Ejercicios del libro

29 a) $A + 5B$

b) $7C - 5D$

c) $\|7C\| - \|5D\|$

d) $\|7C - 5D\|$

$A = (1,2,3) = \hat{i} + 2\hat{j} + 3\hat{k}$

$B = (4,-3,-1) = 4\hat{i} - 3\hat{j} - \hat{k}$

$C = (-5,-3,5) = -5\hat{i} - 3\hat{j} + 5\hat{k}$

$D = (-2,1,6) = -2\hat{i} + \hat{j} + 6\hat{k}$

a) $5B = 5(4\hat{i} - 3\hat{j} - \hat{k})$

$5B = 20\hat{i} - 15\hat{j} - 5\hat{k}$

$A + 5B = (\hat{i} + 2\hat{j} + 3\hat{k}) + (20\hat{i} - 15\hat{j} - 5\hat{k})$

$$\boxed{A + 5B = 21\hat{i} - 13\hat{j} - 2\hat{k}}$$

b) $7C = 7(-5\hat{i} - 3\hat{j} + 5\hat{k})$

$7C = -35\hat{i} - 21\hat{j} + 35\hat{k}$

$5D = 5(-2\hat{i} + \hat{j} + 6\hat{k})$

$5D = -10\hat{i} + 5\hat{j} + 30\hat{k}$

$7C - 5D = -35\hat{i} - 21\hat{j} + 35\hat{k} - 10\hat{i} + 5\hat{j} + 30\hat{k}$

$$\boxed{7C - 5D = -45\hat{i} - 16\hat{j} + 65\hat{k}}$$

c) $|7c| - |5D| = 53.7680 - 32.0156$

$|7c| - |5D| = 21.7524$

$7c = -35\hat{i} - 21\hat{j} + 35\hat{k}$

$|7c| = \sqrt{(-35)^2 + (-21)^2 + (35)^2}$

$|7c| = 53.7680$

$5D = -10\hat{i} + 5\hat{j} + 30\hat{k}$

$|5D| = \sqrt{(-10)^2 + (5)^2 + (30)^2}$

$|5D| = 32.0156$

d) $|7c - 5D| = \sqrt{(-45)^2 + (-16)^2 + (65)^2}$

$7c - 5D = -45\hat{i} - 16\hat{j} + 65\hat{k}$

$|7c - 5D| = 80.6597$

30

a) $2A - C$
b) $|2A| - |C|$
c) $4B + 6C - 2D$
d) $|4B| + |6C| - |2D|$

$A = (1,2,3) = i + 2\hat{j} + 3\hat{k}$
$B = (4,-3,-1) = 4i - 3\hat{j} - \hat{k}$
$C = (-5,-3,5) = -5i + 3\hat{j} + 5\hat{k}$
$D = (-2,1,6) = -2i + \hat{j} + 6\hat{k}$

a) $2A - C = 2i + 4\hat{j} + 6\hat{k} + 5i + 3\hat{j} - 5\hat{k}$

$\boxed{2A - C = 7i + 7\hat{j} + \hat{k}}$

$2A = 2(i + 2\hat{j} + 3\hat{k}) = 2i + 4\hat{j} + 6\hat{k}$

b) $|2A| - |C| = 9.9498 - 7.6811 = 2.2687$

$\boxed{|2A| - |C| = 2.2687}$

$|2A| = \sqrt{(7)^2 + (7)^2 + (1)^2} = \boxed{9.9498}$

$|C| = \sqrt{(5)^2 + (-3)^2 + (5)^2} = \boxed{7.6811}$

c) $4B + 6C - 2D$

$4B = 4(4i - 3\hat{t} - \hat{k})$

$\boxed{4B = 16i - 12\hat{t} - 4\hat{k}}$

$6C = 6(-5i - 3\hat{t} + 5\hat{k})$

$\boxed{6C = -30i - 18\hat{t} + 30\hat{k}}$

$2D = 2(-2i + \hat{t} + 6\hat{k})$

$\boxed{2D = -4i + 2\hat{t} + 12\hat{k}}$

$4B + 6C - 2D = 16i - 12\hat{t} - 4\hat{k} - 30i$
$-18\hat{t} + 30\hat{k} + 4i - 2\hat{t} - 12\hat{k}$

