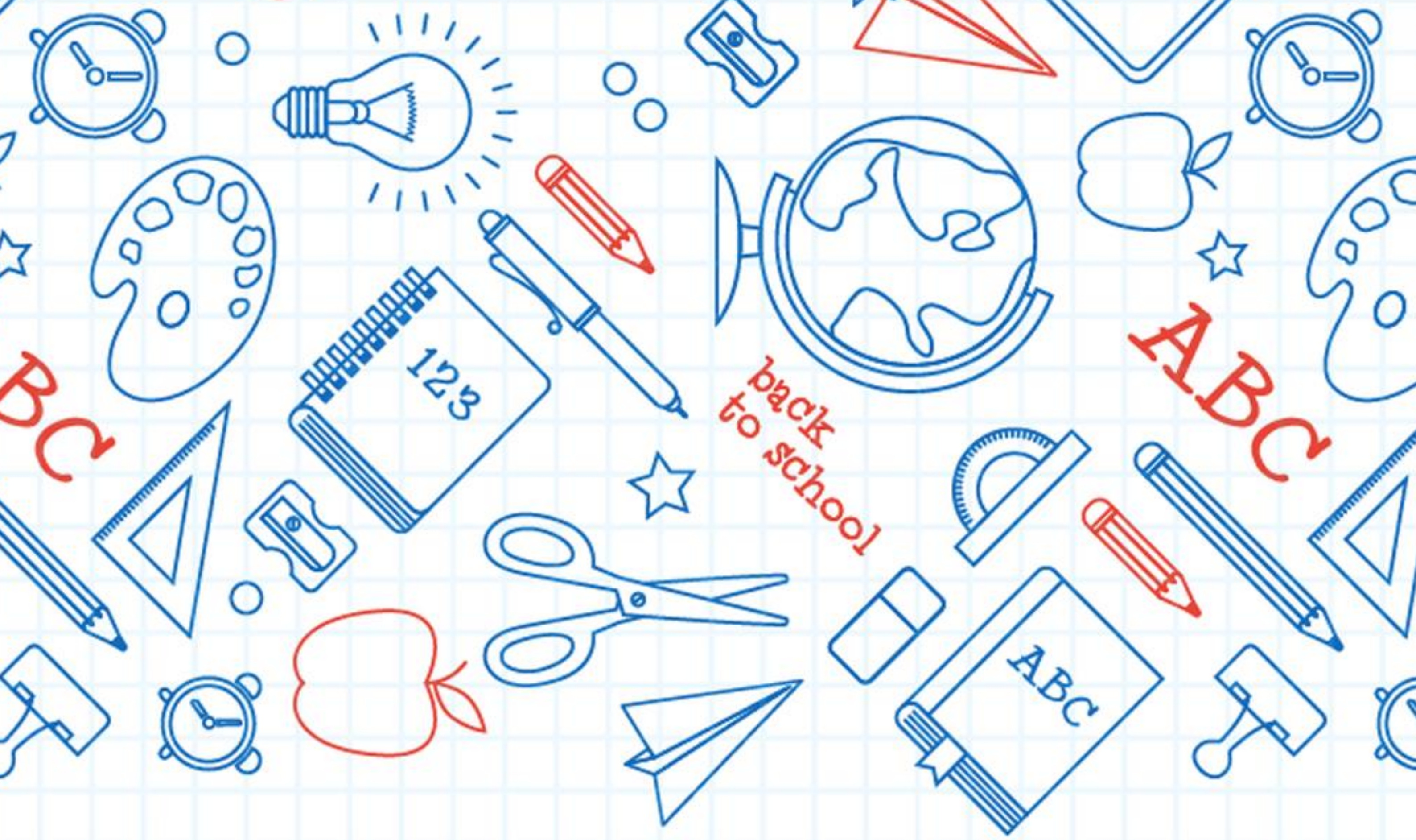

Algebra lineal

Universidad autónoma de Coahuila

TSP

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FACTORIZACION LU

Factorizacion Lu

Problema 10

$$10 = \begin{cases} 2x + y + z = 4 \\ 4x - y + 2z = -1 \\ 2x + 3y + 8z = 3 \end{cases} \qquad \begin{vmatrix} 2 & 1 & 1 \\ 4 & -1 & 2 \\ 2 & 3 & 8 \end{vmatrix} \quad |A| = -42 \neq 0$$

