

Factorizacion Lu

Problema 10

$$10 = \begin{cases} 2x + y + z = 4 \\ 4x - y + 2z = -1 \\ 2x + 3y + 8z = 3 \end{cases} \quad \begin{vmatrix} 2 & 1 & 1 \\ 4 & -1 & 2 \\ 2 & 3 & 8 \end{vmatrix} \quad |A| = -92 \neq 0 //$$

$$\begin{array}{ccc|l} 2 & 1 & 1 & \\ 4 & -1 & 2 & \rightarrow R_2 = R_1(-2) + R_2 \\ 2 & 3 & 8 & \end{array} \quad \begin{array}{ccc|l} 2 & 1 & 1 & \\ 0 & -3 & 0 & \rightarrow R_3 = R_1(-1) + R_3 \\ 2 & 3 & 8 & \end{array}$$

$$\begin{array}{ccc|l} & & & \\ & -4 & -2 & -2 \\ & 4 & -1 & 2 \\ & 0 & -3 & 0 \end{array} \quad \begin{array}{ccc|l} & & & \\ & -2 & -1 & -1 \\ & 2 & 3 & -1 \\ & 0 & 2 & -2 \end{array}$$

$$\begin{array}{ccc|l} 2 & 1 & 1 & \\ 0 & -3 & 0 & \rightarrow R_3 = R_2\left(\frac{2}{3}\right) + R_3 \\ 0 & 2 & -2 & \end{array} \quad \begin{array}{ccc|l} & & & \\ & 0 & -2 & 0 \\ & 0 & 2 & -2 \\ & 0 & 0 & -2 \end{array}$$

$$U = \begin{vmatrix} 2 & 1 & 1 \\ 0 & -3 & 0 \\ 0 & 0 & -2 \end{vmatrix} \quad L = \begin{vmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ -1\frac{2}{3} & 1 & 1 \end{vmatrix} //$$

Resolver $Ly = b$

$$\begin{pmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ -1\frac{2}{3} & 1 & 1 \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} = \begin{pmatrix} 4 \\ -1 \\ 3 \end{pmatrix} \quad \begin{array}{l} y_1 = 4 \\ -2y_1 + y_2 = -1 \\ -y_1 + \frac{2}{3}y_2 + y_3 = 3 \end{array}$$

$$\begin{array}{l} -2(4) + y_2 = -1 \\ y_2 = 8 - 1 = 7 // \end{array}$$

$$-4 + \frac{2}{3}(7) + y_3 = 3$$

$$-4 + \frac{14}{3} + y_3 = 3$$

$$\frac{2}{3} + y_3 = 3$$

$$y_3 = 3 - \frac{2}{3} = \frac{7}{3} //$$

$$\begin{array}{l} y_1 = 4 \\ y_2 = 7 \\ y_3 = \frac{7}{3} // \end{array}$$

Problema 10

$$UX = y$$

$$\frac{4}{7}$$

$$\frac{7}{3}$$

$$\begin{array}{ccc} 2 & 1 & 1 \\ 0 & -3 & 0 \\ 0 & 0 & -2 \end{array}$$

$$\begin{pmatrix} 2 & 1 & 1 \\ 0 & -3 & 0 \\ 0 & 0 & -2 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 4 \\ 7 \\ \frac{7}{3} \end{pmatrix}$$

$$2x_1 + x_2 + x_3 = 4$$

$$-3x_2 = 7$$

$$-2x_3 = \frac{7}{3}$$

$$-2x_3 = \frac{7}{3}$$

$$x_3 = \frac{\frac{7}{3}}{-2} = -\frac{7}{6}$$

$$x_1 = \frac{15}{4}$$

$$x_2 = -\frac{7}{3}$$

$$x_3 = -\frac{7}{6}$$

Solucion
Unica

$$-3x_2 = 7$$

$$x_2 = -\frac{7}{3}$$

$$2x_1 + x_2 + x_3 = 4$$

$$2x_1 + \frac{7}{3} - \frac{7}{6} = 4$$

$$2x_1 - \frac{7}{6} = 4$$

$$x_1 = \frac{4 + \frac{7}{6}}{2} = \frac{15}{4}$$

$$\frac{15}{2}$$

Problema 11

$$11 \begin{cases} x+y+z=2 \\ 4x-8y+3z=-2 \\ 2x-2y+2z=1 \end{cases} \quad \begin{vmatrix} 1 & 1 & 1 \\ 4 & -8 & 3 \\ 2 & -2 & 2 \end{vmatrix} |A| = -4 \neq 0$$

