Post_Clean_KNN

June 28, 2018

1 k-NN on Amazon Reviews Dataset (Part II)

1.1 Data Source:

The preprocessing step has produced final.sqlite file after doing the data preparation & cleaning. The review text is now devoid of punctuations, HTML markups and stop words.

1.2 Objective:

To find accuracy of 10-fold cross validation KNN on vectorized input data, for each of the 4 featurizations, namely BoW, tf-IDF, W2V, tf-IDF weighted W2V. Running time comparison of Brute force vs kd-tree also need to be done.

1.3 At a glance:

Random Sampling is done to reduce input data size and time based slicing to split into training and testing data. The accuracy percentage obtained by applying 10-fold cross validation KNN using 4 Featurizations viz. BoW, tf-idf, W2V, tf-idf W2V are compared. The time taken by brute force and kd-tree methods are plotted and analysed.

2 Preprocessed Data Loading

```
In [34]: #loading libraries for knn
         import numpy as np
         import pandas as pd
         import matplotlib.pyplot as plt
         from sklearn.cross_validation import train_test_split
         from sklearn.neighbors import KNeighborsClassifier
         from sklearn.metrics import accuracy_score
         from sklearn.cross_validation import cross_val_score
         from collections import Counter
         from sklearn.metrics import accuracy_score
         from sklearn import cross_validation

         #loading libraries for scikit learn, nlp, db, plot and matrix.
         import sqlite3
         import pdb
```

```python
import pandas as pd
import numpy as np
import nltk
import string
import matplotlib.pyplot as plt
import seaborn as sns
from sklearn.feature_extraction.text import TfidfTransformer
from sklearn.feature_extraction.text import TfidfVectorizer

from sklearn.feature_extraction.text import CountVectorizer
from sklearn.metrics import confusion_matrix
from sklearn import metrics
from sklearn.metrics import roc_curve, auc
from nltk.stem.porter import PorterStemmer


# using the SQLite Table to read data.
con = sqlite3.connect('./final.sqlite')

#filtering only positive and negative reviews i.e.
# not taking into consideration those reviews with Score=3
final = pd.read_sql_query("""
SELECT *
FROM Reviews
""", con)

print(final.head(3))
print(final.shape)
```

```
    index      Id   ProductId          UserId            ProfileName  \
0  138706  150524  0006641040   ACITT7DI6IDDL        shari zychinski
1  138688  150506  0006641040  A2IW4PEEKO2R0U                  Tracy
2  138689  150507  0006641040  A1S4A3IQ2MU7V4  sally sue "sally sue"

   HelpfulnessNumerator  HelpfulnessDenominator     Score        Time  \
0                     0                       0  positive   939340800
1                     1                       1  positive  1194739200
2                     1                       1  positive  1191456000

                                      Summary  \
0                   EVERY book is educational
1  Love the book, miss the hard cover version
2                chicken soup with rice months

                                                Text  \
0  this witty little book makes my son laugh at l...
1  I grew up reading these Sendak books, and watc...
2  This is a fun way for children to learn their ...
```

```
                                          CleanedText
0  b'witti littl book make son laugh loud recit c...
1  b'grew read sendak book watch realli rosi movi...
2  b'fun way children learn month year learn poem...
(364171, 12)
```

3 Random Sampling & Time Based Slicing

In [35]:
```python
# To randomly sample the data and sort based on time before doing train/ test split.
# The slicing into train & test data will be done later in kfoldcv() function.

num_points = 5000

# you can use random_state for reproducibility
sampled_final = final.sample(n=num_points, random_state=2)

#Sorting data according to Time in ascending order
sorted_final = sampled_final.sort_values('Time', axis=0,
                ascending=True, inplace=False, kind='quicksort', na_position='last')

# fetching the outcome class
y = sorted_final['Score'] # showing you two ways of indexing a pandas df

print(y.shape)
```

```
(5000,)
```

4 Custom Defined Functions

3 user defined functions are written to

```
a) K-fold Cross Validation & estimation of Optimal K.

b) Compute Accuracy of KNN Classifier.

c) k-fold Cross Validation & Accuracy Estimation Timer.
```

