Jose Adrian Olivares Moran 17332507
Tarea departamental.

① Calcular la derivada parcial con el metodo de la cadena $z = x^2 \operatorname{sen} y$ con respecto a s

$x(s,t) = s^2 + t^2$
$y(s,t) = 2st$

$$\frac{\partial z}{\partial s} = \frac{\partial z}{\partial x}\ \frac{\partial x}{\partial y}\ \frac{\partial x}{\partial s} + \frac{\partial z}{\partial y}\ \frac{\partial y}{\partial s}$$

$$\frac{\partial z}{\partial s} = 2x \operatorname{sen} y (2s) + z^2 (\cos y)(2t)$$

$$\frac{\partial z}{\partial s} = 2(s^2+t^2) \operatorname{sen}(2st)(2s) + (s^2+t^2) \cos(2st) 2t$$

② Del problema anterior, Calcular la derivada parcial con respecto a t.

$$\frac{\partial z}{\partial t} = \frac{\partial z}{\partial y}\ \frac{\partial x}{\partial t} + \frac{\partial z}{\partial y}\ \frac{\partial y}{\partial t}$$

$$\frac{\partial z}{\partial t} = 2x \operatorname{sen} y (2t) + x^2 \cos y (2s)$$

$$\frac{\partial z}{\partial t} = 2(s^2+t^2) \operatorname{sen}(2st)(2t) + (s^2+t^2)^2 \cos(2st)(2s)$$

(3) Calcular la derivada parcial con respecto a x $xyz = \cos(x+y+z)$

$xyz = \cos(x+y+z)$

$$\underbrace{\cos(x+y+z) - xyz = 0}_{F(x+y+z)}$$

$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z}$$

$$\frac{\partial z}{\partial y} = -\frac{F_y}{F_z}$$

$F_x = -\operatorname{sen}(x+y+z) - yz$

$F_y = -\operatorname{sen}(x+y+z) - xz$

$F_z = -\operatorname{sen}(x+y+z) - xy$

$$\frac{\partial z}{\partial x} = -\frac{\operatorname{sen}(x+y+z) - yz}{\operatorname{sen}(x+y+z) - xy}$$

(4) Del ejercicio anterior calcular la derecha para con respecto a y

$$\frac{\partial z}{\partial x} = \frac{-F_x}{F_z}$$

$$\frac{\partial z}{\partial y} = -\frac{F_y}{F_z}$$

$$\frac{\partial z}{\partial y} = \frac{-\operatorname{sen}(x+y+z) - xz}{\operatorname{sen}(x+y+z) - xy}$$

(5) $F(x,y) = x^5 + 3x^3y^2 + 3xy^4$

$\frac{\partial f}{\partial x} = f_x \quad \frac{\partial f}{\partial y} = F_y$

$$\frac{\partial f}{\partial x} = 5x^4 + 4x^2y^2 + 3y^4$$

$$\frac{\partial f}{\partial y} = 6x^3y + 12xy^3$$

⑥ $f(x,y) = \frac{x-y}{x+y}$

$$f(x) = \frac{x+y-(x-y)}{(x+y)^2} = \frac{x+y-x+y}{(x+y)^2} = \frac{2y}{(x+y)^2}$$

$$f(y) = \frac{-(x-y)-(x-y)}{(x+y)^2} = \frac{-x+y-x+y}{(x+y)^2} = \frac{-2x+2y}{(x+y)^2}$$

⑦ $f(r,s) = r\ln(r^2+s^2)$

$$\frac{\partial f}{\partial r} = \ln(r^2+s^2) + r\,\frac{1}{r^2+s^2}\cdot 2r = \ln(r^2+s^2) + \frac{2r^2}{r^2+s^2}$$

$$\frac{\partial f}{\partial s} = r\,\frac{1}{r^2+s^2}\,2s = \frac{2rs}{r^2+s^2}$$

