Exe8

CSE 167 (FA 2022) Exercise 8 — Due 11/16/2022

Exercise 8.1 — 2 pts. In ray tracing, suppose we have a camera

- located at **eye position** = $\begin{bmatrix} -5 \\ 1 \\ 0 \end{bmatrix}$
- looking at **target** = $\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$
- with **up vector** = $\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$
- with **field of view**$_y$= 90°
- and an image resolution of **width**= 135 pixels and **height**= 90 pixels.

All positions are relative to a common world coordinate. At pixel $(i, j) = (82, 22)$, what is the ray $(\mathbf{p}_0, \mathbf{d})$ shooting through the center of the pixel? (Here, $\mathbf{p}_0 \in \mathbb{R}^3$ is the source point of the ray, and $\mathbf{d} \in \mathbb{R}^3$ is the unit vector for the direction of the ray; both in the world coordinate.)

Hint *$\mathbf{p}_0$ is trivial. For $\mathbf{d}$, see slides on "RayTracing," page 20 and page 26.*

$$w = \frac{eye - target}{\|eye - target\|} \quad \frac{\begin{bmatrix} -5 \\ 1 \\ 0 \end{bmatrix} - \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}}{\left\| \begin{bmatrix} -5 \\ 1 \\ 0 \end{bmatrix} - \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \right\|} = \frac{\begin{bmatrix} -5 \\ 0 \\ 0 \end{bmatrix}}{\left\| \begin{bmatrix} -5 \\ 0 \\ 0 \end{bmatrix} \right\|} = \begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix} //$$

$$u = \frac{up \times w}{|up \times w|} = \frac{\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix}}{\left| \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix} \right|} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} //$$

$$v = w \times u = \begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix} [0\ 0\ 1] = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} //$$

$$\alpha = 2\frac{i + 1/2}{width} - 1 = 2\frac{82 + 1/2}{135} - 1 = 11/9 - 1 = 2/9 //$$

$$\beta = 1 - 2\frac{j + 1/2}{height} = 1 - 2\frac{22 + 1/2}{90} = 1 - 1/2 = 1/2 //$$

$$d = normalize(\alpha \cdot u \cdot \tan(\tfrac{1}{2} fovy) + \beta \cdot \tan(\tfrac{1}{2} fovy) v - w$$

$$d = \text{normalize}\left(\frac{2}{9}\cdot\frac{35}{90}\cdot\tan\left(\frac{90}{2}\right)[0\,0\,1] + \frac{1}{2}\cdot\tan\left(\frac{90}{2}\right)[0\,1\,0] - \begin{bmatrix}-1\\0\\0\end{bmatrix}\right)$$

$$d = \text{normalize}\left(\begin{bmatrix}0\\0\\1/3\end{bmatrix} + \begin{bmatrix}0\\1/2\\0\end{bmatrix} - \begin{bmatrix}-1\\0\\0\end{bmatrix}\right) = \begin{bmatrix}1\\1/2\\1/3\end{bmatrix} \;//$$

$$(P_0, d) = \left(\begin{bmatrix}-5\\1\\0\end{bmatrix}, \begin{bmatrix}1\\1/2\\1/3\end{bmatrix}\right)$$

Exercise 8.2 — 2 pts. Suppose we have a triangle with its 3 vertex positions given by

$$\mathbf{p}_1 = \begin{bmatrix}4\\0\\0\end{bmatrix}, \quad \mathbf{p}_2 = \begin{bmatrix}0\\2\\0\end{bmatrix}, \quad \mathbf{p}_3 = \begin{bmatrix}0\\0\\8\end{bmatrix}. \tag{1}$$

Now, suppose we have a ray sourced at $\mathbf{p}_0 = \begin{bmatrix}-1\\0\\0\end{bmatrix}$ with direction $\mathbf{d} = \begin{bmatrix}2/3\\1/3\\2/3\end{bmatrix}$. The ray will intersect with the triangle. What is the position $\mathbf{q} \in \mathbb{R}^3$ of this ray-triangle intersection? What is the distance t traveled by the ray (distance between the source and the intersection)? What are the barycentric coordinates $\lambda_1, \lambda_2, \lambda_3$ for $\mathbf{q}$ with respect to the triangle $\mathbf{p}_1\mathbf{p}_2\mathbf{p}_3$?

Hint *Follow page 36 of the slides on "Ray Tracing." You may use symbolic calculator like Wolfram Alpha for solving equations.* ■

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ p_1 & p_2 & p_3 & -d \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} \lambda_1 \\ \lambda_2 \\ \lambda_3 \\ t \end{bmatrix} = \begin{bmatrix} 1 \\ p_0 \\ 1 \\ 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 4 & 0 & 0 & -2/3 \\ 0 & 2 & 0 & -1/3 \\ 0 & 0 & 8 & -1/3 \\ 1 & 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} \lambda_1 \\ \lambda_2 \\ \lambda_3 \\ t \end{bmatrix} = \begin{bmatrix} -1 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 4 & 0 & 0 & -2/3 \\ 0 & 2 & 0 & -1/3 \\ 0 & 0 & 8 & -1/3 \\ 1 & 1 & 1 & 0 \end{bmatrix}^{-1} \begin{bmatrix} -1 \\ 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} \lambda_1 \\ \lambda_2 \\ \lambda_3 \\ t \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 3/20 & -1/5 & -1/20 & 2/5 \\ -1/10 & 3/10 & -1/20 & 2/5 \\ -1/20 & -1/10 & 1/10 & 1/5 \\ -3/5 & -6/5 & -3/10 & 12/5 \end{bmatrix} \begin{bmatrix} -1 \\ 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} \lambda_1 \\ \lambda_2 \\ \lambda_3 \\ t \end{bmatrix} = \begin{bmatrix} 1/4 \\ 1/2 \\ 1/4 \\ 3 \end{bmatrix}$$

$$\boxed{\lambda_1 = 1/4, \quad \lambda_2 = 1/2, \quad \lambda_3 = 1/4, \quad t = 3}$$

$$q = p_0 + td = \begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix} + 3 \begin{bmatrix} 2/3 \\ 1/3 \\ 2/3 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix} \quad //$$