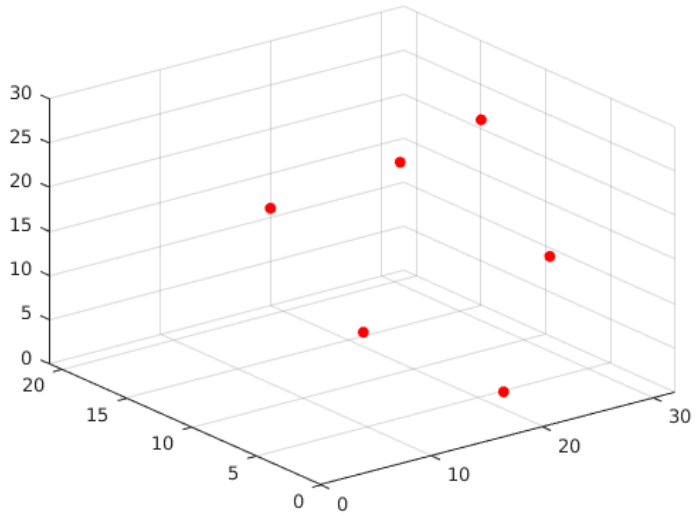
```
13 function [B, N, b] = phase1(B, Cb, N, Cn, b)
14     %k=1
15 end
16
17 function [B, N, b] = phase2(B, Cb, N, Cn, b)
18     %implement by yourself
19 end
```

Display the Convex Hull of following 3D points (1)

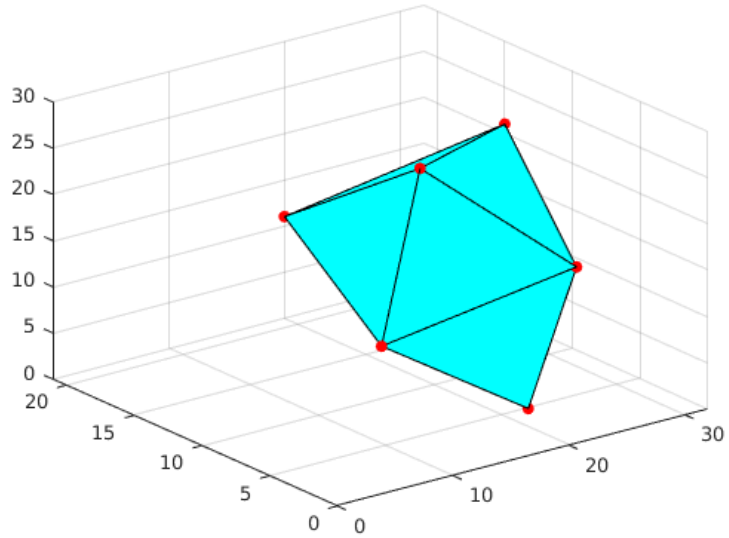
- [5 1 15;23 2 17;32 15 21;20 21 11;13 5 29;20 3 2]
- Display a group of 3D points and their convex hull

```
function conhull()
    clf;
    P = [5 1 15;23 2 17;32 15 21;20 21 11;13 5 29;20 3 2];
    [k, vol] = convhulln(P);
    trisurf(k,P(:,1),P(:,2),P(:,3),'FaceColor','cyan')
end
```

Display the Convex Hull of following 3D points (2)



(a)



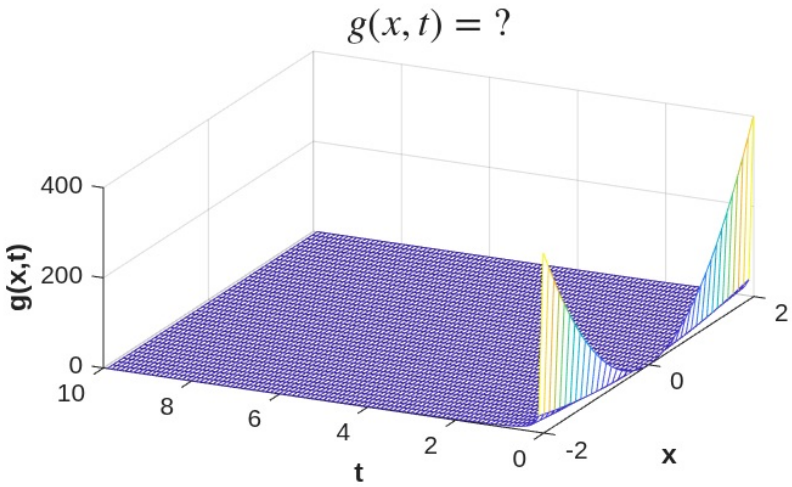
(b)