4.1 a) k-fold Cross Validation & Optimal K estimation

In [36]:
```python
# split the data set into train and test. Do 10-fold cross validation
# X_1, X_test, y_1, y_test = ...
#     cross_validation.train_test_split(X, y, test_size=0.3, random_state=0)

import numpy
```

```
def kfoldcv(X, split_ratio_train = 0.7, algo='auto'):

    # Time based slicing of data into train and test.
    num_train_data = int(split_ratio_train*X.shape[0])

    X_train = X[0:num_train_data]
    y_train = y[0:num_train_data]
    X_test = X[num_train_data+1:]
    y_test = y[num_train_data+1:]

    # creating odd list of K for KNN
    myList = list(range(0,50))
    neighbors = list(filter(lambda x: x % 2 != 0, myList))
#     neighbors = list(range(1,50,2))

    # empty list that will hold cv scores
    cv_scores = []

    # perform 10-fold cross validation
    for k in neighbors:
        knn = KNeighborsClassifier(n_neighbors=k, algorithm=algo)
        scores = cross_val_score(knn, X_train, y_train, cv=10, scoring='accuracy')
        cv_scores.append(scores.mean())

    # changing to misclassification error
    MSE = [1 - x for x in cv_scores]

    # determining best k
    optimal_k = neighbors[MSE.index(min(MSE))]
    print('\nThe optimal number of neighbors is %d.' % optimal_k)

    return optimal_k
```

4.2 b) Compute KNN Classifier Accuracy

In [37]:
```
# ========================= KNN with k = optimal_k ===========================
#To compute the knn classifier accuracy

def compute_accuracy(X, y, optimal_k, split_ratio_train = 0.7, algo='auto'):

    # Time based slicing of data into train and test.
    num_train_data = int(split_ratio_train*X.shape[0])

    X_train = X[0:num_train_data]
    y_train = y[0:num_train_data]
    X_test = X[num_train_data+1:]
    y_test = y[num_train_data+1:]
```

```
    # instantiate learning model k = optimal_k
    knn_optimal = KNeighborsClassifier(n_neighbors=optimal_k, algorithm=algo)

    # fitting the model
    knn_optimal.fit(X_train, y_train)

    # predict the response
    pred = knn_optimal.predict(X_test)

    # evaluate accuracy
    acc = accuracy_score(y_test, pred) * 100
    print(
    '\nThe accuracy of the knn classifier for k = %d is %f%%' % (optimal_k, acc))
    return acc
```

4.3 c) k-fold Cross Validation & Accuracy Estimation Timer

```
In [38]: # To call kfold cv and estimate accuracy with optimal K

         def kfoldknn_timer(X):
             # To run kd-tree knn & time the code
             start_time = time.time()
             k_optimal = kfoldcv(X, algo='kd_tree')

             acc = compute_accuracy(X, y, k_optimal, algo='kd_tree')
             print("Time Taken by KD Tree is {} seconds when dimensionality = {}" \
                   .format(round(time.time() - start_time, 2), X.shape[1]))

             # To run brute force knn & time the code
             start_time = time.time()
             k_optimal = kfoldcv(X, algo='brute')

             acc = compute_accuracy(X, y, k_optimal, algo='brute')
             print("Time Taken by Brute Force is {} seconds when dimensionality = {}" \
                   .format(round(time.time() - start_time, 2), X.shape[1]))
```

5 BoW & KNN

BoW will result in a **sparse matrix with huge number of features** as it creates a feature for each unique word in the review.

If the number of features is very high, it is highly recommended to use another dimensionality reduction method (e.g. **PCA for dense data or TruncatedSVD for sparse data**) to reduce the number of dimensions to a reasonable amount (e.g. 50), before feeding in to KNN. **Otherwise kd-tree will not work with sparse data**).

```
In [39]: #BoW
```

```
import time
from sklearn.decomposition import TruncatedSVD
from sklearn.random_projection import sparse_random_matrix

#BoW
count_vect = CountVectorizer() #in scikit-learn
X = count_vect.fit_transform(sorted_final['CleanedText'].values)
X.get_shape()

# to reduce dimensions using TruncatedSVD.
#kd-tree will not work with sparse matrices. it requires random uniform data.
svd = TruncatedSVD(n_components=5, n_iter=10, random_state=42)
X = svd.fit_transform(X)

# To run brute & kd-tree knn & also time the code
kfoldknn_timer(X)
```

```
The optimal number of neighbors is 27.

The accuracy of the knn classifier for k = 27 is 83.388926%
Time Taken by KD Tree is 7.21 seconds when dimensionality = 5

The optimal number of neighbors is 27.

The accuracy of the knn classifier for k = 27 is 83.388926%
Time Taken by Brute Force is 14.65 seconds when dimensionality = 5
```

