Longitud de arco

1- $r(t) = \langle t, 3\cos t, 3\operatorname{sen} t \rangle, \quad -5 \leq t \leq 5$

$r'(t) = \langle 1, -3\operatorname{sen} t, -3\cos t \rangle$

$|v| = \sqrt{x^2 + y^2 + z^2}$

$|v| = \sqrt{(1)^2 + (-3\operatorname{sen} t)^2 + (-3\cos t)^2}$

$|v| = \sqrt{1 + 9\operatorname{sen}^2 t + 9\cos^2 t}$

$|v| = \sqrt{1 + 9}$

$|v| = \sqrt{10}$

$l = \int_{-5}^{5} = \left[\sqrt{10}\,(-5)\right] - \left[\sqrt{10}\,(5)\right]$

$l \approx 31.62277$

2- $r(t) = \langle 2t, t^2, \frac{1}{3}t^3 \rangle, \quad 0 \leq t \leq 1$

$r'(t) = \langle 2, 2t, t^2 \rangle$

$|v| = \sqrt{(2)^2 + (2t)^2 + (t^2)^2} = \sqrt{4 + 4t^2 + t^4} = \sqrt{4}\,\sqrt{(1 + t^2 + t^4)}$

$|v| = \sqrt{4} + \sqrt{(t^2+1)^2} = 2 + t^2 + 1 = \underline{3 + t^2}$

$\int_0^1 = 3 + t^2 = 3t + \frac{t^3}{3}$

$\int_0^1 = \left[\cancel{3(0) + \frac{(0)^3}{3}}\right]^{1,0} + \left[3(1) + \frac{(1)^3}{3}\right] = \underline{1}$

3: $r(t) = i + t^2 j + t^3 k \quad 0 \leq t \leq 1$

$r(t) = 1 + t^2 + t^3$

$r'(t) = <0, t, t^2>$

$|v| = \sqrt{\cancel{(0)^2} + (t)^2 + (t^2)^2}$

$|v| = \sqrt{t^2 + t^4} =$

$|v| = \sqrt{(t^2) + (1 + t^2)}$

$|v| = t\sqrt{1 + t^2}$

$\int_0^1 t\sqrt{1+t^2} = \frac{2\sqrt{2}}{3} = -\frac{1}{3}$

4: $r(t) = < t^2, t^3, t^4 > \quad 0 < t < 2$

$r'(t) = 2t, 3t^2 + 4t^3$

$|v| = \sqrt{(2t)^2 + (3t^2)^2 + (4t^3)^2} = t\sqrt{16t^4 + 9t^2 + 4}$

$\int_0^2 = t\sqrt{25t + 4}\,dt = \frac{4\,008\sqrt{6} + 129}{9375}$

$\int_0^2 \simeq 12.03457$

5: $r(t) = \langle \text{sen}\,t, \cos t, \tan 1 \rangle, \quad 0 < t < \pi/4$

$r'(t) = \cos t, -\text{sen}\,t, \sec^2(t)$

$|v| = \sqrt{(\cos t)^2 + (-\text{sen}\,t)^2 + (\sec^2(t))^2}$

$|v| = \sqrt{\sec^4(t) + 1}$

$|v| = \sec^2 t + \sqrt{1}$

$$\int_0^{\pi/4} \frac{\tan^3(t)}{3} + \tan(t) + t$$

$$\int_0^{\pi/4} = \left[\frac{\tan^3\left(\frac{\pi}{4}\right)}{3} + \tan\left(\frac{\pi}{4}\right) + \frac{\pi}{4} \right] \approx$$

$0.0045695 + 0.01370 + 0.7853$

$$\int \approx \underline{0.803569}$$

6 = Calcular $T(t)$ y $N(t)$

$R(t) = 3\cos t\, i + 3\operatorname{sen} t\, j \;:\; t1 = \frac{1}{2}\pi$

$R'(t) = -3\operatorname{sen} + 3\cos$

$\|R\| = 3\sqrt{\operatorname{sen}^2 + \cos^2} = 3$

$T = \frac{R'}{|R|} = \frac{-3\operatorname{sen} + 3\cos}{3} = \underline{-\operatorname{sen} + \cos}$

$T' = -\cos - \operatorname{sen}$

$N = -\cos - \operatorname{sen}$

$|T'| = \sqrt{\cos^2 + \operatorname{sen}^2} = 1$

$N = \frac{T}{|T|} = \frac{-\cos - \operatorname{sen}}{1} = \underline{-\cos - \operatorname{sen}}$

aproximadamente.

