# Towards Learning Better and Flexible Representations from Variational Autoencoders

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20th April 2024

# Outline

# Towards **Learning** Better and Flexible **Representations** from Variational Autoencoders

- Representation Learning is concerned with learning representations of the data that make it easier to extract useful information.
- The learned representations can then be used to perform multiple downstream tasks such as classification or retrieval.

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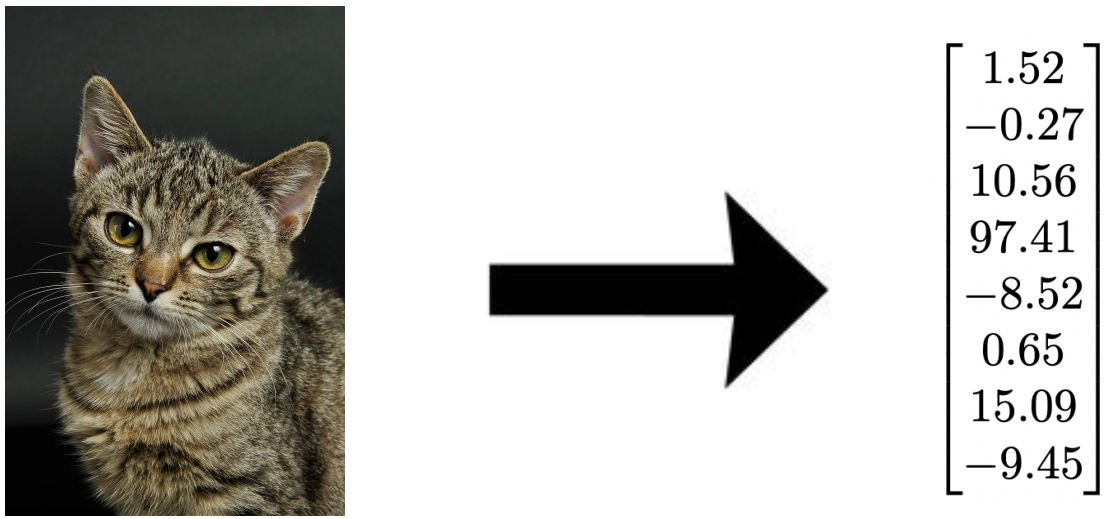


Figure 1: Representating the image of cat as 8-dimensional vector

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- An ideal representation should be expressive, meaning that a reasonably-sized learned representation should capture a huge number of possible input configuration.
- Moreover, we expect the process of learning representations to not be computationally expensive.
- Some examples of models used for representation learning :-
  1. Convolutional Neural Networks for Image Representations
  2. Recurrent Neural Networks or Transformers for Word Representations
- In this project, we focus our attention on learning representations of images using **Variational Autoencoders**

# Towards Learning Better and Flexible Representations from **Variational Autoencoders**

- Variational autoencoders (VAEs) are a deep learning technique for learning latent representations.
- VAEs consist of an encoder network which models the posterior distribution $q_\phi(z|x)$ and a decoder network which models the likelihood distribution $p_\theta(x|z)$
- VAEs have been used for several applications such as dimensionality reduction and image generation.

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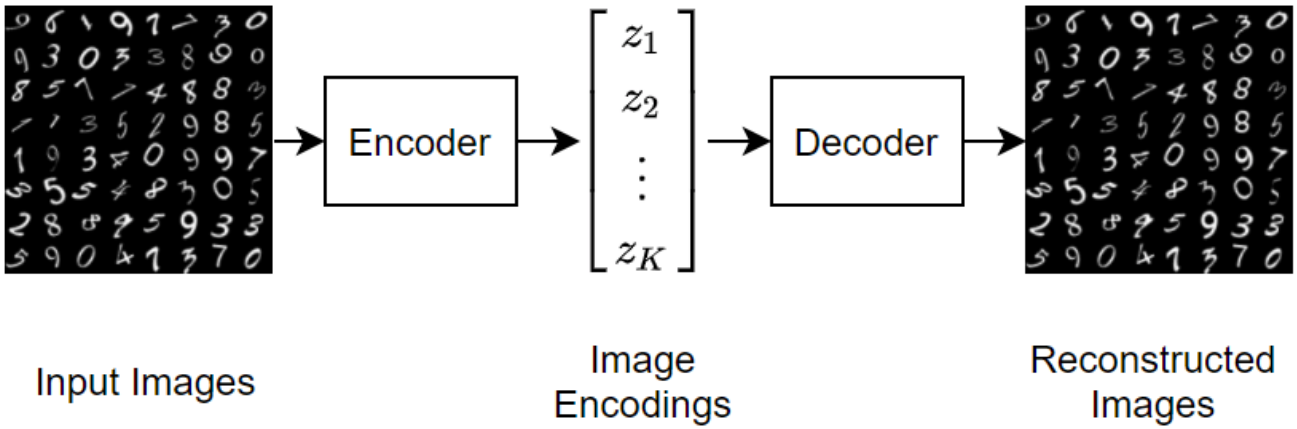