⑧ $f(x,t) = \tan^{-1}(x\sqrt{t})$

$$\frac{\partial f}{\partial x} = \frac{1}{1+x^2t}\cdot\sqrt{t} = \frac{\sqrt{t}}{1+x^2t}$$

$$\frac{\partial f}{\partial t} = \frac{1}{1+x^2t}\cdot\frac{x}{2\sqrt{t}}$$

Escaneado con CamScanner

(9) $f(x, y, z) = x^2 e^{yz}$

$f(x) = 2x e^y$

$f(y) = x^2 e^{yz} z$

$f(z) = x^2 e^{yz} y$

(10) $f(x, y, z, t) = \dfrac{xy^2}{t+2z} = xy^2 (t+2z)^{-1}$

$f(x) = \dfrac{y^2}{t+2z}$

$f(y) = \dfrac{2xy}{t+2t}$

$f(z) = -xy^2 (t+2z)^{-2} 2 = \dfrac{-2xy^2}{(t+2t)^2}$

$f(t) = -xy^2 (t+2z)^{-2} = \dfrac{-xy^2}{(t+2z)^2}$

(11) $z = y \cdot \ln x$

$z_x = y \cdot \dfrac{1}{x} = \dfrac{y}{z}$

$z_y = \ln x$

(12) En una cervecería, fabrican con una h de 10m y un radio de 3cm, considerar como 0.02cm. Calcule el volumen y cambio absoluto.

$V = \pi r^2 h$: $\frac{dv}{dh} = \pi r^2 dh$ $\frac{dv}{dr} = 2\pi r h \, dr$

$$[((\pi r^2)(dh)) + ((2\pi r h)(dr))] =$$

$$[((3^2)(0.02)) + (2\pi(3)(10)(0.02))]$$

$$0.18 + 3.7699 = \frac{3.9494}{282.74} = 0.01407$$

$\pi(3)^2(10)$

1.4%

(13) Dairy Queen esta fabricando conos rellenos con medidas: 15cm de altura y 5cm de radio, con un error de 0.03 cm. Estima el mayor error posible.

$V = \frac{\pi r^2 h}{3}$

$h = 15cm$
$r = 5cm$
$dr = dh = 0.03cm$

$\frac{dv}{dr} = \frac{2\pi r h}{3}$ $\frac{dv}{dh} = \frac{2\pi r^2}{3}$

$$\left[\left(\frac{2\pi r h}{3}\right)(dr)\right] + \left[\left(\frac{\pi r^2}{3}\right)(dh)\right] = \left(\frac{2\pi(5)(15)}{3}\right)(0.03) + \left(\frac{\pi(5)^2}{3}\right)(0.03)$$

$$4.7123 + 0.7853 = \frac{5.4977}{392.70} = 0.013 = 1.39\%$$

Escaneado con CamScanner

(14) Las zucaritas son empacadas en cajas con medidas de 60cm, 20cm y 10cm con un margen de error de 0.5cm

$V = xyz = (60)(20)(10) = 12{,}000$

$\frac{dv}{dx} = yz\,dx \qquad \frac{dv}{dy} = xz\,dy \qquad \frac{dy}{dz} = xy\,dz$

$[(60)(10)(05)] + [(20)(10)(0.5)] + [(20)(60)(0.5)]$

$300 + 100 + 150 = 550$

$\frac{550}{12000} = 0.045 \qquad \underline{4.5\%}$

(15) La caja de dulces que contienen los dulces que poquito vende en el semaforo es de 20cm, 10cm y 5cm cada medida de las cajas es correcto con un margen de error de 0.2cm. Estima el mayor posible.

$V = xyz$

$V = (10)(10)(5) = 1000cm^3$

$\frac{dv}{dx} = yz\,dx \quad \frac{dv}{dy} = xz\,dy \quad \frac{dv}{dz} = xy\,dz$

$[(10)(5)(0.2)] + [(20)(5)(0.2)] + [(20)(10)(0.2)]$

$10 + 20 + 40 = 70$

$\frac{70}{1000} = 0.07 \approx 7\%$

(16) $x^2y + x^2z^2 + y^4$

$$\frac{dz}{dx} = y \cdot \frac{d}{dx} x^2 + z^2 \frac{d}{dx} x^2 + 0 \qquad \frac{dz}{dy} = x^2 \frac{d}{dy} y + 0 + \frac{d}{dy} y^4$$

$$2xy + 2xz^3 \qquad x^2 + 4y^3$$

$$\frac{dz}{dz} = 0 + x^2 \frac{d}{dz}(z^3) + 0 \qquad \underline{3x^2z^2}$$

(17) $e^{2x+3y+4z}$

$$\frac{dz}{dx} = 2e^{2x+3y} + 7e^{6z+7x} \qquad \frac{dz}{dy} = 3e^{2x+3y} + 4e^{2c+3y} + 4e^{4y+5z}$$

