Metodo de inversa Problema 12

$$12 = \begin{cases} 2x - y + z = 0 \\ x + 2y + z = 10 \\ 3x + y = 0 \end{cases} \quad A = \begin{vmatrix} 2 & -1 & +1 \\ 1 & +2 & +1 \\ 3 & +1 & 0 \end{vmatrix} = -10$$

Cofactores

$A11 = (-1)^2 \begin{vmatrix} 2 & 1 \\ 1 & 0 \end{vmatrix} = 1$ $\quad A21 = (-1)^3 \begin{vmatrix} 2 & 1 \\ 1 & 0 \end{vmatrix} \; 1$

$A12 = (-1)^3 \begin{vmatrix} 1 & 1 \\ 3 & 0 \end{vmatrix} -3$ $\quad A22 = (-1)^4 \begin{vmatrix} 2 & +1 \\ 3 & 0 \end{vmatrix} \; -3$

$A13 = (-1)^4 \begin{vmatrix} 1 & 2 \\ 3 & 1 \end{vmatrix} -5$ $\quad A23 = (-1)^5 \begin{vmatrix} 2 & -1 \\ 3 & +1 \end{vmatrix} -5$

$A31 = (-1)^4 \begin{vmatrix} -1 & 1 \\ 2 & 1 \end{vmatrix} -3$ $\quad A = \begin{pmatrix} 1 & -3 & -5 \\ 1 & -3 & -5 \\ -3 & -3 & 5 \end{pmatrix}$

$A32 = (-1)^5 \begin{vmatrix} 2 & 1 \\ 3 & 0 \end{vmatrix} -3$ $\quad A^t \begin{pmatrix} 1 & 1 & -3 \\ -3 & -3 & -3 \\ -5 & -5 & 5 \end{pmatrix} \frac{1}{10}$

$A33 = (-1)^6 \begin{vmatrix} 2 & -1 \\ 1 & 2 \end{vmatrix} \; 5$

$$A^{-1} = \begin{pmatrix} \frac{1}{10} & \frac{1}{10} & -\frac{3}{10} \\ -\frac{3}{10} & -\frac{3}{10} & -\frac{3}{10} \\ -\frac{1}{2} & -\frac{1}{2} & \frac{1}{2} \end{pmatrix}$$

$$\begin{pmatrix} \frac{1}{10} & \frac{1}{10} & -\frac{3}{10} \\ -\frac{3}{10} & -\frac{3}{10} & -\frac{3}{10} \\ -\frac{1}{2} & -\frac{1}{2} & \frac{1}{2} \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} \begin{matrix} 0 \\ 10 \\ 0 \end{matrix} = \begin{matrix} 0 & 1 & 0 \\ 0 & -3 & 0 \\ 0 & -5 & 0 \end{matrix} = (1, -3, -5)$$

Problemu 13

13 - $\begin{cases} x + y - z + w = 4 \\ -x - y = -1 \\ 2x + y - 3w = -4 \\ 2y + z - 2w = -5 \end{cases}$

$$\begin{vmatrix} +1 & +1 & -1 & +1 \\ \cancel{-1} & \cancel{-1} & \cancel{0} & \cancel{0} \\ -2 & +1 & 0 & -3 \\ 0 & 2 & +1 & -2 \end{vmatrix} = \underline{-3}$$

$$-(-1)\cancel{\begin{vmatrix} -1 & -1 & 0 \\ -2 & +1 & -3 \\ 0 & 2 & -2 \end{vmatrix}} + 1 \begin{vmatrix} +1 & +1 & +1 \\ -1 & -1 & 0 \\ -2 & +1 & -3 \end{vmatrix}$$

$$|A| = +1(-3) = \underline{-3}$$

$$A_{11} = (-1)^2 \begin{vmatrix} -1 & 0 & 0 \\ +1 & 0 & -3 \\ 2 & +1 & -2 \end{vmatrix} = \underline{-3}$$

$$A_{21} = (-1)^3 \begin{vmatrix} +1 & -1 & +1 \\ +1 & 0 & -3 \\ 2 & 1 & -2 \end{vmatrix} = \underline{-8}$$

