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$$e^x y y' = e^{-y} + e^{-2x-y}$$

$$e^x y y' = \frac{1}{e^y} + \frac{1}{e^{2x+y}}$$

$$e^x y y' = \frac{1}{e^y}\left(1 + \frac{1}{e^{2x}}\right)$$

$$y e^y y' = \frac{1}{e^x}\left(\frac{e^{2x}+1}{e^{2x}}\right)$$

$$y e^y y' = \frac{e^{2x}+1}{e^{3x}} \Rightarrow \text{EDO de Variables separables}$$

$$y e^y \frac{dy}{dx} = \frac{e^{2x}+1}{e^{3x}}$$

$$y e^y dy = \frac{e^{2x}+1}{e^{3x}} dx$$

$$\begin{array}{ll} u = y & v = e^y \\ du = dy & dv = e^y dy \end{array}$$

$$\int y e^y dy = \int \left(\frac{e^{2x}+1}{e^{3x}}\right) dx$$

$$\int y e^y dy = y e^y - \int e^y dy = y e^y - e^y = e^y(y-1)$$

$$\int \left(\frac{e^{2x}}{e^{3x}} + \frac{1}{e^{3x}}\right) dx = \int e^{2x-3x} dx + \int e^{-3x} dx = \int e^{-x} dx + \int e^{-3x} dx$$

$$= -e^{-x} - \frac{1}{3} e^{-3x} + C$$

$$\boxed{e^y(y-1) = -e^{-x} - \frac{1}{3} e^{-3x} + C}$$

(2) $e^{x+y}\, y' = x$ Condicion inicial $\rightarrow y(0) = \ln 2$

$$e^{x+y}\,\frac{dy}{dx} = x$$

$$e^{x} \cdot e^{y}\,\frac{dy}{dx} = x$$

$$e^{y}\, dy = \frac{x}{e^{x}}\, dx$$

$e^{y}\, dy = x e^{-x}\, dx \Longrightarrow$ EDO de variables separables

$$\int e^{y}\, dy = \int x e^{-x}\, dx$$

$$e^{y} = \int x e^{-x}\, dx = -x e^{-x} + \int e^{-x}\, dx$$

$$\boxed{\begin{aligned} u &= x \\ du &= dx \\ dv &= e^{-x}\, dx \\ v &= -e^{-x} \end{aligned}}$$

$$e^{y} = -x e^{-x} - e^{-x} + C$$

$e^{y} = -e^{-x}(x+1) + C$; si $x = 0 \Rightarrow y = \ln 2$

$$e^{\ln 2} = -e^{-0}(0+1) + C$$

$$2 = (-1)(1) + C \Longrightarrow 2 = -1 + C$$

$$C = 3$$

$$\ln e^{y} = \ln(3 - e^{-x}(x+1))$$

$$\boxed{y = \ln(3 - e^{-x}(x+1))}$$

3 $y' = \dfrac{y^2+x^2}{2xy}$

Cambio de variable

$y = ux \qquad u = \dfrac{y}{x}$

$y' = xu' + u$

$\dfrac{1}{u} = \dfrac{x}{y}$

$y' = \dfrac{y^{\cancel{2}}}{2x\cancel{y}} + \dfrac{x^{\cancel{2}}}{2\cancel{x}y}$

$y' = \dfrac{y}{2x} + \dfrac{x}{2y}$

$xu' + u = \dfrac{1}{2}u + \dfrac{1}{2u}$

$xu' = \dfrac{1}{2}u + \dfrac{1}{2u} - u$

$xu' = \dfrac{1}{2u} - \dfrac{1}{2}u$

$xu' = \dfrac{1}{2}\left(\dfrac{1}{u} - u\right)$

$xu' = \dfrac{1}{2}\left(\dfrac{1-u^2}{u}\right) \Rightarrow \dfrac{2u \cdot u'}{(1-u^2)} = \dfrac{1}{x} \rightarrow$ EDO de variables separables

$\displaystyle\int \frac{2u\,du}{1-u^2} = \int \frac{dx}{x}$

$\boxed{\begin{aligned} z &= 1-u^2 \\ dz &= -2u\,du \\ -dz &= 2u\,du \end{aligned}}$

$\displaystyle -\int \frac{dz}{z} = \ln|x| + C$

$-\ln|z| = \ln|x| + C$

$\ln|z| = -\ln|x| + C$

$\ln|1-u^2| = \ln|x^{-1}| + C$

$\ln\left|1 - \dfrac{y^2}{x^2}\right| = \ln|x^{-1}| + C$

$\cancel{e}^{\cancel{\ln}}\left|\dfrac{x^2-y^2}{x^2}\right| = \cancel{e}^{\ln|x^{-1}| + C}$

$\dfrac{x^2-y^2}{x^2} = x^{-1} \cdot e^C \Rightarrow e^C = K$

$\dfrac{x^2-y^2}{x^{\cancel{2}}} = \dfrac{K}{x}$; si $x \neq 0$.

