

# CSI 3350: PROGRAMMING LANGUAGES

Department of Computer Science &  
Engineering  
Oakland University

## What have we covered so far ?



- Described main quality criteria for high level programming languages such as readability, writability etc. (for example you can add new syntax to the language to facilitate the readability of your program using *define-syntax-rule*, *define-syntax* etc.)



- Described syntax of fundamental program components ( *hw06 loop*, *block structure*, *scoping mechanism*, etc.,)



- Discussed fundamental concepts of operational semantics ( *hw06*, coding the **value-of** function )

yet to cover



- Describe parameter passing and access to non-locals (hw07 – soon! )
- Described data types and type systems ( *hw06*, *grammar*, *hw05*, *define-datatype* )



- Apply major features of functional programming languages ( *hw01~hw04*, *map*, *foldl*, high order functions, *lambda* etc.)



- Described activation records ( Sep 30 lecture notes, slides 43 ~ 66)

## Road Ahead -

**HW05** Due: Nov 12



**HW06** Due: Nov 24



**Exam02** on Nov 27



**HW07** Due: Dec 7 @ 10pm



**Final Exam : 7pm ~10pm : Dec 09, 2019 (same classroom)**

# The LET language

---

How to extend  
**LET** with  
**procedure**  
handling  
capabilities?

---

```
Program ::= Expression
         a-program (exp1)

Expression ::= Number
           const-exp (num)

Expression ::= -(Expression , Expression)
           diff-exp (exp1 exp2)

Expression ::= zero? (Expression)
           zero?-exp (exp1)

Expression ::= if Expression then Expression else Expression
           if-exp (exp1 exp2 exp3)

Expression ::= Identifier
           var-exp (var)

Expression ::= let Identifier = Expression in Expression
           let-exp (var exp1 body)
```

Figure 3.2 Syntax for the LET language

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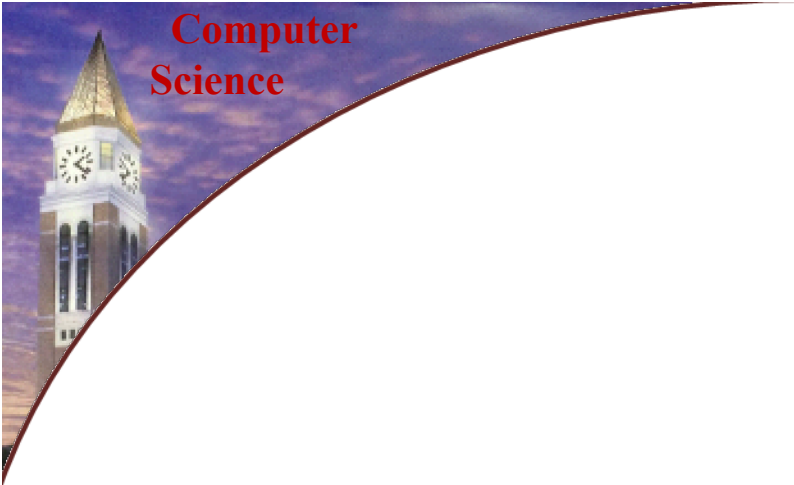
# The Extended LET language

