Exer6

Wednesday, November 2, 2022 7:14 PM

CSE 167 (FA 2022) Exercise 6 — Due 11/2/2022

Exercise 6.1 — 3 pts. (Barycentric coordinates)

In the plane, consider the triangle $\mathbf{p}_1 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$, $\mathbf{p}_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$, $\mathbf{p}_3 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$. What are the barycentric coordinates for points $\mathbf{a} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix} 1/3 \\ 2/3 \end{bmatrix}$ with respect to the triangle $\mathbf{p}_1\mathbf{p}_2\mathbf{p}_3$? From the barycentric coordinates, tell which of $\mathbf{a}$ and $\mathbf{b}$ is/are located in the interior of the triangle.

Hint *The matrix inversion involved in this question is doable by hand. But you may also use symbolic/numerical calculation tool like Wolfram Alpha for computing matrix inversion.* ■

6.2

a)

$$\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} \lambda_1 \\ \lambda_2 \\ \lambda_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} 0 & -1 & 1 \\ -1 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} \lambda_1 \\ \lambda_2 \\ \lambda_3 \end{bmatrix}$$

$$\begin{bmatrix} \lambda_1 \\ \lambda_2 \\ \lambda_3 \end{bmatrix} = \begin{bmatrix} 0 \\ -1 \\ 2 \end{bmatrix}$$

[a] is in the exterior because $\lambda_2 < 0$

b)

$$\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} \lambda_1 \\ \lambda_2 \\ \lambda_3 \end{bmatrix} = \begin{bmatrix} 1/3 \\ 2/3 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} 0 & -1 & 1 \\ -1 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1/3 \\ 2/3 \\ 1 \end{bmatrix} = \begin{bmatrix} \lambda_1 \\ \lambda_2 \\ \lambda_3 \end{bmatrix}$$

$$\begin{bmatrix} \lambda_1 \\ \lambda_2 \\ \lambda_3 \end{bmatrix} = \begin{bmatrix} 1/3 \\ 1/3 \\ 1/3 \end{bmatrix}$$

[b] is in the interior because $0 \leq \lambda_1, \lambda_2, \lambda_3 \leq 1$