Assignment-2

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		z	x	y	s_1	s_2	s_3	
		1	-3	-2	0	0	0	0
	s_1	0	2	1	1	0	0	18
	s_2	0	2	3	0	1	0	42
$*$	s_3	0	3	1	0	0	1	24

		z	x	y	s_1	s_2	s_3	
		1	0	$-2+1=-1$	0	0	$0+1=1$	$0+24=24$
$*$	s_1	0	0	$1-1\times\frac{2}{3}=\frac{1}{3}$	1	0	$0-1\times\frac{2}{3}=-\frac{2}{3}$	$18-24\times\frac{2}{3}=2$
	s_2	0	0	$3-1\times\frac{2}{3}=\frac{7}{3}$	0	1	$0-1\times\frac{2}{3}=-\frac{2}{3}$	$42-24\times\frac{2}{3}=26$
	x	0	3	1	0	0	1	24

		z	x	y	s_1	s_2	s_3	
		1	0	0	$0+1\times3=3$	0	$1+(-\frac{2}{3}\times3)=-1$	$24+2\times3=30$
	y	0	0	$\frac{1}{3}$	1	0	$-\frac{2}{3}$	2
$*$	s_2	0	0	0	$0-1\times7=-7$	1	$-\frac{2}{3}-(-\frac{2}{3})\times7=4$	$26-2\times7=12$
	x	0	3	0	$0-1\times3=-3$	0	$1-(-\frac{2}{3})\times3=3$	$24-2\times3=18$

		z	x	y	s_1	s_2	s_3	
		1	0	0	$3+(-4)\times\frac{1}{4}=2$	$0+1\times\frac{1}{4}=\frac{1}{4}$	0	$30+12\times\frac{1}{4}=33$
	y	0	0	$\frac{1}{3}$	$1+(-7)\times\frac{1}{6}=-\frac{1}{6}$	$0+1\times\frac{1}{6}=\frac{1}{6}$	0	$2+12\times\frac{1}{6}=4$
	s_3	0	0	0	-7	1	4	12
	x	0	3	0	$-3-(-7)\times\frac{3}{4}=\frac{9}{4}$	$0-1\times\frac{3}{4}=-\frac{3}{4}$	$3-4\times\frac{3}{4}=0$	$18-12\times\frac{3}{4}=9$

$$\begin{pmatrix} 0 & \frac{1}{3} & 0 \\ 0 & 0 & 4 \\ 3 & 0 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \\ s_3 \end{pmatrix} = \begin{pmatrix} 4 \\ 12 \\ 9 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \\ s_3 \end{pmatrix} = \begin{pmatrix} 3 \\ 12 \\ 3 \end{pmatrix}$$

$$\therefore \quad x = 3,\ y = 12,\ \max(3x + 2y) = 33$$

2

		z	x_1	x_2	x_3	s_1	s_2	s_3	
		1	1	-3	-1	0	0	0	0
	s_1	0	3	-1	2	1	0	0	7
*	s_2	0	-2	4	0	0	1	0	12
	s_3	0	-4	3	8	0	0	1	10

		z	x_1	x_2	x_3	s_1	s_2	s_3	
		1	$1 + (-2) \times \frac{3}{4} = -\frac{1}{2}$	0	-1	0	$0 + 1 \times \frac{3}{4} = \frac{3}{4}$	0	$0 + 12 \times \frac{3}{4} = 9$
	s_1	0	$3 + (-2) \times \frac{1}{4} = \frac{5}{2}$	0	2	1	$0 + 1 \times \frac{1}{4} = \frac{1}{4}$	0	$7 + 12 \times \frac{1}{4} = 10$
	x_2	0	-2	4	0	0	1	0	12
*	s_3	0	$-4 - (-2) \times \frac{3}{4} = -\frac{5}{2}$	0	8	0	$0 - 1 \times \frac{3}{4} = -\frac{3}{4}$	1	$10 - 12 \times \frac{3}{4} = 1$

		z	x_1	x_2	x_3	s_1	s_2	s_3	
		1	$-\frac{1}{2} + (-\frac{5}{2}) \times \frac{1}{8} = -\frac{13}{16}$	0	0	0	$\frac{3}{4} + (-\frac{3}{4}) \times \frac{1}{8} = \frac{21}{32}$	$0 + 1 \times \frac{1}{8} = \frac{1}{8}$	$9 + 1 \times \frac{1}{8} = \frac{73}{8}$
*	s_1	0	$\frac{5}{2} - (-\frac{5}{2}) \times \frac{1}{4} = \frac{25}{8}$	0	0	1	$\frac{1}{4} - (-\frac{3}{4}) \times \frac{1}{4} = \frac{7}{16}$	$0 - 1 \times \frac{1}{4} = -\frac{1}{4}$	$10 - 1 \times \frac{1}{4} = \frac{39}{4}$
	x_2	0	-2	4	0	0	1	0	12
	x_3	0	$-\frac{5}{2}$	0	8	0	$-\frac{3}{4}$	1	1

