3 Parcial EDO

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① $y'' - 9y = 20\sin x + 3x^2$

$y = y_h + y_p$

y_h = S. homogenea

$y'' - 9y = 0 \qquad y = e^{rt}$

$r^2 - 9 = 0 \qquad r_1 = 3 \quad r_2 = -3$

$y_h = Ce^{3x} + Cze^{-3x}$

(y_p) 2

S. Particular de $3x^2$

$y_1 = Ax^2 + Bx + C$

$y_1' = 2Ax + B$

$y_1'' = 2A$

$y'' - 9y = 3x^2$

$-9Ax^2 - 9B - 9C + 2A = 3x^2$

$-9A = 3 \qquad A = -1/3$

$-9B = 0 \qquad B = 0$

$2A - 9C = 0 \qquad C = -2/27$

$y_1 = \frac{x^2}{3} - \frac{2}{27}$

S. Particular de $20\sin x$

$\alpha + Bi = 1 \quad s = 0$

$y_2 = B\sin x + A\cos x$

$y_2' = -B\cos x + A\sin x$

$y_2'' = -B\sin x - A\cos x$

$y'' - 9y = 20\sin x$

$-10B\sin x - 10A\cos x = 20\sin x$

$-10B = 20 \qquad A = 0$

$-10A = 0 \qquad B = -2$

$y_2 = -2\sin x$

$y_p = -\frac{x^2}{3} - \frac{2}{27} - 2\sin x$

$y = C_1e^{3x} + C_2e^{-3x} - \frac{x^2}{3} - \frac{2}{27} - 2\sin x$

② $y'' - 4y' + 2y = 5e^x$

$y'' - 4y' + 2y = 0 \rightarrow m^2 - 4m + 2 = 0$

$$\frac{4 \pm \sqrt{16 - 4(2)}}{2} = \frac{4 \pm \sqrt{8}}{2} = \frac{4 \pm 2\sqrt{2}}{2}$$

$$= 2 \pm \sqrt{2}$$

$m_1 = 2 + \sqrt{2} \quad m_2 = 2 - \sqrt{2}$

$y_p = Ae^x$

$y'_p = Ae^x$

$y''_p = Ae^x$

$Ae^x - 4(Ae^x) + 2(Ae^x) = 5e^x$

$-4Ae^x + 3Ae^x = 5e^x$

$-1Ae^x = 5e^x$

$-1A = 5$

$A = \frac{5}{-1}$

$A = -5 \qquad y_p = -5e^x$

$y_c = C_1 e^{(2-\sqrt{2})x} + C_2 e^{(2+\sqrt{2})x} - 5e^x$

③ $y'' - 4y' + 4y = t^3$ $\quad y(0) = 1 \quad y'(0) = 0$

$\mathcal{L}\{y'' - 4y' + 4y\} \quad \mathcal{L}\{t^3\}$

$\mathcal{L}\{y''\} - 4\mathcal{L}\{y'\} + 4\mathcal{L}\{y\} = \mathcal{L}\{t^3\}$

$s^2\mathcal{L}\{y\} - s\,y(0) - y'(0) - 4[s\mathcal{L}\{y\} - y(0)] + 4\mathcal{L}\{y\} = 6/s^4$

$s^2\mathcal{L}\{y\} - s(1) - 4s\mathcal{L}\{y\} - 4(-1) + 4\mathcal{L}\{y\} = 6/s^4$

$s^2\mathcal{L}\{y\} - s - 4s\mathcal{L}\{y\} + 4 + 4\mathcal{L}\{y\} = 6/s^4$

$s^2\mathcal{L}\{y\} - 4s\mathcal{L}\{y\} + 4\mathcal{L}\{y\} = 6/s^4 + s - 4$

$[s^2 - 4s + 4] \cdot \mathcal{L}\{y\} = 6s^{-4} + s - 4$

$\mathcal{L}\{y\} = \dfrac{6s^{-4} + s - 4}{s^2 - 4s + 4}$

$\mathcal{L}\{y\}\,\dfrac{6s^5 - 4s^4}{s^4(s^2 - 4s + 4)} = \dfrac{6 + s^5 - 4s^4}{s^4(s-2)^2}$

$y = \mathcal{L}^{-1}\left\{\dfrac{6 + s^5 - 4s^4}{s^4(s-2)^2}\right\}$

Fracciones parciales

$$\frac{A}{s} + \frac{B}{s^2} + \frac{C}{s^3} + \frac{D}{s^4} + \frac{E}{s-2} + \frac{F}{(s-2)^2}$$

$$\frac{6 + s^5 - 4s^4}{s^4(s-2)^2} = \frac{A}{s} + \frac{B}{s^2} + \frac{C}{s^3} + \frac{D}{s^4} + \frac{E}{s-2} + \frac{F}{(s-2)^2}$$

$$6 + s^5 - 4s^4 = As^3(s-2)^2 + Bs^2(s-2)^2 + Cs(s-2)^2 + Ds(s-2)^2 + E(s-2)s^4 + Fs^4$$

$s = 0$

$6 = D(-2)$

$6 = 4D$

$D = \frac{6}{4} = \frac{3}{2}$

$s = 2$

$6 + 5(2)^5 - 4(2)^4 = F(2)^4$

$16F = -26$

$F = -\frac{13}{8}$

$s^5(A+E) + s^4\left(B - 4A - 2E - \frac{13}{8}\right) + s^3(4A + C - 4B) + \rightarrow$

$\rightarrow s^2(4B + 3/2 - 4C) + s(4C - 6) + 6$

$4C - 6 = 0$

$4B - 4C + 3/2 = 0$

$4A - 4B + C = 0$

$-4A + B - 2E - \frac{13}{8} = -4$

$A + E = 1$

$A = 3/4$

$B = 9/8$

$C = 3/2$

$E = 1/4$

$$y = \mathcal{L}^{-1}\left\{\frac{3}{4s} + \frac{9}{8s^2} + \frac{3}{2s^3} + \frac{3}{2s^4} + \frac{1}{4(s-2)} - \frac{13}{8(s-2)^2}\right\}$$

$$y = 3/4\ \mathcal{L}^{-1}\left\{\frac{1}{s}\right\} + \frac{9}{8}\mathcal{L}^{-1}\left\{\frac{1}{s^2}\right\} + \frac{3}{4}\mathcal{L}^{-1}\left\{\frac{2}{s^3}\right\} + \frac{1}{4}\mathcal{L}^{-1}\left\{\frac{6/4}{s^4}\right\}$$

$$+ \frac{1}{4}\mathcal{L}^{-1}\left\{\frac{1}{s-2}\right\} + \mathcal{L}^{-1}\left\{\frac{-13}{8(s-2)^2}\right\}$$

$$y = \frac{3}{4} + \frac{9t}{8} + \frac{3t^2}{4} + \frac{t^3}{4} + \frac{1}{4}e^{2t} - \frac{13}{8}e^{2t}$$