$$A_{12} = (-1)^3 \begin{vmatrix} -1 & 0 & 0 \\ -2 & 0 & -3 \\ 0 & 1 & -2 \end{vmatrix} = \underline{3}$$

$$A_{22} = (-1)^4 \begin{vmatrix} +1 & -1 & +1 \\ -2 & 0 & -3 \\ 0 & 1 & -2 \end{vmatrix} = \underline{5}$$

$$A_{13} = (-1)^4 \begin{vmatrix} -1 & -1 & 0 \\ -2 & +1 & -3 \\ 0 & 2 & -2 \end{vmatrix} = \underline{0}$$

$$A_{23} = (-1)^5 \begin{vmatrix} +1 & +1 & +1 \\ -2 & +1 & -3 \\ 0 & 2 & -2 \end{vmatrix} = \underline{4}$$

$$A_{14} = (-1)^5 \begin{vmatrix} -1 & -1 & 0 \\ -2 & +1 & 0 \\ 0 & 2 & 1 \end{vmatrix} = \underline{3}$$

$$A_{24} = (-1)^6 \begin{vmatrix} +1 & +1 & -1 \\ -2 & +1 & 0 \\ 0 & 2 & +1 \end{vmatrix} = \underline{7}$$

Problema 13

$$\begin{vmatrix} 1 & 1 & -1 & 1 \\ -1 & -1 & 0 & 0 \\ -2 & +1 & 0 & -3 \\ 0 & 2 & +1 & -1 \end{vmatrix}$$

$$A31 = \begin{vmatrix} 1 & -1 & 1 \\ -1 & 0 & 0 \\ 2 & +1 & -2 \end{vmatrix} (-1)^4 = 1$$

$$A33 = (-1)^6 \begin{vmatrix} 1 & 1 & 1 \\ -1 & -1 & 0 \\ 0 & 2 & -2 \end{vmatrix} = -2$$

$$A32 = (-1)^5 \begin{vmatrix} 1 & -1 & 1 \\ -1 & 0 & 0 \\ 0 & +1 & -2 \end{vmatrix} = -1$$

$$A34 = (-1)^7 \begin{vmatrix} 1 & 1 & -1 \\ -1 & -1 & 0 \\ 0 & 2 & +1 \end{vmatrix} = -2$$

$$A41 = (-1)^5 \begin{vmatrix} 1 & -1 & 1 \\ -1 & 0 & 0 \\ +1 & 0 & -3 \end{vmatrix} = -3$$

$$A42 = (-1)^6 \begin{vmatrix} 1 & -1 & 1 \\ -1 & 0 & 0 \\ -2 & 0 & -3 \end{vmatrix} = 3$$

$$A43 = (-1)^7 \begin{vmatrix} 1 & 1 & 1 \\ -1 & -1 & 0 \\ -2 & +1 & -3 \end{vmatrix} = 3$$

$$A44 = (-1)^8 \begin{vmatrix} 1 & 1 & -1 \\ -1 & -1 & 0 \\ -2 & +1 & 0 \end{vmatrix} = 3$$

$$A^+ = \begin{vmatrix} -3 & 3 & 0 & 3 \\ -8 & 5 & 4 & 7 \\ 1 & -1 & -2 & -2 \\ -3 & 3 & 3 & 3 \end{vmatrix}$$

$$A \text{ inversa} \begin{vmatrix} -3 & -8 & 1 & -3 \\ 3 & 5 & -1 & 3 \\ 0 & 4 & -2 & 3 \\ 3 & 7 & -2 & 3 \end{vmatrix}$$

$$A^{-1} \begin{pmatrix} 1 & -\frac{8}{3} & -\frac{1}{3} & -1 \\ -1 & \frac{5}{3} & -\frac{1}{3} & -1 \\ 0 & \frac{4}{3} & +\frac{2}{3} & -1 \\ -1 & -\frac{7}{3} & \frac{2}{3} & -1 \end{pmatrix} \quad \begin{matrix} 4 \\ -1 \\ -4 \\ -5 \end{matrix}$$

