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① Calcular la derivada parcial con respecto a x

$$xyz = \cos(x+y+z) \qquad \frac{\partial z}{\partial x} = -\frac{F_x}{F_z}$$

$$F(x+y+z)$$

$$\frac{\partial z}{\partial y} = \frac{F_y}{F_z}$$

$$\cos(x+y+z) - xyz = 0$$

$$F_x = -\operatorname{Sen}(x+y+z) - yz$$
$$F_y = -\operatorname{Sen}(x+y+z) - xz$$
$$F_z = -\operatorname{Sen}(x+y+z) - xy$$

$$\frac{\partial z}{\partial x} = -\frac{\operatorname{Sen}(x+y+z) - yz}{\operatorname{Sen}(x+y+z) - xy}$$

② Del anterior eje calcular la derivada parcial con respecto a y

$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z} \qquad \frac{\partial z}{\partial y} = -\frac{F_y}{F_z}$$

$$\frac{\partial z}{\partial y} = -\frac{\operatorname{Sen}(x+y+z) - xz}{\operatorname{Sen}(x+y+z) - xy}$$

③ Calcular la derivada parcial con el metodo de la cadena $z = x^2 \operatorname{Sen} y$ con respecto a s

$$x(s,t) = s^2 + t^2 \qquad \frac{\partial z}{\partial s} = \frac{\partial z}{\partial x} \cdot \frac{\partial x}{\partial s} + \frac{\partial z}{\partial y} \cdot \frac{\partial y}{\partial s}$$
$$y(s,t) = 2st$$

$$\frac{\partial z}{\partial s} = 2x \operatorname{Sen} y (2s) + 2^2(\cos y)(2t)$$

$$\frac{\partial z}{\partial s} = 2(s^2+t^2) \operatorname{Sen}(2st)(2s) + (s^2+t^2)^2 \operatorname{Cos}(2st)\, 2t$$

④ Del eje anterior calcular la derivada par. con respecto a t

$$\frac{\partial z}{\partial t} = \frac{\partial z}{\partial y} \cdot \frac{\partial x}{\partial t} + \frac{\partial z}{\partial y} \cdot \frac{\partial y}{\partial t}$$

$$\frac{\partial z}{\partial t} = 2x \operatorname{Sen} y (2t) + x^2 \operatorname{Cos} y (2s)$$

$$\frac{\partial z}{\partial t} = 2(s^2+t^2) \operatorname{Sen}(25t)(2t) + (s^2+t^2)^2 \operatorname{Cos}(25t)(25)$$

⑤ $f(x,t) = \tan^{-1}(x\sqrt{t})$

$$\frac{\partial f}{\partial x} = \frac{1}{1+x^2t} - \sqrt{t} = \frac{\sqrt{t}}{1+x^2t}$$

$$\frac{\partial f}{\partial t} = \frac{1}{1+x^2t} - \frac{x}{2\sqrt{t}}$$

⑥ $F(x, y, z, t) = \frac{xy^2}{t+2z} = xy^2(t+2z)^{-1}$

$f(x) = \frac{y^2}{t+2z}$ $\qquad f(y) = \frac{2xy}{t+2t}$

$F(z) = -xy^2(t+2z)^{-2}(2) = \frac{-2xy^2}{(t+2t)^2}$

$F(t) = -xy^2(t+2z)^{-2} = \frac{-xy^2}{(t+2t)^2}$

⑦ $F(x, y, z) = x^2 e^{yz}$

$F(x) = 2x e^{y}$

$F(y) = x^2 e^{yz} z$

$F(z) = x^2 e^{yz} y$

⑧ $f(r, s) = r \ln(r^2+s^2)$

$\frac{\partial f}{\partial r} = \ln(r^2+s^2) + r \cdot \frac{1}{r^2+s^2} \cdot 2r$

$\frac{\partial f}{\partial r} = \ln(r^2+s^2) + \frac{2r^2}{r^2+s^2}$

$\frac{\partial f}{\partial s} = r \cdot \frac{1}{r^2+s^2} \cdot 2s = \frac{2rs}{r^2+s^2}$