6 tf-IDF & KNN

Sparse matrix generated from tf-IDF is fed in to TruncatedSVD so that kd-tree will work. Then brute & kd-tree knn is run on the the resulting data to find accuracy and to time the code.

```
In [40]: #TF-IDF

         tf_idf_vect = TfidfVectorizer()
         X = tf_idf_vect.fit_transform(sorted_final['CleanedText'].values)
         X.get_shape()

         # to reduce dimensions using TruncatedSVD.
         #kd-tree will not work with sparse matrices. it requires random uniform data.
         svd = TruncatedSVD(n_components=5, n_iter=10, random_state=42)
         X = svd.fit_transform(X)

         # To run brute & kd-tree knn & also time the code
         kfoldknn_timer(X)
```

```
The optimal number of neighbors is 23.

The accuracy of the knn classifier for k = 23 is 83.255504%
Time Taken by KD Tree is 7.17 seconds when dimensionality = 5

The optimal number of neighbors is 23.

The accuracy of the knn classifier for k = 23 is 83.255504%
Time Taken by Brute Force is 15.77 seconds when dimensionality = 5
```

6.1 W2V & KNN

Our own Word2Vec model is trained with input text corpus using your own text corpus. Average W2V is computed for each review & brute force and kd-tree KNN is run.

In [41]:
```
# Train your own Word2Vec model using your own text corpus
import gensim

import re

def cleanhtml(sentence): #function to clean the word of any html-tags
    cleanr = re.compile('<.*?>')
    cleantext = re.sub(cleanr, ' ', sentence)
    return cleantext
#function to clean the word of any punctuation or special characters
def cleanpunc(sentence):
    cleaned = re.sub(r'[?|!|\'|"|#]',r'',sentence)
    cleaned = re.sub(r'[.|,|)|(|\|/]',r' ',cleaned)
    return  cleaned


#select subset of points for fast execution
i=0
list_of_sent=[]
w2v_dim = 100

for sent in sorted_final['CleanedText']:
    sent = str(sent, 'utf-8')
    filtered_sentence=[]
    sent=cleanhtml(sent)
    for w in sent.split():
        for cleaned_words in cleanpunc(w).split():
            if(cleaned_words.isalpha()):
                filtered_sentence.append(cleaned_words.lower())
            else:
                continue
```

```
            list_of_sent.append(filtered_sentence)

    w2v_model=gensim.models.Word2Vec(list_of_sent,min_count=5,size=w2v_dim, workers=4)
```

