HW2

Tuesday, October 4, 2022 9:58 PM

CSE 167 (FA 2022) Exercise 2 — Due 10/5/2022

You've probably seen it when you first learned matrix algebra. The inversion of 2×2 matrices has a simple formula: swap the diagonals, add a minus sign on the off diagonals, and divide the result by the determinant.

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} = \frac{1}{ad - bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}. \quad (1)$$

Exercise 2.1 — 3 pts. Let V be a 2-dimensional vector space. Let $\vec{a}_1, \vec{a}_2 \in V$ be a pair of linearly independent vectors (a basis). Let $\vec{b}_1, \vec{b}_2 \in V$ be another basis satisfying the relation:

$$\vec{b}_1 = -\vec{a}_1 + 2\vec{a}_2, \quad \vec{b}_2 = 3\vec{a}_1 - 5\vec{a}_2. \quad (2)$$

Now, let $\vec{v} \in V$ be a vector with coefficients $\begin{bmatrix} 1 \\ -1 \end{bmatrix}$ under the basis $(\vec{a}_1, \vec{a}_2)$; that is,

$$\vec{v} = \begin{bmatrix} \vec{a}_1 & \vec{a}_2 \end{bmatrix} \begin{bmatrix} 1 \\ -1 \end{bmatrix} = \vec{a}_1 - \vec{a}_2. \quad (3)$$

What is the coefficients of $\vec{v}$ under the basis $(\vec{b}_1, \vec{b}_2)$? That is, what is x, y in

$$\vec{v} = \begin{bmatrix} \vec{b}_1 & \vec{b}_2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = x\vec{b}_1 + y\vec{b}_2? \quad (4)$$

■

Exercise 2.2 — 1 pt. In the lecture we mentioned that matrix multiplication is not commutative. Given an example of $\mathbf{A}, \mathbf{B}$, both square matrices of the same size, so that $\mathbf{AB} \neq \mathbf{BA}$. ■

(2.1) $b_1 = \begin{bmatrix} \vec{a}_1 \\ -2\vec{a}_2 \end{bmatrix}$ $b_2 = \begin{bmatrix} 3\vec{a}_1 \\ -5a_2 \end{bmatrix}$

$\vec{v} = x b_1 + y b_2 = x(-1+2) + y(3-5)$

$$\vec{v} = x - 2y$$

(2.2)

let: $A = \begin{bmatrix} 1 & 2 \\ -2 & 1 \end{bmatrix}$ $B = \begin{bmatrix} 2 & 1 \\ 1 & -2 \end{bmatrix}$

$$A \cdot B = \begin{bmatrix} 4 & -3 \\ -3 & -4 \end{bmatrix}$$

$$B \cdot A = \begin{bmatrix} 0 & 5 \\ 5 & 0 \end{bmatrix}$$

$$A \cdot B \neq B \cdot A$$

Therefor, matrix multiplication is not commutative.