

## Assignment-5

1. Given function  $f(x) = -5x^2 + 5x$ , work out its conjugate function and plot it out
2. Suppose  $f : S \subset R \rightarrow R$  is convex. Let  $a, b \in S$  and  $a < b$ . Show

$$f(x) \leq \frac{b-x}{b-a}f(a) + \frac{x-a}{b-a}f(b), \quad \forall x \in [a, b] \quad (1)$$

3. Given functions  $f(x)$  and  $g(x)$  are convex in  $\text{dom } f$  and  $\text{dom } g$ , please prove  $h(x) = f(x) + g(x)$  is convex in  $\text{dom } h$ .
4. Please prove function  $f(x, y) = x \log(x) + y \log(y)$ , where  $x, y > 0$  is convex.
5. Please convert the following Lasso regression model into a standard QP problem without changing the objective function.

$$\begin{aligned} & \text{Max.} \quad \|y - \theta^T X\|_2^2 \\ \text{s.t.} \quad & \begin{cases} \|\theta\|_1 \leq t \\ y \in R, X \in R^{n+1}, \theta \in R^{n+1} \end{cases} \end{aligned} \quad (2)$$

where  $t$  is a positive constant.

- Hints

1. Submission due: **2025/Dec./5**
2. Submit to [lecwlzhao@163.com](mailto:lecwlzhao@163.com), email title “assignment5-your-name + your student number”