$$\begin{matrix} 2 & 1 & 1 \\ 4 & -1 & 2 \\ 2 & 3 & 8 \end{matrix} \rightarrow R_2 = R_1(-2) + R_2 \qquad \begin{array}{ccc} -4 & -2 & -2 \\ 4 & -1 & 2 \\ \hline 0 & -3 & 0 \end{array}$$

$$\begin{matrix} 2 & 1 & 1 \\ 0 & -3 & 0 \\ 2 & 3 & 8 \end{matrix} \rightarrow R_3 = R_1(-1) + R_3 \qquad \begin{array}{ccc} -2 & -1 & -1 \\ 2 & +3 & -1 \\ \hline 0 & 2 & -2 \end{array}$$

$$\begin{matrix} 2 & 1 & 1 \\ 0 & -3 & 0 \\ 0 & 2 & -2 \end{matrix} \rightarrow R_3 = R_2\left(+\tfrac{2}{3}\right) + R_3 \qquad \begin{array}{ccc} 0 & -2 & 0 \\ 0 & 2 & -2 \\ \hline 0 & 0 & -2 \end{array}$$

$$U = \begin{vmatrix} 2 & 1 & 1 \\ 0 & -3 & 0 \\ 0 & 0 & -2 \end{vmatrix} \qquad L = \begin{vmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ -1 & \tfrac{2}{3} & 1 \end{vmatrix}$$

Resolver $Ly = b$

$$\begin{pmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ -1 & \tfrac{2}{3} & 1 \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} = \begin{matrix} 4 \\ -1 \\ 3 \end{matrix} \qquad \begin{matrix} y_1 = 4 \\ -2y_1 + y_2 = -1 \\ -y_1 + \tfrac{2}{3}y_2 + y_3 = 3 \end{matrix}$$

$$-2(4) + y_2 = -1$$

$$y_2 = 8 - 1 = 7$$

$$-4 + \tfrac{2}{3}(7) + y_3 = 3$$

$$-4 + \tfrac{14}{3} + y_3 = 3$$

$$\tfrac{2}{3} + y_3 = 3$$

$$y_3 = 3 - \tfrac{2}{3} = \tfrac{7}{3}$$

$$y_1 = 4$$
$$y_2 = 7$$
$$y_3 = \tfrac{7}{3}$$

Problema 10

$Ux = y$

$$\begin{matrix} 4 \\ 7 \\ 7/3 \end{matrix} \qquad \begin{matrix} 2 & 1 & 1 \\ 0 & -3 & 0 \\ 0 & 0 & -2 \end{matrix}$$

$$\begin{pmatrix} 2 & 1 & 1 \\ 0 & -3 & 0 \\ 0 & 0 & -2 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{matrix} 4 \\ 7 \\ 7/3 \end{matrix}$$

$$2x_1 + x_2 + x_3 = 4$$
$$-3x_2 = 7$$
$$-2x_3 = 7/3$$

$$-2x_3 = 7/3$$

$$x_3 = \frac{7/3}{-2} = -7/6$$

$$-3x_2 = 7$$

$$x_2 = -\frac{7}{3}$$

$$2x_1 + x_2 + x_3 = 4$$

$$2x_1 - \frac{7}{3} - 7/6 = 4$$

$$2x_1 - \frac{7}{2} = 4$$

$$x_1 = \frac{4 + \frac{7}{2}}{2} = \frac{15}{4}$$

$$\frac{15}{2}$$

$$x_1 = \frac{15}{4}$$

$$x_2 = -\frac{7}{3}$$

$$x_3 = -\frac{7}{6}$$

Solucion unica

Problema 11

11 $\begin{cases} x + y + z = 2 \\ 4x - 8y + 3z = -2 \\ 2x - 2y + 2z = 1 \end{cases}$ $\begin{vmatrix} 1 & 1 & 1 \\ 4 & -8 & +3 \\ 2 & -2 & 2 \end{vmatrix}$ $|A| = -4 \neq 0$ //

$\begin{matrix} 1 & 1 & 1 \\ 4 & -8 & +3 \\ 2 & -2 & 2 \end{matrix} \rightarrow R_2 = R_1(-4) + R_2$

$\begin{matrix} -4 & -4 & -4 \\ 4 & -8 & 3 \\ \hline 0 & -12 & -1 \end{matrix}$

$\begin{matrix} 1 & 1 & 1 \\ 0 & -12 & -1 \\ 2 & -2 & 2 \end{matrix} \rightarrow R_3 = R_2(-2) + R_3$