Figure 2: Variational Autoencoder Architecture

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- After training the VAE on the ELBO objective, the learned $q_\phi(z|x)$ can be used to generate the representation.
- For of a given data $x'$, we can obtain a representation by :-
  1. Sampling from the posterior - $z' \sim q_\phi(z|x')$
  2. Finding the most likely representation (MAP) - $z' = \text{argmax}\ q_\phi(z|x')$
- For this project, we used two popular image datasets :-

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(a) MNIST Dataset


(b) FashionMNIST Dataset

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- For the downstream classification task, we used a feedforward Network consisting of 2 linear layers.
- The encoder and decoder networks of VAE consist of 2 convolutional layers and 2 linear layers.
- The classifier and VAE have been implemented in PyTorch and we have used Adam Optimizer for training the models.
- We varied the size of latent representations as [2, 8, 16, 32, 64, 128].
- VAE was trained for 10 epochs with a batch size of 32 meanwhile the classifier was trained for 20 epochs.
- We implemented an early stopping module to prevent overfitting.
- For evaluation, we have employed 5-fold cross validation.

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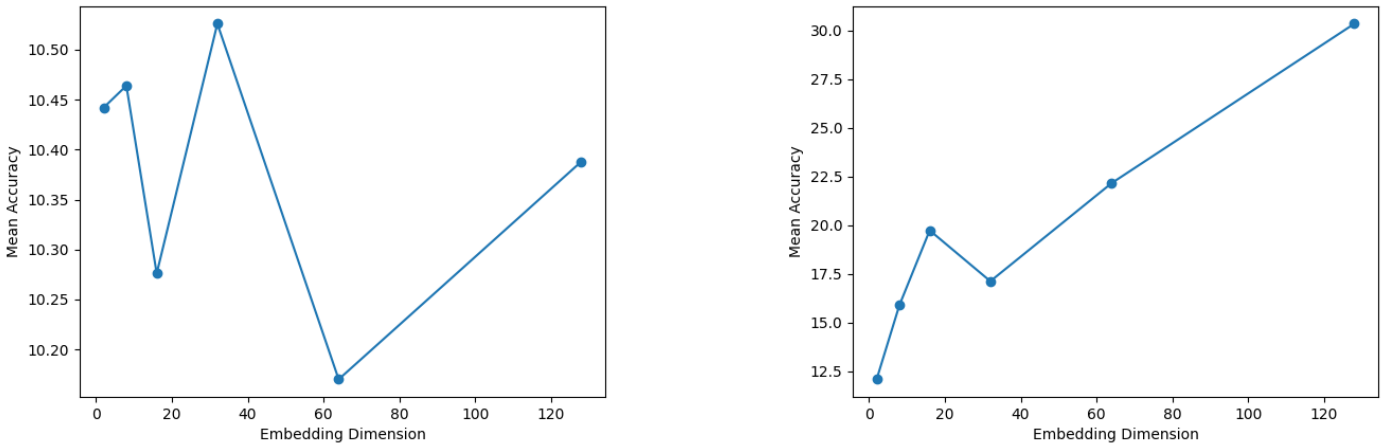
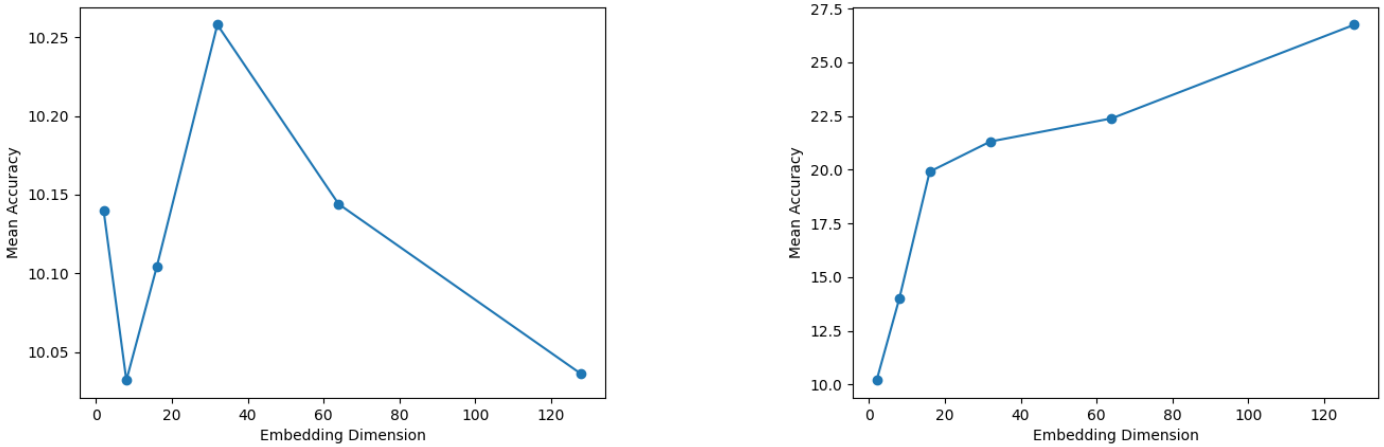
1. MNIST Dataset
   - For Posterior sampling, across all dimensions, the classification accuracy varies around 10 which is comparable to random labeling.
   - For Maximum a posteriori, the classification accuracy increases as we increase the dimensionality of the latent space. However, the best classification accuracy is 30.33 which is low compared to SOTA.




