Assignment-5

1. Given function $f(x) = -5x^2 + 5x$, work out its conjugate function and plot it out
2. Suppose $f : S \subset R \to R$ is convex. Let $a,\ b \in S$ and $a < b$. Show

$$f(x) \leq \frac{b-x}{b-a} f(a) + \frac{x-a}{b-a} f(b), \quad \forall x \in [a, b] \tag{1}$$

3. Given functions $f(x)$ and $g(x)$ are convex in $dom\ f$ and $dom\ g$, please prove $h(x) = f(x) + g(x)$ is convex in $dom\ h$.
4. Prove function $f(x, y) = xlog(x) + ylog(y)$, where $x, y > 0$, is convex.
5. Please convert the following Lasso regression model into a standard QP probblem without changing the objective function.

$$\begin{aligned} &\text{Max.} \quad \| Y - \theta^T X \|_2^2 \\ &\text{s.t.} \begin{cases} \| \theta \|_1 \leq t \\ \theta \in R^{n+1}, X \in R^n, Y \in R \end{cases} \\ &\text{where t is a positive constant} \end{aligned} \tag{2}$$

- Hints
  1. Submission due: 2025/Dec./5
  2. Submit to lecwlzhao@163.com, email title "assigment5_your-name + your student number"