EXAMEN 2DO PARCIAL. 18258709

16/07/21.

① $(2y-2xy^3+4x+6)dx+(2x-3x^2y^2-1)dy$ con la condición inicial $y(-1)=0$

$M=2y-2xy^3+4x+6$

$N=2x-3x^2y^2-1$

$\frac{\partial M}{\partial y}=2-6y^2x \qquad \frac{\partial N}{\partial x}=2-6y^2x \Big\}$ E.D. exacta

$f(x,y)=\int N dx=\int(-2xy^3+2y+4x+6)dx$

$=-2y^3\int x dx+(2y+6)\int dx+4\int x dx$

$=-y^3x^2+2x^2+2xy+6x+h(y)$

$(F(x,y))'=2x-3x^2y^2$

$((x)\quad -y^3x^2+2x^2+2xy+6x)=2x-3x^2y^2$

$h(y)=\int M(x,y)-(2x-3x^2y^2)dy$

$=\int -dy=-y+c$

$h(y)=-y+c$

$F(x,y)=-y^3x^2+2x^2+2xy+6x+h(y)$

$F(x,y)=-y-y^3x^2+2x^2+2xy+6x+c$

$=-y-y^3x^2+2x^2+2xy+6x=c$

$=-(0)-(0)^3(-1)^2+2(-1)^2+2(-1)(0)+6(-1)=c$

$2-6=c$

$c=-4$

$-y-y^3x^2+2x^2+2xy+6x=-4$

② $xy' + 2y + 2x^2y^2 = 0$

$y = z^\lambda$ $\qquad$ $y = \frac{1}{z^2}$

$x\frac{dy}{dx} + 2y + 2x^2y^2 = 0$

$\lambda^2 + 2\lambda + 2$

$2\lambda = 2\lambda + 2$

$\lambda = -2$

$dy = \frac{-2\,dz}{z^3}$

$\frac{2z^2 + 2x^2}{z^4} - \frac{2x\,dz}{z^3\,dx} = 0$

$(2z^2 + 2x^2)dx - 2xz\,dz = 0$

$(z^2 + x^2)dx - xz\,dz$

$-xz\,dz = (-x^2 - z^2)dx$

$u = \frac{z}{x}$ $\qquad$ $z = ux$ $\qquad$ $dz = u\,dx + x\,du$

① $-ux^2(u\,dx + x\,du)$ $-$

② $= (-u^2 - 1)x^2\,dx$

③ $= -u^2x^2\,dx - ux^3\,du$

④ $= -u^2x^2\,dx - x^2\,du$

⑤ $-ux^3\,du = -x^2\,dx$

$u\,du = \frac{dx}{x}$

$\int u\,du = \int \frac{1}{x}\,dx$ $\qquad$ $u = \frac{z}{x}$ $\qquad$ $z = \frac{1}{\sqrt{y}}$

$\frac{u^2}{2} = \ln(x) + C$

$\frac{z^2}{2x^2} = \ln(x) + C = \frac{1}{2x^2y} = \ln(x) + C$

③ $(2x^2+y)\,dx + (x^2y-x) = 0$

$M = 2x^2+y$
$N = x^2y-x$

$\frac{\partial M}{\partial y} = 1$ $\quad \frac{\partial N}{\partial x} = 2xy-1$

$\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}$ E.D. INEXACTA

$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x} = \frac{1}{x^2}$

F.I $M(x)$

$M'(x) = \frac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}{N} M(x) = \frac{1-(2xy-1)}{x^2y-x} M(x)$

$M'(x) = -\frac{2}{x} M(x) \quad M(x) = e^{\int -2/x\,dx} = e^{-2\ln x} = \frac{1}{x^2}$

$\frac{1}{x^2}\left[(2x^2+y)\,dx + (x^2y-x)\,dy\right] = 0 \quad \frac{2x^2+y}{x^2}dx \quad \frac{(xy-1)}{x}dy = 0$

$M = \frac{2x^2+y}{x^2} \quad N = \frac{xy-1}{x}$

$f(x,y) = \int M(x,y)\,dx \quad \int N(x,y)\,dy$

$= \int \frac{xy-1}{x}\,dy = \frac{1}{x}\int (xy-1)\,dy$

$= \frac{1}{x}\left(\frac{xy^2}{2} - y\right) = \frac{y^2}{2} - \frac{y}{x} + c = \frac{y^2}{2} - \frac{y}{x} + h(x)$

Calcular $h(x)$

$\left(\frac{y^2}{2} - \frac{y}{x} + h(x)\right) = \frac{2x^2+y}{x^2}$

$\frac{y}{x^2} + h(x) = \frac{2x^2+y}{x^2}$

$h'(x) = 2$
$h(x) = 2x + c$

$f(x,y) = \frac{y^2}{2} - \frac{y}{x} + 2x + c$

$= -\frac{y}{x} + \frac{y^2}{2} + 2x + c$

④ Al apagar un motor su temperatura es de 98°C y el medio que se encuentra se conserva a 21°C. Si despues de 10 min el motor se ha enfriado a 88°C, encuentre el instante en el cuál su temperatura es de 35°C

$T(t) = c°$
$t = $ tiempo

$T'(t) = k[T(t) - 21°]$ $T(0) = 98°$ $T(10) = 88°C$

$\frac{dT}{dt} = k(T - 21°) = \frac{dT}{T-21°} = k\,dt = \int \frac{dT}{T-30} = k\int dt$

$\ln(T - 21°) = kt + c = e^{\ln|T-21°|} = e^{|kt+c|} = ce^{kt}$

$T - 21° = e^{kt} = ce^{kt} = T - 21° = ce^{kt} \rightarrow T(t) = 21° + ce^{kt}$

$T(0) = 98° = 21° + ce^{kt}$

$= 98 = 21° + C$
$= C = 98 - 21$
$= 77$

$C = 77$ $T(t) = 21 + 77e^{kt}$

$T(10) = 88°C \rightarrow 21 + 77e^{k(10)}$

$= e^{10k} = \frac{88-21}{77} = \frac{67}{77}$

$e^{10k} = \frac{67}{77} = \ln e^{10k} = \ln\left(\frac{67}{77}\right)$

$10k = \ln\left(\frac{67}{77}\right)$

$k = \frac{1}{10}\ln\left(\frac{67}{77}\right) = -0.013911281$

$T(t) = 21 + 77e^{(-0.0139128)t}$

$T(10) = 35° \rightarrow 21 + 77e^{(-0.0139128)t}$

$\frac{35° - 21}{77} = e^{(-0.0139128)t}$

$= \frac{14}{77} = \frac{2}{11}$

$\ln e^{(-0.0139128)t}$ $\qquad \ln\frac{2}{11} = (-0.0139128)t$

$= \ln\left(\frac{2}{11}\right)$

$t = \frac{\ln\left(\frac{2}{11}\right)}{-0.0139128} = \frac{17047}{(0.0139128)} = 122.5443016 \text{ Min}$

$\frac{122.5443016}{60} = 2.01240 \text{ hrs}$