(a) Posterior Sampling (b) Maximum a posteriori

Figure 4: Classification Accuracy of VAE on MNIST Dataset

2. FashionMNIST Dataset
   - Similar to MNIST Dataset, the classification accuracy for posterior sampling varies around 10 which is comparable to random labeling.
   - For Maximum a posteriori, the classification accuracy increases as we increase the dimensionality of the latent space. However, the best classification accuracy is 26.74 which is low compared to SOTA.




(a) Posterior Sampling (b) Maximum a posteriori

Figure 5: Classification Accuracy of VAE on FashionMNIST Dataset

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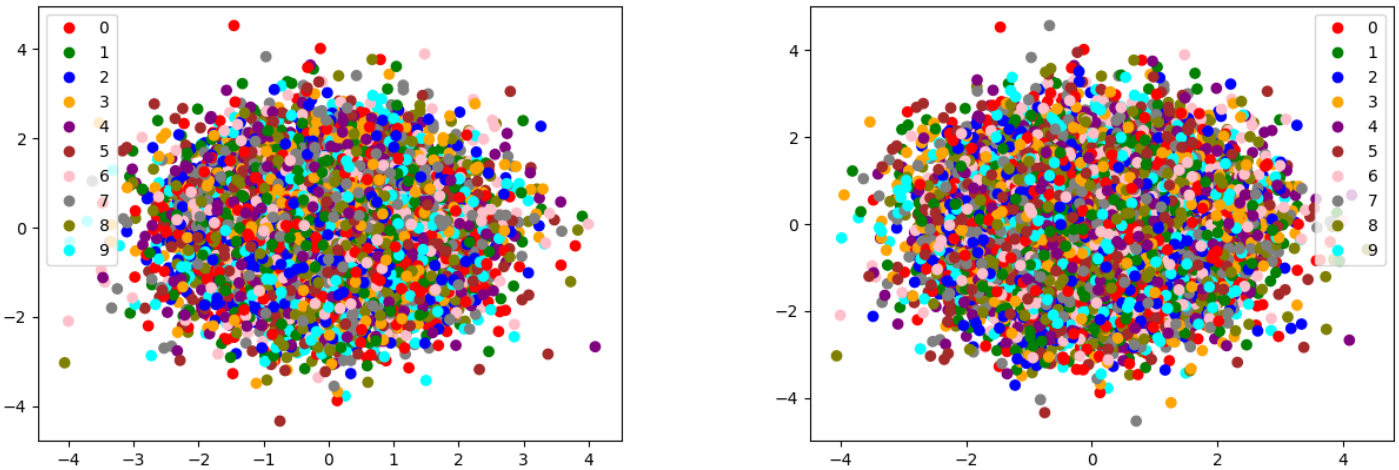
- In order to understand why the classification accuracy was so low, we plotted the latent space representations.
- It was observed that input data belonging to the different classes have similar representations in latent space.




