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CSI 5810 Assignment 3

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# Question 1

For this question I looked at the Perceptron Lerning Rule to apply to my script. Since the Perctron has two functions

1. Calculate the Weighted Sum.
2. Produce the output.

I created two functions that correspond to each of the functions above.

Running the script:

| Iteration | Weight |
|---|---|
| 1 | [1. 1. 1.] |
| 2 | [-1. 0. -1.] |
| 3 | [-1. 1. -4.] |
| 4 | [-1. 1. -4.] |
| 5 | [-1. 1. -4.] |

The Percentron then gave the output:

| OUTPUT |
|---|
| [-1. 1. -4.] |

# Question 2

For this question we will be using the Majority Rule or more fundamentaly in math the Binomial Distribution. Which states that the Probability where **X** is eqaul to **x** given a probability **p** and some quantity of **n** samples is equal to:

$P(X = x \mid p, n) = ( n! / r! ( n - x ) ) ( p^{x} ( 1 - p )^{n - x} )$

Where **X** is the set of all testing learners and **x** is one of the learners.

This can be futher expressed in statistical form as $P(X = x \mid p, n) = (nCr) ( p^{x} ( 1 - p )^{n - x} )$ Where **nCr** Means 'n Choose r' which is just the binomial coefficient or the 'combinations' of n choosing a value r.

$$\binom{n}{k} = \frac{n!}{k!(n-k)!}$$

```
We are given that p = 55% = .55
and we have 7 learners = n
```

To calculate the accuracy of the ensemble, we will need to calculate the summation of the probabilities of **x**. So by applying the binomial distribution for the first iteration:

| **n** | **nCr** | $p^x$ | $( 1 - p )^{n - x}$ | **P(X = x \ n, p)** |
|---|---|---|---|---|
| 1 | 7C1 => 7 | $.55^{1}$ => 0.55 | $(1 - .55)^{7 - 1}$ => $(.45)^6$ => 0.008303766 | 0.031969499 |
| 2 | 7C2 => 21 | $.55^{2}$ => 0.3025 | $(1 - .55)^{7 - 2}$ => $(.45)^5$ => 0.018452813 | 0.1172214946 |
| 3 | 7C3 => 35 | $.55^{3}$ => 0.166375 | $(1 - .55)^{7 - 3}$ => $(.45)^4$ => 0.04100625 | 0.2387845195 |
| 4 | 7C4 => 35 | $.55^{4}$ => 0.09150625 | $(1 - .55)^{7 - 4}$ => $(.45)^3$ => 0.091125 | 0.2918477461 |
| 5 | 7C5 => 21 | $.55^{5}$ => 0.050328438 | $(1 - .55)^{7 - 5}$ => $(.45)^2$ => 0.2025 | 0.2140216826 |
| 6 | 7C6 => 7 | $.55^{6}$ => 0.027680641 | $(1 - .55)^{7 - 6}$ => $(.45)^1$ => 0.45 | 0.0871940192 |
| 7 | 7C7 => 1 | $.55^{7}$ => 0.015224352 | $(1 - .55)^{7 - 7}$ => $(.45)^0$ => 1 | 0.015224352 |

Resulting with an **accuracy** of **.608 = 60.8%** When P(X > 3) == P(X >= 4)

To find the minimum learners to achieve 90% accuracy we can run the binomial distribution function until we reach an n value where the result equal 90%. The amount of learners needed to achieve 90% is: **163 Learners.**

## Question 3

The Fisher's linear discriminant function is in the code.

C1 Projection: -0.647594985288474

C2 Projection: -2.6790008954842017

Determination of [3,3] Class Label: -1.201995650505309

Z-cut: -1.663297940386338

So the predicted value is: **Class One.**

## Question 4

GINI Value for **Color**:

Red = 3/6 | Blue = 1/6 | Green = 2/6

Red and + = 2/3, Red and - = 1/3, Blue and + = 1/1, Green and - = 2/2

G(red) = 1 - [ $(2/3)^2$+$(1/3)^2$ ]= 1 - (11/18) = **0.388**

G(blue) = 1 - ($1^2$ + $0^2$) = **0**

G(green) = 1 - ($0^2$ + $1^2$) = **0**

weighted_gini(Color) = ((3/6) * .388) + ((1/6) * 0 )+ ((2/6) * 0 ) = **0.0277**

GINI Value for **Shape**:

Square = 4/6 | Round = 2/6

Square and + = 2/4, Square and - = 2/4, Round and + = 1/2, Round and - = 1/2

G(square) = 1 - [ $(2/4)^2$+$(2/4)^2$ ]= 1 - (1/2) = **0.50**

G(round) = 1 - [$(1/2)^2$ + $(1/2)^2$] = **.5**

weighted_gini(Shape) = ((4/6) * .5) + ((2/6) * .5 )) = **0.5**

GINI Value for **Size**:

Big = 4/6 | Small = 2/6

Big and + = 3/4, Big and - = 1/4, Small and - = 2/2

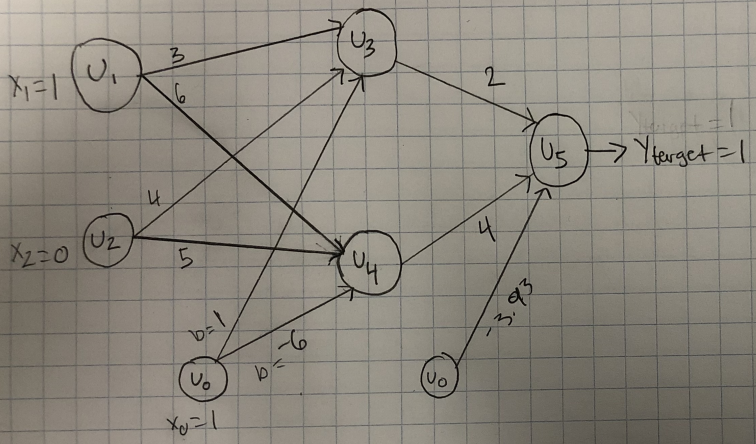
G(big) = 1 - [ $(3/4)^2$+$(1/4)^2$ ]= **0.3750**