How to extend  
**LET** with  
**procedure**  
handling  
capabilities?

```
Expression ::= proc (Identifier) Expression  
              proc-exp (var body)  
  
Expression ::= (Expression Expression)  
              call-exp (rator rand)
```

```
Program ::= Expression  
          a-program (exp1)  
  
Expression ::= Number  
             const-exp (num)  
  
Expression ::= -(Expression , Expression)  
             diff-exp (exp1 exp2)  
  
Expression ::= zero? (Expression)  
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             var-exp (var)  
  
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```

Figure 3.2 Syntax for the LET language



## Suggested reading:

- EOPL: 3.1-3.2 (implementation of LET language)
- EOPL: 3.3 (Extend the LET language with Procedures p74- p82)

# Fold Functions

---

- Two variants: `foldl` & `foldr`
  - applies a procedure to the elements of one or more lists.
  - ( `foldl` `proc` `default` `list`<sub>1</sub> ... `list`<sub>n</sub>)
    - the # of parameters that `proc` have must be **n+1**
    - following left to right order
    - the result of ( `proc` `x`<sub>1</sub> ... `x`<sub>n</sub> `default` ) is stored and was used as the `default` for the next round

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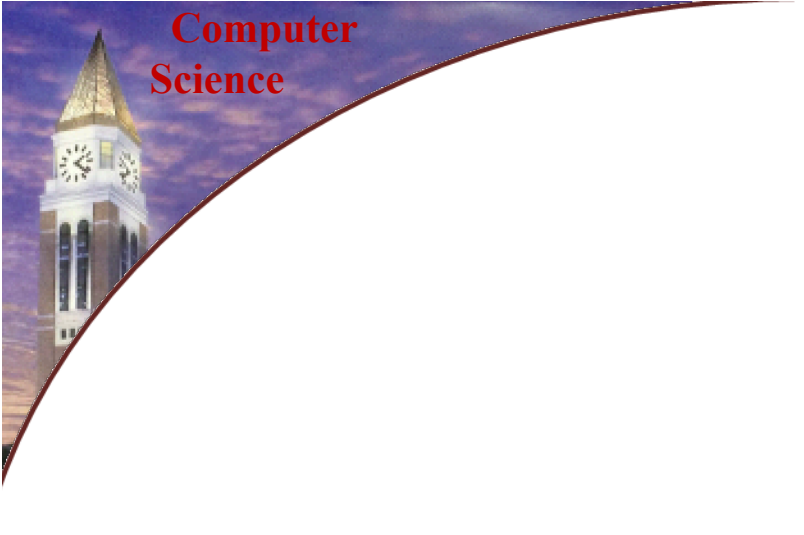


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  - (`foldl proc default list1 ... listn`)
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- ( foldl proc **default** list<sub>1</sub> ... list<sub>n</sub>)
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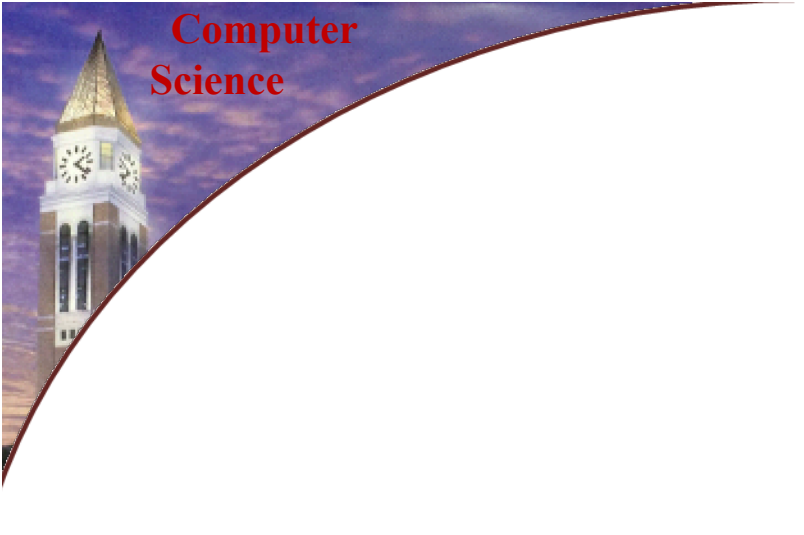


