Exercises Data Science with R

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Chapter 1

There are quite a lot standard basic mathematical functions. Try to google for R cheat sheets if you are interested in a comprehensive overview.

```
3^2     #1
## [1] 9
sqrt(9) #2
## [1] 3
9^.5
## [1] 3
pi^2    #3
## [1] 9.869604
(abs(3^2-4^2))^.5 #4
## [1] 2.645751
log(exp(4), base = exp(1)) #5
## [1] 4
log(exp(4))
## [1] 4
log(100, base = 10) #6
## [1] 2
log10(100)
## [1] 2
factorial(8) #7
## [1] 40320
exp(factorial(3)) #8
## [1] 403.4288
```

Chapter 2

The chapter intends to focus on writing simple functions in R.

1. Formulate the EOQ formula in R

```
cost_eoq_fun <- function(q, d, co, cl) {
  # returns total cost per period
  # d...demand
  # q...lot size
  # co...ordering cost
  # cl...stock holding cost
  ((1/2)*cl*q)+((d/q)*co)
```

```
}

# test cost function
cost_eoq_fun(d=100, q=20, cl = .1, co = 50)
## [1] 251

eoq_fun <- function(co, d, cl) {
  # return optimal lot size
  # d...demand
  # co...ordering cost
  # cl...stock holding cost
  sqrt((2*co*d)/cl)
}
# optimal lot size
q.star <-  eoq_fun(d = 100, cl = .1, co = 50)
# optimal cost
cost_eoq_fun(d=100, q=q.star, cl = .1, co = 50)
## [1] 31.62278
```

2. Derive a function for calculating weighted Euclidean distance between two points.

```
weuc_d2_func <- function(x,y,w) {
  # calculates weighted Euclidean distance between x and y
  # y,x...vectors
  # w.. weight vector
  sqrt(sum(w*(x-y)^2))
}
# test distacne function
weuc_d2_func(x = c(1,2,3), y= c(3,2,1), w=c(1,1,1) )
## [1] 2.828427
# result should be sqrt(8)
```

3. Alter your EOQ function by checking whether all arguments are supplied and stop execution while displaying a error message. -> postponed

4. Formulate a function for the Geometric Poisson density distribution.

```
geom_pois_dens_fun <- function(n, lambda, theta){
  # calculates density value of geometric Poisson distribution
  # n...integer, demand/successes, theta,lambda..parameters
  k.vec <- 1:n
  sum(exp(-lambda)*lambda^k.vec/factorial(k.vec)*(1-theta)^(n-k.vec)*choose(n-1, k.vec-1))
}
# test function
geom_pois_dens_fun(n=3, lambda=.5, theta = 2)
## [1] 0.1642687
```

Chapter 3

Basics on data types and data manipulation

1. Calculate the outer product of two vectors (without outer())

```
x <- 1:5
y <- 10:6
as.matrix(x) %*% t(as.matrix(y))
##      [,1] [,2] [,3] [,4] [,5]
## [1,]   10    9    8    7    6
## [2,]   20   18   16   14   12
## [3,]   30   27   24   21   18
## [4,]   40   36   32   28   24
## [5,]   50   45   40   35   30
```

2. Define a function that calculates the trace of a matrix.

```
trace_func <- function(z){
  # calculates trace of z
  # z...matrix
  sum(diag(z))
}
A <- matrix(rnorm(9) , ncol=3)
trace_func(A)
## [1] 0.9503803
```

