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TAREA departamental 19/09/2020

① $\int (2x-3)\,dx$

$\int 2x\,dx \quad \int -3\,dx$

prop 2 prop 1, iii

$\int \frac{2x^{1+1}}{1+1} - 3\int x\,dx = x + c$

$\frac{2x^2}{2} - 3x + c$

$\boxed{x^2 - 3x + c}$

i) $\int k\,f(x)\,dx = k\int f(x)\,dx$

constant (función) = cuando hay una constante sale del factor de integrac

ii) $\int x^n\,dx = \frac{x^{n+1}}{n+1} + c$

$n \neq -1$

iii) $\int dx = x + c$

iv) $\int \frac{dx}{x} = \ln|x| + c$

v) $\int [f(x) \pm g(x)]\,dx = \int f(x)\,dx \pm \int g(x)\,dx$

② $\int \sqrt{x}\,dx = \int (x)^{1/2}\,dx$

$\int \frac{(x)^{1/2+1}}{\frac{1}{2}+1}\,dx$

$\frac{(x)^{3/2}}{\frac{3}{2}} = \frac{x^{3/2}}{\frac{3}{2}} = \frac{2}{3(x)^{3/2}} + c$

③ $\int \sqrt[3]{x}\, dx = \int (x)^{1/3}$

$\int (x)^{1/3} = \frac{(x)^{1/3+1}}{\frac{1}{3}+1} = \frac{x^{4/3}}{\frac{4}{3}}$ $\boxed{\frac{3}{4(x)^{4/3}} + C}$

④ $\int 2\sqrt{x-1}\, dx$

$2\int \sqrt{x-1}\, dx$

$2\int (x-1)^{1/2}\, dx$

$2\int \frac{(x-1)^{1/2+1}}{\frac{1}{2}+1} + C$

$2\int \frac{(x-1)^{3/2}}{\frac{3}{2}} + C$

$\boxed{2\int \frac{2}{3(x-1)^{3/2}} + C}$

$\int x\,dx = \frac{x^2}{2}$ $\int dx = x + c$

⑤ $\int \left(\frac{3}{x^3} + 5x\right) dx$

$\int \frac{3}{x^3}\,dx + \int 5x\,dx$

$\int 3x^{-3}\,dx + \int 5x\,dx$

$3\int x^{-3}\,dx + 5\int x\,dx$

$3\,\frac{x^{-3+1}}{-3+1} + \frac{5x^2}{2}$

$\frac{3x^{-2}}{-2} + \frac{5x^2}{2}$

constante sale cuando esta multiplicando

⑥ $\int (x+1)(x-2)\,dx$

$$\begin{array}{r} x^2 - 2x \\ 1x - 2 \\ \hline x^2 - x - 2 \end{array}$$

$\int x^2 - x - 2\,dx$

$\int \frac{x^{2+1}}{2+1} - \frac{x^{1+1}}{1+1}$

$\frac{x^3}{3} - \frac{x^2}{2}$

$\int x^2 - \int x - \int 2\,dx$

$\frac{x^3}{3} - \frac{x^2}{2} - 2x + c$

⑦ $\int \left(3t + \frac{4}{t}\right) dt$

$3\int t \, dt \quad \int \frac{4}{t} dt$ $\rightarrow \frac{du}{u} = \ln|u| + c$

$\rightarrow \frac{dt}{t} = \ln$

$\boxed{\frac{3t^2}{2} \quad 4 \ln|t| + c}$

⑧ $\int_{-1}^{2} (x-2)^2 dx$

$(x-2)(x-2)$

$$\begin{array}{r} x^2 - 2x \\ -2x + 4 \\ \hline x^2 - 4x + 4 \end{array}$$

$\int x^2 - 4x + 4 \rightarrow -1^2 - 4(-1) + 4 +$

$\int_{-1} \frac{x^3}{3} - 4\frac{x^2}{2} + 4x + c \Big|_{-1}^{2} \rightarrow \frac{(-1)^3}{3} - \frac{4(-1)^2}{2} +$