```
; (expr ("move" "(" expr (arbno expr) ")") move-expr)
(move-expr
 (point-expr first-move rest-of-moves)
 (letrec
  ([start-p (point-val->p (value-of point-expr env))]
   [all-moves-as-expr (map (lambda (ex) (value-of ex env)) (flat-list
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  )
 )
```

**(define (move st start-p)**

    ; to save space we omit the function body here  
    ; which you can find on page 4 of the code brochure.

**)**

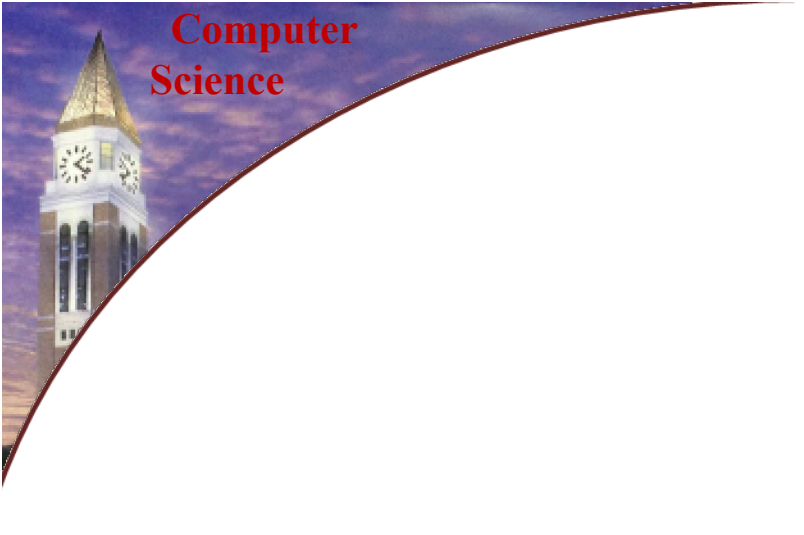


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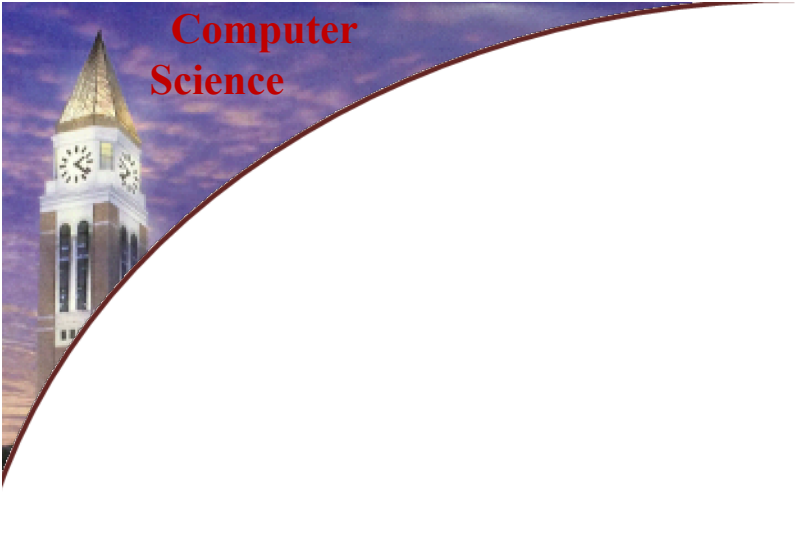


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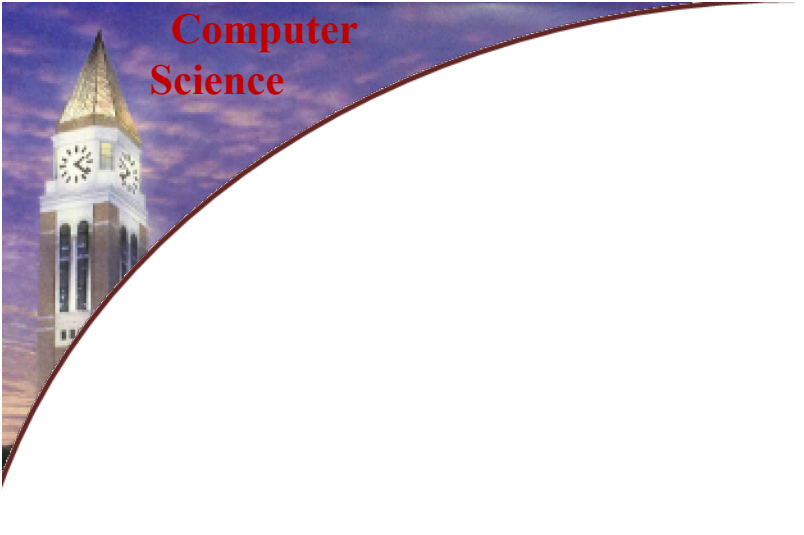
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swap **st** and **start-p** causes a problem ?



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swap **st** and **start-p** causes a  
problem ? **YES !!**

# Fold Functions

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- Two variants: `foldl` & `foldr`
  - applies a procedure to the elements of one or more lists.
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- `( foldl proc default list1 ... listn)`
  - the # of parameters that proc have must be **n+1**
  - following left to right order
  - the result of `(proc x1 ... xn default)` is stored and was used as the default for the next round

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   [final-p (foldl move start-p all-moves-step)])
  (point-val final-p)
  )
 )
```

(define (move st start-p)

; to save space we omit the function body here  
; which you can find on page 4 of the code brochure.