$x^2 - y^2 = xK$

$x^2 - xK = y^2$

$\boxed{y^2 = x - xK}$

4) $y' = \dfrac{x+y-1}{3x-y+5}$; $\begin{vmatrix} 1 & 1 \\ 3 & -1 \end{vmatrix} = -1-3 = -4 \neq 0$

- Se determina el sistema de ecuaciones:

$$\left.\begin{array}{l} x+y-1=0 \\ 3x-y+5=0 \end{array}\right\} \quad \begin{array}{r} x+y=1 \\ 3x-y=-5 \\ \hline 4x=-4 \end{array}$$

$x = \dfrac{-4}{4}$; $x=-1$

$y = 1-x$

$y = 1-(-1)$ $\quad$ $y=2$

$y = 1+1$

- Con cambio de variable

$$\begin{cases} x = u-1 \\ y = v+2 \end{cases}$$

Se sustituye en las EDO iniciales

$$\frac{dy}{dx} = \frac{dy}{dv}\,\frac{dv}{du}\,\frac{du}{dx} = (1)\frac{dv}{du}(1) \Rightarrow \frac{dy}{dx} = \frac{dv}{du};$$

$$\frac{dv}{du} = \frac{(u-1)+(v+2)-1}{3(u-1)-(v+2)+5} = \frac{u\cancel{-1}+v\cancel{+2}\cancel{-1}}{3u\cancel{-3}-v\cancel{-2}\cancel{+5}} = \frac{u+v}{3u-v}$$

$$\frac{dv}{du} = \frac{u\left(1+\frac{v}{u}\right)}{u\left(3-\frac{v}{u}\right)} = \frac{1+\frac{v}{u}}{3-\frac{v}{u}} ; \; z = \frac{v}{u} \Rightarrow$$

$v = zu$

$$\frac{dv}{du} = \frac{dz}{du}u + z\frac{du}{du}$$

$$\frac{dv}{du} = \frac{u\,dz}{du} + z$$

$$\frac{u\,dz}{du} + z = \frac{1+z}{3-z}$$

$$\frac{u\,dz}{du} = \frac{1+z}{3-z} - z$$

$$\frac{u\,dz}{du} = \frac{1+z-3z+z^2}{3-z}$$

$$u \cdot \frac{dz}{du} = \frac{1-2z+z^2}{3-z}$$ $\Rightarrow$ EDO de variables separables

$$\int \frac{(3-z)}{z^2-2z+1}\,dz = \int \frac{du}{u} \; ; (z^2-2z+1)' = 2z-2$$

$$\boxed{\begin{array}{c} t = z^2-2z+1 \\ dt = (2z-2)\,dz \\ s = z-1 \\ ds = dz \end{array}}$$

$$\int \frac{-\frac{1}{2}(2z-2)+3-1}{z^2-2z+1}\,dz = \ln|u| + C$$

$$-\frac{1}{2}\int \frac{(2z-2)}{z^2-2z+1}\,dz + 2\int \frac{dz}{z^2-2z+1} = \ln|u|+C$$

$$-\frac{1}{2}\int \frac{dt}{t} + 2\int \frac{dz}{(z-1)^2} = \ln|u| + C$$

$$-\frac{1}{2}\ln|t| + 2\int \frac{ds}{s^2} = \ln|u| + C$$

$$-\frac{1}{2}\ln|t| + 2\frac{s^{-2+1}}{(-2+1)} = \ln|u| + C$$

$$-\frac{1}{2}\ln|t| - \frac{2}{s} = \ln|u| + C$$

$$\frac{1}{2}\ln|t| + \frac{2}{s} = -\ln|u| + C$$

$$\frac{1}{2}\ln\left|\left(\frac{y-x-3}{x+1}\right)^2\right| + \frac{2(x+1)}{y-x-3} = \ln|(x+1)^{-1}| + C$$

$$\frac{\cancel{2}}{\cancel{2}}\ln\left|\frac{y-x-3}{x+1}\right| + \frac{2(x+1)}{y-x-3} = \ln\left|\frac{1}{x+1}\right| + C$$

$$\boxed{\ln\left|\frac{y-x-3}{x+1}\right| + \frac{2(x+1)}{y-x-3} - \ln\left|\frac{1}{x+1}\right| = C}$$

Devolviendo todos los cambios de variable:

$$t = z^2-2z+1$$

$$t = (z-1)^2 = \left(\frac{v}{u}-1\right)^2$$

$$t = \left(\frac{v-u}{u}\right)^2 ; \; x = u-1 \Rightarrow u = x+1$$

$$y = v+2 \Rightarrow v = y-2$$

$$t = \left(\frac{y-2-(x+1)}{x+1}\right)^2$$

$$t = \left(\frac{y-2-x-1}{x+1}\right)^2$$

$$t = \left(\frac{y-x-3}{x+1}\right)^2 \; //$$

$$s = z-1 \Rightarrow s = \frac{v}{u}-1$$

$$s = \frac{v-u}{u}$$

$$s = \frac{y-2-(x+1)}{x+1} = \frac{y-2-x-1}{x+1}$$

$$s = \frac{y-x-3}{x+1} \Rightarrow \frac{1}{s} = \frac{x+1}{y-x-3}$$