3.Create a vector containing the first 100 Fibonacci numbers. Most commonly, the Fibonacci numbers are defined recursively by $F_n = F_{n-1} + F_{n-2}$ whereby $F_0 = 0$ and $F_1 = 1$. However, there is also an explicit formulation: $F_n = \sum_{k=0}^{\lfloor \frac{n-1}{2} \rfloor} \binom{n-k-1}{k}$ (ceck here)

```
fib_num_fun <- function(n){
  # calculate nth Fibonacci number
  # n...number
  k.vec <- 0:floor((n-1)/2)
  sum(choose(n-k.vec-1, k.vec))
}
# vectorize fib_num_fun such that it accepts input vectors
vfib_num_fun <- Vectorize(fib_num_fun)
# doesn't work
fib_num_fun(1:100)
## Warning in 0:floor((n - 1)/2): numerischer Ausdruck hat 100 Elemente: nur erstes
## wird genutzt
## [1] 100
# works
vfib_num_fun(1:100)
##   [1] 1.000000e+00 1.000000e+00 2.000000e+00 3.000000e+00 5.000000e+00
##   [6] 8.000000e+00 1.300000e+01 2.100000e+01 3.400000e+01 5.500000e+01
##  [11] 8.900000e+01 1.440000e+02 2.330000e+02 3.770000e+02 6.100000e+02
##  [16] 9.870000e+02 1.597000e+03 2.584000e+03 4.181000e+03 6.765000e+03
##  [21] 1.094600e+04 1.771100e+04 2.865700e+04 4.636800e+04 7.502500e+04
##  [26] 1.213930e+05 1.964180e+05 3.178110e+05 5.142290e+05 8.320400e+05
##  [31] 1.346269e+06 2.178309e+06 3.524578e+06 5.702887e+06 9.227465e+06
##  [36] 1.493035e+07 2.415782e+07 3.908817e+07 6.324599e+07 1.023342e+08
##  [41] 1.655801e+08 2.679143e+08 4.334944e+08 7.014087e+08 1.134903e+09
##  [46] 1.836312e+09 2.971215e+09 4.807527e+09 7.778742e+09 1.258627e+10
```

```
##  [51] 2.036501e+10 3.295128e+10 5.331629e+10 8.626757e+10 1.395839e+11
##  [56] 2.258514e+11 3.654353e+11 5.912867e+11 9.567220e+11 1.548009e+12
##  [61] 2.504731e+12 4.052740e+12 6.557470e+12 1.061021e+13 1.716768e+13
##  [66] 2.777789e+13 4.494557e+13 7.272346e+13 1.176690e+14 1.903925e+14
##  [71] 3.080615e+14 4.984540e+14 8.065155e+14 1.304970e+15 2.111485e+15
##  [76] 3.416455e+15 5.527940e+15 8.944394e+15 1.447233e+16 2.341673e+16
##  [81] 3.788906e+16 6.130579e+16 9.919485e+16 1.605006e+17 2.596955e+17
##  [86] 4.201961e+17 6.798916e+17 1.100088e+18 1.779979e+18 2.880067e+18
##  [91] 4.660047e+18 7.540114e+18 1.220016e+19 1.974027e+19 3.194043e+19
##  [96] 5.168071e+19 8.362114e+19 1.353019e+20 2.189230e+20 3.542248e+20
```

4. Create a matrix containing the all binominal coefficients up to n = 50

```
pas <- outer(1:50, 1:50, choose)
```

5. Preference matrices in the Analystical Hirachy Process (AHP) show a form of (inverted) symmetry. How can you check this in R

The AHP method requires for a matrix $A = \{a_{ij}|i, j = 1, ..., n\}$ that $a_{i,j} = \frac{1}{a_{j,i}}$. Thereby, diagonal elements are always 1 (i.e., $a_{ii} = 1$)

```
# create 3x3 matrix
A <- matrix(sample(1:10 , 9), ncol=3)
# diagonals to 1
diag(A) <- 1
# checks matrix elementwise
A == t(1/A) # -> problem double counting of elements
##       [,1]  [,2]  [,3]
## [1,]  TRUE FALSE FALSE
## [2,] FALSE  TRUE FALSE
## [3,] FALSE FALSE  TRUE
# Alternative: compare just elements of triangle sub-matrices
A[lower.tri(A)] == 1/A[upper.tri(A)]
## [1] FALSE FALSE FALSE
```

6. Calculate the synthesis of an AHP preference matrix

The synthesis is calculated in two steps: (a) normalizing A by dividing with column sums (b) calculate row means of normalized matrix

```
# convert A (chunk before) into a consistent matrix
A[lower.tri(A)] <- 1/A[upper.tri(A)]
# step (a) of synthesis
dev <- colSums(A) # calculate colSums
A.prime <- t(t(A) / dev) # Alternative a: using that R divides columnwise
A.prime <- A %*% diag(1 / dev) # Alternative b: use matrix calculation
# step (b) of synthesis
rowMeans(A.prime)
## [1] 0.71734015 0.22739983 0.05526002
```