)

replace `proc` with `move`

- ( foldl `proc` **default**  $list_1 \dots list_n$ )
  - the # of parameters that `proc` have must be  **$n+1$**
  - following left to right order
  - the result of (`proc`  $x_1 \dots x_n$  **default**) is stored and was used as the default for the next round

```
; (expr ("move" "(" expr (arbno expr)")) move-expr)
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   [final-p (foldl move start-p all-moves-step)])
  (point-val final-p)
  )
 )
```

(define (**move st start-p**)

; to save space we omit the function body here  
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)

for example if `start-p` is (point 0 0)

```
(foldl move start-p ` ( (left-step 4) (up-step 2) (right-step 3) ) )
```

for example if start-p is (point 0 0)

```
(foldl move (point 0 0) ` ( (left-step 4) (up-step 2) (right-step 3) ) )
```

```
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)
```

```
(foldl move (point 0 0) ` ( (left-step 4) (up-step 2) (right-step 3) ) )
```

```
(define (move 1st 2start-p)  
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  ; which you can find on page 4 of the code brochure.  
)
```

```
(foldl move (point 0 0) '( (left-step 4) (up-step 2) (right-step 3) ) )
```

```
(foldl move (point 0 0) \ ( left-step 4) (up-step 2) (right-step 3) ) )
```

(define (<sup>1</sup>move <sup>2</sup>st start-p)  
; to save space we omit the function body here  
; which you can find on page 4 of the code brochure.  
)



```
(define (move 1st 2start-p)  
  ; to save space we omit the function body here  
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```
(foldl move (point 0 0) '( (left-step 4) (up-step 2) (right-step 3) ) )
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```

```
(foldl move (point 0 0) '( (left-step 4) (up-step 2) (right-step 3) ) )
```



```
(foldl move (point -4 0) '( (up-step 2) (right-step 3) ) )
```

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(foldl move (point 0 0) '( (left-step 4) (up-step 2) (right-step 3) ) )
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```
(foldl move (point -4 2) '( (right-step 3) ) )
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```

```
(foldl move (point 0 0) '( (left-step 4) (up-step 2) (right-step 3) ) )
```

Diagram: Arrows point from the <sup>2</sup>0 in (point 0 0) to the <sup>2</sup> in the second 0, and from the <sup>1</sup> in (left-step 4) to the 4.



```
(foldl move (point -4 0) '( (up-step 2) (right-step 3) ) )
```

Diagram: Arrows point from the <sup>2</sup>-4 in (point -4 0) to the 4, and from the <sup>1</sup> in (up-step 2) to the 2.



```
(foldl move (point -4 2) '( (right-step 3) ) )
```

Diagram: Arrows point from the <sup>2</sup>-4 in (point -4 2) to the 4, and from the <sup>1</sup> in (right-step 3) to the 3.



```
(define (move 1st 2start-p)
  ; to save space we omit the function body here
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)
```

```
(foldl move (point 0 0) '( (left-step 4) (up-step 2) (right-step 3) ) )
```

Diagram: A horizontal line with a vertical line in the middle. An arrow labeled '2' points down to '0' in '(point 0 0)'. An arrow labeled '1' points down to '(left-step 4)'.



```
(foldl move (point -4 0) '( (up-step 2) (right-step 3) ) )
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Diagram: A horizontal line with a vertical line in the middle. An arrow labeled '2' points down to '-4' in '(point -4 0)'. An arrow labeled '1' points down to '(up-step 2)'.



```
(foldl move (point -4 2) '( (right-step 3) ) )
```

Diagram: A horizontal line with a vertical line in the middle. An arrow labeled '2' points down to '2' in '(point -4 2)'. An arrow labeled '1' points down to '(right-step 3)'.



```
(foldl move (point -1 2) '( ) )
```



```
(define (move 1st 2start-p)
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)
```

```
(foldl move (point 0 0) '( (left-step 4) (up-step 2) (right-step 3) ) )
```

Diagram: A horizontal line with a vertical line in the middle. An arrow labeled '2' points down to '0' in '(point 0 0)'. An arrow labeled '1' points down to '(left-step 4)'.

↓

```
(foldl move (point -4 0) '( (up-step 2) (right-step 3) ) )
```

Diagram: A horizontal line with a vertical line in the middle. An arrow labeled '2' points down to '-4' in '(point -4 0)'. An arrow labeled '1' points down to '(up-step 2)'.

