① $\begin{cases} 3x_1 + x_2 - 2x_3 = 5 \\ 2x_1 + x_2 - 3x_3 = 5 \\ 5x_1 - x_2 + x_3 = 2 \end{cases}$ $A = \begin{pmatrix} 3 & 1 & -2 \\ 2 & 1 & -3 \\ 5 & -1 & 1 \end{pmatrix}$

$|A| = \begin{vmatrix} 3 & 1 & -2 \\ 2 & 1 & -3 \\ 5 & -1 & 1 \end{vmatrix} \begin{matrix} 3 & 1 \\ 2 & 1 \\ 5 & -1 \end{matrix} = (3 - 15 + 4) - (-10 + 9 + 2) = \boxed{-9} \neq 0$

$\boxed{A \rightarrow U}$

$\begin{pmatrix} 3 & 1 & -2 \\ 2 & 1 & -3 \\ 5 & -1 & 1 \end{pmatrix}$ $R_2 \rightarrow R_1(-\frac{2}{3}) + R_2$ $R_3 \rightarrow R_1(-\frac{5}{3}) + R_3$

$\begin{array}{ccc} -2 & -\frac{2}{3} & \frac{4}{3} \\ 2 & 1 & -3 \\ \hline NR_2 = 0 & \frac{1}{3} & -\frac{5}{3} \end{array}$ $\begin{array}{ccc} -5 & -\frac{5}{3} & \frac{10}{3} \\ 5 & -1 & 1 \\ \hline NR_3 = 0 & -\frac{8}{3} & \frac{13}{3} \end{array}$

$\begin{pmatrix} 3 & 1 & -2 \\ 0 & \frac{1}{3} & -\frac{5}{3} \\ 0 & -\frac{8}{3} & \frac{13}{3} \end{pmatrix}$ $R_3 \rightarrow R_2(8) + R_3$

$\begin{array}{ccc} 0 & \frac{8}{3} & -\frac{40}{3} \\ 0 & -\frac{8}{3} & \frac{13}{3} \\ \hline NR_3 = 0 & 0 & -9 \end{array}$

$U = \begin{pmatrix} 3 & 1 & -2 \\ 0 & \frac{1}{3} & -\frac{5}{3} \\ 0 & 0 & -9 \end{pmatrix}$ $L = \begin{pmatrix} 1 & 0 & 0 \\ \frac{2}{3} & 1 & 0 \\ \frac{5}{3} & -8 & 1 \end{pmatrix}$

$Ly = b$; $\begin{pmatrix} 1 & 0 & 0 \\ \frac{2}{3} & 1 & 0 \\ \frac{5}{3} & -8 & 1 \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} = \begin{pmatrix} 5 \\ 5 \\ 2 \end{pmatrix}$ $\begin{cases} y_1 = 5 \\ \frac{2}{3}y_1 + y_2 = 5 \\ \frac{5}{3}y_1 - 8y_2 + y_3 = 2 \end{cases}$ $\boxed{y_1 = 5}$

$\frac{2}{3}(5) + y_2 = 5$ $\frac{5}{3}(5) - 8(\frac{5}{3}) + y_3 = 2$

$\frac{10}{3} + y_2 = 5$ $\frac{25}{3} - \frac{40}{3} + y_3 = 2$ $\boxed{y_3 = 7}$

$\boxed{y_2 = \frac{5}{3}}$ $-\frac{15}{3} + y_3 = 2$

$Ux = y$; $\begin{pmatrix} 3 & 1 & -2 \\ 0 & \frac{1}{3} & -\frac{5}{3} \\ 0 & 0 & -9 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 5 \\ \frac{5}{3} \\ 7 \end{pmatrix}$ $\begin{cases} 3x_1 + x_2 - 2x_3 = 5 \\ \frac{1}{3}x_2 - \frac{5}{3}x_3 = \frac{5}{3} \\ 9x_3 = 7 \end{cases}$

$9x_3 = 7$ $\frac{1}{3}x_2 - \frac{5}{3}(-\frac{7}{9}) = \frac{5}{3}$ $3x_1 + \frac{10}{9} - 2(\frac{7}{9}) = 5$

$\boxed{x_3 = -\frac{7}{9}}$ $\frac{1}{3}x_2 + \frac{35}{27} = \frac{5}{3}$ $3x_1 + \frac{10}{9} + \frac{14}{9} = 5$ Solución =

$\frac{1}{3}x_2 = \frac{30}{27}$ $3x_1 + \frac{8}{3} = 5$ $X = \begin{pmatrix} \frac{7}{9} \\ \frac{10}{9} \\ -\frac{7}{9} \end{pmatrix}$