(a) MNIST Dataset (b) FashionMNIST Dataset

Figure 6: Latent Space of VAE for Posterior Sampling

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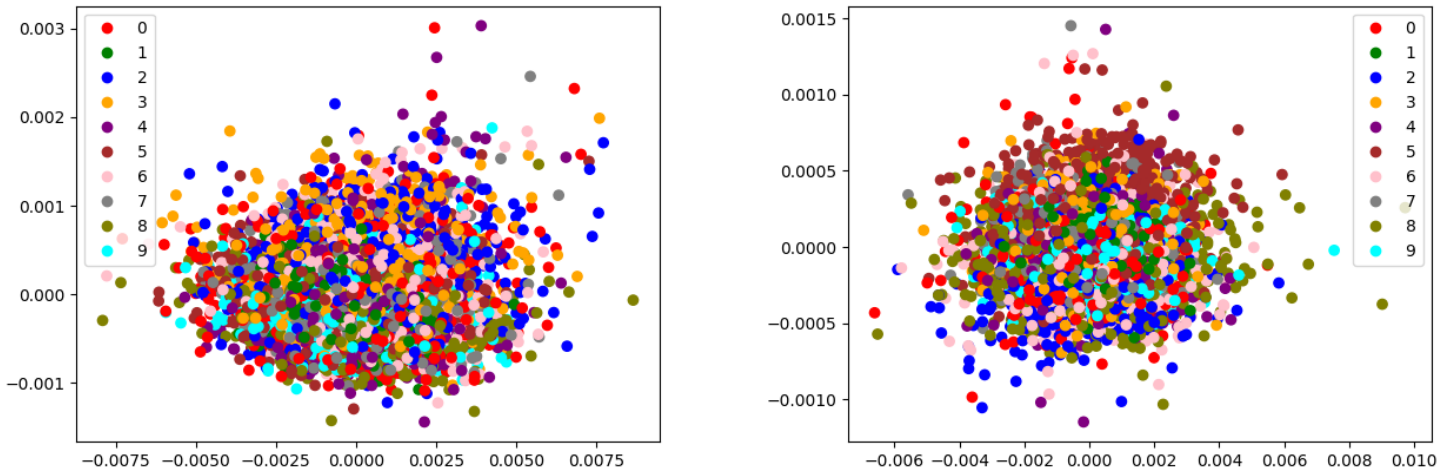
- Due to similar representations, the classifier wasn't able to distinguish between two classes thus leading to poor classification.
- For higher dimensional latent representations, we used Principal Component Analysis (PCA) to project down the representations to 2-dimensional space for plotting the latent space.




(a) MNIST Dataset (b) FashionMNIST Dataset

Figure 7: Latent Space of VAE for Maximum a posteriori

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- Our next objective was to improve these latent space representations for better performance in downstream classification task.
- For this, we referred to Zhao et. al., 2017 which proposed a new family of VAEs called InfoVAE.
- The paper discusses about two major problems of VAE :-
  1. The approximate inference distribution is often significantly different from the true posterior.
  2. When the conditional distribution is sufficiently expressive, the latent variables are often ignored.
- The paper argues that the above two problems arise due to ELBO objective used to train VAEs and proposes a new loss function.
- The following tutorial serves as a good start for understanding InfoVAE.

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- In particular, we used Maximum Mean Discrepancy (MMD) instead of KL Divergence to quantify the distance between two distributions.
- Maximum Mean Discrepancy can be efficiently implemented using the kernel trick. Let $k(.\,,.)$ be any positive definite kernel then,

$$\begin{aligned}\mathcal{L}_{\text{MMD}}(q\,||\,p) = &\mathbb{E}_{p(z),\,p(z')}[k(z,\,z')] \\ &+ \mathbb{E}_{q(z),\,q(z')}[k(z,\,z')] \\ &- 2\mathbb{E}_{q(z),\,p(z')}[k(z,\,z')]\end{aligned}$$

- In our project, we utilized Gaussian Kernel with $\sigma = 1$ to implement Maximum Mean Discrepancy.

$$k(z,\,z') = e^{-\frac{||\,z - z'\,||^2}{2\sigma^2}}$$

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- For computing the MMD distance, we first simply generated $n$ samples from the prior distribution $p(z)$ and compared these generated samples with the encoder output.
- For training InfoVAE, we used the same hyperparameters as that of standard VAE. The additional hyperparameter $n$ was set to 200.
- We also implemented an early stopping module to prevent overfitting and employed 5-fold cross validation for evaluation.
- We used the following Github repository for reference while implementing our model.