$\begin{matrix} -2 & -2 & -2 \\ 2 & -2 & 2 \\ \hline 0 & -4 & 0 \end{matrix}$

$\begin{matrix} 1 & 1 & 1 \\ 0 & -12 & -1 \\ 0 & -4 & 0 \end{matrix} \rightarrow R_3 = R_2\left(-\frac{1}{3}\right) + R_3$

$\begin{matrix} 0 & 4 & +\frac{1}{3} \\ 0 & -4 & 0 \\ \hline 0 & 0 & \frac{1}{3} \end{matrix}$

$U = \begin{matrix} 1 & 1 & 1 \\ 0 & -12 & -1 \\ 0 & 0 & \frac{1}{3} \end{matrix}$ $L = \begin{matrix} 1 & 0 & 0 \\ -4 & 1 & 0 \\ -2 & -\frac{1}{3} & 1 \end{matrix}$

Resolver $Ly = b$

$y_1 = 2$

$-4y_1 + y_2 = -2$

$-2y_1 - \frac{1}{3}y_2 + y_3 = 1$

$-4(2) + y_2 = -2$

$-8 + y_2 = -2$

$y_2 = -2 + 6 = 4$ //

$-2y_1 + \frac{1}{3}y_2 + y_3 = 1$

$-2(2) + \frac{1}{3}(4) + y_3 = 1$

$-\frac{8}{3} + y_3 = 1$

$y_3 = 1 + \frac{8}{3} = \frac{11}{3}$ //

$y_1 = 2$

$y_2 = 4$

$y_3 = \frac{11}{3}$ //

Problema 11

$$UX = \begin{matrix} 1 & 1 & 1 \\ 0 & -12 & -1 \\ 0 & 0 & 1/3 \end{matrix}$$

$$x_1 + x_2 + x_3 = 2$$
$$-12x_2 - x_3 = 4$$
$$1/3 x_3 = 11/5$$

$$x_3 = \frac{\frac{11}{5}}{\frac{1}{3}} = \frac{33}{5}$$

$$-12x_2 - x_3 = 4$$

$$-12x_2 - \frac{33}{5} = 4$$

$$x_2 = \frac{4 + \frac{33}{5}}{-12}$$

$$x_2 = \frac{-53}{60}$$

$$x_1 + x_2 + x_3 = 2$$

$$x_1 + \left(-\frac{53}{60}\right) + \frac{33}{5} = 2$$

$$x_1 + \frac{343}{60} = 2$$

$$x_1 = 2 - \frac{343}{60} = \frac{-223}{60}$$

$$x_1 = \frac{-223}{60}$$

$$x_2 = \frac{-53}{60}$$

$$x_3 = \frac{33}{5}$$

Solución única

Problema 12 Factorización LU

$$12.\ \begin{cases} 2x - y + z = 0 \\ x + 2y + z = 10 \\ 3x + y = 0 \end{cases} \qquad \begin{vmatrix} 2 & -1 & 1 \\ 1 & 2 & 1 \\ 3 & 1 & 0 \end{vmatrix} \quad |A| = -10 \neq 0$$

$$\begin{vmatrix} 2 & -1 & 1 \\ 1 & 2 & 1 \\ 3 & 1 & 0 \end{vmatrix} \to R_2 = R_1(-\tfrac{1}{2}) + R_2$$

$$\begin{array}{ccc} -1 & \tfrac{1}{2} & -\tfrac{1}{2} \\ 1 & 2 & 1 \\ \hline 0 & \tfrac{5}{2} & \tfrac{1}{2} \end{array}$$

$$\begin{matrix} 2 & -1 & 1 \\ 0 & \tfrac{5}{2} & \tfrac{1}{2} \\ 3 & 1 & 0 \end{matrix} \to R_3 = R_1(-\tfrac{6}{4}) + R_3$$

$$\begin{array}{ccc} -3 & +\tfrac{6}{4} & -\tfrac{6}{4} \\ 3 & 1 & 0 \\ \hline 0 & \tfrac{5}{2} & -\tfrac{6}{4} \end{array}$$

$$\begin{matrix} 2 & -1 & 1 \\ 0 & \tfrac{5}{2} & \tfrac{1}{2} \\ 0 & \tfrac{5}{2} & -\tfrac{6}{4} \end{matrix} \to R_3 = R_2(-1) + R_3$$

$$\begin{array}{ccc} 0 & -\tfrac{5}{2} & -\tfrac{1}{2} \\ 0 & \tfrac{5}{2} & -\tfrac{6}{4} \\ \hline 0 & 0 & 2 \end{array}$$

$$U = \begin{matrix} 2 & -1 & 1 \\ 0 & \tfrac{5}{2} & \tfrac{1}{2} \\ 0 & 0 & 2 \end{matrix} \qquad L = \begin{matrix} 1 & 0 & 0 \\ -\tfrac{1}{2} & 1 & 0 \\ -\tfrac{6}{4} & -1 & 1 \end{matrix}$$