In [42]:
```
# average Word2Vec
# compute average word2vec for each review.
sent_vectors = []; # the avg-w2v for each sentence/review is stored in this list

for sent in sorted_final['CleanedText']: # for each review/sentence
    sent_vec = np.zeros(w2v_dim) # as word vectors are of zero length
    cnt_words =0; # num of words with a valid vector in the sentence/review
    sent = str(sent, 'utf-8')
    sent = re.sub("[^\w]", " ",  sent).split()

    for word in sent: # for each word in a review/sentence
        try:
            vec = w2v_model.wv[word]
            sent_vec += vec
            cnt_words += 1
        except:
            pass
    sent_vec /= cnt_words
    sent_vectors.append(sent_vec)

print(sent_vectors[0:2])

sent_df = pd.DataFrame(np.nan_to_num(sent_vectors))

# To run brute & kd-tree knn & also time the code
kfoldknn_timer(sent_df)
```

```
[array([ 0.14096403,  0.15196943, -0.0840353 ,  0.13165361, -0.35688924,
       -0.02161835,  0.1535544 , -0.28388014,  0.36978848, -0.14056698,
        0.06698793,  0.1785222 , -0.15523394,  0.50568303,  0.71111251,
       -0.04802473, -0.10913318, -0.06762949,  0.06191286, -0.39890753,
       -0.11614569, -0.2162448 ,  0.32639967,  0.54663147,  0.06329022,
       -0.03752851,  0.12258716,  0.16566674, -0.07008869, -0.16885243,
        0.20653615, -0.08211443,  0.07522928,  0.03821804, -0.11691547,
        0.18917599, -0.15604629, -0.42077018,  0.16732795, -0.13304788,
       -0.12058857, -0.2646071 ,  0.06772086, -0.26104035, -0.1916956 ,
        0.05214952, -0.05506117,  0.04836735, -0.03821971,  0.04785469,
       -0.16660572,  0.18515983,  0.12132749,  0.12231749, -0.16537742,
       -0.11631031,  0.09249982,  0.14332525, -0.08784672,  0.05710632,
        0.21051363, -0.06665659,  0.39793519, -0.01907207,  0.11012624,
       -0.01890125,  0.16413239, -0.0374707 , -0.44964036,  0.13633638,
        0.65436258, -0.01178733, -0.15413906, -0.22194976,  0.04722565,
       -0.15363131, -0.15634076,  0.17935718, -0.03464829, -0.30687417,
        0.61355281, -0.00911581,  0.08730479, -0.12300512,  0.00604612,
```

```
 0.02960093,  0.11609484,  0.05567315, -0.02400271,  0.05302361,
 0.33259143,  0.17456012, -0.12392631, -0.04545302, -0.06463347,
 0.19466834,  0.03619871,  0.00299885, -0.09803861,  0.00877023]), array([ 0.15388207,
-0.01138705,  0.14698216, -0.25585839,  0.33567225, -0.11939835,
 0.07216691,  0.15643564, -0.1708553 ,  0.49969259,  0.66549695,
-0.0343489 , -0.10684035, -0.05250593,  0.04911484, -0.35639421,
-0.10879123, -0.20179508,  0.34142168,  0.55294895,  0.04318049,
-0.02295859,  0.11535637,  0.15874915, -0.07737031, -0.17549721,
 0.20316829, -0.05303845,  0.06189512,  0.0132671 , -0.13294441,
 0.15925501, -0.12788824, -0.39758771,  0.15565328, -0.1066956 ,
-0.11802546, -0.24376911,  0.08050644, -0.2328947 , -0.1999753 ,
 0.06314848, -0.0198259 ,  0.04216687, -0.02828741,  0.00559923,
-0.16688622,  0.16641217,  0.12286145,  0.09395997, -0.15866378,
-0.11984431,  0.11673733,  0.13357522, -0.08876404,  0.08041362,
 0.22698358, -0.07781131,  0.38174612, -0.01151328,  0.08405738,
-0.01136235,  0.14029506, -0.04082814, -0.41567415,  0.10160016,
 0.62275785, -0.03750381, -0.16618943, -0.21209588,  0.07845303,
-0.12988073, -0.17298796,  0.17538829,  0.01874316, -0.27970698,
 0.58708844, -0.00370657,  0.09662686, -0.10822236,  0.03201176,
 0.02515504,  0.11637905,  0.03262628, -0.04056843,  0.07033372,
 0.32096252,  0.15506069, -0.09207555, -0.01752648, -0.08143727,
 0.18154041,  0.05188356, -0.00623841, -0.09701755,  0.0021875 ])]

The optimal number of neighbors is 25.

The accuracy of the knn classifier for k = 25 is 83.522348%
Time Taken by KD Tree is 42.91 seconds when dimensionality = 100

The optimal number of neighbors is 25.

The accuracy of the knn classifier for k = 25 is 83.522348%
Time Taken by Brute Force is 18.51 seconds when dimensionality = 100
```

7 TF-IDF weighted W2V

The tf-IDF vector is multiplied to W2V vector as a weightage parameter.