$$x = 4 + \frac{8}{3} + \frac{4}{3} + 5 = 13$$

$$y = -4 + \frac{5}{3} + \frac{4}{3} + 5 = 4$$

$$z = 0 + -\frac{4}{3} - \frac{8}{3} + 5 = 1$$

$$w = -4 + \frac{7}{3} - \frac{8}{3} + 5 = \frac{2}{3}$$

x	y	z	w
13	4	1	$\frac{2}{3}$

Problema 17

$$14: \begin{cases} x + y - z = 1 \\ -x + y + z + w = 3 \\ 2x - 2y + w = 4 \\ 2y - z + 3w = 4 \end{cases} \qquad \begin{vmatrix} 1 & +1 & -1 & 0 \\ -1 & +1 & +1 & +1 \\ 2 & -2 & 0 & +1 \\ 0 & +2 & -1 & +3 \end{vmatrix} \qquad |A| = 26$$

$$|A| = +1 \begin{vmatrix} -1 & 1 & 1 \\ 2 & -2 & +1 \\ 0 & 2 & 3 \end{vmatrix} - 1 \begin{vmatrix} 1 & 1 & 0 \\ 2 & -2 & 1 \\ 0 & 2 & 3 \end{vmatrix} + 1 \begin{vmatrix} 1 & 1 & 0 \\ -1 & 1 & 1 \\ 2 & -2 & 1 \end{vmatrix} = 1(6) - 1(-14) + 1(6) = 26$$

$$A_{11} = (-1)^2 \begin{vmatrix} +1 & +1 & +1 \\ -2 & 0 & +1 \\ 2 & -1 & +3 \end{vmatrix} = 11 \qquad A_{21} = (-1)^3 \begin{vmatrix} 1 & -1 & 0 \\ -2 & 0 & 1 \\ 2 & -1 & +3 \end{vmatrix} = 7 \qquad A_{31} = (-1)^4 \begin{vmatrix} +1 & -1 & 0 \\ +1 & +1 & 1 \\ +2 & -1 & 3 \end{vmatrix} = 5$$

$$A_{12} = (-1)^3 \begin{vmatrix} -1 & 1 & 1 \\ 2 & 0 & 1 \\ 0 & -1 & 3 \end{vmatrix} = 9 \qquad A_{22} = (-1)^4 \begin{vmatrix} 1 & +1 & 0 \\ 2 & 0 & 1 \\ 0 & -1 & 3 \end{vmatrix} = 7 \qquad A_{32} = (-1)^5 \begin{vmatrix} -1 & -1 & 0 \\ -1 & +1 & 1 \\ 0 & -1 & 3 \end{vmatrix} = -1$$

$$A_{13} = (-1)^4 \begin{vmatrix} -1 & +1 & 1 \\ 2 & -2 & 1 \\ 0 & +2 & 3 \end{vmatrix} = 6 \qquad A_{23} = (-1)^5 \begin{vmatrix} 1 & 1 & 0 \\ 2 & -2 & +1 \\ 0 & 2 & +3 \end{vmatrix} = +14 \qquad A_{33} = (-1)^6 \begin{vmatrix} 1 & 1 & 0 \\ -1 & 1 & 1 \\ 0 & 2 & 3 \end{vmatrix} = 4$$

$$A_{14} = (-1)^5 \begin{vmatrix} -1 & 1 & 1 \\ 2 & -2 & 0 \\ 0 & +2 & -1 \end{vmatrix} = +1 \qquad A_{24} = (-1)^6 \begin{vmatrix} 1 & 1 & -1 \\ 2 & -2 & 0 \\ 0 & 2 & -1 \end{vmatrix} = 0 \qquad A_{34} = (-1)^8 \begin{vmatrix} 1 & 1 & -1 \\ -1 & 1 & 1 \\ 0 & 2 & -1 \end{vmatrix} = -2$$

$$A_{41} = (-1)^5 \begin{vmatrix} 1 & -1 & 0 \\ 1 & 1 & 1 \\ -2 & 0 & 1 \end{vmatrix} = 4 \qquad A_{42} = (-1)^6 \begin{vmatrix} 1 & -1 & 0 \\ -1 & 1 & 1 \\ 2 & 0 & 1 \end{vmatrix} = -2 \qquad A_{43} = (-1)^7 \begin{vmatrix} 1 & 1 & 0 \\ -1 & 1 & 1 \\ 2 & -2 & 1 \end{vmatrix} = -6$$

$$A_{44} = (-1)^8 \begin{vmatrix} 1 & 1 & -1 \\ -1 & 1 & 1 \\ 2 & -2 & 0 \end{vmatrix} = 4$$