$Ly = b$

$y_1 = 0$

$-\tfrac{1}{2}y_1 + y_2 = 10$

$-\tfrac{6}{4}y_1 - y_2 + y_3 = 0$

$-\tfrac{1}{2}(0) + y_2 = 10$

$y_2 = 10$

$-\tfrac{6}{4}(0) - 10 + y_3 = 0$

$-10 + y_3 = 0$

$y_3 = 10$

$y_1 = 0$

$y_2 = 10$

$y_3 = 10$

$Ux = y$

$2x_1 - x_2 + x_3 = 0$

$\tfrac{5}{2}x_2 + \tfrac{1}{2}x_3 = 10$

$2x_3 = 10$

$2x_3 = 10$

$x_3 = \dfrac{10}{2}$

$x_3 = 5$

$\tfrac{5}{2}x_2 + \tfrac{1}{2}(5) = 10$

$\tfrac{5}{2}x_2 + \tfrac{5}{2} = 10$

$x_2 = \dfrac{10 - \tfrac{5}{2}}{\tfrac{5}{2}}$

$x_2 = 9$

$2x_1 - x_2 + x_3 = 0$

$2x_1 - 9 + 5 = 0$

$x_1 = \dfrac{+9-5}{2} = 2$

$x_1 = 2$

$x_2 = 9$

$x_3 = 5$

Solución única

METODO DE INVERSA

Metodo de inversa Problema 12

$$12 = \begin{cases} 2x - y + z = 0 \\ x + 2y + z = 10 \\ 3x + y = 0 \end{cases} \quad A = \begin{vmatrix} 2 & -1 & +1 \\ 1 & 2 & +1 \\ 3 & +1 & 0 \end{vmatrix} = \underline{-10}$$

Cofactores

$A11 = (-1)^2 \begin{vmatrix} 2 & 1 \\ 1 & 0 \end{vmatrix} = 1 \qquad A21 = (-1)^3 \begin{vmatrix} 2 & 1 \\ 1 & 0 \end{vmatrix} \; 1$

$A12 = (-1)^3 \begin{vmatrix} 1 & 1 \\ 3 & 0 \end{vmatrix} -3 \qquad A22 = (-1)^4 \begin{vmatrix} 2 & +1 \\ 3 & 0 \end{vmatrix} \; -3$

$A13 = (-1)^4 \begin{vmatrix} 1 & 2 \\ 3 & 1 \end{vmatrix} -5 \qquad A23 = (-1)^5 \begin{vmatrix} 2 & -1 \\ 3 & +1 \end{vmatrix} \; -5$

$A31 = (-1)^4 \begin{vmatrix} -1 & 1 \\ 2 & 1 \end{vmatrix} -3 \qquad A = \begin{pmatrix} 1 & -3 & -5 \\ 1 & -3 & -5 \\ -3 & -3 & 5 \end{pmatrix}$

$A32 = (-1)^5 \begin{vmatrix} 2 & 1 \\ 3 & 0 \end{vmatrix} -3 \qquad A^{+} \begin{pmatrix} 1 & 1 & -3 \\ -3 & -3 & -3 \\ -5 & -5 & 5 \end{pmatrix} \frac{1}{10}$

$A33 = (-1)^6 \begin{vmatrix} 2 & -1 \\ 1 & 2 \end{vmatrix} \; 5 \qquad A^{-1} = \begin{pmatrix} \frac{1}{10} & \frac{1}{10} & -\frac{3}{10} \\ -\frac{3}{10} & -\frac{3}{10} & -\frac{3}{10} \\ -\frac{1}{2} & -\frac{1}{2} & \frac{1}{2} \end{pmatrix}$

$$\begin{pmatrix} \frac{1}{10} & \frac{1}{10} & -\frac{3}{10} \\ -\frac{3}{10} & -\frac{3}{10} & -\frac{3}{10} \\ -\frac{1}{2} & -\frac{1}{2} & \frac{1}{2} \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} \begin{matrix} 0 \\ 10 \\ 0 \end{matrix} = \begin{matrix} 0 & 1 & 0 \\ 0 & -3 & 0 \\ 0 & -5 & 0 \end{matrix} = (1, -3, -5)$$