In [43]:
```python
# TF-IDF
tf_idf_vect = TfidfVectorizer()
final_tf_idf = tf_idf_vect.fit_transform(final['CleanedText'].values)

# TF-IDF weighted Word2Vec
tfidf_feat = tf_idf_vect.get_feature_names() # tfidf words/col-names
# final_tf_idf is the sparse matrix with
# row= sentence, col=word and cell_val = tfidf
```

```
# the tfidf-w2v for each sentence/review is stored in this list
tfidf_sent_vectors = [];
row=0;
for sent in sorted_final['CleanedText']: # for each review/sentence
    sent_vec = np.zeros(w2v_dim) # as word vectors are of zero length
    cnt_words =0; # num of words with a valid vector in the sentence/review
    sent = str(sent, 'utf-8')
    sent = re.sub("[^\w]", " ",  sent).split()
    #print(sent)
    weight_sum =0; # num of words with a valid vector in the sentence/review
    for word in sent: # for each word in a review/sentence
        try:
            vec = w2v_model.wv[word]
            # obtain the tf_idfidf of a word in a sentence/review
            tf_idf = final_tf_idf[row, tfidf_feat.index(word)]
            sent_vec += (vec * tf_idf)
            weight_sum += tf_idf
        except:
            pass
    sent_vec /= weight_sum
    tfidf_sent_vectors.append(sent_vec)
    row += 1


print(sent_vectors[0:2])

sent_df = pd.DataFrame(np.nan_to_num(sent_vectors))

# To run brute & kd-tree knn & also time the code
kfoldknn_timer(sent_df)
```

```
C:\Anaconda\lib\site-packages\ipykernel_launcher.py:27: RuntimeWarning: invalid value encounte


[array([ 0.14096403,  0.15196943, -0.0840353 ,  0.13165361, -0.35688924,
       -0.02161835,  0.1535544 , -0.28388014,  0.36978848, -0.14056698,
        0.06698793,  0.1785222 , -0.15523394,  0.50568303,  0.71111251,
       -0.04802473, -0.10913318, -0.06762949,  0.06191286, -0.39890753,
       -0.11614569, -0.2162448 ,  0.32639967,  0.54663147,  0.06329022,
       -0.03752851,  0.12258716,  0.16566674, -0.07008869, -0.16885243,
        0.20653615, -0.08211443,  0.07522928,  0.03821804, -0.11691547,
        0.18917599, -0.15604629, -0.42077018,  0.16732795, -0.13304788,
       -0.12058857, -0.2646071 ,  0.06772086, -0.26104035, -0.1916956 ,
        0.05214952, -0.05506117,  0.04836735, -0.03821971,  0.04785469,
       -0.16660572,  0.18515983,  0.12132749,  0.12231749, -0.16537742,
       -0.11631031,  0.09249982,  0.14332525, -0.08784672,  0.05710632,
        0.21051363, -0.06665659,  0.39793519, -0.01907207,  0.11012624,
       -0.01890125,  0.16413239, -0.0374707 , -0.44964036,  0.13633638,
```

```
 0.65436258, -0.01178733, -0.15413906, -0.22194976,  0.04722565,
-0.15363131, -0.15634076,  0.17935718, -0.03464829, -0.30687417,
 0.61355281, -0.00911581,  0.08730479, -0.12300512,  0.00604612,
 0.02960093,  0.11609484,  0.05567315, -0.02400271,  0.05302361,
 0.33259143,  0.17456012, -0.12392631, -0.04545302, -0.06463347,
 0.19466834,  0.03619871,  0.00299885, -0.09803861,  0.00877023]), array([ 0.15388207,
-0.01138705,  0.14698216, -0.25585839,  0.33567225, -0.11939835,
 0.07216691,  0.15643564, -0.1708553 ,  0.49969259,  0.66549695,
-0.0343489 , -0.10684035, -0.05250593,  0.04911484, -0.35639421,
-0.10879123, -0.20179508,  0.34142168,  0.55294895,  0.04318049,
-0.02295859,  0.11535637,  0.15874915, -0.07737031, -0.17549721,
 0.20316829, -0.05303845,  0.06189512,  0.0132671 , -0.13294441,
 0.15925501, -0.12788824, -0.39758771,  0.15565328, -0.1066956 ,
-0.11802546, -0.24376911,  0.08050644, -0.2328947 , -0.1999753 ,
 0.06314848, -0.0198259 ,  0.04216687, -0.02828741,  0.00559923,
-0.16688622,  0.16641217,  0.12286145,  0.09395997, -0.15866378,
-0.11984431,  0.11673733,  0.13357522, -0.08876404,  0.08041362,
 0.22698358, -0.07781131,  0.38174612, -0.01151328,  0.08405738,
-0.01136235,  0.14029506, -0.04082814, -0.41567415,  0.10160016,
 0.62275785, -0.03750381, -0.16618943, -0.21209588,  0.07845303,
-0.12988073, -0.17298796,  0.17538829,  0.01874316, -0.27970698,
 0.58708844, -0.00370657,  0.09662686, -0.10822236,  0.03201176,
 0.02515504,  0.11637905,  0.03262628, -0.04056843,  0.07033372,
 0.32096252,  0.15506069, -0.09207555, -0.01752648, -0.08143727,
 0.18154041,  0.05188356, -0.00623841, -0.09701755,  0.0021875 ])]

The optimal number of neighbors is 25.

The accuracy of the knn classifier for k = 25 is 83.522348%
Time Taken by KD Tree is 48.36 seconds when dimensionality = 100

The optimal number of neighbors is 25.

The accuracy of the knn classifier for k = 25 is 83.522348%
Time Taken by Brute Force is 19.38 seconds when dimensionality = 100
```