	z	x_1	x_2	x_3	s_1	s_2	s_3	
	1	0	0	0	$0 + 1 \times \frac{13}{50} = \frac{13}{50}$	$\frac{21}{32} + \frac{7}{16} \times \frac{13}{50} = \frac{77}{100}$	$\frac{1}{8} + (-\frac{1}{4}) \times \frac{13}{50} = \frac{3}{50}$	$\frac{73}{8} + \frac{39}{4} \times \frac{13}{50} = \frac{583}{50}$
x_1	0	$\frac{25}{8}$	0	0	1	$\frac{7}{16}$	$-\frac{1}{4}$	$\frac{39}{4}$
x_2	0	0	4	0	$0 + 1 \times \frac{16}{25} = \frac{16}{25}$	$1 + \frac{7}{16} \times \frac{16}{25} = \frac{32}{25}$	$0 + (-\frac{1}{4}) \times \frac{16}{25} = -\frac{4}{25}$	$12 + \frac{39}{4} \times \frac{16}{25} = \frac{456}{25}$
x_3	0	0	0	8	$0 + 1 \times \frac{4}{5} = \frac{4}{5}$	$-\frac{3}{4} + \frac{7}{16} \times \frac{4}{5} = -\frac{2}{5}$	$1 + (-\frac{1}{4}) \times \frac{4}{5} = \frac{4}{5}$	$1 + \frac{39}{4} \times \frac{4}{5} = \frac{44}{5}$

$$\begin{pmatrix} \frac{25}{8} & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 8 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} \frac{39}{4} \\ \frac{456}{25} \\ \frac{44}{5} \end{pmatrix}$$

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} \frac{78}{25} \\ \frac{144}{25} \\ \frac{11}{10} \end{pmatrix}$$

$$\therefore \quad x_1 = \frac{78}{25},\ x_2 = \frac{144}{25},\ x_3 = \frac{11}{10},\ \max\left(-x_1 + 3x_2 + x_3\right) = \frac{583}{50}$$

3

		z	x_1	x_2	x_3	x_4	s_1	s_2	s_3	
		1	3	−5	−2	−1	0	0	0	0
*	s_1	0	1	1	1	0	1	0	0	4
	s_2	0	4	−1	1	2	0	1	0	12
	s_3	0	−1	1	2	3	0	0	1	12

		z	x_1	x_2	x_3	x_4	s_1	s_2	s_3	
		1	$3 + 1 \times 5 = 8$	0	$-2 + 1 \times 5 = 3$	−1	$0 + 1 \times 5 = 5$	0	0	$0 + 4 \times 5 = 20$
	x_2	0	1	1	1	0	1	0	0	4
	s_2	0	$4 + 1 = 5$	0	$1 + 1 = 2$	2	$0 + 1 = 1$	1	0	$12 + 4 = 16$
*	s_3	0	$-1 - 1 = -2$	0	$2 - 1 = 1$	3	$0 - 1 = -1$	0	1	$12 - 4 = 8$

	z	x_1	x_2	x_3	x_4	s_1	s_2	s_3	
	1	$8 + (-2) \times \frac{1}{3} = \frac{22}{3}$	0	$3 + 1 \times \frac{1}{3} = \frac{10}{3}$	0	$5 + (-1) \times \frac{1}{3} = \frac{14}{3}$	0	$0 + 1 \times \frac{1}{3} = \frac{1}{3}$	$20 + 8 \times \frac{1}{3} = \frac{68}{3}$
x_2	0	1	1	1	0	1	0	0	4
s_2	0	$5 - (-2) \times \frac{2}{3} = \frac{19}{3}$	0	$2 - 1 \times \frac{2}{3} = \frac{4}{3}$	0	$1 - (-1) \times \frac{2}{3} = \frac{5}{3}$	1	$0 - 1 \times \frac{2}{3} = -\frac{2}{3}$	$16 - 8 \times \frac{2}{3} = \frac{32}{3}$
x_4	0	−2	0	1	3	−1	0	1	8

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 3 & 0 \end{pmatrix} \begin{pmatrix} x_2 \\ x_4 \\ s_2 \end{pmatrix} = \begin{pmatrix} 4 \\ \frac{32}{3} \\ 8 \end{pmatrix}$$

$$\begin{pmatrix} x_2 \\ x_4 \\ s_2 \end{pmatrix} = \begin{pmatrix} 4 \\ \frac{8}{3} \\ \frac{32}{3} \end{pmatrix}$$

$$\therefore \quad x_1 = 0,\ x_2 = 4,\ x_3 = 0,\ x_4 = \frac{8}{3},\ \max\left(-3x_1 + 5x_2 + 2x_3 + x_4\right) = \frac{68}{3}$$