Problema 13

13: $\begin{cases} x + y - z + w = 4 \\ -x - y = -1 \\ 2x + y - 3w = -4 \\ 2y + z - 2w = -5 \end{cases}$

$$\begin{vmatrix} +1 & +1 & -1 & +1 \\ -1 & -1 & 0 & 0 \\ -2 & +1 & 0 & -3 \\ 0 & 2 & +1 & -2 \end{vmatrix} = -3$$

$$\cancel{-(-1)\begin{vmatrix} -1 & -1 & 0 \\ -2 & +1 & -3 \\ 0 & 2 & -2 \end{vmatrix}}^{0} + 1 \begin{vmatrix} +1 & +1 & +1 \\ -1 & -1 & 0 \\ -2 & +1 & -3 \end{vmatrix}$$

$$|A| = +1(-3) = -3$$

$$A_{11} = (-1)^2 \begin{vmatrix} -1 & 0 & 0 \\ +1 & 0 & -3 \\ 2 & +1 & -2 \end{vmatrix} = -3$$

$$A_{21} = (-1)^3 \begin{vmatrix} +1 & -1 & +1 \\ +1 & 0 & -3 \\ 2 & 1 & -2 \end{vmatrix} = -8$$

$$A_{12} = (-1)^3 \begin{vmatrix} -1 & 0 & 0 \\ -2 & 0 & -3 \\ 0 & 1 & -2 \end{vmatrix} = 3$$

$$A_{22} = (-1)^4 \begin{vmatrix} +1 & -1 & +1 \\ -2 & 0 & -3 \\ 0 & 1 & -2 \end{vmatrix} = 5$$

$$A_{13} = (-1)^4 \begin{vmatrix} -1 & -1 & 0 \\ -2 & +1 & -3 \\ 0 & 2 & -2 \end{vmatrix} = 0$$

$$A_{23} = (-1)^5 \begin{vmatrix} +1 & +1 & +1 \\ -2 & +1 & -3 \\ 0 & 2 & -2 \end{vmatrix} = 4$$

$$A_{14} = (-1)^5 \begin{vmatrix} -1 & -1 & 0 \\ -2 & +1 & 0 \\ 0 & 2 & 1 \end{vmatrix} = 3$$

$$A_{24} = (-1)^6 \begin{vmatrix} +1 & +1 & -1 \\ -2 & +1 & 0 \\ 0 & 2 & +1 \end{vmatrix} = 7$$

Problema 13

$$\begin{vmatrix} 1 & 1 & -1 & 1 \\ -1 & -1 & 0 & 0 \\ -2 & +1 & 0 & -3 \\ 0 & 2 & +1 & -1 \end{vmatrix}$$

$$A31 = \begin{vmatrix} 1 & -1 & 1 \\ -1 & 0 & 0 \\ 2 & +1 & -2 \end{vmatrix} (-1)^4 = 1$$

$$A33 = (-1)^6 \begin{vmatrix} 1 & 1 & 1 \\ -1 & -1 & 0 \\ 0 & 2 & -2 \end{vmatrix} = -2$$

$$A32 = (-1)^5 \begin{vmatrix} 1 & -1 & 1 \\ -1 & 0 & 0 \\ 0 & +1 & -2 \end{vmatrix} = -1$$

$$A34 = (-1)^7 \begin{vmatrix} 1 & 1 & -1 \\ -1 & -1 & 0 \\ 0 & 2 & +1 \end{vmatrix} = -2$$

$$A41 = (-1)^5 \begin{vmatrix} 1 & -1 & 1 \\ -1 & 0 & 0 \\ +1 & 0 & -3 \end{vmatrix} = -3$$

$$A42 = (-1)^6 \begin{vmatrix} 1 & -1 & 1 \\ -1 & 0 & 0 \\ -2 & 0 & -3 \end{vmatrix} = 3$$

$$A43 = (-1)^7 \begin{vmatrix} 1 & 1 & 1 \\ -1 & -1 & 0 \\ -2 & +1 & -3 \end{vmatrix} = 3$$

$$A44 = (-1)^8 \begin{vmatrix} 1 & 1 & -1 \\ -1 & -1 & 0 \\ -2 & +1 & 0 \end{vmatrix} = 3$$