↓

```
(foldl move (point -4 2) '( (right-step 3) ) )
```

Diagram: A horizontal line with a vertical line in the middle. An arrow labeled '2' points down to '2' in '(point -4 2)'. An arrow labeled '1' points down to '(right-step 3)'.

↓

```
(foldl move (point -1 2) '( ) )
```

Diagram: A horizontal line with a vertical line in the middle. No arrows are present.

```
(define (move 1st 2start-p)
  ; to save space we omit the function body here
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)
```

```
(foldl move (point 0 0) '( (left-step 4) (up-step 2) (right-step 3) ) )
```



```
(foldl move (point -4 0) '( (up-step 2) (right-step 3) ) )
```



```
(foldl move (point -4 2) '( (right-step 3) ) )
```



```
(foldl move (point -1 2) ' ( ) )
```



```
(point -1 2)
```

```
(define (move 1st 2start-p)
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)
```

```
(foldl move (point 0 0) '( (left-step 4) (up-step 2) (right-step 3) ) )
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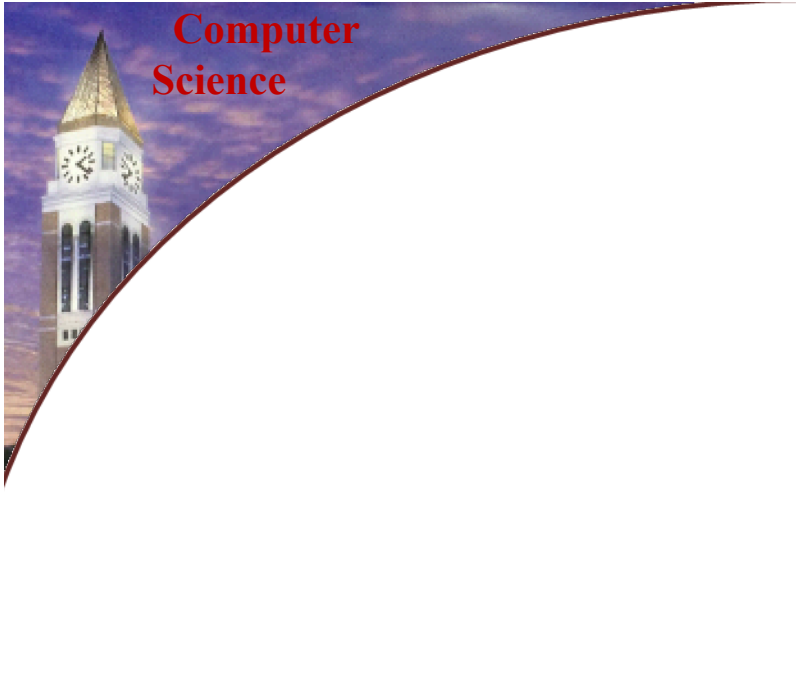


```
(foldl move (point -1 2) ' ( ) )
```



```
(point -1 2)
```

each intermediate  
point is decided by  
**move** function !



# EXTEND LET LANGUAGE WITH PROCEDURES

# The LET language

---

---

```
Program ::= Expression
         a-program (exp1)

Expression ::= Number
           const-exp (num)

Expression ::= -(Expression , Expression)
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```