Derivada parcial

7= Calcular la derivada parcial con el método de la cadena
$z = x^2 \operatorname{sen} y$ con respecto a s

$x(s,t) = s^2 + t^2$
$y(s,t) = 2st$

$$\frac{\partial z}{\partial s} = \frac{\partial z}{\partial x} \frac{\partial x}{\partial y} \frac{\partial x}{\partial s} + \frac{\partial z}{\partial y} \frac{\partial y}{\partial s}$$

$$\frac{\partial z}{\partial s} = 2x \operatorname{sen} y (2s) + z^2 (\cos y)(2t)$$

$$\frac{\partial z}{\partial s} = 2(s^2+t^2) \operatorname{sen}(2st)(2s) + (s^2+t^2) \cos(2st) 2t$$

8= Del problema anterior, Calcular la derivada parcial con respecto a t.

$$\frac{\partial z}{\partial t} = \frac{\partial z}{\partial y} \frac{\partial x}{\partial t} + \frac{\partial z}{\partial y} \frac{\partial y}{\partial t}$$

$$\frac{\partial z}{\partial t} = 2x \operatorname{sen} y (2t) + x^2 \cos y (2s)$$

$$\frac{\partial z}{\partial t} = 2(s^2+t^2) \operatorname{sen}(2st)(2t) + (s^2+t^2)^2 \cos(2st)(2s)$$

9 = Calcular la derivada parcial con respecto a x $xyz = \cos(x+y+z)$

$xyz = \cos(x+y+z)$

$$\overbrace{\cos(x+y+z) - xyz = 0}^{F(x+y+z)}$$

$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z}$$

$$\frac{\partial t}{\partial y} = -\frac{F_y}{F_z}$$

$F_x = -\operatorname{sen}(x+y+z) - yz$

$F_y = -\operatorname{sen}(x+y+z) - xz$

$F_z = -\operatorname{sen}(x+y+z) - xy$

$$\frac{\partial z}{\partial x} = -\frac{\operatorname{sen}(x+y+z) - yz}{\operatorname{sen}(x+y+z) - xy}$$

10 = Del ejercicio anterior calcular la derecha para con respecto a y

$$\frac{\partial z}{\partial x} = \frac{-F_x}{F_z}$$

$$\frac{\partial z}{\partial y} = -\frac{F_y}{F_z}$$

$$\frac{\partial z}{\partial y} = \frac{-\operatorname{sen}(x+y+z) - xz}{\operatorname{sen}(x+y+z) - xy}$$

11 = $F(x,y) = x^5 + 3x^3y^2 + 3xy^4$

$$\frac{\partial f}{\partial x} = f_x \qquad \frac{\partial f}{\partial y} = F_y$$

$$\frac{\partial f}{\partial x} = 5x^4 + 4x^2y^2 + 3y^4$$

$$\frac{\partial F}{\partial y} = 6x^3y + 12xy^3$$

12= $f(x,y) = \frac{x-y}{x+y}$

$$f(x) = \frac{x+y-(x-y)}{(x+y)^2} = \frac{x+y-x+y}{(x+y)^2} = \frac{2y}{(x+y)^2}$$

$$f(y) = \frac{-(x-y)-(x-y)}{(x+y)^2} = \frac{-x+y-x+y}{(x+y)^2} = \frac{-2x+2y}{(x+y)^2}$$

13= $f(r,s) = r\ln(r^2+s^2)$

$$\frac{\partial f}{\partial r} = \ln(r^2+s^2) + r\frac{1}{r^2+s^2}\cdot 2r = \ln(r^2+s^2) + \frac{2r^2}{r^2+s^2}$$

$$\frac{\partial f}{\partial s} = r\frac{1}{r^2+s^2}\,2s = \frac{2rs}{r^2+s^2}$$

14= $f(x,t) = \tan^{-1}(x\sqrt{t})$

$$\frac{\partial f}{\partial x} = \frac{1}{1+x^2t}\cdot\sqrt{t} = \frac{\sqrt{t}}{1+x^2t}$$

$$\frac{\partial f}{\partial t} = \frac{1}{1+x^2t}\cdot\frac{x}{2\sqrt{t}}$$

15= $f(x,y,z) = x^2 e^{yz}$

$f(x) = 2x\, e^{y}$

$f(y) = x^2 e^{yz} z$

$f(z) = x^2 e^{yz} y$

16= $f(x,y,z,t) = \frac{xy^2}{t+2z} = xy^2(t+2z)^{-1}$

$f(x) = \frac{y^2}{t+2z}$

$f(y) = \frac{2xy}{t+2t}$

$f(z) = -xy^2(t+2z)^{-2}2 = \frac{-2xy^2}{(t+2t)^2}$

$f(t) = -xy^2(t+2z)^{-2} = \frac{-xy^2}{(t+2z)^2}$

17= $z = y \cdot \ln x$

$z_x = y \cdot \frac{1}{x} = \frac{y}{z}$

$z_y = \ln x$

18= En una cervecería, fabrican con una h de 10m y un radio de 3cm, considerar como 0.02cm. Calcule el volumen y cambio absoluto.