$\boxed{4B + 6C - 2D = -10i - 32\hat{t} + 14\hat{k}}$

d) $|4B| + |6C| - |2D|$

$|4B| = \sqrt{(16)^2 + (-12)^2 + (-4)^2}$

$|4B| = 20.3960$

$|6C| = \sqrt{(-30)^2 + (-18)^2 + (30)^2}$

$|6C| = 46.0868$

$|2D| = \sqrt{(-4)^2 + (+2)^2 + (12)^2}$

$|2D| = 12.8062$

$|4B| + |6C| - |2D| = 20.3960 + 46.0868 - 12.8062$

$|4B| + |6C| - |2D| = 53.6766$

31

a) $C + 3D - 8A$

b) $|A|\,|B|(C-D)$

$A = (1,2,3) = i + 2j + 3k$

$B = (4,-3,-1) = 4i + 3j - k$

$C = (-5,-3,5) = -5i - 3j + 5k$

$D = (-2,1,6) = -2i + j + 6k$

a) $C + 3D - 8A$

$3D = 3(-2i + j + 6k)$ $\quad 3D = -6i + 3j + 18k$

$-8A = -8(i + 2j + 3k)$

$-8A = -8i - 16j - 24k$

$C + 3D - 8A = -5i - 3j + 5k - 6i + 3j + 18k$
$- 8i - 16j - 24k$

$C + 3D - 8A = -19i - 16j - k$

b) $|A||B|(C-D)$

$C-D = -5\hat{i} - 3\hat{j} + 5\hat{k} + 2\hat{i} - \hat{j} - 6\hat{k}$

$C-D = -3\hat{i} - 4\hat{j} + \hat{k}$

$|A| = \sqrt{(1)^2 + (2)^2 + (3)^2}$

$|A| = 3.6055$

$|B| = \sqrt{(4)^2 + (-3)^2 + (-1)^2}$

$|B| = 5.0990$

b) $|A||B|(C-D) = (3.6055)(5.0990)(-3\hat{i} - 4\hat{j} - \hat{k})$

$|A||B|(C-D) = -55.1533\hat{i} - 73.5377\hat{j} - 18.3844\hat{k}$

32

a) $3A - 2B + C - 12D$

b) $|A| C - |B| D$

$A = (1,2,3) = (i + 2j + 3k)$

$B = (4,-3,-1) = (4i - 3j - k)$

$C = (-5,-3,5) = (-5i - 3j + 5k)$

$D = (-2,1,6) = (-2i + j + 6k)$

a) $3A - 2B + C - 12D$

$3A = 3(i + 2j + 3k) = 3i + 6j + 9k$

$-2B = -2(4i - 3j - k) = -8i + 6j + 2k$

$12D = 12(-2i + j + 6k) = -24i + 12j + 72k$

$3A - 2B + C - 12D = 3i + 6j + 9k - 8i + 6j + 2k$
$-5i - 3j + 5k - 24i - 12j - 72k$

$3A - 2B + C - 12D = -34i - 3j + 56k$

b) $|A|C - |B|D$

$|A| = 3.6055$

$|B| = 5.0990$

$|A|C - |B|D = (3.6055)(-5i - 3\hat{j} + 5\hat{k}) - (5.0990)(-2i + \hat{j} + 6\hat{k})$

$|A|C - |B|D = (-18.0275i - 10.8165\hat{j} + 18.0275\hat{k}) - (-10.198i + 5.0990\hat{j} + 30.594\hat{k})$

$|A|C - |B|D = -7.8295i - 15.9155\hat{j} - 12.5665\hat{k}$

(39)

$P_1(3,-1,-4)$ $P_2(7,2,4)$ $Q(x,y,z)$

$Q = ?$ $V(\overrightarrow{P_1P_2}) = 3V(P_1Q)$

$7i+2j+4k$ $\quad$ $xi+yj+zk$

$-(3i-j-4k)$ $\quad$ $-(3i-j-4k)$

$4i+3j+8k$ $\quad$ $3[(x-3)i+3(y+1)j+3(z-4)k$

$4i+3j+8k = 3(x-3)i + 3(y+1)j + 3(z-4)k$

$4i+3j+8k = 3x-9i+3y+3j+3z-12k$

$4i+3j+8k+9i-3j+12k = 3x+3y+3z$

$13i+0j+20k = 3(x+y+z)$

$\frac{13i}{3} + \frac{0j}{3} + \frac{20}{3}k = x+y+z$

$Q(13/3, 0, 20/3)$

(40)

