Assignment-1

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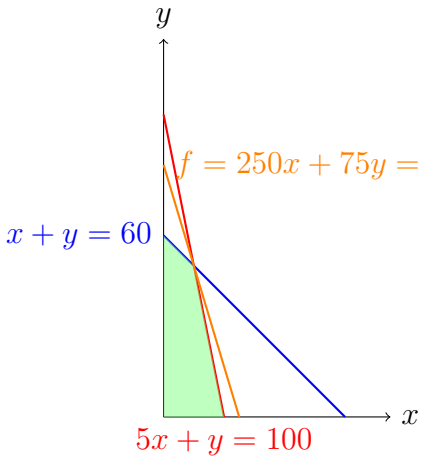
1

$$
\begin{aligned}
\begin{vmatrix} 1 \end{vmatrix} &= 1 > 0 \\
\begin{vmatrix} 1 & \alpha \\ \alpha & 4 \end{vmatrix} &= 4 - \alpha^2 > 0,\ \alpha \in (-2, 2) \\
\begin{vmatrix} 1 & \alpha & -1 \\ \alpha & 4 & 2 \\ -1 & 2 & 4 \end{vmatrix} &= 1 \times 4 \times 4 + (-1) \times 2\alpha + (-1) \times 2\alpha \\
&\quad - (-1) \times 4 \times (-1) - 4\alpha^2 - 1 \times 2 \times 2 \\
&= -4\alpha^2 - 4\alpha + 8 \\
&= -4(\alpha^2 + \alpha - 2) > 0,\ \alpha \in (-2, 1)
\end{aligned}
$$

$$
\therefore \alpha \in (-2, 1)
$$

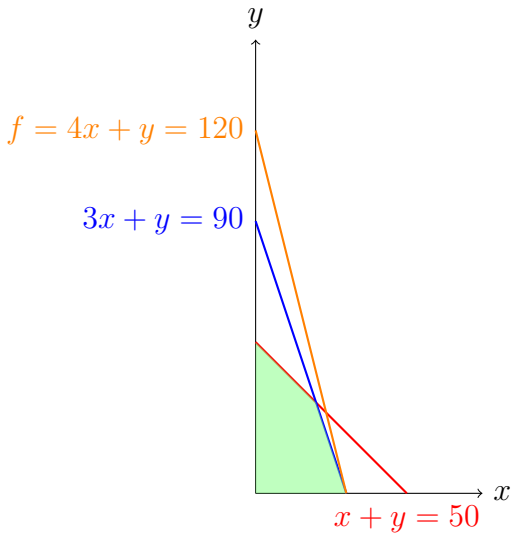
2

(a)



$$x = 10,\ y = 50,$$
$$\max f = 250 \times 10 + 75 \times 50$$
$$= 6250$$

(b)



$$x = 30,\ y = 0,$$
$$\max f = 4 \times 30 + 0$$
$$= 120$$

3

$$\begin{aligned} \text{minimize} \quad & f = 5x + 7y \\ \text{s.t.} \quad & \begin{cases} 2x + y & \geq 8 \\ x + 2y & \geq 10 \\ x, y & \geq 0 \end{cases} \end{aligned}$$

4

$$\begin{aligned} \text{minimize} \quad & f = (16, 10, 15, 10, 12, 10)(x_{P,A}, x_{P,B}, x_{P,C}, x_{Q,A}, x_{Q,B}, x_{Q,C})^T \\ \text{s.t.} \quad & \begin{cases} x_{P,A} + x_{Q,A} & \geq 5 \\ x_{P,B} + x_{Q,B} & \geq 5 \\ x_{P,C} + x_{Q,C} & \geq 4 \\ x_{P,A} + x_{P,B} + x_{P,C} & \leq 8 \\ x_{Q,A} + x_{Q,B} + x_{Q,C} & \leq 6 \\ x_{P,A}, x_{P,B}, x_{P,C}, x_{Q,A}, x_{Q,B}, x_{Q,C} & \geq 0 \end{cases} \end{aligned}$$