9) $F(x,t) = \tan^{-1}(x\sqrt{t})$

$$\frac{\partial f}{\partial x} = \frac{1}{1+x^2t} \cdot \sqrt{t} = \frac{\sqrt{t}}{1+x^2t}$$

$$\frac{\partial f}{\partial t} = \frac{1}{1+x^2t} \cdot \frac{x}{2\sqrt{t}}$$

10) $F(x,y) = x^5 + 3x^3y^2 + 3xy^4$

$$\frac{\partial F}{\partial x} = F_x \qquad \frac{\partial f}{\partial y} = F_y$$

$$\frac{\partial f}{\partial x} = 5x^4 + 9xy^2 + 3y^4$$

$$\frac{\partial f}{\partial y} = 6x^3y + 12xy^3 + x^5$$

11) $F(x,y) = \dfrac{x-y}{x+y}$

$$F(x) = \frac{x+y-(x-y)}{(x+y)^2} = \frac{2y}{(x+y)^2}$$

$$F(y) = \frac{-(x-y)-(x-y)}{(x+y)^2} = \frac{-2x+2y}{(x+y)^2}$$

(12) $z = y \ln(x)$

$z_x = y \cdot \frac{1}{x} = \frac{y}{x}$

$z_y = \ln x$

(13) $\text{Sen}(x^2 y^5) + 4x^2 + 3y^2 + 3$

$\frac{dz}{dx} = 2xy^3 \cos(x^2 y^5) + 8x$

$\frac{dz}{dy} = 5x^2 y^4 \cos(x^2 y^5 + 6y^2$

(14) $e^{2x+3y} + e^{4y+5z} + e^{6z+7x}$

$\frac{dz}{dx} = 2e^{2x+3y} + 7e^{6z+7x}$

$\frac{dz}{dy} = 3e^{2x+3y} + 4e^{4y+5z}$

$\frac{dz}{dz} = 5e^{4y+5z} + 6e^{6z+7x}$

(15) $x^2y + x^2z^3 + y^4$

$$\frac{dz}{dx} = 2xy + 2xz^3$$

$$\frac{dz}{dy} = x^2 + 4y^3$$

$$\frac{dz}{dz} = 3x^2z^2$$

(16) $e^{2x+3y+4z}$

$$\frac{dz}{dx} = 2e^{2x+3y+4z}$$

$$\frac{dz}{dy} = 3e^{2x+3y+4z}$$

$$\frac{dz}{dz} = 4z^{2x+3y+4z}$$

(17) Una empresa fabrica 2 tipos de zapatos, los formales y los deportivos. Suponga que la función de costos condjuntos de producir "x" pares del modelo formal y "y" pares del deportivo semanalmente es:

$$C(x, y) = 0.07x^2 + 75x + 85y + 6000$$

Determine costos cuando $x = 100$ $y = 50$

$$\frac{dC}{dx} = 2(0.07x) + 75 = 0.14x + 75$$

$$= 0.14(100) + 75(100) + 85(50) + 6000$$

$$= 14 + 7500 + 4250 + 6000$$

$$= \underline{17\,764}$$

(18) La demanda de un bien "D" esta en función de su precio "P" y del nivel de renta "R", estan relacionados por el modelo

$$D(P, R) = \ln(3P + R) + \sqrt[3]{2P + R^2}$$

Cuando $P = 12$ y $R = 20$

¿Cual es la D integral del precio y la demanda marginal de la renta?

$$\frac{dD}{dP} = \frac{1}{3P + R} \cdot 3 + \frac{1}{3(2P + R^2)^{2/3}} \cdot 2$$

$$= \frac{3}{3P + R} + \frac{2}{3(P + R^2)^{2/3}}$$

$$\frac{dD}{dP} = \frac{3}{3(12) + (20)} + \frac{2}{3(12) + (20^2)^{2/3}} = \underline{0.029}$$

$$\frac{dD}{dR} = \frac{1}{3(12) + (20)} + \frac{2(20)}{3(2(12) + (20)^2)^{2/3}}$$

$$\frac{dD}{dR} = \underline{0.25}$$

(19) Un Fabricante estima que la produccion anual de cierta fabrica esta dada por $P(L, K) = 120 K^{1/5} L^{1/5} + 10K$ donde K es el gasto del capital en dolores y L el tamaño de la fuerza loboral en horas - trabajador. Halle la productividad morginal de mono de obra P_L cuando el gasto de Capital es de 243 dlls y el nivel de trabajo es de 256 hrs

$$\frac{dP}{dK} = 120 L^{1/5} \frac{d}{dK} K^{1/5} + 10 \frac{d}{dK} K$$

$$\frac{\partial P}{\partial L} = 120 K^{1/5} \frac{d}{\partial L} L^{1/5}$$

$$= \frac{1}{5} \cdot \frac{120 L^{1/5}}{K^{1/5}} = 24 \frac{L^{1/5}}{K^{1/5}} + 10.$$