Draw following functions by Matlab

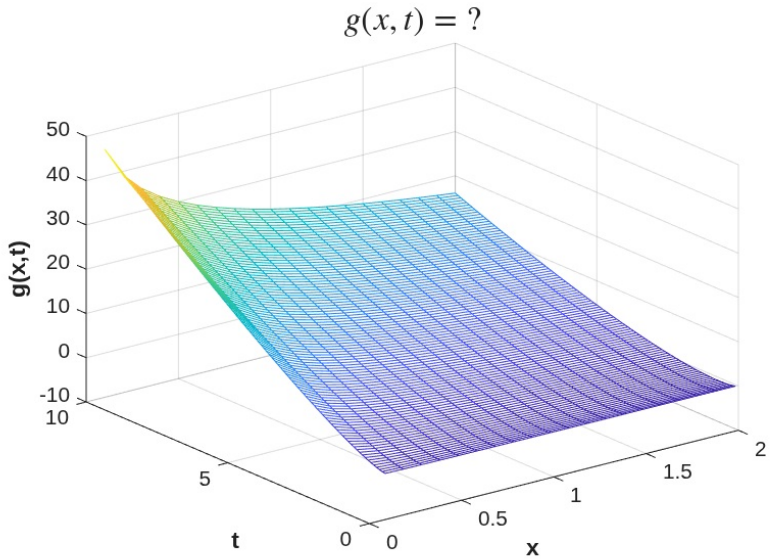
1. Exponential function: $f(x) = e^{2x}$, $x \in [-5, 5]$
2. Power function: $f(x) = x^{1.1}$, $x \in (0, 5]$
3. Power function: $f(x) = x^{0.5}$, $x \in (0, 5]$
4. Absolute power: $f(x) = |x|^{1.5}$, $x \in [-5, 5]$
5. Logarithmic: $f(x) = ln(x)$, $x \in (0, 50]$
6. Negative entropy: $f(x) = xlog_2(x)$, $x \in (0, 10]$

Draw Perspective Projection functions by Matlab

1. $f(x) = x^2$, Perspective Proj. $g(x,t) = ?$, $x \in [-2, 2], t(0, 10]$
2. $f(x) = -log(x)$, Perspective Proj. $g(x,t) = ?$, $x \in (0, 2], t(0, 10]$



(c)



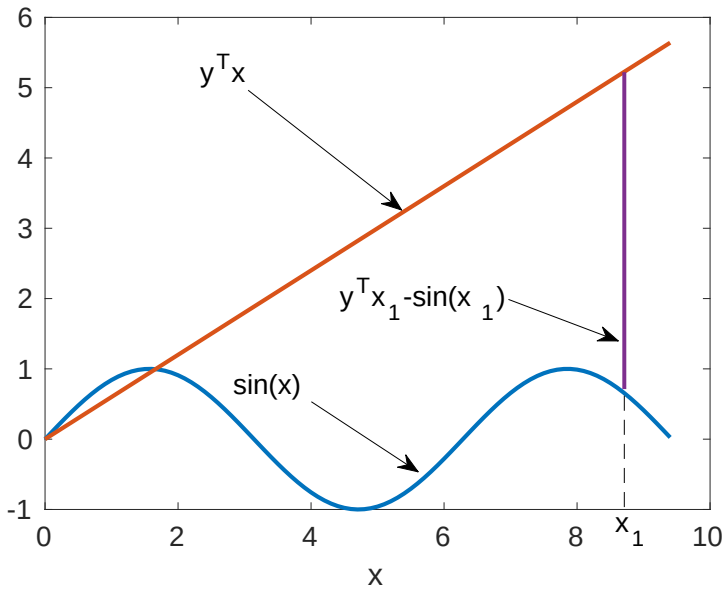
(d)

Draw Conjugate functions by Matlab (1)

- Given function $f(x), R^n \rightarrow R$, the conjugate function is defined as

$$f^*(y) = sup_x(y^T x - f(x)), R^n \rightarrow R \tag{1}$$

1. Given $f(x) = x^2$
2. Given $f(x) = xln(x)$



Draw Conjugate functions by Matlab (2)

- Given function $f(x), R^n \rightarrow R$, the conjugate function is defined as

$$f^*(y) = sup_x(y^T x - f(x)), R^n \rightarrow R \tag{2}$$

1. Given $f(x) = x^2 \quad \longrightarrow \quad f^*(y) = y \cdot x - x^2$
2. Given $f(x) = xln(x) \quad \longrightarrow \quad f^*(y) = y \cdot x - xln(x)$

