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CSI 3350

Homework Question 1:

(I assumed “n ∈ N” was indicating n is a natural number thus I used S to indicate a subset. If “n ∈ N” was meant to indicate n is a natural number in subset of N then replace N for S.)

1.

Top-Down:

A natural number *n* is in S if and only if

1. *n* = 2, or
2. *n*-3 ∈ S.

Bottom-Up:

Define the set S to be the smallest set contained in N and satisfying the following two properties:

1. 2 ∈ S, and
2. If n ∈ S, then n+3 ∈ S.

Rules of Inference:

Derivation: { 2, 5, 8, … }

2 ∈ N 2 ∈ S

5 ∈ N (2) ∈ S

8 ∈ N ( 5 . (2) ) ∈ S

( 8 . ( 5 . ( 2 ) ) ) ∈ S

2.

(n and m are both ∈ N so I generalized both to just one variable (n) to reduce confusion)

Top-Down:

A natural number *n* is in S if and only if

1. *n* = 1, or
2. *n*-3 ∈ S, or
3. *n-2* ∈ S.

Bottom-Up:

Define the set S to be the smallest set contained in N and satisfying the following three properties:

1. 1 ∈ S, and
2. If n ∈ S, then n+2 ∈ S, and
3. If n ∈ S, then n+3 ∈ S.

Rules of Inference:

Derivation:

{ {

1, 3, 5, 7, 9 … = 1,3,4,5,6,7,8,9,10,11, …

4, 6, 8, 10, 11 … }

}

1 ∈ N 1 ∈ S

3 ∈ N (1) ∈ S

4 ∈ N ( 3 . (1) ) ∈ S

5 ∈ N ( 4 . (3 . (1) )) ∈ S

6 ∈ N (5 . ( 4 . (3 . (1) ))) ∈ S

7 ∈ N (6 . (5 . ( 4 . (3 . (1) )))) ∈ S

8 ∈ N (7. (6 . (5 . ( 4 . (3 . (1) ))))) ∈ S

( 8 . (7. (6 . (5 . ( 4 . (3 . (1) )))))) ∈ S

3.

Top-Down:

A pair of natural numbers (n,m) is in S if and only if

1. (n,m) = (0,1), or
2. (n-1, m-2) ∈ S

Bottom-Up:

Define the set S to be the smallest set contained in N and satisfying the following two properties:

1. (0,1) ∈ S, and
2. If (n,m) ∈ S, then (n+1, m+2) ∈ S.

Rules of Inference:

Derivation: { (0,1) , (1,3), (2,5), …}

(0,1) ∈ N (0,1) ∈ S

(1,3) ∈ N (0,1) ∈ S

(2,5) ∈ N ( (1,3) . (0,1) ) ∈ S

( (2,5) . ( (1,3) . ( 0,1 ) ) ) ∈ S

4.

Top-Down:

A pair of natural numbers (n,m) is in S if and only if

1. (n, m) = (0,0), or
2. (n-1, m-2n+1) ∈ S.

Bottom-Up:

Define the set S to be the smallest set contained in N and satisfying the following two properties:

1. (0,0) ∈ S, and
2. If (n,m) ∈ S, then (n+1, m+2n+1) ∈ S.

Rule of Inference:

Derivation: { (0,0), (1,1), (2,4), (3,9), … }

(0,0) ∈ N (0,0) ∈ S

(1,1) ∈ N (0,0) ∈ S

(2,4) ∈ N ( (1,1) . (0,0) ) ∈ S

(3,9) ∈ N ( (2,4). ( (1,1) . (0,0) ) )∈ S

( (3,9) . ( (2,5) . ( (1,3) . ( 0,1 ) ) ) ) ∈ S