(20) $w = xy^2 + x^2 z + yz^2$ $\quad x = t^2 \quad y = 2t \quad z = 2$

$\frac{dw}{dx} = y^2 + 2xz (2t) \qquad \frac{dw}{dy} = 2xy + z^2 (2)$

$$[(y^2 + 2xz)(2t)] + [(2xy + z^2)(2)]$$

(21) $z = \sqrt{x^2 + y^2} \quad x \quad e^{2t} \quad y = e^{-2t}$

$$\frac{dz}{dx} = \frac{x}{(x^2 + y^2)} \qquad \frac{dz}{dy} = \frac{y}{(x^2 + y^2)}$$

$$\left(\frac{x}{(x^2 + y^2)} \cdot 2e^{2t}\right) + \left(\frac{y}{(x^2 + y^2)} \cdot -2e^{-2t}\right)$$

(22) $z = 2xy \quad x = e^{4s} + t^2 \quad y = s^3 + \operatorname{sen}(5t)$

$$\frac{dz}{dx} = 2y \qquad \frac{dz}{dy} = 2x \qquad \frac{dx}{ds} = 4e^{4s} \qquad \frac{dx}{dt} = 2t$$

$$\frac{dz}{ds} = (2y \cdot 4e^{4s}) + (2x \cdot 3s^2)$$

$$\frac{dz}{dt} = (2y \cdot 2t) + (2x \cdot 5 \cos(5t))$$

(23) $F(x, y, z) = x^3 e^{yw}$

$F(x) = 3x e^{yw}$

$F(y) = x^3 eyw$

$F(w) = x^3 e^{yw}$

(24) $f(x,y) = x \ln(x^2 + w^2)$

$$\frac{\partial f}{\partial x} = \ln(x^2 + w^2) + x \cdot \frac{1}{x^2 + w^2} \cdot 2x$$

$$= \ln(x^2 + w^2) + \frac{2x^2}{x^2 + w^2}$$

$$\frac{\partial f}{\partial w} = \frac{2xw}{x^2 + w^2}$$

(25) $w = x/y \quad x = se^t \quad y = 1 + se^{-t}$

$$\frac{dz}{dx} = \frac{1}{y} \quad \frac{dz}{dy} = xy^{-1} \quad \frac{dx}{ds} = e^t \quad \frac{dx}{dt} = se^t$$

$$\frac{dz}{ds} = \left(\frac{1}{y} \cdot e^t\right) + \left(-\frac{x}{y^2} \cdot e^{-t}\right)$$

$$\frac{dz}{dt} = \left(\frac{1}{y} \cdot se^t\right) + \left(-\frac{x}{y^2} \cdot -se^{-t}\right)$$

(27) Las dimensiones de una caja rectangular son: x, y, z cada medida es correcta con un margen de error de 0.2cm
Encuentra el mayor error

$V = xyz$

$x = 55cm$ $= (70 \cdot 30 \cdot 0.2) + (55 \cdot 30 \cdot 0.2) + (55 \cdot 70 \cdot 0.2)$

$y = 70cm$ $= 420 + 330 + 770$

$z = 30cm$

$= 1520\ cm^3$ $V = 55 \cdot 70 \cdot 30 = 115\,500$

$= \frac{1520}{115\,500} = 0.013 \rightarrow \underline{1.3\%}$

(28) Calcular el volumen de un cono con medidas 1.7m de h y 0.5m de radio con un error de 30cm. Estima el mayor error

$V = \frac{\pi r^2 h}{3} = \frac{\pi (0.5)^2 (1.7)}{3} = 0.4450 = V$

$h = 1.7m$ $\left(\frac{2\pi (0.5)(1.7)(0.3)}{3}\right) + \left(\frac{\pi (0.5)^2}{3} \cdot 0.3\right)$

$r = 0.5m$

$0.534 + 0.0785 = 0.6125$

$= \underline{61.25\%}$

(29) Una compañia de cajas fabrica cajas con medidas de $x = 30cm$, $y = 40cm$ y $z = 10cm$ con un margen de error = 5cm

$V = xyz = (30)(40)(10) = 12000$

$\frac{dv}{dx} = yz \quad \frac{dv}{dy} = xz \quad \frac{dv}{dz} = xy$

$\frac{9500}{12000} = 0.79$

$= 79\%$

$= (40)(10)(5) + (30)(10)(5) + (30)(40)(5)$

$2000 + 1500 + 6000 = 9500$

(30) Una fabrica de tanques de gas tienen $h = 40m$ y $5m$ de radio, error = 0.07m
Calcular vol y cambio absoluto

$V = \pi r^2 h \to \pi(5)^2(40) = 3141.60$

$\frac{dv}{dh} = \pi r^2$

$\frac{dv}{dr} = 2\pi r h$

$= (\pi(5)^2(0.07)) + (2\pi(5)(40)(0.07))$

$= 5.49 + 87.96$

$= 93.45$

$= \frac{93.45}{3141.6} = 0.0297$

$= 2.97\%$