$\frac{(2)^3}{3} - \frac{4(2)^2}{2} = -49.666$

9. $\int \left(\frac{x^3+1}{x^5}\right) dx$

$\int \frac{x^3}{x^5} dx \quad \int \frac{1}{x^5} dx$

$\int \frac{1}{x^2} \quad \int \frac{1}{x^5} dx$

$\int \frac{x^{-2+1}}{-2+1}$

$-\frac{1}{x} - \frac{1}{4x^4} + C$

10. $\int (x+1)^{17} dx$

$\frac{(x+1)^{17+1}}{17+1} = \frac{(x+1)^{18}}{18} \rightarrow \frac{1}{18}(x+1)^{18} + C$

11. $\int x^2(x-1) \, dx$

$\int x^3 - x^2$

$\frac{x^4}{4} - \frac{x^3}{3} + C$

(12) $\int_{0}^{2\pi} \sin x \, dx$

$-\cos x + c \Big|_{0}^{2\pi}$

$-\cos 0 + c \quad -\cos 2\pi + c \quad = -0.9998$

(13) $\int \csc x(\cot x - 3\csc x)\, dx$ Clase (duda)

$\int \csc x \cdot \cot x \, dx - 3\int \csc^2 x \, dx =$

$= -\csc x - 3\cot x + c$

(14) $\int_{\pi/6 = 0.5235}^{\pi/3 = 1.0471} \sec x \tan x$ calcular con radianes duda

$= \sec u + c \to \sec \pi/6 + \sec \pi/3 =$

$1/\cos(\frac{\pi}{3}) \quad 1/\cos(\frac{\pi}{6})$

$\to \sec x \Big|_{\pi/6}^{\pi/3} = \sec(\frac{\pi}{3}) - \sec(\frac{\pi}{6})$

$= \frac{1}{\cos\frac{\pi}{3}} - \frac{1}{\cos\frac{\pi}{6}} = 0.8452$

(15) $\int (\sqrt{a} - \sqrt{x})^2 \, dx$

$a^2 + 2ab + b^2$

$a - 2\sqrt{a}\sqrt{x} + \sqrt{x}$

$\int a - 2\sqrt{a}\sqrt{x} + \cancel{\sqrt{x}}$

$\int a\,dx - \int 2\sqrt{a}\sqrt{x}\,dx + \int x\,dx$

$a\int dx \quad -2\sqrt{a}\int (x)^{1/2+1}dx$

$\frac{1}{2}+1$

$-2\sqrt{a}\int \frac{(x)^{3/2}}{\frac{3}{2}} \qquad \frac{2(x)^{3/2}}{3}$

$\frac{(x)^{1/2}}{\frac{3}{2}} \qquad \frac{2(x)^{3/2}}{3}$

$-2\sqrt{a}\left[\frac{2(x)^{3/2}}{3}\right] = -\frac{4\sqrt{a}(x)^{3/2}}{3}$

$$ax - \frac{4\sqrt{a}(x)^{3/2}}{3} + \frac{x^2}{2} + c$$

(16) $\int_0^a (a^2 x - x^3)\,dx$

$\int_0^a a^2 x\,dx - \int x^3\,dx$

$a^2\int \frac{x^2}{2} \qquad -\frac{x^4}{4}$

$$\frac{a^2x^2}{2} - \frac{x^4}{4} + c = \frac{a^2(0)^2}{2} - \frac{(0)}{4} + \frac{a^2(a)}{2} - \frac{(a)}{4} = \frac{a^2(a)}{2} - \frac{a}{4}$$

(17) $\int \left(\sqrt{x} - \frac{1}{\sqrt{x}}\right) dx$

$\int \sqrt{x}\, dx \quad - \int \frac{1}{\sqrt{x}}\, dx$

$\frac{x^{3/2}}{\frac{3}{2}} \quad - \frac{x^{-1/2+1}}{-\frac{1}{2}+1} = \frac{-x^{1/2}}{\frac{1}{2}} \quad \frac{-2x^{1/2}}{1} = -2x^{1/2}$

$\boxed{\frac{2x^{3/2}}{3} - 2x^{1/2} + c}$

(18) $\int \frac{3x^2}{(x^3+1)^2}\, dx$

$3\int \frac{x^2}{(x^3+1)^2}$

$\frac{x^2}{x^5+2x^3+1}$

$3\int \frac{x^2}{x^5} + \frac{x^2}{2x^3} + \frac{x^2}{1}$

$3\int \frac{1}{x^3}\, dx + \int \frac{1}{2x}\, dx + \int x^2\, dx$

$3\int x^{-3} \quad \frac{1}{2}\int \frac{dx}{x}$

$3\frac{x^{-2}}{-2} + \frac{1}{2}\ln|x| + \frac{x^3}{3} + c$

Trinomio
cambio variable

II Integrales por cambio de variable

1) $\int \frac{t}{(4t^2+9)^2} dt = \frac{1}{8}\int \frac{du}{u^2} dt = \frac{1}{8}\int u^{-2} dt$

