Convex Optimization

Lab 9: Integer Programming

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Autumn Semester 2025

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Outline

The integer programming problem (1)

$$\text{s. t.} \begin{cases} \text{Min. } 5x_1 - x_2 + 2x_3 \\ 2x_1 - x_2 + 4x_3 \leq 4 \\ x_1 + x_2 + 2x_3 \leq 5 \\ 2x_1 - x_2 + x_3 \geq 1 \\ x_1, x_2 \in I^+ \end{cases} \tag{1}$$

$$\Downarrow$$

$$\text{s. t.} \begin{cases} \text{Min. } 5x_1 - x_2 + 2x_3 \\ 2x_1 - x_2 + 4x_3 \leq 4 \\ x_1 + x_2 + 2x_3 \leq 5 \\ -2x_1 + x_2 - x_3 \leq -1 \\ x_1, x_2 \in I^+ \end{cases} \tag{2}$$

$$\text{Min. } 5x_1 - x_2 + 2x_3$$

$$\text{s. t. } \begin{cases} 2x_1 - x_2 + 4x_3 \leq 4 \\ x_1 + x_2 + 2x_3 \leq 5 \\ -2x_1 + x_2 - x_3 \leq -1 \\ x_1, x_2 \in I^+ \end{cases}$$

```
c = [5 -1 2]; %coefficient of target func.
A = [2 -1 4;1 1 2;-2 1 -1]; %inequations
b = [4 5 -1]'; %right side of inequations
lb = [0 0 0]'; %lower bound
intcon = [1 2 3]; %index of integer var.
[fval, x] = intlinprog(c, intcon, A, b,[],[], lb,[])
```

Problem-1

$$\text{Min.} \quad \sum_{i=1}^{9} x_i$$

$$\text{s. t.} \begin{cases} x_1 + x_4 + x_5 + x_8 + x_9 \geq 1 \\ x_2 + x_3 + x_4 \geq 1 \\ x_1 + x_2 + x_3 + x_4 \geq 1 \\ x_1 + x_5 + x_6 + x_7 \geq 1 \\ x_5 + x_6 + x_7 \geq 1 \\ x_1 + x_8 + x_9 \geq 1 \\ x_{1\cdots 9} \in \{0, 1\} \end{cases} \tag{3}$$

- Please solve above problem by calling intlinprog in Matlab

Problem-2

$$\begin{aligned} &\text{Max.} \quad 5x_1 + 8x_2 + 6x_3 \\ &\text{s. t.} \begin{cases} 9x_1 + 6x_2 + 10x_3 \leq 14 \\ 20x_1 + 63x_2 + 10x_3 \leq 110 \\ x_{1\cdots 3} \in I^+ \end{cases} \end{aligned} \tag{4}$$

- Solve it by calling intlinprog in Matlab

Problem-3

- An electric utility must determine which generators to start up at the beginning of each day. They have three generators with capacities, operating cost, and start-up costs shown in the following table. A day is divided into two periods, and each generator may be started at the beginning of each period. A generator started in period-1 may be used in period-2 without incurring an additional start-up cost. All generators are turned off at the end of the day. Demand for power is 2500 megawatts (MW) in period-1 and 3500 MW in period-2. Formulate and solve this problem as a mixed-integer linear program.

Generator	start-up cost ($)	cost/megawatt per period	Generator capacity (MW)
1	2800	5	1900
2	2000	3	1700
3	1900	8	2900

Problem-4

- The company **A** runs two refineries supplying three markets M_1, M_2 and M_3, using a pipeline owned by the company **B**. The basic charge for pipeline use is \$80 per 1000 barrels/mile. The objective is to meet demands at M_1, M_2, and M_3 (shown on table below), using supplies from R_1, and R_2, and minimizes the overall costs for company **A**.

Table: Distance (miles) between markets and refineries, and demands

R_i-to-M_j	M_1	M_2	M_3	Refinery cost
R_1	26	25	22	2.5\$/barrel
R_2	21	28	30	2.8\$/barrel
Demands	5000	7000	4000	

Problem-5

- Solve the following linear integer programming problem by Gomory cutting-plane method with Excel and verify your answer with intlinprog() in Matlab.

$$\text{s. t.} \begin{cases} \text{Max.} \quad 2x_1 + x_2 + 3x_3 \\ x_1 + x_2 + 3x_3 \leq 17 \\ 3x_1 + 2x_2 + 2x_3 \leq 11 \\ x_1, x_2, x_3 \geq 0 \quad \& \quad \textit{integer} \end{cases} \tag{5}$$