8 Accuracy % Comparison across 4 Featurizations

The accuracy percentage obtatined using 4 Featurizations viz. BoW, tf-idf, W2V, tf-idf W2V are plotted as a bar diagram below.

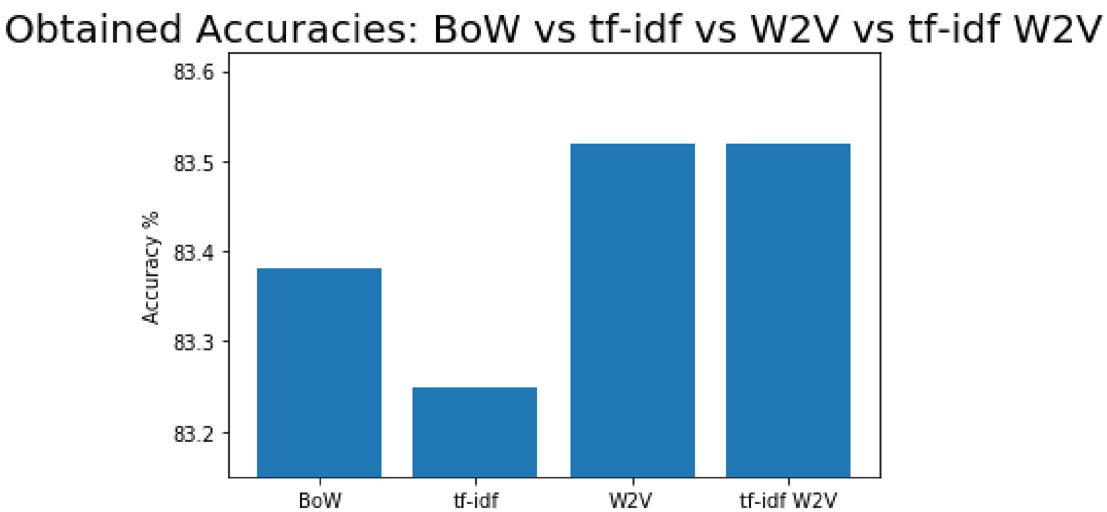
In [63]:
```
# To plot accuracy percentrage obtatined using 4 Featurizations

data = [83.38,83.25,83.52, 83.52]

x = np.arange(4)
```

```
plt.bar(x, height= data)
plt.xticks(x, ['BoW','tf-idf','W2V', 'tf-idf W2V']);
plt.ylim([min(data)-.1, max(data)+.1])

plt.ylabel('Accuracy %')
plt.title('Obtained Accuracies: BoW vs tf-idf vs W2V vs tf-idf W2V', fontsize=20)
plt.show()
```