$V = \pi r^2 h \quad \frac{dv}{dh} = \pi r^2 dh \quad \frac{dv}{dr} = 2\pi r h \, dr$

$$[((\pi r^2)(dh)) + ((2\pi r h)(dr))] =$$

$$[((3^2)(0.02)) + (2\pi(3)(10)(0.02))]$$

$$0.18 + 3.7699 = \frac{3.9494}{282.74} = 0.01407$$

$\pi(3)^2(10)$

$\underline{1.4\%}$

19= Dairy Queen esta fabricando conos refenemos con medidas: 15 cm de altura y 5cm de radio, con un error de 0.03 cm. Estima el mayor error posible.

$V = \frac{\pi r^2 h}{3}$

$h = 15cm$
$r = 5cm$
$dr = dh = 0.03cm$

$\frac{dv}{dr} = \frac{2\pi r h}{3} \quad \frac{dv}{dh} = \frac{2\pi r^2}{3}$

$$\left[\left(\frac{2\pi r h}{3}\right)(dr)\right] + \left[\left(\frac{\pi r^2}{3}\right)(dh)\right] = \left(\frac{2\pi(5)(15)}{3}\right)(0.03) + \left(\frac{\pi(5)^2}{3}\right)(0.03)$$

$$4.7123 + 0.7853 = \frac{5.4977}{392.70} = 0.013 = \underline{1.39\%}$$

20= Las zucaritas son empacadas en cajas con medidas de 60cm, 20cm y 10cm con un margen de error de 0.5cm

$V = xyz = (60)(20)(10) = 12{,}000$

$\frac{dv}{dx} = yz\,dx \quad \frac{dv}{dy} = xz\,dy \quad \frac{dy}{dz} = xy\,dz$

$[(60)(10)(0.5)] + [(20)(10)(0.5)] + [(20)(60)(0.5)]$

$300 + 100 + 150 = 550$

$\frac{550}{12000} = 0.045 \quad \underline{4.5\%}$

21= La caja de dulces que contienen los dulces que poquito vende en el semaforo es de 20cm, 10cm y 5cm cada medida de las cajas es correcto con un margen de error de 0.2cm. Estima el mayor posible.

$V = xyz$

$V = (20)(10)(5) = 1000cm^3$

$\frac{dv}{dx} = yz\,dx \quad \frac{dv}{dy} = xz\,dy \quad \frac{dv}{dz} = xy\,dz$

$[(10)(5)(0.2)] + [(20)(5)(0.2)] + [(20)(10)(0.2)]$

$10 + 20 + 40 = 70$

$\frac{70}{1000} = 0.07 \approx 7\%$

Escaneado con CamScanner

22= La produccion de una fabrica es $P(L,K)=120K^{1/5}L^{1/5}+C$ donde K es el gasto del capital en dolares y L es el tamaño de la fuerza laboral Calcule cuanto = PL=243 y PK=256

$$\frac{\partial P}{\partial K}=120L^{1/5}\frac{\partial}{\partial K}K^{1/5}+10\frac{\partial}{\partial K}K \qquad \frac{\partial P}{\partial L}=120K^{1/5}\frac{\partial}{\partial L}L^{1/5}$$

$$\frac{1}{5}\frac{120L^{1/5}}{K^{4/5}}=24\frac{L^{1/5}}{K^{4/5}}+10 \qquad \frac{1}{5}\frac{120K^{1/5}}{L^{4/5}}=24\frac{K^{1/5}}{L^{4/5}}$$

$$\frac{24(243)^{1/5}}{(254)^{4/5}}+10=10.85 \qquad \frac{24(256)^{1/5}}{(243)^{4/5}}=\underline{0.89}$$

