1. $(2y - 2xy^3 + 4x + 6)dx + (2x - 3x^2y^2 - 1)dy = 0$

$M = \frac{\partial f}{\partial x}$ $\quad N = \frac{\partial f}{\partial y}$ $\quad C.I = y(-1) = 0$

$x = -1 \quad y = 0$

$\frac{\partial M}{\partial y} = 2 - 6xy^2 \qquad \frac{\partial N}{\partial x} = 2 - 6xy^2$ si es exacta

$\frac{\partial f}{\partial x} = 2y - 2xy^3 + 4x + 6 \rightarrow$ se integra

$$\int df = \int (2y - 2xy^3 + 4x + 6)\,dx$$

$$f(x,y) = 2xy - x^2y^3 + 2x^2 + 6x + \phi(y)$$

-Ahora derivamos con respecto a "y"

$$\frac{\partial f}{\partial y} = 2x - 3x^2y^2 + \phi'(y)$$

$$2x - 3x^2y^2 - 1 = 2x - 3x^2y^2 + \phi'(y) \rightarrow \int 1\,dy \Rightarrow -y + C$$

$$\cancel{2x}^{0} - \cancel{3x^2y^2}^{0} - 1 - \cancel{2x}^{0} + \cancel{3x^2y^2}^{0} = \phi'(y)$$

$$-1 = \phi'(y)$$

La solucion es:

$$\boxed{2xy + x^2y^3 + 2x^2 + 6x - y = C}$$

$$\cancel{2(-1)(0)}^{0} + \cancel{(-1)^2(0)^3}^{0} + 2(-1)^2 + 6(-1) + 0 = C$$

$$2 - 6 = C \Rightarrow \boxed{C = -4}$$

$$\boxed{2xy + x^2y^3 + 2x^2 + 6x - y = -4}$$

2 $xy' + 2y + 2x^2y^2 = 0 \quad / \cdot \left(\frac{1}{x}\right)$

$$y' + \frac{2y}{x} + \frac{2x^2y^2}{x} = 0$$

$$y' + \frac{2}{x}y + 2xy^2 = 0 \Rightarrow \text{Ecuacion bernoulli: con } n = 2$$

-multiplicamos la ecuacion por y^{-2}:

$$y'y^{-2} + \frac{2}{x}yy^{-2} + 2xy^2y^{-2} = 0$$

$$y'y^{-2} + \frac{2}{x}y^{-2} + 2x = 0$$

$$-z' + \frac{2}{x}z + 2x = 0$$

$$\Rightarrow -z' + \frac{2}{x}z = -2x \quad / \cdot (-1)$$

$$z' - \frac{2}{x}z = 2x \rightarrow \text{Ecuacion diferencial Lineal de Primer orden}$$

Factor integrante: $\mu(x) = e^{\int -\frac{2}{x}dx}$

$$\mu(x) = e^{-2\ln x} = e^{\ln x^{-2}} = x^{-2} = \frac{1}{x^2}$$

$$\frac{1}{x^2}z' - \frac{2}{x^3}z = \frac{2\cancel{x}}{x^{\cancel{2}}}$$

$$\frac{d}{dx}\left(\frac{1}{x^2}z\right) = \frac{2}{x}$$

$$\int\left(\frac{1}{x^2}z\right)\frac{d}{dx} = \frac{2}{x}$$

$$\int \frac{d}{dx}\left(\frac{1}{x^2}z\right)dx = \int \frac{2}{x}dx$$

$$\frac{1}{x^2}\cdot z = 2\ln|x| + C$$

$$z = 2x^2\ln|x| + Cx^2$$

$$\boxed{y = \frac{1}{2x^2\ln|x| + Cx^2}}$$

③ $(2x^2+y)dx + (x^2y-x)\,dy = 0$

$M = \frac{\partial f}{\partial x}$ $\quad N = \frac{\partial f}{\partial y}$

$\frac{\partial M}{\partial y} = 1 \qquad \frac{\partial N}{\partial x} = 2xy-1 \qquad \frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}$ No es exacta

$$\frac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}{N} = \frac{1-(2xy-1)}{x^2y-x} = \frac{1-2xy+1}{x(xy-1)} = \frac{2-2xy}{x(xy-1)} = \frac{-2\cancel{(1-xy)}}{x\cancel{(1-xy)}}$$

$$g(x) = -\frac{2}{x} \Rightarrow \mu = e^{\int -\frac{2}{x}dx} = e^{-2\ln|x|} = e^{\ln x^{-2}} = x^{-2} = \frac{1}{x^2} = \frac{-2}{x}$$