$u = 4t^2+9$

$du = 8t\ dt$

$\frac{du}{8} = t\,dt$

$\frac{1}{8} \quad \frac{u^{-2+1}}{-2+1} \quad \frac{u^{-1}}{-1}$

$\frac{1}{8} \cdot \frac{-1}{u} = -\frac{1}{8}u$

2) $\int 2ax(ax^2+b)^4 dx$

$u = ax^2+b$

$du = 2ax\ dx$

$\int \sqrt{u}\ dx = \int u^4\ dx = \int \frac{u^5}{5} + c = \frac{(ax^2+b)^5}{5} + c$

$\int 2ax(ax^2+b)^4\ dx = 2\int ax(ax^2+b)^4\ dx$

$u = ax^2+b$

$u = 2ax$

$\frac{u}{2} = ax$

$\int u^4 \frac{dx}{2} = \frac{1}{2}\int u^{4+1} \over {4+1}$ $= \frac{1}{2}\cdot\frac{1}{5}\quad \frac{1}{10}u^5 + c$

$= \frac{1(ax^2+b)^5}{10} + c$

3) $\int x^2 (1+x^3)^{1/4}\, dx \quad \int u^{1/4} \frac{du}{3} = \frac{1}{3}\int u^{1/4}\, du$

$u = 1+x^3$

$du = 3x^2$

$\frac{du}{3} = x^2$

$\frac{1}{3} \frac{u^{5/4}}{\frac{5}{4}} + c$

$\frac{1}{3} \frac{4u^{5/4}}{5} + c$

$\boxed{\frac{4(1+x^3)^{5/4}}{15} + c}$

4) $\int \frac{2s}{\sqrt[3]{6-5s^2}}\, ds \quad = \int \frac{du}{5u^{1/3}} \quad -\frac{1}{5}\int u^{-1/3}\, du$

$u = 6-5x^2$

$du = -10x\, dx$

$-\frac{du}{5} = 2x\, dx$

$= -\frac{1}{5}\left(\frac{u^{2/3}}{2/3}\right) + c$

$= -\frac{1}{5} \cdot \frac{3}{2}\, u^{2/3} + c$

$\boxed{= -\frac{3}{10}(6-5x^2)^{2/3} + c}$

5) $\int 2x^3 (1-x^4)^{-1/4} dx$

$u = 1-x^4$

$du = -4x^3$

$\frac{du}{-4} = x^3 \, dx$

$\int 2u^{-1/4} \frac{du}{-4} \int = -\frac{1}{4} \int 2 \frac{u^{-1/4+1}}{-\frac{1}{4}+1} dx$

$\left[\frac{2(u)^{3/4}}{\frac{3}{4}}\right] dx$

$\frac{1}{4} \quad \frac{-8u^{3/4}}{3} dx$

$-\frac{8(1-x^4)^{3/4}}{12} + c$

$-\frac{4}{6}(1-x^4)^{3/4} + c$

$\boxed{-\frac{2}{3}(1-x^4)^{3/4} + c}$

6) $\int \cos(3x+1) dx$

$u = 3x+1$

$du = 3 \, dx$

$\frac{du}{3} = dx$

$\int \cos u = \sin u + c$

$\frac{1}{3} \int \cos u =$

$\boxed{\frac{1}{3} \operatorname{sen}(3x+1) + c}$

7) $\int \sin(2\pi x) dx$

$u = 2\pi x \quad \Rightarrow \frac{1}{2\pi} \int \sin(u) dx \rightarrow = \cos x$

$\frac{1}{2\pi} \int \sin x \, dx = -\frac{\cos(u)}{2\pi} = \boxed{-\frac{\cos(2\pi x)}{2\pi} + c}$

(8) $\int x \sec x^2 \, dx$

$u = x^2$

$du = 2x$

(9) $\int \frac{\sec^2\left(\frac{1}{x}\right)}{x^2} dx = -\int \sec^2 u \, du = -\tan u + c$