$$A^{+} = \begin{vmatrix} -3 & 3 & 0 & 3 \\ -8 & 5 & 4 & 7 \\ 1 & -1 & -2 & -2 \\ -3 & 3 & 3 & 3 \end{vmatrix}$$

$$A\text{ inversa } \begin{vmatrix} -3 & -8 & 1 & -3 \\ 3 & 5 & -1 & 3 \\ 0 & 4 & -2 & 3 \\ 3 & 7 & -2 & 3 \end{vmatrix}$$

$$A^{-1} \begin{pmatrix} 1 & -\frac{8}{3} & -\frac{1}{3} & -1 \\ -1 & \frac{5}{3} & -\frac{1}{3} & -1 \\ 0 & \frac{4}{3} & +\frac{2}{3} & -1 \\ -1 & -\frac{7}{3} & \frac{2}{3} & -1 \end{pmatrix} \begin{matrix} 4 \\ -1 \\ -4 \\ -5 \end{matrix}$$

$x = 4 + \frac{8}{3} + \frac{4}{3} + 5 = 13$

$y = -4 + \frac{5}{3} + \frac{4}{3} + 5 = 4$

$z = 0 + -\frac{4}{3} - \frac{8}{3} + 5 = 1$

$w = -4 + \frac{7}{3} - \frac{8}{3} + 5 = \frac{2}{3}$

x	y	z	w
13	4	1	$\frac{2}{3}$

Problema 17

14: $\begin{cases} x+y-z = 1 \\ -x+y+z+w = 3 \\ 2x-2y+w = 4 \\ 2y-z+3w = 4 \end{cases}$ $\begin{vmatrix} 1 & +1 & -1 & 0 \\ -1 & +1 & +1 & +1 \\ 2 & -2 & 0 & +1 \\ 0 & +2 & -1 & +3 \end{vmatrix}$ $|A| = 26$

$$|A| = +1\begin{vmatrix} -1 & 1 & 1 \\ 2 & -2 & +1 \\ 0 & 2 & 3 \end{vmatrix} - 1\begin{vmatrix} 1 & 1 & 0 \\ 2 & -2 & 1 \\ 0 & 2 & 3 \end{vmatrix} + 1\begin{vmatrix} 1 & 1 & 0 \\ -1 & 1 & 1 \\ 2 & -2 & 1 \end{vmatrix} = 1(6) - 1(-14) + 1(6) = 26$$

$A_{11} = (-1)^2\begin{vmatrix} +1 & +1 & +1 \\ -2 & 0 & +1 \\ 2 & -1 & +3 \end{vmatrix} = 17$ $A_{21} = (-1)^3\begin{vmatrix} 1 & -1 & 0 \\ -2 & 0 & 1 \\ 2 & -1 & +3 \end{vmatrix} = 7$ $A_{31} = (-1)^4\begin{vmatrix} +1 & -1 & 0 \\ +1 & +1 & 1 \\ +2 & -1 & 3 \end{vmatrix} = 5$

$A_{12} = (-1)^3\begin{vmatrix} -1 & 1 & 1 \\ 2 & 0 & 1 \\ 0 & -1 & 3 \end{vmatrix} = 9$ $A_{22} = (-1)^4\begin{vmatrix} 1 & +1 & 0 \\ 2 & 0 & 1 \\ 0 & -1 & 3 \end{vmatrix} = 7$ $A_{32} = (-1)^5\begin{vmatrix} -1 & -1 & 0 \\ -1 & +1 & 1 \\ 0 & -1 & 3 \end{vmatrix} = -1$

$A_{13} = (-1)^4\begin{vmatrix} -1 & +1 & 1 \\ 2 & -2 & 1 \\ 0 & +2 & 3 \end{vmatrix} = 6$ $A_{23} = (-1)^5\begin{vmatrix} 1 & 1 & 0 \\ 2 & -2 & +1 \\ 0 & 2 & +3 \end{vmatrix} = +14$ $A_{33} = (-1)^6\begin{vmatrix} 1 & 1 & 0 \\ -1 & 1 & 1 \\ 0 & 2 & 3 \end{vmatrix} = 4$

$A_{14} = (-1)^5\begin{vmatrix} -1 & 1 & 1 \\ 2 & -2 & 0 \\ 0 & +2 & -1 \end{vmatrix} = +1$ $A_{24} = (-1)^6\begin{vmatrix} 1 & 1 & -1 \\ 2 & -2 & 0 \\ 0 & 2 & -1 \end{vmatrix} = 0$ $A_{34} = (-1)^8\begin{vmatrix} 1 & 1 & -1 \\ -1 & 1 & 1 \\ 0 & 2 & -1 \end{vmatrix} = -2$