$\frac{1}{x^2}$ Factor integrante

$$\frac{1}{x^2}(2x^2+y)dx + \frac{1}{x^2}(x^2y-x)dy = 0$$

$$\left(2+\frac{y}{x^2}\right)dx + \left(y-\frac{1}{x}\right)dy = 0$$

$M(x,y) = 2 + \frac{y}{x^2} \Rightarrow \frac{\partial M}{\partial y} = \frac{1}{x^2}$

$N(x,y) = y - \frac{1}{x} \Rightarrow \frac{\partial N}{\partial x} = -\left(-\frac{1}{x^2}\right) = \frac{1}{x^2}$

Ya es exacta

$$\frac{\partial f}{\partial x} = M(x,y) = 2+\frac{y}{x^2} \Rightarrow f(x,y) = \int\left(2+\frac{y}{x^2}\right)dx$$

$$2x + y\frac{x^{-2+1}}{(-2+1)} + g(y) \qquad f(x,y) = 2x - \frac{y}{x} + g(y)$$

$$\frac{\partial f}{\partial y} = N(x,y) = y - \frac{1}{x}\,;\ \frac{\partial f}{\partial y} = -\cancel{\frac{1}{x}} + g'(y) = y - \cancel{\frac{1}{x}}$$

$\int y\,dy$

$g(y) = \int y\,dy$

$g(y) = \frac{y^2}{2} + C$

$$f(x,y) = 2x - \frac{y}{x} + \frac{y^2}{2} + C$$

$$\boxed{2x - \frac{y}{x} + \frac{y^2}{2} = C}$$

4

$T_F = 98°C$

$T_A = 21°C$

$T(10) = 88°C$

$T(?) = 35°C$

$T(0) = 98°C$

$$\frac{dT}{dt} = k(T - T_A)$$

$$\frac{dT}{dt} = k(T - 21)$$

$$\frac{dT}{T-21} = k\,dt \Rightarrow \int \frac{dT}{T-21} = \int k\,dt$$

$$\ln|T-21| = kt + c$$

$$\cancel{e}^{\ln|T-21|} = e^{kt+c}$$

$$T - 21 = e^{kt} \cdot e^{c}$$

$$\boxed{T(t) = Ce^{kt} + 21}$$

$T(0) = 98$

$$98 = Ce^{k(0)} + 21$$

$$77 = C + 21$$

$$\boxed{C = 77}$$

$\boxed{\text{Tomamos las C.I.}}$

$T(10) = 88$

$$88 = Ce^{k(10)} + 21$$

$$Ce^{10k} = 67$$

$$\ln C + \cancel{\ln} e^{10k} = \ln|67|$$

$$\ln C + 10k = \ln|67|$$

$$\ln 77 + 10k = \ln|67|$$

$$10k = \ln|67| - \ln|77|$$

$$10k = -0.139112$$

$$k = \frac{-0.139112}{10} = \boxed{-0.013911}$$

a) $T(t) = 35$

$$35 = Ce^{kt} + 21$$

$$14 = Ce^{(-0.013911)t}$$

$$14 = Ce^{-0.013911t}$$

$$\ln 14 = \ln^{C} + \cancel{\ln} e^{-0.013911t}$$

$$\ln 14 = \ln^{77} - 0.013911t$$

$$2.639057 = 4.343805 - 0.013911t$$

$$-0.013911t = -1.704748$$

$$t = \frac{-1.704748}{-0.013911}$$

$$\boxed{t = 122.6 \text{ min}}$$