G(samll) = 1 - [$1^2$ + $1^2$] = **1**

weighted_gini(size) = ((4/6) * 0.3750) + ((2/6) * 1 )) = **0.5833**

**So are best attribute for the root is SIZE**

# Question 5




Sigmoid activation function: $\sigma(x)=\frac{1}{(1+e^{-x})}$

$(3)(1)+(4)(0)+1 = z_{U_3}=4$

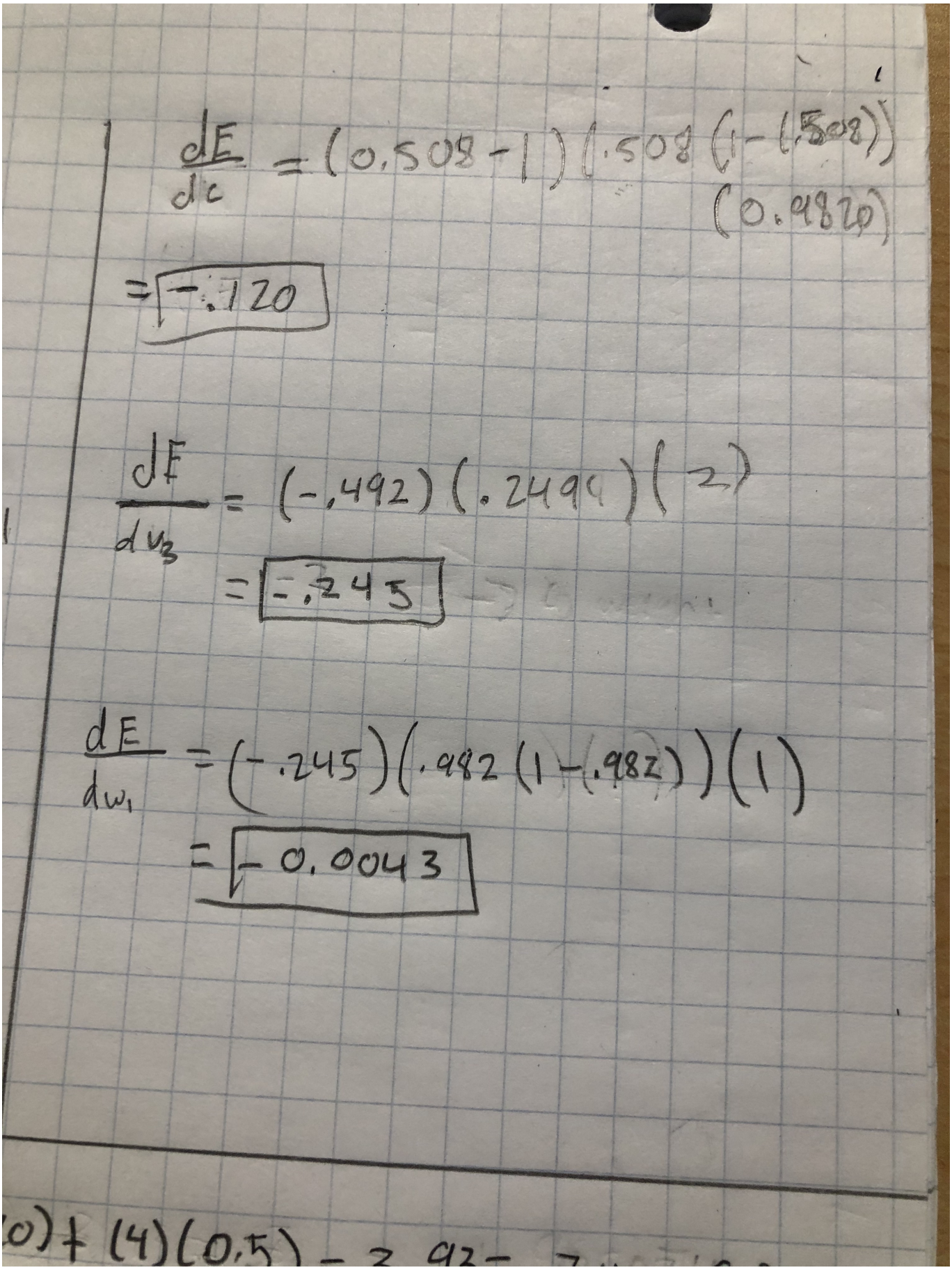
$(6)(1)+(5)(0)-6 = z_{U_4}=0$

$U_3=\sigma(4)=0.9820$

$U_4=\sigma(0)=0.5$

$(2)(0.9820)+(4)(0.5)-3.93 = z_{U_5}=-0.034$

$U_5=\sigma(z_{U_5})=\frac{1}{(1+e^{-.034})}=\boxed{0.508}$




## Reference's

Q1 Code:

```
import numpy as np
import matplotlib as plt


data = np.array([[2,2,-1], [3,5,-1], [1,3,-1], [-1, -0.5, -1]]) #Given
Class 1 and Class 2
classes = np.array([1,1,2,2]) #Classes
aug_vec = np.array([1,1,1]) #weights
d = 4 #how deep the summation will go

#Perceptron that takes in the two classes data and the classes map

print(aug_vec)
def perceptron(input, class_label, to_d):
    global aug_vec
    if to_d == 0:
        print('OUTPUT: ')
        print(aug_vec)
    else:
        cal_weight(input, class_label)
        to_d -= 1
        perceptron(input, class_label, to_d)

#Calculates the weight
def cal_weight(dt, labels):
    global aug_vec
    if dt.size == 0:
        print(aug_vec)
    else:
        if ((np.dot(dt[0], aug_vec))* labels[0]) <= 0:
            aug_vec = aug_vec + (dt[0] * labels[0])
        cal_weight(dt[1:], labels[1:])

#calls the perceptron with overloaded parameters
perceptron(data, classes, d)
```

## Question 2 Code

```
from scipy.stats import binom
import numpy as np

#Find learners
found = False
accuracy = 0
learners = 7 + 1

while not found:
    i = learners/2
    if( 1 - binom.cdf(i, learners, .55)) >= .9:
```

```
            found = True
            accuracy = (1 - binom.cdf(i, learners, .55))
        else:
            learners += 1
```

## Question 3 Code

```
import numpy as np

#class vectors
c1 = np.array([[2,10], [2,5], [1,2], [4,9]])
c2 = np.array([[8,4], [5,8], [7,5], [6,4]])
test = [3,3]
#means
c1_mean = np.mean(c1, 0)
c2_mean = np.mean(c2, 0)

#matrix
c1_mat = 3 * np.cov(c1.T)
c2_mat = 3 * np.cov(c2.T)

mat = c1_mat + c2_mat

mat_inv = np.linalg.inv(mat)

#weight vector

weight_vec = np.matmul(mat_inv, (c1_mean - c2_mean))

#mat mult to get classification
c1_pro = np.matmul(weight_vec.T, c1_mean)
c2_pro = np.matmul(weight_vec.T, c2_mean)

#cut in half
z = .5 * (c1_pro + c2_pro)

print("C1 Projection: {}".format(c1_pro))
print("C2 Projection: {}".format(c2_pro))
var = np.matmul(weight_vec.T, test)
print("Determination of [3,3] Class Label: {}".format(var))
print("Z-cut: {}".format(z) )
print("So the predicted value is: Class One.")
```