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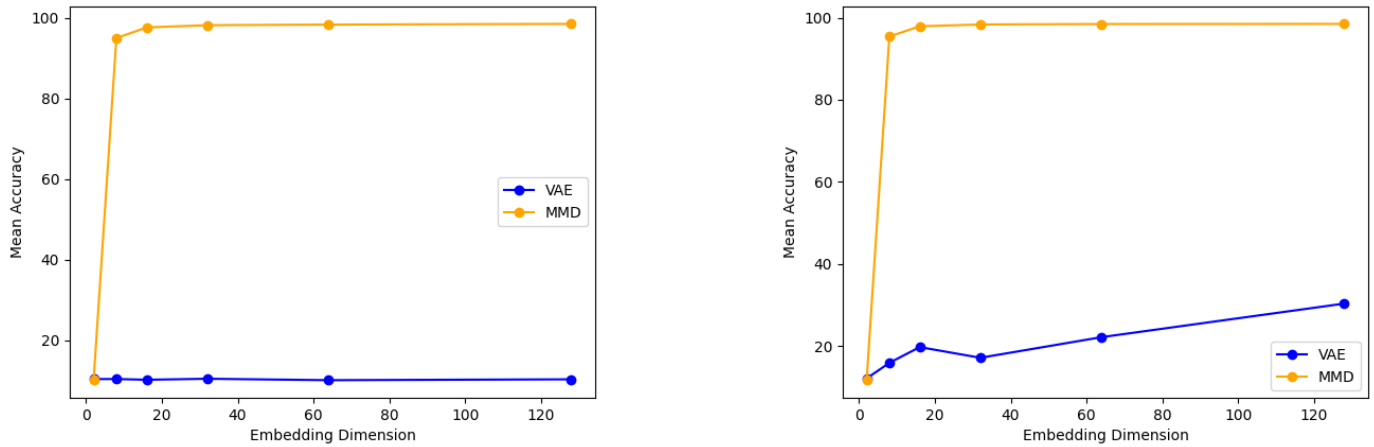
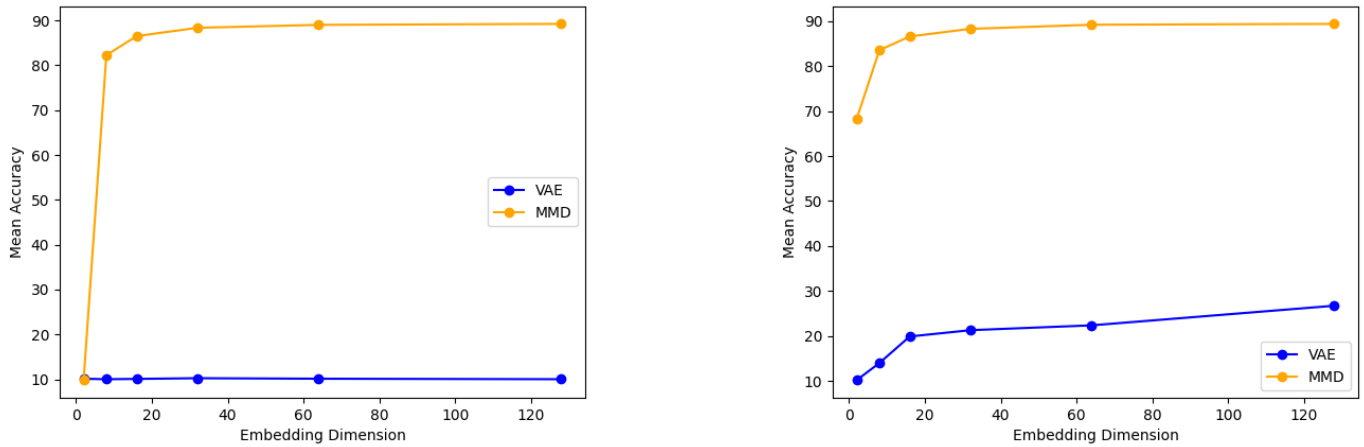
1. MNIST Dataset
   - Barring the 2-dimensional case, the classification accuracy for posterior sampling approaches 98% which is comparable to SOTA.
   - The classification accuracy for Maximum a posteriori follows a similar pattern as that of Posterior Sampling.