$A_{41} = (-1)^5\begin{vmatrix} 1 & -1 & 0 \\ 1 & 1 & 1 \\ -2 & 0 & 1 \end{vmatrix} = 4$ $A_{42} = (-1)^6\begin{vmatrix} 1 & -1 & 0 \\ -1 & 1 & 1 \\ 2 & 0 & 1 \end{vmatrix} = -2$ $A_{43} = (-1)^7\begin{vmatrix} 1 & 1 & 0 \\ -1 & 1 & 1 \\ 2 & -2 & 1 \end{vmatrix} = -6$

$A_{44} = (-1)^8\begin{vmatrix} 1 & 1 & -1 \\ -1 & 1 & 1 \\ 2 & -2 & 0 \end{vmatrix} = 4$

Escaneado con CamScanner

Problema 14

$$A = \begin{matrix} 11 & 9 & 6 & -1 \\ 7 & 7 & 14 & 0 \\ 5 & -1 & 4 & -2 \\ -4 & -2 & -6 & +4 \end{matrix} \qquad A^{t} = \begin{matrix} 11 & 7 & 5 & -4 \\ 9 & 7 & -1 & -2 \\ 6 & 14 & 4 & -6 \\ -1 & 0 & -2 & +4 \end{matrix}$$

$$A^{-1} = \begin{matrix} \frac{11}{26} & \frac{7}{26} & \frac{5}{26} & -\frac{2}{13} \\ \frac{9}{26} & \frac{7}{26} & \frac{-1}{26} & \frac{-1}{13} \\ \frac{3}{13} & \frac{7}{13} & \frac{2}{13} & \frac{-3}{13} \\ \frac{-1}{13} & 0 & \frac{-1}{13} & +\frac{2}{13} \end{matrix} \Rightarrow \begin{matrix} 1 \\ 3 \\ 4 \\ 4 \end{matrix} \quad \begin{matrix} \frac{11}{26} + \frac{21}{26} + \frac{10}{13} - \frac{8}{13} = \frac{343}{208} \\ \frac{9}{26} + \frac{21}{26} - \frac{2}{13} - \frac{4}{13} = \frac{9}{13} \\ \frac{3}{13} + \frac{21}{13} + \frac{8}{13} - \frac{12}{13} = \frac{20}{13} \\ -\frac{1}{3} + 0 + \frac{4}{13} + \frac{8}{13} = -\frac{1}{39} \end{matrix}$$

x	y	z	w
$\frac{243}{208}$	$\frac{9}{13}$	$\frac{20}{13}$	$\frac{-1}{39}$

Problema 15

$$15 = \begin{cases} x - y + 2z = 5 \\ 3x + 2y + z = -1 \\ x + 3y - z = -5 \end{cases} \quad \begin{vmatrix} 1 & -1 & 2 \\ 3 & 2 & 1 \\ 1 & 3 & -1 \end{vmatrix} = \underline{5}$$

Cofactores

$A_{11} = (-1)^2 \begin{vmatrix} 2 & 1 \\ 3 & -1 \end{vmatrix} = (1)(-2-3)$
$(1)(-5) = -5$

$A_{21} = (-1)^3 \begin{vmatrix} -1 & 2 \\ 3 & -1 \end{vmatrix} -5(-1) = \underline{5}$

$A_{12} = (-1)^3 \begin{vmatrix} 3 & 1 \\ 1 & -1 \end{vmatrix} = (-1)(-3-1)$
$(-1)(-4) = 4$

$A_{22} = (-1)^4 \begin{vmatrix} 1 & 2 \\ 1 & -1 \end{vmatrix} -2(1) = -2$

$A_{13} = (-1)^4 \begin{vmatrix} 3 & 2 \\ 1 & 3 \end{vmatrix} = (1)(9-2)$
$(1)(7) = 7$

$A_{23} = (-1)^5 \begin{vmatrix} 1 & -1 \\ 1 & 3 \end{vmatrix} \; 2(-1) = -2$

$A_{31} = (-1)^4 \begin{vmatrix} -1 & 2 \\ 2 & 1 \end{vmatrix} = -5(-1) = 5$

$A = \begin{matrix} -5 & 4 & 7 \\ 5 & -2 & -2 \\ 5 & 3 & 6 \end{matrix} \quad A^t = \begin{matrix} -5 & 5 & 5 \\ 4 & -2 & 3 \\ 7 & -2 & 6 \end{matrix}$