$= -\tan\left(\frac{1}{x}\right) + c$

$u = \frac{1}{x}$

$du = -\frac{1}{x^2} dx$

(10) $\int \frac{\sin\sqrt{x}}{\sqrt{x}} dx$

$\int 2 \sin(x) \, dx$

$= 2\int \sin x \, dx$

$u = \sqrt{x}$

$du = \frac{1}{2\sqrt{x}} dx$

$\int \sin x \, dx$

$= -\cos x$

$2\sqrt{x} = dx$

$2\int \sin x \, dx$

$= -2 \cos x$

$= -2 \cos\sqrt{x} + c$

(11) $\int x^{-3/4}(x^{1/4}+1)^{-2}\,dx \quad \int u^{-2}\frac{du}{4} = \frac{1}{4}\int \frac{u^{-1}}{-1} \to -\frac{1}{u}$

$u = x^{1/4}+1$ $\quad \frac{1}{4}x^{1/4-1}$

$du = \frac{x^{-3/4}}{4}$

$\frac{u}{4} = x^{-3/4}\,dx$

$\boxed{-\frac{1}{x^{1/4}+1}+c}$

(12) $\int \frac{4x+6}{\sqrt{x^2+3x+1}}\,dx$

$u = x^2+3x+1$

$du = 2x+3$

(13) $\int \frac{(\ln x)^2}{x}\,dx \quad \int \frac{u^2}{x}\,dx \quad \int \frac{(x)^2}{x} \to \frac{\frac{x^3}{3}}{x}$

$u = \ln x$

$du \; \frac{x}{1}$

$1(du) = x$

$= \frac{\frac{x^3}{3x}}{x}+c$

$\frac{x^3}{3x^2} = \boxed{\frac{x}{3}+c}$

14) $\int (x+1)\sqrt{2x+x^2}\,dx$

$u = 2x + x^2$
$du = 2 + 2x$
$du = 2x + 2$

III) Calcula las sig integrales

1) $\int \frac{dx}{x+1} = \frac{du}{u} = \ln|u| + c$

$\boxed{\ln|x+1| + c}$

2) $\int \frac{x}{3-x^2}\,dx \quad \frac{-1}{2}\int \frac{1}{u}\,du$

$u = 3 - x^2$
$du = -2x\,dx$
$\frac{du}{-2x} = dx$
$dx = -\frac{1}{2x}$

$= \frac{-\ln(u)}{2}$

$\boxed{= -\frac{\ln(|3-x^2|)}{2} + c}$

③ $\int \frac{\sin x}{2 + \cos x} dx$ $\quad -\int \sin x$

$-\int \cos x + C$

$u = 2 + \cos x$
$du = -\sin x$

$= \cos x + C$

$-du = \sin x$

$\int -\frac{1}{u} du = -\int \frac{1}{u} du = \ln(|u|)$
$= \ln|2 + \cos(x)| + C$

④ $\int \left(\frac{1}{x+2} - \frac{1}{x-2}\right) dx$

$\int \frac{1}{x+2} dx - \int \frac{1}{x-2} dx$

$\ln|x+2| - \ln|x-2| + C$

⑤ $\int \frac{\sin x - \cos x}{\sin x + \cos x}$

$u = \sin x + \cos x$
$du = \cos x - \sin x \quad = \frac{1}{\cos x - \sin x} du = -\int \frac{1}{u} du$

$= -\ln|\sin(x) + \cos(x)| + C$

(6) $\int_0^1 \frac{dx}{1+x^2}$ = (9) $\frac{dx}{(\sqrt{1})^2+x^2} = \frac{1}{1}\ arc\tan \frac{x^2}{1} + c$

$arc\tan (0)^2 + c \Big|_0^1 + arc\tan (1)^2 + c$

$= 0.6168$

(7) $\int_0^{1/\sqrt{2}} \frac{dx}{\sqrt{1-x^2}}$ (18) $\frac{du}{\sqrt{a^2-u^2}} = \frac{dx}{\sqrt{(\sqrt{1})^2-x^2}} =$