(a) Posterior Sampling (b) Maximum a posteriori

Figure 8: Classification Accuracy of InfoVAE on MNIST Dataset

2. FashionMNIST Dataset
   - Similar to MNIST Dataset, classification accuracy for posterior sampling approaches 90%.
   - The classification accuracy for Maximum a posteriori follows a similar pattern as that of Posterior Sampling.




(a) Posterior Sampling (b) Maximum a posteriori

Figure 9: Classification Accuracy of InfoVAE on FashionMNIST Dataset

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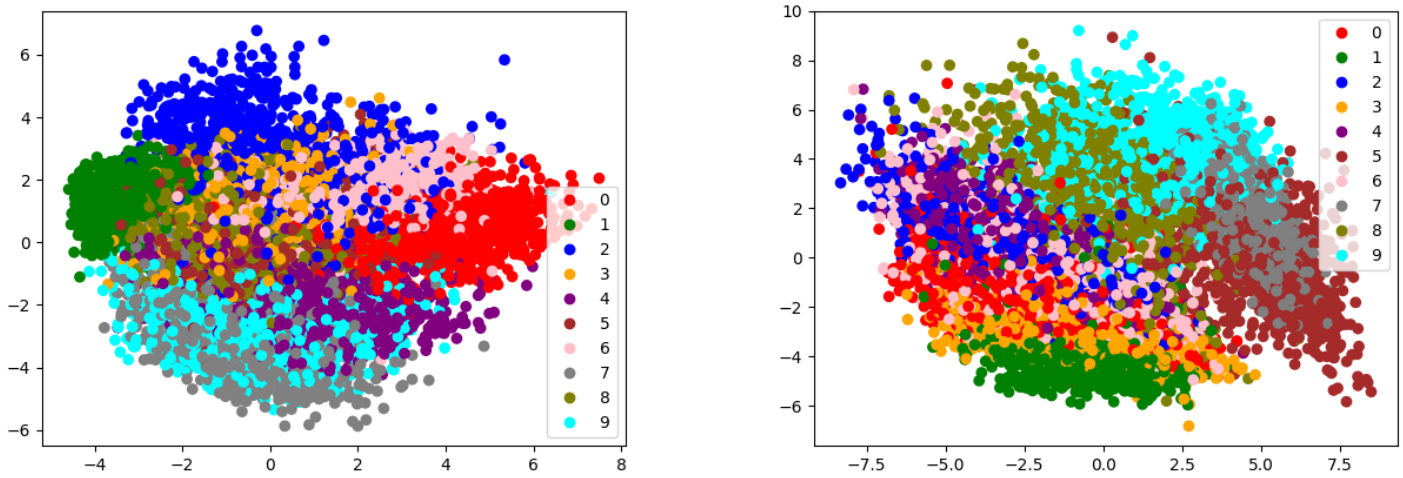
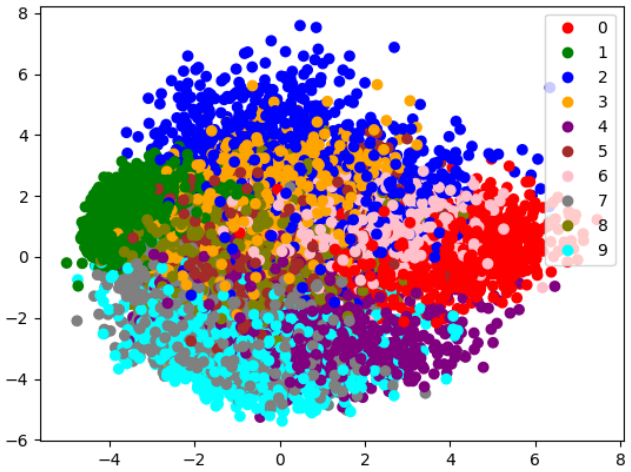
- After plotting the latent space of InfoVAE, we observed the formation of distinct clusters in the latent space corresponding to each class.
- Due to the formation of distinct clusters in the latent space, we are able to achieve such good performance in the downstream classification task.