Figure 3.2 Syntax for the LET language

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# The LET language

How to extend  
**LET** with  
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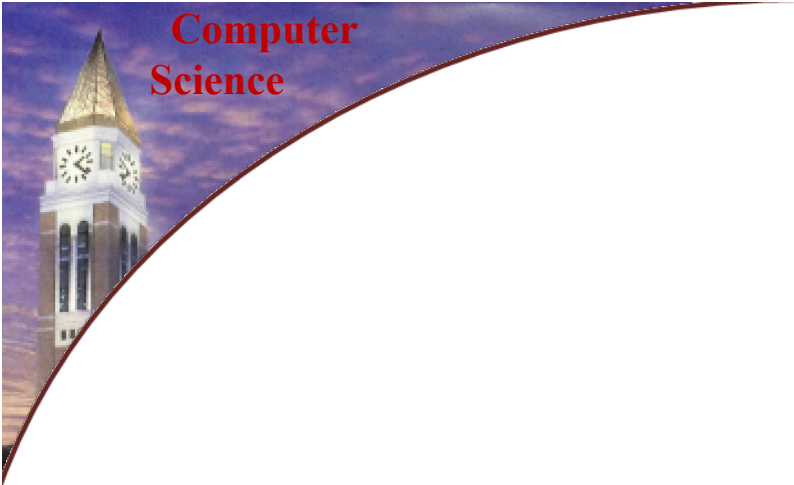
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Expression ::= let Identifier = Expression in Expression
            let-exp (var exp1 body)
```

Figure 3.2 Syntax for the LET language



## Suggested reading:

- EOPL: 3.1-3.2 (implementation of LET language)
- EOPL: 3.3 (Extend the LET language with Procedures p74- p82)

# Extend the language with Procedures

---

- Concrete Syntax rules -

Expression ::= `proc` (Identifier) Expression  
| (Expression Expression)

Procedure Definition

Procedure Call



# The Extended LET language

*Program* ::= *Expression*

`a-program (exp1)`

*Expression* ::= *Number*

`const-exp (num)`

*Expression* ::= *-(Expression , Expression)*

`diff-exp (exp1 exp2)`

*Expression* ::= *zero? (Expression)*

`zero?-exp (exp1)`

*Expression* ::= *if Expression then Expression else Expression*

`if-exp (exp1 exp2 exp3)`

*Expression* ::= *Identifier*

`var-exp (var)`

*Expression* ::= *let Identifier = Expression in Expression*

`let-exp (var exp1 body)`

*Expression* ::= *proc (Identifier) Expression*

`proc-exp (var body)`

*Expression* ::= *(Expression Expression)*

`call-exp (rator rand)`

Figure 3.2 Syntax for the LET language

# Examples

---

```
let f = proc (x) -(x,11)
      in (f (f 77))
```

# Examples

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```

←(num-val 55)

- EOPL: 3.1-3.2 (implementation of LET language)
  - EOPL: 3.3 (Extend the LET language with Procedures p74- p82)
- 

To Define a Procedure in Scheme –

```
(define a  
  (lambda (x) (+ x 1) ) )
```

- EOPL: 3.1-3.2 (implementation of LET language)
  - EOPL: 3.3 (Extend the LET language with Procedures p74- p82)
- 

To Define a Procedure in Scheme –

```
(define a whole thing is one value, given to a  
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To Define a Procedure in Scheme –

```
(define a whole thing is one value, given to a  
  (lambda (x) (+ x 1) ) )
```

```
(define-datatype expval expval?  
  (num-val  
    (num number?) )  
  (bool-val  
    (bool boolean?))  
  (proc-val  
    (proc proc?))  
  )
```

- EOPL: 3.1-3.2 (implementation of LET language)
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To Define a Procedure in Scheme –

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(define a whole thing is one value, given to a  
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```
(define-datatype expval expval?  
  (num-val  
    (num number?) )  
  (bool-val  
    (bool boolean?))  
  (proc-val  
    (proc proc?))  
  )
```

```
(define-datatype proc proc?  
  (procedure  
    (var identifier?)  
    (body expression?)  
    (saved-env environment?)))
```




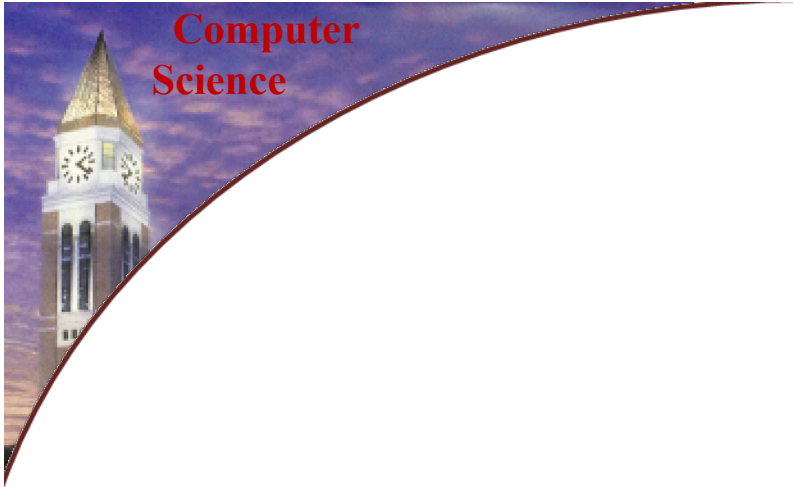
Found at procedure  
creation-time, **not**  
procedure call-time !!

- EOPL: 3.1-3.2 (implementation of LET language)
  - EOPL: 3.3 (Extend the LET language with Procedures p74- p82)
- 