Now, let's try to find out the superum of the functions w.r.t x

$$\Downarrow$$

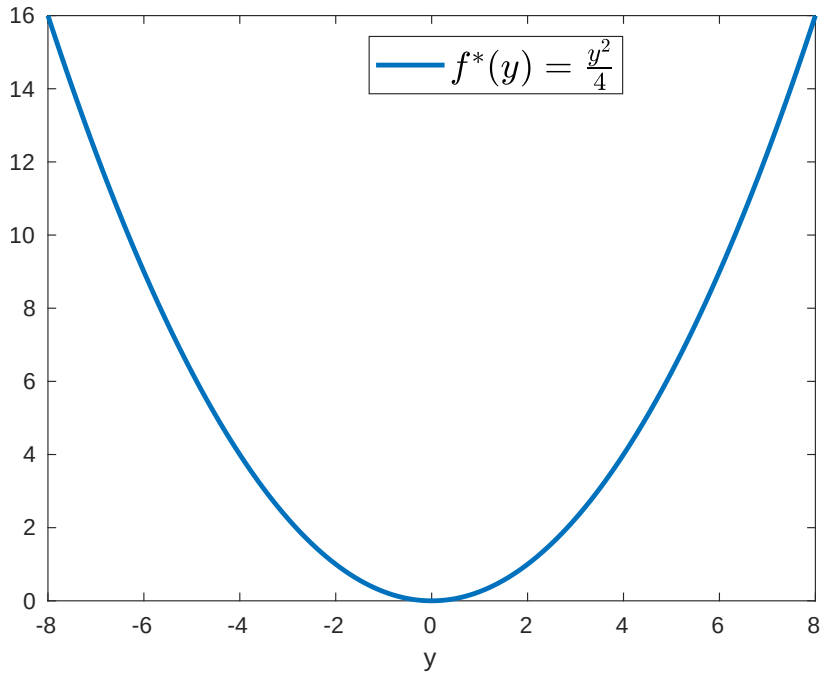
3. $\frac{\partial f^*(y)}{\partial x} = y - 2x = 0 \quad \longrightarrow \quad x = \frac{y}{2}$
4. $\frac{\partial f^*(y)}{\partial x} = y - 1 - ln(x) = 0 \quad \longrightarrow \quad x = e^{y-1}$

$$\Downarrow$$

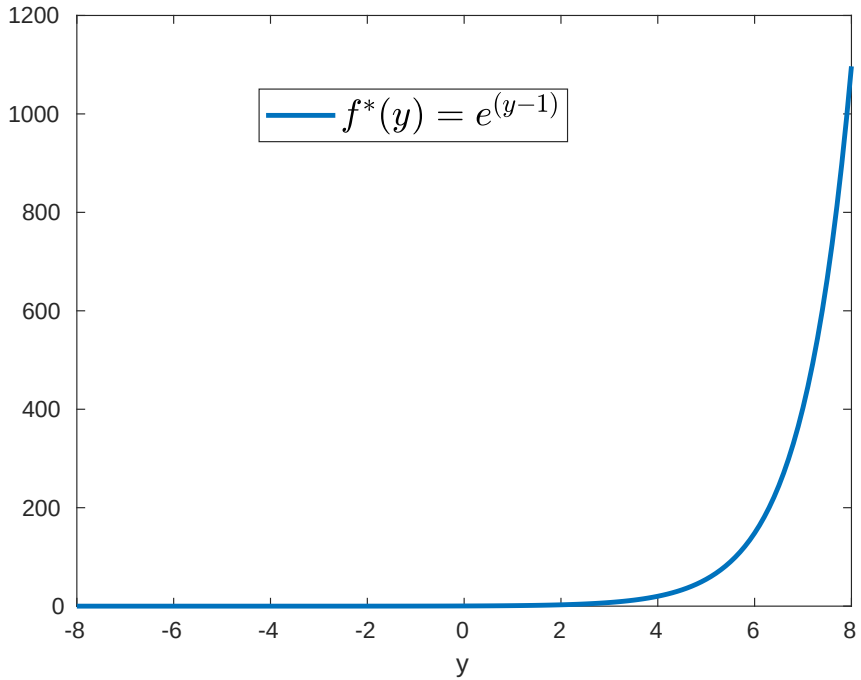
5. $f^*(y) = \frac{y^2}{4}$
6. $f^*(y) = e^{y-1}$