(a) MNIST Dataset (b) FashionMNIST

Figure 10: Latent Space of InfoVAE for Posterior Sampling




(a) MNIST Dataset

Figure 11: Latent Space of InfoVAE for Maximum a posteriori

# Towards Learning Better and **Flexible Representations** from Variational Autoencoders

- So far, we utilized InfoVAE to improve the representations learnt from Variational Autoencoder architecture.
- However, when training these representations, we didn't consider the computational and statistical constraints of the downstream task.
- In practice, these representations would be shared with different clients with varying computational resources.
- In this context, rigid fixed-capacity representations can be either over or under-accommodating to the task at hand.

# Towards Learning Better and **Flexible Representations** from Variational Autoencoders

- Our objective in the latter half of the project would be to make the representations learnt from VAEs flexible.
- In order to achieve this goal, we refer to Kusupati et al., 2022 which proposes **Matryokshka Representation Learning**.

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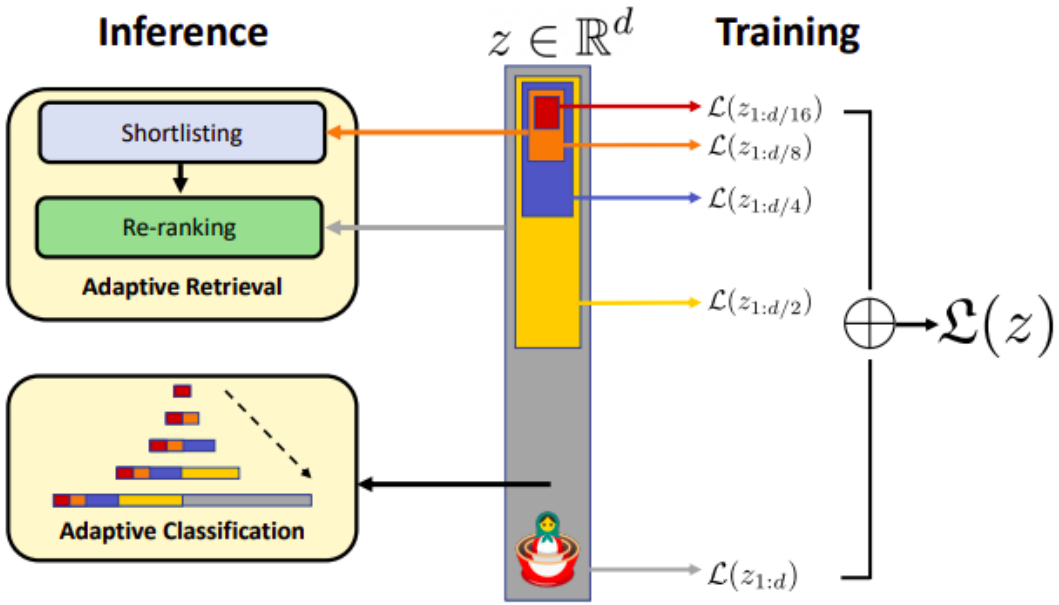


Figure 12: Matryoshka Representation Learning

# Towards Learning Better and **Flexible Representations** from Variational Autoencoders

- The paper argues that gradient-based training of deep learning models tends to diffuse information across the entire representation vector leading to rigidity.
- To induce flexibility in the learned representation, Matryoshka Representation Learning encodes information at different granularities
- In this way, a single embedding can adapt to the computational constraints of multiple downstream tasks.
- There have been several applications of Matryoshka Representation Learning :-
  1. MatFormer (Kudugunta, Sneha, et al., 2023)
  2. AdANNS (Rege, Aniket, et al., 2024)
- In this project, we intend to implement Matryokshka Representation Learning in the VAE algorithm in order to obtain flexible latent representations.

# Future Roadmap

1. Implement Kusupati et al., 2022 paper.
   - Particularly, we explore the application of MRL in downstream classification task.
   - We will use Resnet-50 models and ImageNet-1K dataset for evaluating this task.
2. We integrate Matryokshka Representation Learning in VAE algorithm.
3. Compare the results of MRL with individually trained models.