$A_{32} = (-1)^5 \begin{vmatrix} 1 & 2 \\ 1 & -1 \end{vmatrix} -3(-1) = 3$

$A^{-1} = \frac{1}{5} \begin{pmatrix} -5 & 5 & 5 \\ 4 & -2 & 3 \\ 7 & -2 & 6 \end{pmatrix}$

$A_{33} = (-1)^6 \begin{vmatrix} 1 & -1 \\ 3 & 2 \end{vmatrix} \; 5(1) = 6$

$A^{-1} = \begin{pmatrix} -1 & 1 & 1 \\ 4/5 & -2/5 & 3/5 \\ 7/5 & -2/5 & 6/5 \end{pmatrix}$

$$\begin{pmatrix} -1 & 1 & 1 \\ 4/5 & -2/5 & 3/5 \\ 7/5 & -2/5 & 6/5 \end{pmatrix} \begin{vmatrix} 3 \\ -1 \\ 5 \end{vmatrix} = \begin{matrix} -3-1+5 = 1 \\ 12/5 + 2/5 + 3 = \frac{29}{5} \\ 21/5 + 2/5 + 6 = \frac{53}{5} \end{matrix}$$

$$\underline{1, \frac{29}{5}, \frac{53}{5}}$$

Escaneado con CamScanner

Problema 16

$$16 = \begin{cases} x - y + z = -2 \\ 2x + y - 2z = 5 \\ 4x + y + 2z = 0 \end{cases} \qquad \begin{vmatrix} 1 & -1 & 1 \\ 2 & 1 & -2 \\ 4 & 1 & 2 \end{vmatrix} = 14$$

Cofactores

$$A_{11} = (-1)^2 \begin{vmatrix} 1 & -2 \\ 1 & 2 \end{vmatrix} \; 4 \qquad A21 = (-1)^3 \begin{vmatrix} -1 & 1 \\ 1 & 2 \end{vmatrix} \; 3$$

$$A12 = (-1)^3 \begin{vmatrix} 2 & -2 \\ 4 & 2 \end{vmatrix} \; -12 \qquad A22 = (-1)^4 \begin{vmatrix} 1 & 1 \\ 4 & 2 \end{vmatrix} \; -2$$

$$A13 = (-1)^4 \begin{vmatrix} 2 & 1 \\ 4 & 1 \end{vmatrix} \; -2 \qquad A23 = (-1)^5 \begin{vmatrix} 1 & -1 \\ 4 & 1 \end{vmatrix} \; -5$$

$$A31 = (-1)^4 \begin{vmatrix} -1 & 1 \\ 1 & -2 \end{vmatrix} \; 1$$

$$A = \begin{matrix} 4 & -12 & -2 \\ 3 & -2 & -5 \\ 1 & 4 & -1 \end{matrix} \qquad A^T = \begin{matrix} 4 & 3 & 1 \\ -12 & -2 & 4 \\ -2 & -5 & -1 \end{matrix}$$

$$A32 = (-1)^5 \begin{vmatrix} 1 & 1 \\ 2 & -2 \end{vmatrix} \; 4$$

$$A^{-1} = \begin{matrix} \frac{4}{14} & \frac{3}{14} & \frac{1}{14} \\ -\frac{6}{7} & -\frac{1}{7} & -\frac{5}{14} \\ \frac{1}{14} & \frac{4}{14} & -\frac{1}{14} \end{matrix} \qquad \frac{-12}{14} = \frac{-6}{7}$$

$$A33 = (-1)^6 \begin{vmatrix} 1 & -1 \\ 2 & 1 \end{vmatrix} \; -1$$

$$\begin{matrix} 4/14 & 3/14 & 1/14 \\ -6/7 & -1/7 & -5/14 \\ 1/14 & 4/14 & -1/14 \end{matrix} \cdot \begin{pmatrix} 2 \\ 5 \\ 0 \end{pmatrix} = \begin{matrix} \frac{4}{7} & \frac{15}{14} & +0 = & \frac{23}{14} \\ -\frac{12}{7} & -\frac{5}{7} & 0 & -\frac{17}{7} \\ \frac{1}{7} & \frac{5}{7} & 0 & \frac{6}{7} \end{matrix}$$

$$\frac{23}{14}, \; \frac{-17}{7}, \; \frac{6}{7}$$