$\boxed{x_2 = \frac{10}{9}}$ $3x_1 = \frac{7}{3}$

$\boxed{x_1 = \frac{7}{9}}$

②

$$\begin{cases} A + B + C = 100 \\ 20A + 30B + 50C = 4450 \\ 22A + 33B + 60C = 5270 \end{cases} \qquad \begin{bmatrix} 1 & 1 & 1 \\ 20 & 30 & 50 \\ 22 & 33 & 60 \end{bmatrix} = A$$

$$|A| = \begin{vmatrix} 1 & 1 & 1 \\ 20 & 30 & 50 \\ 22 & 33 & 60 \end{vmatrix} \begin{matrix} 1 & 1 \\ 20 & 30 \\ 22 & 33 \end{matrix} = (1800 + 1100 + 660) - (660 + 1650 + 1200)$$

$$\boxed{|A| = 50} \qquad A^{-1} = \frac{1}{|A|} = \frac{1}{|A|} \, adj\, A$$

$A_{11} = (-1)^2 \begin{vmatrix} 30 & 50 \\ 33 & 60 \end{vmatrix} = (1)(1800) - (1650) = \boxed{150}$

$A_{12} = (-1)^3 \begin{vmatrix} 20 & 50 \\ 22 & 60 \end{vmatrix} = (-1)(1200) - (1100) = \boxed{-100}$

$A = \begin{pmatrix} 150 & -100 & 0 \\ -27 & 38 & -8 \\ 20 & -30 & 10 \end{pmatrix}$

$A_{13} = (-1)^4 \begin{vmatrix} 20 & 30 \\ 22 & 33 \end{vmatrix} = (1)(660) - (660) = \boxed{0}$

$A^T = \begin{pmatrix} 150 & -27 & 20 \\ -100 & 38 & -30 \\ 0 & -8 & 10 \end{pmatrix}$

$A_{21} = (-1)^3 \begin{vmatrix} 1 & 1 \\ 33 & 60 \end{vmatrix} = (-1)(60) - (33) = \boxed{-27}$

$A_{22} = (-1)^4 \begin{vmatrix} 1 & 1 \\ 22 & 60 \end{vmatrix} = (1)(60) - (22) = \boxed{38}$

$A^{-1} = \frac{1}{50} \begin{pmatrix} 150 & -27 & 20 \\ -100 & 38 & -30 \\ 0 & -8 & 10 \end{pmatrix}$

$A_{23} = (-1)^5 \begin{vmatrix} 1 & 1 \\ 22 & 30 \end{vmatrix} = (-1)(30) - (22) = \boxed{-8}$

$$\begin{pmatrix} 3 & \frac{-27}{50} & \frac{20}{50} \\ -2 & \frac{38}{50} & \frac{-30}{50} \\ 0 & \frac{-4}{25} & \frac{1}{5} \end{pmatrix} \begin{pmatrix} 100 \\ 4450 \\ 5270 \end{pmatrix}$$

$A_{31} = (-1)^4 \begin{vmatrix} 1 & 1 \\ 30 & 50 \end{vmatrix} = (1)(50) - (30) = \boxed{20}$

$3 \times ③ \quad ③ \times 1$

$A_{32} = (-1)^5 \begin{vmatrix} 1 & 1 \\ 20 & 50 \end{vmatrix} = (-1)(50) - (20) = \boxed{-30}$

$A_{33} = (-1)^6 \begin{vmatrix} 1 & 1 \\ 20 & 30 \end{vmatrix} = (1)(30) - (20) = \boxed{10}$

$X = \begin{pmatrix} A_{11} \\ A_{21} \\ A_{31} \end{pmatrix}$

Solucion=

$A_{11} = 3(100) - \frac{27}{50}(4450) + \frac{20}{50}(5270) = 5$

$A_{21} = -2(100) + \frac{38}{50}(4450) - \frac{30}{50}(5270) = 20$

$X = \begin{pmatrix} 5 \\ 20 \\ 75 \end{pmatrix}$

$A_{31} = 0(100) - \frac{4}{25}(4450) + \frac{1}{5}(5270) = 75$

③

$X =$ sofas $\longrightarrow 8h \longrightarrow \$1000$ $342h \longrightarrow$ trabajo/semana

$Y =$ sillones $\longrightarrow 6h \longrightarrow \$600$ $\$40200 \longrightarrow$ Material

$$8h + 6h = 342h$$
$$\$1000 + \$600 = \$40200$$

$$A = \begin{pmatrix} 8 & 6 \\ 1000 & 600 \end{pmatrix};$$

$$\begin{cases} 8x + 6y = 342 \\ 1000x + 600y = 40200 \end{cases}$$

$$|A| = (4800) - (6000) = \boxed{-1200}$$

$$|Ax| = \begin{pmatrix} 342 & 6 \\ 40200 & 600 \end{pmatrix} = (205200) - (241200) = \boxed{-36000}$$

$$|Ay| = \begin{pmatrix} 8 & 342 \\ 1000 & 40200 \end{pmatrix} = (321600) - (342000) = \boxed{-20400}$$

$$X = \frac{|Ax|}{|A|} = \frac{-36000}{-1200} = \boxed{30}$$

$$y = \frac{|Ay|}{|A|} = \frac{-20400}{-1200} = \boxed{17}$$

Solucion=

30 sofas.

17 sillones reclinables.