(18) $e^{2x3y} + e^{4y+5z} + e^{6z+7y}$

$$\frac{dz}{dx} = 2e^{2x+3y} + 7e^{6z+7x} \qquad \frac{dz}{dy} = 3e^{2x+3y} + 4e^{4y+5z}$$

$$\frac{dz}{dx} = 5e^{4y+5z} + 6e^{6z+7x}$$

(19) La produccion de una fabrica es $P(L,K) = 120K^{1/5} L^{1/5} + C$ donde K es el gasto del capital en dolares y L es el tamaño de la fuerza laboral. Calculo cuanto: $PL = 243$ y $PK = 256$

$$\frac{\partial P}{\partial K} = 120L^{1/5} \frac{\partial}{\partial K} K^{1/5} + 10 \frac{\partial}{\partial K} K \qquad \frac{\partial P}{\partial L} = 120K^{1/5} \frac{\partial}{\partial L} L^{1/5}$$

$$\frac{1}{5} \frac{120L^{1/5}}{K^{4/5}} = 24 \frac{L^{1/5}}{K^{4/6}} + 10 \qquad \frac{1}{5} \frac{120K^{1/5}}{L^{4/6}} = 24 \frac{K^{1/5}}{L^{4/5}}$$

$$\frac{24(243)^{1/5}}{(254)^{4/5}} + 10 = 10.85 \qquad \frac{24(256)^{1/5}}{(243)^{4/5}} = \underline{0.89}$$

(20) La demanda de un bien D, esta en funsion de su precio "p" y del nivel de la renta B, estan relacionados por el modelo $D(P \cdot R) = \ln(3p + B) + \sqrt[3]{2p + B^2}$ cuanto $p = 12$ y $B = 20$

$$\frac{\partial D}{\partial p} = \frac{3}{3p + B} + \frac{2}{3(2p + B^2)^{2/3}} \Big|_{(12,20)} = \frac{1}{3(12) + 20} + \frac{2}{3(2(12) + 20^2)^{1/3}} = \underline{0.024}$$

$$\frac{\partial P}{\partial p} = \frac{1}{(3p + B)} + \frac{2B}{3(2p + B^2)^{2/3}} \Big|_{(12,20)} = \frac{1}{(3(12) + 20)} + \frac{2(20)}{3(2(12) + (20)^2)^{2/3}} = \underline{0.25}$$

(21) $\operatorname{sen}(x^2 y^5) + 4x^2 + 3y^2 + 3$

$$\frac{dz}{dx} = 2xy^3 \cos(x^2 y^5) + 8x \qquad \frac{dz}{dy} = 5x^2 y^4 \cos(x^2 y^5 + 6y)$$

(22) Una empresa fabrica balones, el modelo Champion (x) y el modelo supercopa (y) el costo de la función semanal es:

$c(x, y) = 6.07x^2 + 75x + 85y + 6000$ determine costo

$x = 100 \quad y = 50$

$$\frac{\partial c}{\partial x} = 2(0.07x) + 75 = 0.14x + 75 \Big|_{(100,50)} = 84$$

$$\frac{\partial c}{\partial y} = 85 \Big|_{(100,50)} = 85$$

(23) La dimensiones de una caja rectangular son; 50, 20 y 30 cm, el margen de error 0.2 cm estima el mayor error posible.

$V = xyz \qquad v = (55)(70)(30) = 15,500 \text{ cm}^3$

$$\frac{dv}{dx} = yz\,dx + xz\,dy + xy\,dz = [(70)(30)(.2)] + [(55)(30)(.2)] + [(55)(70)(.2)]$$

$$\frac{dv}{dx} = 1520$$

$$\frac{dv}{v} = \frac{1520}{115,500} = 0.013 \quad 1.3\%$$

(24) Calcular el volumen de un cono con medidas 1.2m de altura y de 0.5m de radio con un error de 30cm. Estimar el mayor error posible cuando el cono se encuentra esas medidas.

