Assignment-6

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1

$$
\begin{aligned}
L &= xy - \lambda(4x^2 + y^2 - 8),\ \ \lambda > 0 \\
\nabla L &= (y - 8\lambda x, x - 2\lambda y, 4x^2 + y^2 - 8)^T
\end{aligned}
$$

$$\begin{cases} y - 8\lambda x &= 0 \\ x - 2\lambda y &= 0 \\ 4x^2 + y^2 - 8 &= 0 \end{cases}$$

$$\begin{cases} y &= 8\lambda x \\ x &= 2\lambda y \\ 4x^2 + y^2 &= 8 \end{cases}$$

$$\begin{cases} y &= 16\lambda^2 y \\ x &= 16\lambda^2 x \\ 4x^2 + y^2 &= 8 \end{cases}$$

$$\begin{aligned} \because 4 \times 0^2 + 0^2 &\neq 8 \\ \therefore 16\lambda^2 &= 1 \\ \therefore \lambda &= \frac{1}{4} \\ \therefore y &= 8 \times \frac{1}{4}x \\ &= 2x \\ \therefore 4x^2 + (2x)^2 &= 8 \\ 8x^2 &= 8 \\ \therefore x &= \pm 1 \\ \therefore y &= \pm 2 \end{aligned}$$

$$f(1,2) = 2,\ f(1,-2) = -2$$

$$f(-1,2) = -2,\ f(-1,-2) = 2$$

$$\therefore \text{Min.}\, f = f(1,-2) = f(-1,2) = -2$$

2

$$L = 4y - 2z - \lambda_1(2x - y - z - 1) - \lambda_2(y - z - 1),\ \ \lambda_1, \lambda_2 > 0$$
$$\nabla L = (-2\lambda_1, 4 + \lambda_1 - \lambda_2, -2 + \lambda_1 - \lambda_2, 2x - y - z - 1, y - z - 1)^T$$

$$\begin{cases} -2\lambda_1 & = 0 \\ 4 + \lambda_1 - \lambda_2 & = 0 \\ -2 + \lambda_1 - \lambda_2 & = 0 \\ 2x - y - z - 1 & = 0 \\ y - z - 1 & = 0 \end{cases}$$
$$\because \lambda_1 > 0 \wedge \lambda_1 = 0$$

$\therefore$ Can't solve by Lagrange multiplier.