8.1 Running Time Analysis: kd-tree vs Brute Force

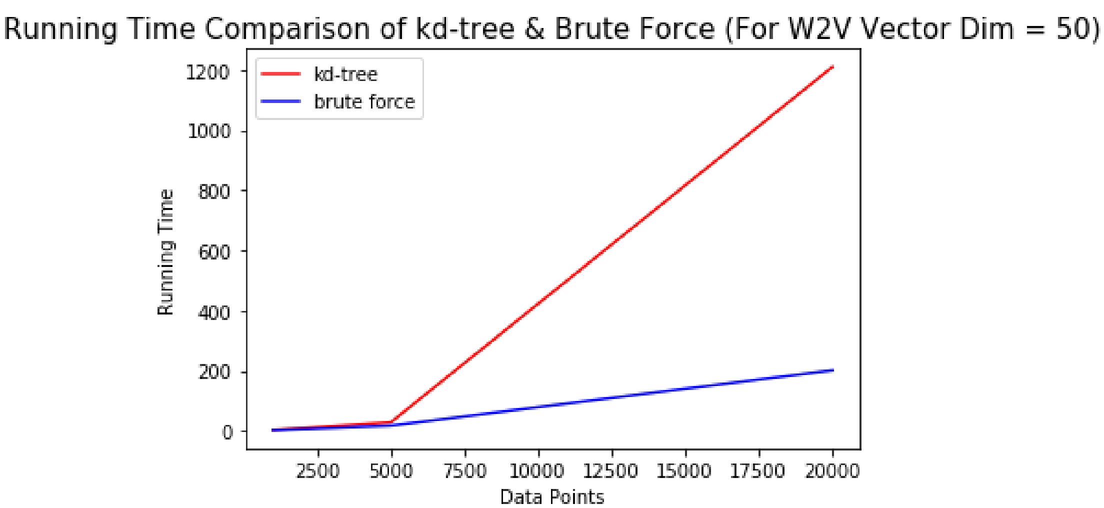
The comparison of running time between kd-tree and brute force knn for varying dataset sizes is plotted below. The values hardcoded are noted after timing the code run multiple times with varying parameters.

```
In [44]: # with w2v dimension = 5

         plt.plot([1000,5000,20000], [1.92,7.92,39.35], 'r-', label='kd-tree')
         plt.plot([1000,5000,20000], [2.11,16.74,194.46], 'b-', label='brute force')
         plt.xlabel('Data Points')
         plt.ylabel('Running Time')
         plt.title('Running Time Comparison of kd-tree & Brute Force (For W2V Vector Dim = 5)'
         plt.legend()
         plt.show()
```

In [45]:
```
# with w2v dimension = 50

plt.plot([1000,5000,20000], [2.94,28.24,1210.01], 'r-', label='kd-tree')
plt.plot([1000,5000,20000], [2.14,17.29,200.99], 'b-', label='brute force')
plt.xlabel('Data Points')
plt.ylabel('Running Time')
plt.title('Running Time Comparison of kd-tree & Brute Force (For W2V Vector Dim = 50)
plt.legend()
plt.show()
```



8.2 Observations

1. The Accuracy obtained using **W2V featurization is slightly higher** than BoW and tf-IDF methods, though not significantly higher.

2. **When dimension is small (dim=5), the increase in running time of kd-tree knn is linear but the brute force algorithm is exponential. Hence, at lower dimensions, kd-tree performs better.**

3. **When dimension is high (dim=50), the increase in running time of kd-tree knn is exponential, but brute force algorithm is comparatively linear. Hence, brute force is better when dimension is high.**

4. **Explanation of 2 & 3:** It is known that kD-Trees **don't scale very well with high dimensionality**. When d is not small, the time complexity of knn becomes, O (2ˆd*log n). When 2ˆd = n, then time complexity = O (n log n) which is more than brute force (O (n)). This explains (3).

5. Even when d is small, **time complexity of kd-tree would be O (log n) only when data is uniformly distributed**. When data is not uniform, then kd-tree would move towards complexity of simple implementation, O (n)

6. It has been noticed that the **general rules given in (2) and (3) are not hard and fast**. For the same number of datapoints, kd-tree is seen much efficient at very high dimensions.

   Timing Results: (W2V dimension = 50) Brute Force = 2.16 seconds; kd-tree = 4.06 seconds (W2V dimension = 500) Brute Force = 26.55 seconds; kd-tree = 25.14 seconds.

   Reason: The performance may depend a lot on the characteristics of the data. For example, are the data points evenly distributed, clustered or otherwise arranged?

Note: What happens for kd-tree at high dimensions? To evaluate a query point at boundary, the circle drawn would intersect all adjoining regions. Thus, we have to look at all 2ˆd adjoining regions. when d = 20 then there are 1 million regions, which is huge! Thus, when dimensionality is 10K or 20K, then kd-tree is useless, as shown in the timing results.