$P_1(1,3,5)$ $P_2(2,-1,4)$ $A(x,y,z)$

$R = ?$ $V(P_1R) = -2V(P_2R)$

$$\overrightarrow{P_1R} = x\hat{i} + y\hat{j} + z\hat{k}$$
$$- (1\hat{i} + 3\hat{j} + 5\hat{k})$$
$$(x-1)\hat{i} + (y-3)\hat{j} + (z-5)\hat{k}$$
$$x - i + y - 3\hat{j} + z - 5\hat{k}$$

$$\overrightarrow{P_2R} = x\hat{i} + y\hat{j} + z\hat{k}$$
$$-(2\hat{i} - \hat{j} + 4\hat{k})$$
$$-2\left[(x-2)\hat{i} + (y+1)\hat{j} + (z-4)\hat{k}\right]$$

$$-2V(P_2R) = -2x + 4i - 2y - 2\hat{j} - 2z + 8\hat{k}$$

$$x - i + y - 3\hat{j} - 7 - 5\hat{k} = -2x + 4i - 2y - 2z - 8\hat{k}$$

$$x + y + z + 2x + 2y + 2z = i + 3\hat{j} + 5\hat{k} + 4i - 2\hat{j} + 8\hat{k}$$

$$3x + 3y + 3z = 5i + \hat{j} + 13\hat{k}$$

$$3(x+y+z) = 5i + \hat{j} + 13\hat{k}$$
$$x+y+z = \frac{5}{3}i + \frac{\hat{j}}{3} + \frac{13\hat{k}}{3}$$

$R\left(5/3, 1/3, 13/3\right)$

41

$P_1(3,2,-4)$ $P_2(-5,4,2)$ $P_3(x,y,z)$

$4V(\overrightarrow{P_1P_2}) = -3V(\overrightarrow{P_2P_3})$

$$\overrightarrow{P_1P_2} = -5i + 4j + 2k - (3i + 2j - 4k)$$

$$4(-8i + 2j + 6k)$$

$$\overrightarrow{P_1P_2} = -32i + 8j + 24k$$

$$\overrightarrow{P_3P_2} = xi + yj + 2k - (-5i + 4j + 2k)$$

$$-3((x+5)i + (y-4)j + (z+2)k)$$

$$\overrightarrow{P_3P_2} = -3x - 15i - 3y + 12j - 3z + 6k$$

$$-32i + 8j + 24k = -3x - 15i - 3y + 12j - 3z + 6k$$

$$-32i + 8j + 24k + 18i - 12j - 6k = -3(x + y + z)$$

$$\frac{-17i}{-3} - \frac{4j}{-3} + \frac{18k}{-3} = x + y + z$$

$$P_3\left(17/3, 4/3, -6\right)$$

42

$P_1(7,0,-2)$ $P_2(2,-3,5)$ $P_3(x,y,z)$

$$\overrightarrow{V(P_1P_3)} = 5\overrightarrow{V(P_2P_3)}$$

$$\overrightarrow{P_3P_1} = xi + y\hat{j} + z\hat{k}$$
$$-(7i + 0\hat{j} - 2\hat{k})$$
$$(x+7)i + (y-0)\hat{j} + (z+2)\hat{k}$$
$$x-7i + y-0\hat{j} + z + 2\hat{k}$$

$$\overrightarrow{P_3P_2} = xi + y\hat{j} + z\hat{k}$$
$$-(2i - 3\hat{j} + 5\hat{k})$$
$$5((x-2)i + (y+3)\hat{j} + (z-5)k)$$
$$5x - 10i + 5y + 15\hat{j} + 5z - 25\hat{k}$$

$$x-7i+y-\hat{j}+z+2k = 5x-10i+5y+15\hat{j}+5z-25\hat{k}$$
$$x+y+z-5x-5y-5z = -10i+15\hat{j}-25\hat{k}+7i+2\hat{k}$$