$$\begin{array}{ccc|ccc} 1 & 1 & 1 & & & \\ 4 & -8 & 3 & \rightarrow R_2 = R_1(-4) + R_2 & 1 & 1 & 1 \\ 2 & -2 & 2 & -4 & -4 & -4 & \\ & & & 4 & -8 & 3 & \\ & & & \hline & & & 0 & -12 & -1 & \end{array} \quad \begin{array}{ccc|ccc} 1 & 1 & 1 & & & \\ 0 & -12 & -1 & \rightarrow R_3 = R_2(-2) + R_3 & 1 & 1 & 1 \\ 2 & -2 & 2 & -2 & -2 & -2 & \\ & & & 2 & -2 & 2 & \\ & & & \hline & & & 0 & -4 & 0 & \end{array}$$

$$\begin{array}{ccc|ccc} 1 & 1 & 1 & & & \\ 0 & -12 & -1 & \rightarrow R_3 = R_2(-\frac{1}{3}) + R_3 & 1 & 1 & 1 & L = 1 & 0 & 0 \\ 0 & -4 & 0 & 0 & 4 & +\frac{1}{3} & -4 & 1 & 0 \\ & & & 0 & -4 & 0 & 0 & 0 & \frac{1}{3} & -2 & -\frac{1}{3} & 1 \\ & & & \hline & & & 0 & 0 & \frac{1}{3} & \end{array} \quad U = \begin{array}{ccc|ccc} 1 & 1 & 1 & & & \\ 0 & -12 & -1 & & & \\ 0 & 0 & \frac{1}{3} & & & \end{array}$$

Resolver $Ly=b$

$$\begin{aligned} y_1 &= 2 \\ -4y_1 + y_2 &= -2 \\ -2y_1 - \frac{1}{3}y_2 + y_3 &= 1 \\ -4(2) + y_2 &= -2 \\ -8 + y_2 &= -2 \\ y_2 &= -2 + 6 = 4 \end{aligned}$$

$$\begin{aligned} -2y_1 + \frac{1}{3}y_2 + y_3 &= 1 \\ -2(2) + \frac{1}{3}(4) + y_3 &= 1 \\ -\frac{8}{3} + y_3 &= 1 \\ y_3 &= 1 + \frac{8}{3} = \frac{11}{3} \end{aligned}$$

$$\begin{aligned} y_1 &= 2 \\ y_2 &= 4 \\ y_3 &= \frac{11}{3} \end{aligned}$$

Problema 11

$$UX = \begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 0 & -12 & -1 & 4 \\ 0 & 0 & \frac{1}{3} & \frac{1}{5} \end{array}$$

$$\begin{aligned} x_1 + x_2 + x_3 &= 2 \\ -12x_2 - x_3 &= 4 \\ \frac{1}{3}x_3 &= \frac{1}{5} \end{aligned}$$

$$x_3 = \frac{\frac{11}{5}}{\frac{1}{3}} = \frac{33}{5} \checkmark$$

$$-12x_2 - x_3 = 4$$

$$-12x_2 - \frac{33}{5} = 4$$

$$x_2 = \frac{4 + \frac{33}{5}}{-12}$$

$$x_2 = \frac{-53}{60} \checkmark$$

$$x_1 + x_2 + x_3 = 2$$

$$x_1 + \left(-\frac{53}{60}\right) + \frac{33}{5} = 2$$

$$x_1 + \frac{343}{60} = 2$$

$$x_1 = 2 - \frac{343}{60} = \frac{-223}{60} \checkmark$$

$$x_1 = \frac{-223}{60}$$

$$x_2 = \frac{-53}{60}$$

Solución única

$$x_3 = \frac{33}{5} \checkmark$$

Problema 12 Factorización LU

$$12 = \begin{cases} 2x - y + z = 0 \\ x + 2y + z = 10 \\ 3x + y = 0 \end{cases}$$

