CSE 357 - HW 1

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1. **Dice Dependence (16 points).** You roll two fair 6-sided dice ($D1$ and $D2$).

   (a) Which of the following are independent?
   $D1 = 6, D1 + D2 = 5$
   $D1 = 5, D1 + D2 = 6$
   $D1 = 4, D1 + D2 = 7$
   None of the above
   Answer: $D1 = 4, D1 + D2 = 7$ are independent events.

   (b) Why? (show your work)
   Answer: For the first case $D1 = 6, D1 + D2 = 5$, $D1$ is already greater than 5, and $D2$ cannot be negative. Therefore the two events here are mutually exclusive and thus cannot be independent.

   For the second case $D1 = 5, D1 + D2 = 6$, the probability that $D1 + D2 = 6$ is equal to $\frac{5}{36}$. However, given that $D1$ has to be equal to 5, the probability that $D1 + D2 = 6$ becomes $\frac{1}{6}$ as there is only 1 value $D2$ can take - 1 - in order to satisfy the equality. Therefore since the probability changes when $D1$ is set to 5, the two events cannot be independent.

   For the third case $D1 = 4, D1 + D2 = 7$, the probability that $D1 + D2 = 7$ is equal to $\frac{6}{36} = \frac{1}{6}$. However, given that $D1$ has to be equal to 4, the probability that $D1 + D2 = 6$ remains the same at $\frac{1}{6}$ due to 1 out of six possible values for $D2$ - 3 - satisfying the equality $D1 + D2 = 7$. Therefore since the probability remains the same when $D1$ is set to 4, the two events are independent.

2. **Conditionally Independent Mining? (24 points)** Suppose you are given five mines that may or may not contain gold, and based on stories you hear of all mines in the area you assume there is a 50% probability that any single mine has gold.

   (a) Under this assumption, what is the probability that all five mines have gold?
   Answer: The probability that all five mines have gold is equal to $(0.5)^5 = 0.03125$.

   (b) Suppose you learn that at least one mine has gold. Given only this additional information, what is the probability that all five contain gold? (show your work)

Answer: We can use Bayes' theorem: $P(A|B) = P(A \cap B)/P(B)$. We let event A be the event where all five mines contain gold. We let event B be the event where at least one mine has gold. $P(A \cap B) = P(A)$ because $P(A) \subset P(B)$, or the set of all events where at least one mine has gold includes all the events where all 5 mines have gold. We then divide $P(A)$, 0.03125, by $P(B) = 1 - 0.03125$ (the probability that no mines have gold subtracted from 1), to get our final answer of 0.03225806451612903.

(c) Your friend is also given 5 mines (ignore all information above). They are all along a road leading out of town. You learn that _only one of these mines has gold. Your friend knows which mine it is and asks you to guess. suggesting he might share some of it if you get it right (fun friend!). You guess the mine closest to town, what is the probability that you are right?

Answer: The probability that the mine closest to town has gold is equal to $\frac{1}{5} = 0.2$.

(d) Annoyingly, your friend won't tell you whether you're right! Instead, he reveals that it's not the mine furthest from town. Should you change your answer? Why or why not? (explain probabilistically)

Answer: The answer should **not** be changed because given the removal of the mine furthest from the town, the probabilities that each remaining mine is the one that contains gold remain the same with respect to each other, and thus it would not necessarily be advantageous if the answer was changed.

It would only make sense to change the answer if it was *explicitly* stated that there was a shift in probabilities of the remaining mines such that one had a greater probability of containing gold than the others.