$$-4(x+y+z) = -3i+15\hat{j}-23\hat{k}$$

$$x+y+z = \frac{-3i}{-4} + \frac{15\hat{j}}{-4} - \frac{23\hat{k}}{-4}$$

$$x+y+z = \frac{3}{4}i - \frac{15}{4}\hat{j} + \frac{23}{4}\hat{k}$$

$$\boxed{P_3\left(\frac{3}{4}, -\frac{15}{4}, \frac{23}{4}\right)}$$

45 Vectores Unitarios

a) $P_1(4,-1,-6)$ y $P_2(5,7,-2)$

$\overrightarrow{V(P_1P_2)}$ La misma direccion

$$\begin{array}{r} 5i+7j-2k \\ -(4i-j-6k) \\ \hline \end{array}$$

$$\overrightarrow{V(P_1P_2)} = i+8j+4k$$

$$|P_1P_2| = \sqrt{(1)^2+(8)^2+(4)^2} = \sqrt{81} = 9$$

Vector Unitario $= i+8j+4k$

$$\boxed{U_{P_1P_2} = \frac{1}{9}i + \frac{8}{9}j + \frac{4}{9}k}$$

b) $P_1(-2,5,3)$ $P_2(-4,7,5)$

$$\overrightarrow{V(P_1P_2)} = \begin{array}{r} -4i+7j+5k \\ -(-2i+5j+3k) \\ \hline -2i+2j+2k \end{array}$$

$$|P_1P_2| = \sqrt{(-2)^2+(2)^2+(2)^2} = \sqrt{12}$$

$$|P_1P_2| = \sqrt{12} = \sqrt{4\cdot 3} = 2\sqrt{3}$$

$$U_{P_1P_2} = \frac{2i}{2\sqrt{3}} + \frac{2j}{2\sqrt{3}} + \frac{2k}{2\sqrt{3}}$$

$$\boxed{U_{P_1P_2} = -\frac{\sqrt{3}}{2}i + \frac{\sqrt{3}}{2}j + \frac{\sqrt{3}}{3}k}$$

46

a) $P_1(3,0,-1)$ y $P_2(-3,8,-1)$

$$\overrightarrow{V(P_1P_2)} = \begin{array}{r} -3i + 8j - k \\ -(3i + 0j - k) \\ \hline -6i + 8j + 0k \end{array}$$

$$|P_1P_2| = \sqrt{(-6)^2 + (8)^2} = \sqrt{100} = 10$$

$$U_{P_1P_2} = \frac{-6i}{10} + \frac{8}{10}j + \frac{0}{10}k$$

$$\boxed{U_{P_1P_2} = -\tfrac{3}{5}i + \tfrac{4}{5}j}$$

b) $P_1(-8,-5,2)$ y $P_2(-3,-9,4)$

$$\overrightarrow{V(P_1P_2)} = \begin{array}{r} -3i + 9j + 4k \\ -(-8i - 5j + 2k) \\ \hline 5i - 4j + 2k \end{array}$$

$$|P_1P_2| = \sqrt{(5)^2 + (-4)^2 + (2)^2} = \sqrt{25 + 16 + 4}$$

$$|P_1P_2| = \sqrt{45} = \sqrt{9 \times 5} = 3\sqrt{3}$$

$$U_{P_1P_2} = \frac{5i}{3\sqrt{5}} - \frac{4j}{3\sqrt{5}} + \frac{2k}{3\sqrt{5}}$$

$$\boxed{U_{P_1P_2} = \frac{\sqrt{5}}{15}i - \frac{4\sqrt{5}}{15}j + \frac{2\sqrt{5}}{15}k}$$

31

Determine el Area del triangulo cuyos Vertices son $(-2,3,1)$, $(1,2,3)$ y $(3,-1,2)$

$P_1(-2,3,1)$ $P_2(1,2,3)$ $P_3(3,-1,2)$

$$A = \overrightarrow{P_1P_2} = \begin{array}{r} i + 2\hat{j} + 3\hat{k} \\ -(-2i + 3\hat{j} + \hat{k}) \\ \hline 3i - \hat{j} + 2\hat{k} \end{array}$$

$$B = \overrightarrow{P_1P_3} = \begin{array}{r} 3i - \hat{j} + 2\hat{k} \\ -(-2j + 3\hat{j} + \hat{k}) \\ \hline 5i - 4\hat{j} + \hat{k} \end{array}$$

$$+ \quad - \quad +$$

$$A \times B \begin{vmatrix} i & \hat{j} & \hat{k} \\ 3 & -1 & +2 \\ 5 & -4 & +2 \end{vmatrix}$$