Problema 17

$$A = \begin{matrix} 11 & 9 & 6 & -1 \\ 7 & 7 & 14 & 0 \\ 5 & -1 & 4 & -2 \\ -4 & -2 & -6 & +4 \end{matrix} \qquad A^t = \begin{matrix} 11 & 7 & 5 & -4 \\ 9 & 7 & -1 & -2 \\ 6 & 14 & 4 & -6 \\ -1 & 0 & -2 & +4 \end{matrix}$$

$$A^{-1} = \begin{matrix} \frac{11}{26} & \frac{7}{26} & \frac{5}{26} & -\frac{2}{13} \\ \frac{9}{26} & \frac{7}{26} & \frac{-1}{26} & \frac{-1}{13} \\ \frac{3}{13} & \frac{7}{13} & \frac{2}{13} & \frac{-3}{13} \\ \frac{-1}{13} & 0 & \frac{-1}{13} & +\frac{2}{13} \end{matrix} \Rightarrow \begin{matrix} 1 & \frac{11}{26} + \frac{21}{26} + \frac{10}{13} - \frac{8}{13} = \frac{343}{208} \\ 3 & \frac{9}{26} + \frac{21}{26} - \frac{2}{13} - \frac{4}{13} = \frac{9}{13} \\ 4 & \frac{3}{13} + \frac{21}{13} + \frac{8}{13} - \frac{12}{13} = \frac{20}{13} \\ 4 & \frac{-1}{3} + 0 + \frac{4}{13} + \frac{8}{13} = -\frac{1}{39} \end{matrix}$$

$$\begin{matrix} x & y & z & w \\ \frac{243}{208} & \frac{9}{13} & \frac{20}{13} & \frac{-1}{39} \end{matrix}$$

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Problema 15

$$15=\begin{cases} x-y+2z=3 \\ 3x+2y+z=-1 \\ x+3y-z=-5 \end{cases} \quad \begin{vmatrix} 1 & -1 & 2 \\ 3 & 2 & 1 \\ 1 & 3 & -1 \end{vmatrix} = \underline{5}$$

Cofactores

$$A_{11}=(-1)^2\begin{vmatrix} 2 & 1 \\ 3 & -1 \end{vmatrix}=(1)(-2-3)$$
$$(1)(-5)=-5$$

$$A_{21}=(-1)^3\begin{vmatrix} -1 & 2 \\ 3 & -1 \end{vmatrix}-5(-1)=\underline{5}$$

$$A_{12}=(-1)^3\begin{vmatrix} 3 & 1 \\ 1 & -1 \end{vmatrix}=(-1)(-3-1)$$
$$(-1)(-4)=4$$

$$A_{22}=(-1)^4\begin{vmatrix} 1 & 2 \\ 1 & -1 \end{vmatrix}-2(1)=-2$$

$$A_{13}=(-1)^4\begin{vmatrix} 3 & 2 \\ 1 & 3 \end{vmatrix}=(1)(9-2)$$
$$(1)(7)=7$$

$$A_{23}=(-1)^5\begin{vmatrix} 1 & -1 \\ 1 & 3 \end{vmatrix}\ 2(-1)=-2$$

$$A_{31}=(-1)^4\begin{vmatrix} -1 & 2 \\ 2 & 1 \end{vmatrix}=-5(-1)=5$$

$$A=\begin{matrix} -5 & 4 & 7 \\ 5 & -2 & -2 \\ 5 & 3 & 6 \end{matrix} \qquad A^t=\begin{matrix} -5 & 5 & 5 \\ 4 & -2 & 3 \\ 7 & -2 & 6 \end{matrix}$$

