# Project Summary

*This project analyzes a chessboard using standard chess rules to determine whether a single King, controlled by player Black, is in checkmate, stalemate, or neither during player Black’s turn. Player White controls any number of pieces, of any rank, in any configuration.*

# Propositions

*Checkmate – Evaluates whether the Black King is in Checkmate, meaning it has no legal moves, and is currently in check. True if the Black King is checkmated.*

*Stalemate – Evaluates if the Black King is in Stalemate, meaning that it has no legal moves, however it is not currently in check. Stalemate additionally stipulates that the Black player has no other legal moves; in the model the only piece player Black controls is the King, meaning it only analyzes the King’s moves. True if the Black King is stalemated.*

*Check – Evaluates if the Black King is in Check, meaning that a White-controlled piece can move to its current location and capture it. True if the Black King is in check.*

*BK\_No\_Moves – Evaluates whether the Black King can make any legal moves. True if the Black King has no legal moves.*

*XX\_Space\_Occupied – Evaluates the presence of a given piece on a given space on the chessboard. “XX” is replaced by a code relating to each piece; “BK” represents the Black King, “WK” represents White’s King, “WQ” represents the White Queen, “WR” the Rook, “WB” the Bishop, “WH” the Knight (WH representing “White Horse”, an alias of the Knight), and “WP” the White Pawn. These codes carry to other propositions as well.*

*XX\_Total\_Count – Uses propositional logic to evaluate the number of a given piece on the board. “XX” code carries from XX\_Space\_Occupied.*

# Constraints

*List of constraint types used in the model and their (English) interpretation. You only need to provide one example for each constraint type: e.g., if you have constraints saying “cars have one colour assigned” in a car configuration setting, then you only need to show the constraints for a single car. Essentially, we want to see the pattern for all of the types of constraints, and not every constraint enumerated.*

*¬(Checkmate)∨ ¬(Stalemate)*

*The Black King cannot both be in Checkmate and Stalemate; the game-ending conditions are mutually exclusive.*

*BK\_No\_Moves ⬄ (Checkmate ∨ Stalemate)*

*If and only if the Black King has no legal moves, the game has come to an end, either in Checkmate or Stalemate.*

# Model Exploration

*List all the ways that you have explored your model – not only the final version, but intermediate versions as well. See (C3) in the project description for ideas.*

*In an intermediate version of the model, we tried entering a real-life chessboard that was in checkmate into the model only to discover that the model believed this board-state to still be playable. After analyzing the real-life board state ourselves, we realized that the stalemate was a case of discovered mate: one of the moves the Black King could take allowed it to capture a threatening piece, only to be threatened itself by another of White’s pieces, making the move illegal. We therefore had to reconsider the code of our model to allow White’s pieces to project threat even to locations currently occupied by other allied pieces.*

*Once we had a working version of the model complete, one of the explorations we tried was setting the model to either be in Checkmate, Stalemate, or neither and then minimizing or maximizing the number of a singular given piece on the chessboard, in order to find out how many of a piece were the minimum to create a checkmate or stalemate condition, and the maximum of a piece that could exist on the board while avoiding both conditions. There were our results:*

*Queen – Minimum of 1 for Stalemate, 2 for Checkmate. 43 White Queens and 1 Black King can exist on the board without ending the game.*

*Rook – Minimum of 2 for Stalemate, 2 for Checkmate. 50 White Rooks can coexist with a Black King without ending the game.*

*Bishop – Minimum of 3 for Stalemate, 3 for Checkmate. 57 White Bishops can coexist with a Black King without ending the game.*

*Knight – Minimum of 2 for Stalemate, 3 for Checkmate. 61 White Knights can coexist with a Black King without ending the game.*

*Pawn – Minimum of 3 for Stalemate, 4 for Checkmate. 63 White Pawns (an entire board full!) can coexist with a Black King without ending the game.*

*King – Minimum of 2 for Stalemate, and no number of White Kings can checkmate a Black King. 58 White Kings can coexist with a Black King without ending the game, possibly for either side!*

# First-Order Extension

*Describe how you might extend your model to a predicate logic setting, including how both the propositions and constraints would be updated.* ***There is no need to implement this extension!***

*Using predicate logic would eliminate the need for the huge number of constraints implemented in our model; using first-order logic we could use a domain that constrains piece positions to the board (rather than manually checking to ensure pieces cannot leave the board, or be placed outside of it), n-ary function symbols to represent and verify the locations and types of pieces, and even logically quantify the threat locations of pieces rather than computing the threatened locations and then logically operating on that information.*

*For example, the function QueenThreat(x, y) is a binary function that maps a single x, y location (presumably that of a Queen on the coordinate plane that is the chessboard) onto a set of x, y coordinates, specifically all of those that are on the same x, y, x=y or x=-y axes that the Queen inhabits. This logical function, along with functions corresponding to the other movement-patterns of the remaining chess pieces, would replace the enormously complex board-parsing system used in our model to determine similar information about the model that informs whether or not the Black King is in a game-ending situation.*

# Useful Notation

*Feel free to copy/paste the symbols here and remove this section before submitting.*