EXAMEN 1er parcial. 08/07/21

① $e^{x} yy' = e^{-y} + e^{-2x-y}$

$e^{-y} + e^{-2x-y}$ $\quad e^{x} y \frac{dy}{dx} = e^{-y} + e^{-2x-y}$

$e^{x} y\, dy = (e^{-y} + e^{-2x-y})\, dx$

$e^{x} y\, dy = (e^{-y} + e^{-2x} \cdot e^{-y})\, dx$

$e^{x} y\, dy = e^{-y}(1 + e^{-2x})\, dx$

$y\, dy = \frac{e^{-y}(1+e^{-2x})\, dx}{e^{x}}$

$\frac{y\, dy}{e^{-y}} = \frac{(1+e^{-2x})\, dx}{e^{x}}$

$e^{y} y\, dy = e^{x}(1+e^{-2x})\, dx$

$e^{y} y\, dy = (e^{-x} + e^{-3x})\, dx$

$$\int e^{y} y\, dy = \int (e^{-x} + e^{-3x})\, dx$$

$$= \int u e^{au+b}\, du = \frac{1}{a} u e^{au+b} - \frac{1}{a^2} e^{au+b} + C \qquad \int e^{au}\, du = \frac{1}{a} e^{au} + C$$

$y e^{y} - e^{y} + C_1 = -e^{-x} - \frac{1}{3} e^{-3x} + C_2$

$y e^{y} - e^{y} = -e^{-x} - \frac{1}{3} e^{-3x} + C$

$y e^{y} - e^{y} + e^{-x} + \frac{1}{3} e^{-3x} = C$

2° $e^{x+y}y' = x$ con condicion inicial

Condición inicial.
$y(0) = \ln 2$

$$e^{x+y}\frac{dy}{dx} = x$$

$$e^x \cdot e^y \frac{dy}{dx} = x$$

$$e^y\,dy = \frac{x}{e^x}\,dx$$

$$\int e^y\,dy = \int \frac{x}{e^x}\,dx$$

$$\int e^y\,dy = \int xe^{-x}\,dx$$

$u = x$
$du = dx$
$dv = e^{-x}\,dx$
$v = -e^{-x}$

$$\int v = \int e^{-x}\,dx$$

$$e^y = x(-e^{-x}) - \int -e^{-x}\,dx$$

$$e^y = -xe^{-x} + \int e^{-x}\,dx$$

$$e^y = -xe^{-x} + (-e^{-x}) + C$$

$$e^y = -xe^{-x} - e^{-x} + C$$

→ Condición

$$e^y = -xe^{-x} - e^{-x} + C$$

$$e^{\ln(2)} = -(0)e^{-0} - e^{-0} + C$$

$2 = 0 - 1 + C$

$2 = -1 + C$

$2 + 1 = C$

$3 = C$

$$e^y = -xe^{-x} - e^{-x} + 3$$

③

$$y' = \frac{y^2+x^2}{2xy}$$

$M(x,y) = y^2+x^2$

$N(x,y) = 2xy$

$u = \frac{y}{x}$

$y = ux$

$dy = u\,dx + x\,du$

$$\frac{dy}{dx} = \frac{y^2+x^2}{2xy}$$

$$dy = \frac{y^2+x^2}{2xy}\,dx$$

$2xy\,dy = y^2+x^2\,dx$

$-(y^2+x^2)\,dx + 2xy\,dy = 0$

$(-(ux)^2 - x^2)\,dx + (2x(ux))\,(u\,dx + x\,du) = 0$

$(-ux^2)\,dx - x^2\,dx + 2xu\,dx + 2x^2\,du + u^2x\,dx + ux^2\,du = 0$

$-2\cancel{ux}\,dx - \cancel{x^2}\,dx + 2\cancel{xu}\,dx + 2\cancel{x^2}\,du + u^2x\,dx + \cancel{ux^2}\,du = 0$

$-dx + 2du + u^2x\,dx + u\,du = 0$

$(-1+u^2x)\,dx + (2+1)\,du = 0$

$(-1+u^2x)\,dx = (-2-1)\,du$

$$dx = \frac{-3}{-1+u^2x}\,du$$

$$\frac{dx}{x} = \frac{-3}{-1+u^2}\,du$$

$$\int \frac{dx}{x} = -3\int \frac{1}{-1+u^2}\,du$$

$\ln|x| = -3\ln|-1+u^2| + C$

$\ln|x| + 3\ln|-1+u^2| + C$

$e^{\ln|x|} + e^{3\ln|-1+u^2|} = C$

$x - 1u^2 = C$

$x - 1\left(\frac{x}{y}\right)^2 = C$

4 $y = \frac{x+y-1}{3x-y+5}$

$\frac{dy'}{dx}$ $y' = ux'$

$dy' = x\,du + u\,dx$

$\frac{dy'}{dx'} = \frac{x'+h+y+k-1}{3(x+h)+y+k+5}$ $\frac{h+y-1}{3h-y+5}$

$y = -\frac{8}{-4}$ $y = 2$ $x = -1$

$dy\,(3x-ux) = dx\,(x+y)$

$dy\,(3x-y) - dx\,(x+y)$

$(x'du+u\,dx)(3x-ux) - dx(x+ux')$

$3x^2du + 3xu\,dx - x^2u\,du - u^2x\,dx - x\,dx + ux\,dx$

$3x\,du + 3u\,dx - xu\,du - u^2dx - dx + u\,dx$

$(3u\,dx - u^2dx - u\,dx - dx)\ (3x\,du + xu\,du)$

$(3u+u^2-u-1)\,dx\ \ (3x-xu)\,du$

$(3u+u^2+u-1)$

$(7u+u^2-1)\,dx\ \ \ x(3-u)\,du = 0$

$\frac{dx}{x} = \frac{-(3-u)}{(u^2+2u+1)}$

$\int \frac{dx}{x} = \int \frac{3-u}{(u-1)(u-1)}$

$3 - u = A(u+1) + B(u-1)$

$3 - u = Au + A + Bu - B$

$-u = Au + Bu$ $\qquad$ $B = -\frac{1}{2}$

$3 = A - B$ $\qquad$ $A = \frac{7}{2}$

$$\int \left(\frac{7}{2} \frac{1}{(u-1)} + \frac{1}{2} \frac{1}{(u-1)} \right) du$$

$\ln|x| - \frac{1}{2} \ln|u+1| + \frac{1}{2} \ln|(u-1)| + c$

$\ln|x| = \frac{7}{2} \ln \left| \frac{(y+2)}{x-1} \right| - \frac{1}{2} \ln \left| \frac{y+2}{x-1} \right| + c$