$arc\sin \frac{u}{a} + c = arc\sin \frac{-x^2}{1}$

$= arc\sin -(0)^2 + arc\sin -\left(\frac{1}{\sqrt{2}}\right)^2$

(8) $\int \frac{1}{\sqrt{x}(1+\sqrt{x})} dx$

$= 2\int \frac{1}{u} du = \ln(u)$

$u = 1+\sqrt{x}$

$du = \frac{1}{2\sqrt{x}}$

$2\int \frac{1}{u} du$

$dx = 2\sqrt{x}\, dx$

$= 2 \ln(\sqrt{x}+1) + c$

(9) $\int \frac{\tan(\ln x)}{x} dx$

$u = \ln x$

$\frac{du}{dx} = \frac{1}{x} \cdot x$

$dx = x\,du$

aplica identidad

$\tan\theta = \frac{\sin\theta}{\cos\theta} = \int \frac{\sin(u)}{\cos(u)} = -\int \frac{1}{u} du$

$\int \frac{1}{u} du = \ln(u)$

$-\int \frac{1}{u} du = -\ln(\cos(u)) = -\ln(\cos \ln(x))$

$\int \tan(\ln(x)) = -\ln(|\cos \ln(x)|) + c$

(10) $\int e^{2x} dx \quad \frac{1}{2}\int e^{2x} + c$

$u = 2x$
$du = 2\,dx$
$\frac{du}{2} = dx$

(11) $\int x e^{x^2} dx$

$\frac{1}{2}\int e^u du$

$\int a^u du = \frac{a^u}{\ln(a)}$ $a = e$

$= e^u$

$u = x^2$

$du = 2x \ dx$

$\frac{du}{2x} = dx$

$\frac{1}{2}\int e^u du = \frac{e^u}{2} = \frac{e^{x^2}}{2}$

$\int x e^{x^2} dx = \frac{e^{x^2}}{2} + C$

$n = \frac{1}{2}$

(12) $\int \frac{e^x}{\sqrt{e^x+1}} dx$ $\int u^n du$ $\int \frac{1}{\sqrt{u}} du = 2\sqrt{u}$

$2\sqrt{e^x+1}$

$u = e^x + 1$

$du = e^x \ dx$

$\int \frac{e^x}{\sqrt{e^x+1}} dx = 2\sqrt{e^x+1} + C$

(13) $\int_0^{3/2} \frac{dx}{9+4x^2} = \frac{du}{(3)^9+(2x)^2} = \frac{1}{2} \arctan \frac{2x}{3} + C$

$= \frac{1}{2} \arctan \frac{2x}{3} + C$

(14) $\int_{3/4}^{3} \frac{dx}{\sqrt{16x^2-9}}$

(15) $\int \cos x \, e^{\sin} dx$

$\int e^u \, du$

$u = \sin x$

$du = \cos x$

$dx = \frac{1}{\cos x}$

$\int a^u du = \frac{a^u}{\ln(a)}$ $a = e$

$= e^{\sin x}$

$\int e^{\sin x} \cos x \, dx$

$= e^{\sin x} + c$

(16) $\int_0^1 x\left(e^{x^2}+2\right)dx$

$u = x^2$
$du = 2x$
$= \frac{1}{2x}$

$\frac{1}{2}\int\left(e^u+2\right)du$

$= \frac{e^u}{2} + u$

$= \frac{e^{x^2}}{2} + x^2$

$\int x\left(e^{x^2}+2\right)dx = \boxed{\frac{e^{x^2}}{2} + x^2 + c}$

(17) $\int 2^x\,dx = \frac{a^u}{\ln|a|} + c$

$\boxed{\frac{2^x}{\ln|2|} + c}$

(18) $\int x\,5^{-x^2}$

(19) $\int_0^{\ln 2} \frac{e^x}{1+e^{2x}}\,dx$

$u = e^x$
$du = e^x$

$= \int \frac{1}{u^2+1} = \arctan(u)$

$\int \frac{e^x}{e^{2x}+1} = \boxed{\arctan(e^x) + C}$

(20) $\int \frac{\arcsin x}{\sqrt{1-x^2}\,\cancel{dx}}\,dx$

$u = \arcsin x$
$du = \frac{u'}{\sqrt{1-x^2}} = \frac{1}{\sqrt{1-x^2}}$

$dx = \sqrt{1-x^2}$

$= \int u\,du = \frac{u^2}{2} = \boxed{\frac{\arcsin^2(x)}{2} + C}$