```
(define-datatype expval expval?  
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    (num number?) )  
  (bool-val  
    (bool boolean?))  
  (proc-val  
    (proc proc?))  
  )
```

```
(define-datatype proc proc?  
  (procedure  
    (var identifier?)  
    (body expression?)  
    (saved-env environment?)))
```






# Closure (in hw07)

- EOPL: 3.1-3.2 (implementation of LET language)
  - EOPL: 3.3 (Extend the LET language with Procedures p74- p82)
- 

```
(define-datatype expval expval?  
  (num-val  
    (num number?) )  
  (bool-val  
    (bool boolean?))  
  (proc-val  
    (proc proc?))  
  )
```

```
(define-datatype proc proc?  
  (procedure  
    (var identifier?)  
    (body expression?)  
    (saved-env environment?)))
```



- EOPL: 3.1-3.2 (implementation of LET language)
- EOPL: 3.3 (Extend the LET language with Procedures p74- p82)

```
(define-datatype expval expval?  
  (num-val  
    (num number?) )  
  (bool-val  
    (bool boolean?))  
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```

```
(define-datatype proc proc?  
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    (var identifier?)  
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    (saved-env environment?)))
```



Procedures have to be  
associated with an  
environment !

- EOPL: 3.1-3.2 (implementation of LET language)
- EOPL: 3.3 (Extend the LET language with Procedures p74- p82)

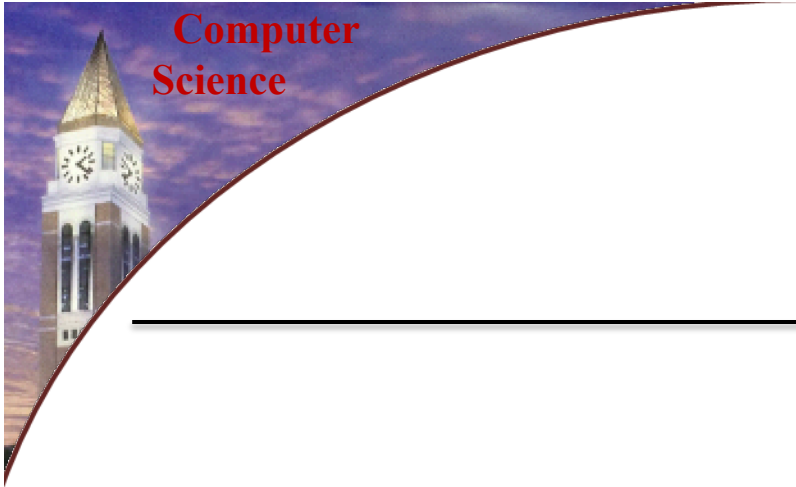
```
(define-datatype expval expval?  
  (num-val  
    (num number?) )  
  (bool-val  
    (bool boolean?))  
  (proc-val  
    (proc proc?))  
  )
```

```
(define-datatype proc proc?  
  (procedure  
    (var identifier?)  
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    (saved-env environment?)))
```



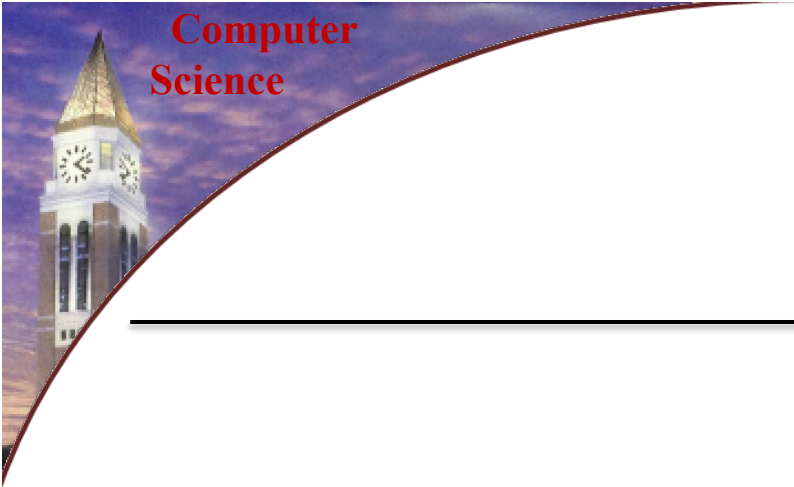
Procedures have to be  
associated with an  
environment !

