Exer7

Exercise 7.1 — 1 pt. (Bézier)

Let $\mathbf{F}(u, v)$ be a symmetric bi-affine $\mathbb{R}^2$-valued function with

$$\mathbf{F}(0,0) = \begin{bmatrix} 4 \\ 8 \end{bmatrix}, \quad \mathbf{F}(1,0) = \begin{bmatrix} 4 \\ 4 \end{bmatrix}, \quad \mathbf{F}(1,1) = \begin{bmatrix} 8 \\ 4 \end{bmatrix}. \tag{1}$$

What is $\mathbf{F}(\frac{1}{2}, \frac{1}{2})$? ∎

$$f(0.5, 0) = \frac{1-0.5}{1-0} \times \begin{bmatrix} 4 \\ 8 \end{bmatrix} + \frac{0.5-0}{1-0} \begin{bmatrix} 4 \\ 4 \end{bmatrix}$$

$$= 0.5 \begin{bmatrix} 4 \\ 8 \end{bmatrix} + 0.5 \begin{bmatrix} 4 \\ 4 \end{bmatrix} = \begin{bmatrix} 2 \\ 4 \end{bmatrix} + \begin{bmatrix} 2 \\ 2 \end{bmatrix}$$

$$\Rightarrow f(0.5, 0) = \begin{bmatrix} 4 \\ 6 \end{bmatrix}$$

$$f(0.5, 1) = \frac{1-0.5}{1-0} \times \begin{bmatrix} 4 \\ 4 \end{bmatrix} + \frac{0.5-0}{1-0} \times \begin{bmatrix} 8 \\ 4 \end{bmatrix}$$

$$= 0.5 \begin{bmatrix} 4 \\ 4 \end{bmatrix} + 0.5 \begin{bmatrix} 8 \\ 4 \end{bmatrix}$$

$$\Rightarrow f(0.5, 1) = \begin{bmatrix} 6 \\ 4 \end{bmatrix}$$

$$f(0.5, 0.5) = \frac{1-0.5}{1-0} \times \begin{bmatrix} 4 \\ 6 \end{bmatrix} + \frac{0.5-0}{1-0} \times \begin{bmatrix} 6 \\ 4 \end{bmatrix}$$

$$= 0.5 \begin{bmatrix} 4 \\ \cdot \end{bmatrix} + 0.5 \begin{bmatrix} 6 \\ \end{bmatrix}$$

$$\Rightarrow f(0.5, 0.5) = \begin{bmatrix} 5 \\ 5 \end{bmatrix} \quad \text{//}$$

Exercise 7.2 — 2 pts. (B-spline)
 Let $\mathbf{F}(u, v)$ be a symmetric bi-affine $\mathbb{R}^2$-valued function with

$$\mathbf{F}(-1, 0) = \begin{bmatrix} 4 \\ 8 \end{bmatrix}, \quad \mathbf{F}(0, 1) = \begin{bmatrix} 4 \\ 4 \end{bmatrix}, \quad \mathbf{F}(1, 2) = \begin{bmatrix} 8 \\ 4 \end{bmatrix}. \tag{2}$$

$$x_1, y_2 \qquad\qquad x_2 y_3 \qquad\qquad x_3 y_4$$

What is $\mathbf{F}(\frac{1}{2}, \frac{1}{2})$?

$$u_1 = -1 \qquad\qquad v_1 = -1$$

$$u_2 = 0 \qquad\qquad v_2 = 0$$

$$u_3 = 1 \qquad\qquad v_3 = 1$$

$$u_4 = 2 \qquad\qquad v_4 = 2$$

$$f(0, 0.5) = \frac{1-0}{1-0} \begin{bmatrix} 4 \\ 8 \end{bmatrix} + \frac{0-0}{1-0} \begin{bmatrix} 4 \\ 4 \end{bmatrix}$$

$$\Rightarrow f(0, 0.5) = \begin{bmatrix} 4 \\ 8 \end{bmatrix}$$

$$f(0.5, 1) = \frac{1-0.5}{1-0.5} \begin{bmatrix} 8 \\ 4 \end{bmatrix} + \frac{0.5-1}{1-0.5} \begin{bmatrix} 4 \\ 4 \end{bmatrix}$$

$$= \begin{bmatrix} 8 \\ 4 \end{bmatrix} - \begin{bmatrix} 4 \\ 4 \end{bmatrix}$$

$$f(0.5, 1) = \begin{bmatrix} 4 \\ 0 \end{bmatrix}$$

$$f(0.5, 0.5) = \frac{1 - 0.5}{1 - 0.5} \begin{bmatrix} 4 \\ 8 \end{bmatrix} + \frac{0.5 - 0}{1 - 0.5} \begin{bmatrix} 4 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} 4 \\ 8 \end{bmatrix} + \begin{bmatrix} 4 \\ 0 \end{bmatrix}$$

$$\Rightarrow f(0.5, 0.5) = \begin{bmatrix} 8 \\ 8 \end{bmatrix} \; //$$