Draw Conjugate functions by Matlab (3)

5. $f^*(y) = \frac{y^2}{4}$
6. $f^*(y) = e^{y-1}$



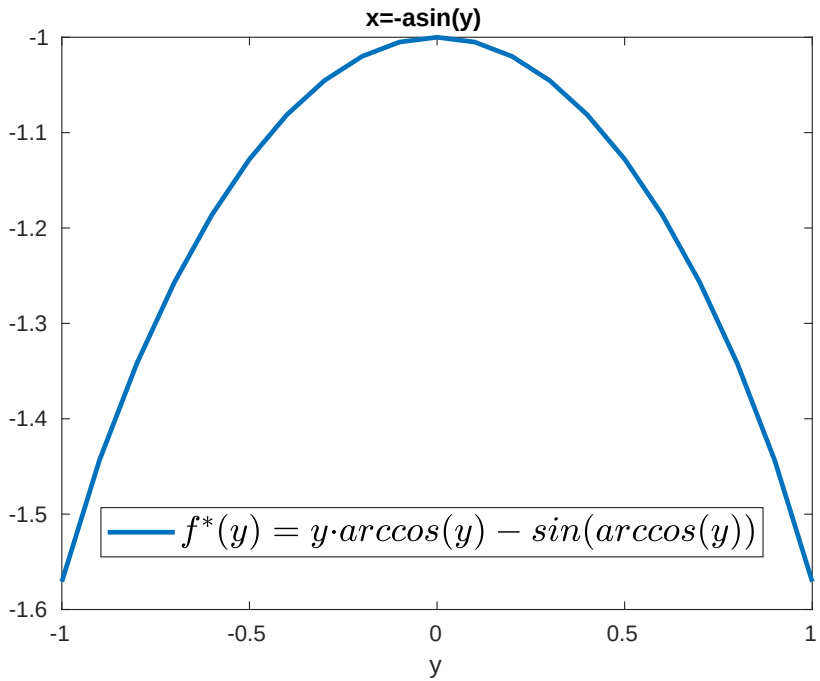
(e)



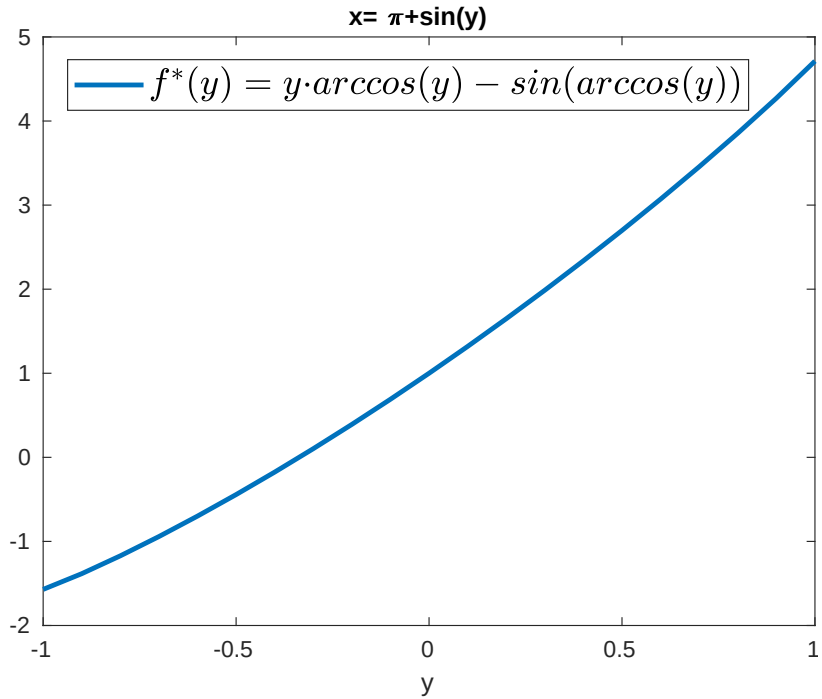
(f)

Draw Conjugate functions by Matlab (4)

- Given $f(x) = cos(x)$, plot out $f^*(y) = sup_x(y \cdot x - cos(x))$
- Given $f(x) = sin(x)$, plot out $f^*(y) = sup_x(y \cdot x - sin(x))$
- Given $f(x) = e^{-\frac{x^2}{2}}$, plot out $f^*(y) = sup_x(y \cdot x - e^{-\frac{x^2}{2}})$



(g)



(h)