Extra Calculo integral 18287836

2/12/20

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① $\int \frac{dx}{x^2\sqrt{9-x^2}}$ caso ①

$x = a\sin\theta$

$dx = a\cos\theta\, d\theta$

$\sqrt{a^2-x^2} = a\cos\theta$

$x = 3\,\text{sen}\,\theta$

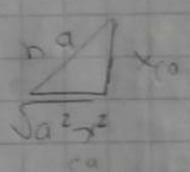
$dx = 3\cos\theta\, d\theta$

$\sqrt{a^2-}\ \ 3\cos\theta$

$$\int \frac{dx}{x^2\sqrt{9-x^2}} = \int \frac{3\cos\theta\, d\theta}{(3\,\text{sen}\,\theta)^2\, 3\cos\theta} = \int \frac{1}{9\,\text{sen}^2\theta}\, d\theta$$

$\csc\theta = \frac{1}{\text{sen}\,\theta}$ $\qquad \frac{1}{9}\int \csc^2\theta\, d\theta = -\cot u + c$

$$= \frac{1}{9} \cdot -\cot\theta + c$$



$$\boxed{= -\frac{1}{9}\ \frac{\sqrt{9-x^2}}{x} + c}$$

$\cot = \frac{ca}{co}$

⑥ $\int_{\pi/2}^{3\pi/4} \sin^5 x \cos^3 x \, dx$

$\int \text{sen}^5 x \cos^2 x \cos x \, dx$

$\int \text{sen}^5 x (1-\text{sen}^2 x) \cos x \, dx$

$u = \text{sen}\, x$
$du = \cos x \, dx$

$\int u^5 (1-u^2) \, du$

$\int u^5 - u^7 \, du$

$= \frac{1}{6} u^6 - \frac{1}{8} + C$

$u = \text{sen} \frac{\pi}{2}$

$u = 1$

$u = \text{sen}\ 3\pi/4$
$u \ \frac{\sqrt{2}}{2} = 0.7071$

$\frac{1}{6} \text{sen}^6 x - \frac{1}{8} \text{sen}^8 x \Big|_{1}^{0.7071} =$

$\left[\frac{1}{6} \text{sen}^6\left(\frac{\sqrt{2}}{2}\right) - \frac{1}{8} \text{sen}^8\left(\frac{\sqrt{2}}{2}\right)\right] - \left[\frac{1}{6} \text{sen}^6(1) - \frac{1}{8} \text{sen}^8(1)\right]$

$= -0.0192$

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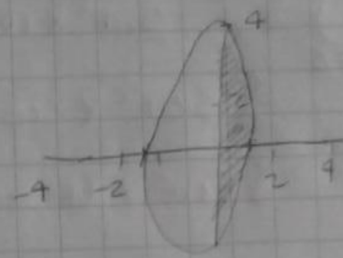
(5) Volumen

$y = 4 - x^2$, $x = 0$, $y = 0$



$x^2 + 4 = 0$

$x^2 = \frac{-4}{-1}$

$x = \sqrt{4}$

$x_1 = 2$ $\quad x_2 = 2$

$V = \int_{-2}^{2} \pi r^2 \, dx$

$V = \int \pi (4 - x^2)^2$

$V = \pi \int (x^4 - 8x^2 + 16) \, dx$

$V = \pi \left[\frac{x^5}{5} - \frac{8x^3}{3} + 16x \right]_{-2}^{2}$

$\pi \left[\left(\frac{1}{5}(2)^5 - \frac{8}{3}(2)^3 + 16(2) \right] - \left[\frac{1}{5}(-2)^5 - \frac{8}{3}(-2)^3 + 16(-\right.\right.$

$\pi \left[17.06 - (-17.06) \right) = \pi (34.12)$ $= \frac{853}{25} \pi$

$= 107.1911$

④ area

$y = -x^2 + 4x + 2 \; ; \; y = x - 2$

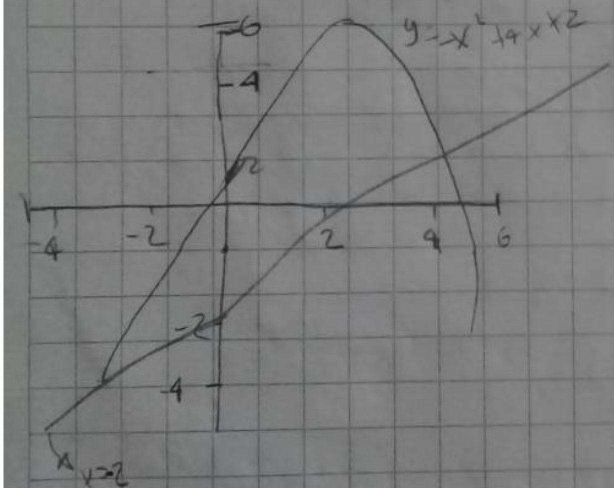
$-x^2 + 4x + 2 = x - 2$

$x - 2 + x^2 - 4x - 2 = 0$

$(x+1) \quad (x-4) = 0$

$x + 1 = 0 \qquad\qquad x - 4 = 0$

$x = -1 \qquad\qquad x = 4$



$A = \int_{-1}^{4} [-x^2 + 4x + 2 - (x-2)] \, dx$

$= \int_{-1}^{4} (-x^2 + 4x + 2 - x + 2) \, dx]$

$= \int (-x^2 + 3x + 4) \, dx$

$-\int x^2 dx + 3\int x dx + 4\int dx$

$-\frac{x^3}{3} + \frac{3x^2}{2} + 4x + c \Big|_{-1}^{4}$

$\left[-\frac{(4)^3}{3} + \frac{3(4)^2}{2} + 4(4)\right] - \left[-\frac{(-1)^3}{3} + \frac{3(-1)^2}{2} + 4(-1)\right]$

$\frac{56}{3} - \left(-\frac{13}{6}\right) = 18.66 - (-2.166) = \underline{20.8333}$

(3) $\int_1^9 \frac{dx}{\sqrt{x}(1+\sqrt{x})^2}$

$u = 1+\sqrt{x}$ $\quad 1+(x)^{1/2}$

$du = \frac{1}{2}x^{-1/2}$

$du = \frac{1}{2\sqrt{x}}$

$2\int \frac{1}{u^2}\, du$

$\frac{-2}{u}$

$\boxed{-\frac{2}{\sqrt{x}+1} + C}$

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(2) $\int \frac{6x^3+3x^2+3}{x^2+x^4}\,dx$

$3(2x^3+x^2+1)$

$3\int \frac{2x^3+x^2+1}{x^2+x^4}$ $x^2(1+x^2)$

$3\int \frac{2x^3+x^2+1}{x^2(1+x^2)}$

$3\int\left(\frac{2x}{x^2+1}+\frac{1}{x^2}\right)dx$

$3\int \frac{x}{x^2+1}\,dx + \int \frac{1}{x^2}\,dx$

$u=x^2+1$

$\frac{du}{dx}=2x \rightarrow \frac{1}{2x}du$

$=\frac{1}{2}\int \frac{1}{u}\,du$

$\int \frac{1}{u}\,du=\ln|u|$

$=\frac{\ln(u)}{2}$

$\frac{\ln(x^2+1)}{2}$

$\int \frac{1}{x^2}$

$\int x^n\,dx=\frac{x^{n+1}}{n+1}=-\frac{1}{x}$

$\ln(x^2+1)=-\frac{1}{x}$

$3\int \frac{2x^3+x^2+1}{x^2+x^4}\,dx = 3\ln(x^2+1)-\frac{3}{x}+C$