$v = \frac{\pi r^2 h}{3}$ $h = 1.7m$ $r = 0.5m$ $dr = dh = 0.3m$

$\frac{dv}{dr} = \frac{2\pi r h}{3}$ $\frac{dv}{dh} = \frac{2\pi r^2}{3}$

$$\left[\left(\frac{2\pi r h}{3}\right)(dr)\right] + \left[\left(\frac{\pi r^2}{3}\right)(dh)\right] = \left(\frac{2\pi(0.5)(1.m)}{3}\right)(0.3) +$$

$$\left(\frac{\pi(0.5)^2}{3}\right)(0.3) = 0.534 + 0.157 = 0.691$$

$$\frac{0.691}{0.445} = 1.552 = 155.2\%$$

(25) Una compañía de cajas fabricaron cajas, con medidas de 30, y 40cm y 10 cm y 10cm y una margen de error de 5cm

$v = xyz = (30)(40)(10) = 12,000$

$\frac{dv}{dx} = yz\,dx$ $\frac{dv}{dy} = xz\,dy$ $\frac{dv}{dz} = xy\,dz$

$$[(40)(10)(5)] + [(30)(10)(5)] + [(30)(40)(5)]$$

$$2000 + 1500 + 6000 = 9500$$

(26) Una fabrica de tanques tiene $h=90m$ y $r=5m$ con un margen de error de $0.07m$

$V=\pi r^2 h$ $\quad \frac{dv}{dh}=\pi r^2 dh$ $\quad \frac{dv}{dr}=2\pi r h\, dr$

$((\pi r^2)(dh)) + [(2\pi r h)(dr)] = (\pi(5^2)(0.07)) + (2\pi(5)(40)(0.07))$

$=93.43$

$\frac{dv}{v} = \frac{93.43}{3141.59} = 0.029$ $\quad \underline{2.9\%}$ ✓

(27) $w = xy^2 + x^2z + yz^2$

$x=t^2 \quad y=2t \quad z=2$

$\frac{dw}{dx} = y^2 \frac{d}{dy}[y] + z\frac{d}{dx}[x^2] + 0$ $\quad \frac{dw}{dy} = 2xy + z^2$ $\quad \frac{dw}{dz} = x^2 + 2yz$

$[(y^2+2x^2)(2t)] + [(2x+2^2)(2)] + [\cancel{(x^2+2y^2)}(0)]$ $\to 0$

(28) $z = 2xy \quad x = e^{4s} + t^2 \quad y = s^3 + \sin 5t$

$\frac{dz}{dx} = 2y$ $\quad \frac{dz}{dy} = 2x$ $\quad \frac{dx}{ds} = 4e^{4s}$ $\quad \frac{dx}{dt} = 2t$ $\quad \frac{dy}{ds} = 3s^2$ $\quad \frac{dy}{dt} = 5\cos 5t$

$\frac{dz}{ds} = [(2y)(4e^{4s})] + [(2x)(3s^2)]$ $\quad \frac{dz}{dt} = [(2y)(2t)] + [(2y)(5\cos 5t)]$

Escaneado con CamScanner

(29) $z = \sqrt{x^2+y^2}$ $\quad x = e^{2x}$ $\quad y = e^{-2t}$

$\frac{dz}{dx} = \frac{2y}{2(x^2+y^2)}$ $\quad \frac{dz}{dy} = \frac{y}{(x^2+y^2)}$ $\quad \frac{dy}{dt} = 2e^{2t}$ $\quad \frac{dy}{dt} = -2e^{-2t}$

$$\left[\left(\frac{y}{x^2+y^2}\right)(2e^{2t}) + \left(\frac{x}{(x^2+y^2}\right)\left(-2e^{-2t}\right)\right]$$

(30) $w = \frac{x}{y}$ $\quad x = se^{t}$ $\quad y = 1+se^{-t}$

$\frac{dz}{dx} = \frac{1}{y}$ $\quad \frac{dz}{dy} = xy^{-1} = \frac{-x}{y^2}$ $\quad \frac{dx}{dt} = se^{t}$ $\quad \frac{dy}{dt} = -se^{t}$

$\frac{dx}{ds} = e^{t}$ $\quad \frac{dy}{ds} = e^{-t}$

$$\frac{dz}{ds} = \left[\left(\frac{1}{y}\right)(e^{t})\right] + \left[\left(-\frac{x}{y^2}\right)(e^{-t})\right]$$

$$\frac{dz}{dt} = \left[\left(\frac{1}{y}\right)(se^{t})\right] + \left[\left(-\frac{x}{y^2}\right)\left(-se^{-t}\right)\right]$$