$$\begin{vmatrix} 2 & -1 & 1 \\ 1 & 2 & 1 \\ 3 & 1 & 0 \end{vmatrix} |A| = -10 \neq 0$$

$$\begin{vmatrix} 2 & -1 & 1 \\ 1 & 2 & 1 \\ 3 & 1 & 0 \end{vmatrix} \rightarrow R_2 = R_1(-\frac{1}{2}) + R_2$$

$$\begin{vmatrix} 2 & -1 & 1 \\ 0 & \frac{5}{2} & \frac{1}{2} \\ 3 & 1 & 0 \end{vmatrix}$$

$$\begin{vmatrix} 1 & 2 & 1 \\ 0 & \frac{5}{2} & \frac{1}{2} \end{vmatrix}$$

$$\begin{vmatrix} 2 & -1 & 1 \\ 0 & \frac{5}{2} & \frac{1}{2} \\ 3 & 1 & 0 \end{vmatrix} \rightarrow R_3 = R_1(-\frac{6}{5}) + R_3$$

$$\begin{vmatrix} 2 & -1 & 1 \\ 0 & \frac{5}{2} & \frac{1}{2} \\ 0 & \frac{5}{2} & -\frac{6}{5} \end{vmatrix}$$

$$\begin{vmatrix} 2 & -1 & 1 \\ 0 & \frac{5}{2} & \frac{1}{2} \\ 0 & 0 & 2 \end{vmatrix}$$

$$\begin{vmatrix} 2 & -1 & 1 \\ 0 & \frac{5}{2} & \frac{1}{2} \\ 0 & \frac{5}{2} & -\frac{6}{5} \end{vmatrix} \rightarrow R_3 = R_2(-1) + R_3$$

$$\begin{vmatrix} 2 & -1 & 1 \\ 0 & \frac{5}{2} & \frac{1}{2} \\ 0 & 0 & 2 \end{vmatrix}$$

$$U = \begin{vmatrix} 2 & -1 & 1 \\ 0 & \frac{5}{2} & \frac{1}{2} \\ 0 & 0 & 2 \end{vmatrix} \quad L = \begin{vmatrix} 1 & 0 & 0 \\ -\frac{1}{2} & 1 & 0 \\ -\frac{6}{5} & -1 & 1 \end{vmatrix}$$

$$Ly = b$$

$$y_1 = 0$$

$$-\frac{1}{2}y_1 + y_2 = 10$$

$$-\frac{6}{5}y_1 + y_2 + y_3 = 0$$

$$Ux = y$$

$$2x_1 - x_2 + x_3 = 0$$

$$\frac{5}{2}x_2 + \frac{1}{2}x_3 = 10$$

$$2x_3 = 10$$

$$2x_3 = 10 \quad \frac{5}{2}x_2 + \frac{1}{2}(5) = 10$$

$$x_3 = \frac{10}{2} \quad \frac{5}{2}x_2 + \frac{5}{2} = 10$$

$$x_3 = 5 \quad x_2 = \frac{10 - \frac{5}{2}}{\frac{5}{2}}$$

$$x_2 = 9$$

$$2x_1 - x_2 + x_3 = 0$$

$$2x_1 - 9 + 5 = 0$$

$$x_1 = \frac{9 - 5}{2} = 2$$

$$x_1 = 2 \quad x_2 = 9 \quad x_3 = 5$$

Solución única

$$y_1 = 0$$

$$y_2 = 10$$

$$y_3 = 10$$