

Assignment-5

1. Given function $f(x) = -5x^2 + 5x$, work out its conjugate function and plot it out
2. Suppose $f : S \subset R \rightarrow R$ is convex. Let $a, b \in S$ and $a < b$. Show

$$f(x) \leq \frac{b-x}{b-a}f(a) + \frac{x-a}{b-a}f(b), \quad \forall x \in [a, b] \quad (1)$$

3. Given functions $f(x)$ and $g(x)$ are convex in $\text{dom } f$ and $\text{dom } g$, please prove $h(x) = f(x) + g(x)$ is convex in $\text{dom } h$.
4. Please prove function $f(x, y) = x \log(x) + y \log(y)$, where $x, y > 0$ is convex.
5. Please convert the following Lasso regression model into a standard QP problem without changing the objective function.

$$\begin{aligned} & \text{Min.} \quad \|y - \theta^T X\|_2^2 \\ \text{s.t.} \quad & \begin{cases} \|\theta\|_1 \leq t \\ y \in R, X \in R^{n+1}, \theta \in R^{n+1} \end{cases} \end{aligned} \quad (2)$$

where t is a positive constant.

- Hints

1. Submission due: 2025/Dec./5
2. Submit to lecwlzhao@163.com, email title “assignment5-your-name + your student number”