23- La demanda de un bien D, esta en funsion de su presio "p" y del nivel de la renta B, estan relacionadas por el modelo $D(P\cdot K)=\ln(3p+B)+\sqrt[3]{2p+B^2}$ cuanto p=12 y B=20

$$\frac{\partial D}{\partial p}=\frac{3}{3p+B}+\frac{2}{3(2p+B^2)^{2/3}}\bigg|_{(12,20)}=\frac{1}{3(12)+20}+\frac{2}{3(2(12)+20^2)^{1/3}}=\underline{0.029}$$

$$\frac{\partial D}{\partial p}=\frac{1}{(3p+B)}+\frac{2B}{3(2p+B^2)^{2/3}}\bigg|_{(12,20)}=\frac{1}{(3(12)+20))}+\frac{2(20)}{3(2(12)+(20)^{2/5}}=\underline{0.25}$$

24= $\operatorname{sen}(x^2y^5)+4x^2+3y^2+3$

$$\frac{dz}{dx}=2xy^3\cos(x^2y^3)+8x \qquad \frac{dz}{dy}=5x^2y^4\cos(x^2y^5+6y)$$

Escaneado con CamScanner

25= Una empresa fabrica balones, el modelo Champion (x) y el modelo supercopa (y) el costo de la función semanal es:

$c(x, y) = 6.07x^2 + 75x + 85y + 6000$ determine costo

$x = 100 \quad y = 50$

$$\frac{\partial c}{\partial x} = 2(0.07x) + 75 = 0.14x + 75 \Big|_{(100,50)} = 84$$

$$\frac{\partial c}{\partial y} = 85 \Big|_{(100,50)} = \underline{85}$$

26= La dimensiones de una caja rectangular son ; 50, 20 y 30 cm, el margen de error 0.2 cm estima el mayor error posible.

$V = xyz \qquad v = (55)(70)(30) = 115,500\ cm^3$

$$\frac{dv}{dx} = yz\,dx + xz\,dy + xy\,dz = \left[(70)(30)(.2)\right] + \left[(55)(30)(.2)\right] + \left[(55)(70)(.2)\right]$$

$$\frac{dv}{dx} = 1520$$

$$\frac{dv}{v} = \frac{1520}{115,500} = 0.013 \quad \underline{1.3\%}$$

En los siguientes ejercicios, el vector posición r describe la trayectoria de un objeto que se mueve en el espacio. Hallar velocidad, rapidez y aceleración

27= $r(t) = \langle ti, 5tj + 3tk \rangle$

velocidad $= r'(t) \langle 1, 5, 3 \rangle$

Rapidez $= |r'(t)| = \sqrt{(1)^2 + (5)^2 + (3)^2} = \sqrt{35} \approx \underline{5.9160}$

$a(t) = r''(t) = \langle 0, 0, 0 \rangle$ aceleración

28= $r(t) = ti + t^2 j + \frac{t^2}{4} k$, $t = 2$

velocidad $= r'(t) = 1 + 2t + t$

Rapidez $= |r'(t)| = \sqrt{(1)^2 + (2(2))^2 + (2)^2} = \sqrt{21} \approx \underline{4.582}$

Vector aceleración $= r''(t) = \langle 0, 2, 1 \rangle$

29= $r(t) = t^2 i + tj + 2t^{3/2} k$ $t = 4$

velocidad $= r'(t) = 2t + 1 + 3\sqrt{t}$

Rapidez $= |r'(t)| = \sqrt{(2(4))^2 + (1)^2 + (3\sqrt{4})^2}$

$\sqrt{64 + 1 + 36} = \sqrt{101} \approx \underline{10.049}$

Vector aceleración $= r''(t) = \langle 2, 0, \frac{3}{2\sqrt{t}} \rangle$

Escaneado con CamScanner

$30 = r(t) = \langle \ln t, \frac{1}{t}, t^4 \rangle$; $r = 2$

velocidad $= r'(t) = \langle \frac{1}{t}, -\frac{1}{t^2}, 4t^3 \rangle$

Rapidez $= |r'(t)| = \sqrt{\left(\frac{1}{t}\right)^2 + \left(-\frac{1}{t^2}\right)^2 + \left(4t^3\right)^2}$

$= \sqrt{\left(\frac{1}{2}\right)^2 + \left(-\frac{1}{2^2}\right)^2 + \left(4(2)^3\right)^2}$

$= \sqrt{\frac{1}{4} + \frac{1}{16} + 1024}$

$= \sqrt{\frac{16389}{16}} = \sqrt{1024.3125}$

≈ 32.0048882

$\frac{1}{4} + \frac{1}{16} = \frac{16+4}{64} = \frac{20}{64} = \frac{10}{32} = \frac{5}{16}$

$\frac{5}{16} + \frac{1024}{1} = \frac{5 + 16384}{16} = \frac{16389}{16}$

Vector aceleración $= r''(t) = \langle -\frac{1}{t^2}, \frac{2}{t^3}, 12t^2 \rangle$