$$A_{32}=(-1)^5\begin{vmatrix} 1 & 2 \\ 1 & -1 \end{vmatrix}-3(-1)=3$$

$$A^{-1}=\frac{1}{5}\begin{pmatrix} -5 & 5 & 5 \\ 4 & -2 & 3 \\ 7 & -2 & 6 \end{pmatrix}$$

$$A_{33}=(-1)^6\begin{vmatrix} 1 & -1 \\ 3 & 2 \end{vmatrix}\ 5(1)=6$$

$$A^{-1}=\begin{pmatrix} -1 & 1 & 1 \\ 4/5 & -2/5 & 3/5 \\ 7/5 & -2/5 & 6/5 \end{pmatrix}$$

$$\begin{pmatrix} -1 & 1 & 1 \\ 4/5 & -2/5 & 3/5 \\ 7/5 & -2/5 & 6/5 \end{pmatrix}\begin{vmatrix} 3 \\ -1 \\ 5 \end{vmatrix} = \begin{matrix} -3-1+5=1 \\ 12/5+2/5+3=\frac{29}{5} \\ 21/5+2/5+6=\frac{53}{5} \end{matrix}$$

$$\underline{1,\ \frac{29}{5},\ \frac{53}{5}}$$

Problema 16

$$16 = \begin{cases} x - y + z = -2 \\ 2x + y - 2z = 5 \\ 4x + y + 2z = 0 \end{cases} \qquad \begin{vmatrix} 1 & -1 & 1 \\ 2 & 1 & -2 \\ 4 & 1 & 2 \end{vmatrix} = 14$$

Cofactores

$A_{11} = (-1)^2 \begin{vmatrix} 1 & -2 \\ 1 & 2 \end{vmatrix}\ 4 \qquad A21 = (-1)^3 \begin{vmatrix} -1 & 1 \\ 1 & 2 \end{vmatrix}\ 3$

$A12 = (-1)^3 \begin{vmatrix} 2 & -2 \\ 4 & 2 \end{vmatrix}\ -12 \qquad A22 = (-1)^4 \begin{vmatrix} 1 & 1 \\ 4 & 2 \end{vmatrix}\ -2$

$A13 = (-1)^4 \begin{vmatrix} 2 & 1 \\ 4 & 1 \end{vmatrix}\ -2 \qquad A23 = (-1)^5 \begin{vmatrix} 1 & -1 \\ 4 & 1 \end{vmatrix}\ -5$

$A31 = (-1)^4 \begin{vmatrix} -1 & 1 \\ 1 & -2 \end{vmatrix}\ 1$

$A = \begin{matrix} 4 & -12 & -2 \\ 3 & -2 & -5 \\ 1 & 4 & -1 \end{matrix} \qquad A^T = \begin{matrix} 4 & 3 & 1 \\ -12 & -2 & 4 \\ -2 & -5 & -1 \end{matrix}$

$A32 = (-1)^5 \begin{vmatrix} 1 & 1 \\ 2 & -2 \end{vmatrix}\ 4$

$A^{-1} = \begin{matrix} \frac{4}{14} & \frac{3}{14} & \frac{1}{14} \\ -\frac{6}{7} & -\frac{1}{7} & -\frac{5}{14} \\ \frac{1}{14} & \frac{4}{14} & -\frac{1}{14} \end{matrix} \qquad \frac{-12}{14} = -\frac{6}{7}$

$A33 = (-1)^6 \begin{vmatrix} 1 & -1 \\ 2 & 1 \end{vmatrix}\ -1$

$$\begin{matrix} \frac{4}{14} & \frac{3}{14} & \frac{1}{14} \\ -\frac{6}{7} & -\frac{1}{7} & -\frac{5}{14} \\ \frac{1}{14} & \frac{4}{14} & -\frac{1}{14} \end{matrix} \cdot \begin{pmatrix} 2 \\ 5 \\ 0 \end{pmatrix} = \begin{matrix} \frac{4}{7} & \frac{15}{14} & +0 = & \frac{23}{14} \\ -\frac{12}{7} & -\frac{5}{7} & 0 & -\frac{17}{7} \\ \frac{1}{7} & \frac{5}{7} & 0 & \frac{6}{7} \end{matrix}$$

$$\frac{23}{14}, -\frac{17}{7}, \frac{6}{7}$$