$(-1+8)i - (3-10)\hat{j} + (-12+5)\hat{k}$

$7i + 7\hat{j} - 7\hat{k}$ = Area del Paralelepipedo

calculamos el Area $= \frac{AP}{2}$ = Area del triangulo

$$\frac{\sqrt{(7)^2 + (7)^2 + (-7)^2}}{2} = \frac{12.124355\,65}{2}$$

$$\boxed{A = 6.062}$$

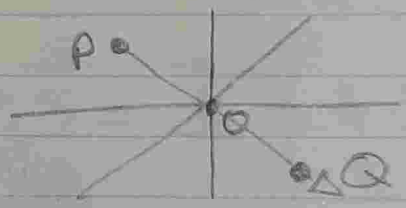
(2) Indique cuales son los componentes del vector que va del punto P(2, -1, 3) al punto Q(-1, 2, -3), si este se translada al origen.

$$\overrightarrow{V(PQ)} = (-1, 2, -3) - (2, -1, 3)$$

$$-3, 3, -6$$

$$(-3, 3, -6) + (0, 0, 0)$$

$$-3, 3, -6$$



$$X = -3, \quad Y = 3, \quad Z = -6$$

(3) Dado el vector en representacion de posicion del ejercicio anterior, obtenga el vetor unitario y expreselo en terminos de los vectores unitarios

$P(2, -1, 3) \quad Q(-1, 2, -3)$

$$\overrightarrow{V(PQ)} = (-1\hat{i} + 2\hat{j} - 3\hat{k}) - (2\hat{i} - \hat{j} + 3\hat{k})$$

$$-3\hat{i} + 3\hat{j} - 6\hat{k}$$

$$|PQ| = \sqrt{(-3)^2 + (3)^2 + (-6)^2} = \sqrt{54} = 3\sqrt{6}$$

$$U_{QP} = \frac{-3\hat{i}}{\sqrt{54}} + \frac{3\hat{j}}{\sqrt{54}} - \frac{6\hat{k}}{\sqrt{54}}$$

$$U_{QP} = \frac{-9\sqrt{6}}{54}\hat{i} + \frac{3\sqrt{6}\hat{j}}{54} - \frac{18\sqrt{6}}{54}$$

(4) Si el vector anterior se desea transladarlo al Punto R(-3, -2, 1), cual debera ser el Punto S Para que este tenga la misma magnitud, direccion y sentido.

$$\overrightarrow{V(PQ)} = (-3, 3, -6)$$

$$\overrightarrow{V(PQR)} = (-3, -2, 1) + (-3, 3, -6)$$

$$S = -6, 1, -5$$

$$\overrightarrow{V(PQRS)} = (-6, 1, -5) - (-3, -2, 1)$$

$$-3, 3, -6$$

El Punto $S = (-6, 1, -5)$

$$|\overrightarrow{V(PQRS)}| = \sqrt{(-3)^2 + (3)^2 + (-6)^2}$$

$$|\overrightarrow{V(PQRS)}| = \sqrt{54} \approx 7.3484$$

(5) Considere al vector en representacion de posicion del ejercicio 2, denominelo A, si ademas se le da el vector B = (4, 2, 1) obtenga ||A| - |3B||, escriba el resultado de la operacion como se le indique.

$A = (-3, 3, -6) = -3i + 3j - 6k$

$B = (4, 2, 1) = 4i + 2j + k$

a) ||A| - |3B||

$3B = 3(4i + 2j + k)$

$3B = 12i + 6j + 3k$

$|3B| = \sqrt{(12)^2 + (6)^2 + (3)^2} = \sqrt{189}$

$|A| = \sqrt{(-3)^2 + (3)^2 + (-6)^2} = \sqrt{54}$

$|\sqrt{54} - \sqrt{189}| = |-6.3992|$

Respuesta = 6.3992