Together they are called  
a **closure** .



Computer  
Science

# SEMANTICS



# SEMANTICS

## (operational semantics)



# Extend (value-of ...)

For procedure definition!

```
(define (value-of ex env)
  (cases expression ex
    ...
    (proc-exp (var body)
      (proc-val (procedure var body env) ))
  )
)
```

AST  
returned  
by SLLGEN  
parser

Expression ::= proc (Identifier) Expression  
proc-exp (var body)

Concrete  
Syntax

Abstract  
Syntax  
( SLLGEN ... )

# Extend (value-of ...)

For procedure call!

```
(define (value-of ex env)
  (cases expression ex
    ...
```

```
    (call-exp (rator rand)
```

```
      (let ((proc (expval->proc (value-of rator env)))
```

```
        (arg (value-of rand env)))
```

```
        (apply-procedure proc arg)) )))
```

Expression ::= (Expression Expression)  
call-exp (rator rand)

```
( proc (x) -(x, 11) 12)
```

```
(      Expression      Expression )
```

SLLGEN  
Parser

Expression ::= proc (Identifier) Expression  
proc-exp (var body)

# Extend (value-of ...)

For procedure call!

```
(define (value-of ex env)
  (cases expression ex
    ...
```

Expression ::= (Expression Expression)  
call-exp (rator rand)

```
    (call-exp (rator rand)
      (let ((proc (expval->proc (value-of rator env)))
            (arg (value-of rand env)))
        (apply-procedure proc arg)) )))
```

```
( (proc (x) - (x, 11) 12)
  rator rand → (value-of rand env) → (num-val 12)
  ↓
(value-of rator env)
  ↓
(proc-val (expval->proc (value-of rator env))
  (procedure (var-exp 'x) (diff-exp (var-exp 'x) (const-exp 11) ) env )
)
```

# Extend (value-of ...)

For procedure call!

```
(define (value-of ex env)
  (cases expression ex
    ...
```

Expression ::= (Expression Expression)  
call-exp (rator rand)

```
    (call-exp (rator rand)
      (let ((proc (expval->proc (value-of rator env)))
            (arg (value-of rand env)))
        (apply-procedure proc arg) ) ) )
```

Diagram illustrating the evaluation of a procedure call expression:

```
( proc (x) - (x, 11) 12 )
```

The diagram shows the evaluation process:

- The **proc** (procedure) is evaluated to a **proc-val** object: `(procedure (var-exp 'x) (diff-exp (var-exp 'x) (const-exp 11) ) env )`.
- The **rand** (right-hand side) is evaluated to a **num-val** object: `(num-val 12)`.
- The **arg** (argument) is passed to the **proc** (procedure).

# Extend (value-of ...)

For procedure call!

```
(define (value-of ex env)
  (cases expression ex
    ...
```

Expression ::= (Expression Expression)  
call-exp (rator rand)

```
    (call-exp (rator rand)
      (let ((proc (expval->proc (value-of rator env)))
            (arg (value-of rand env)))
        (apply-procedure proc arg)))))
```

Diagram illustrating the evaluation of a procedure call expression:

```
( proc (x) - (x, 11) 12 )
```

The expression is evaluated as follows:

- The **proc** part is evaluated to a procedure object: `(proc-val (procedure (var-exp 'x) (diff-exp (var-exp 'x) (const-exp 11)) env))`.
- The **rand** part is evaluated to a value: `(value-of rand env) → (num-val 12)`.
- The procedure object is then applied to the value: `(apply-procedure (proc-val ...) (num-val 12))`.

# Extend (value-of ...)

For procedure call!

```
(define (apply-procedure proc1 val)
  (cases proc proc1
    (procedure (var body saved-env)
      (value-of body (extend-env var val saved-env))
    )
  )
)
```

```
( proc (x) - (x, 11) 12)
```



```
(apply-procedure
  proc (procedure (var-exp 'x) (diff-exp (var-exp 'x) (const-exp 11) ) (empty-env) )
  arg (num-val 12)
)
```



```
(value-of
  (diff-exp (var-exp 'x) (const-exp 11) )
  (extend-env 'x (num-val 12) (empty-env))
)
```