Chapter 4

1. Construct a list with 100 entries. Afterwards, display only entries with odd indices.

```
x <- sample(x = 1:100, size = 100)
l1 <- as.list(x)
l1.odd <- l1[1:100 %% 2 == 1]
```

2. Construct a tibble or data frame consisting of 100 columns and 1 row. Extract only every 3rd column.

```
z <- matrix(sample(x = 1:1000, size = 100), nrow=1)
z <- as.data.frame(z)
z.sub <- z[, col(z) %% 3 == 0]
```

3. Construct a matrix of size 6×6 and fill it by sampling numbers between 1 and 100. Retain indices of all entries ≤ 50.

```
A <- matrix(sample(x = 1:100, 36), ncol = 6)
which(A <= 50)
##  [1]  1  2  5  6  7  8 11 14 15 22 23 25 26 29 30 31 32 35
```

4. Formulate a function for calculating the moving average over a vector.

```
ma_fun <- function(x, n){
  # calculates moving average
  # x...vector to be averaged, n...time window width
  cx <- c(0,cumsum(x))
  (cx[(n+1):length(cx)] - cx[1:(length(cx) - n)]) / n
}
# test function
ma_fun(x = 1:10 , n=4)
## [1] 2.5 3.5 4.5 5.5 6.5 7.5 8.5
```

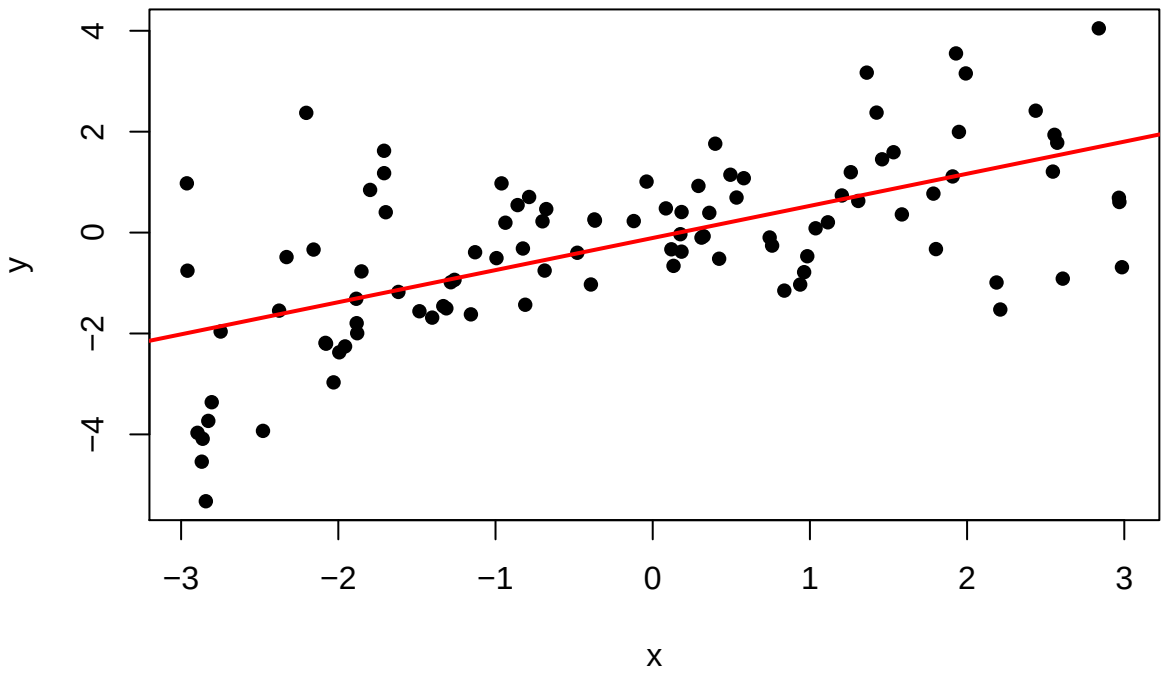
Chapter 5

1. Sample 100 observations in the range $[-3, 3]$ from the follwing model and plot the sample:

$$y = \sin(3 \cdot x^2 - 4) \cdot x + 0.5 \cdot x + \epsilon$$

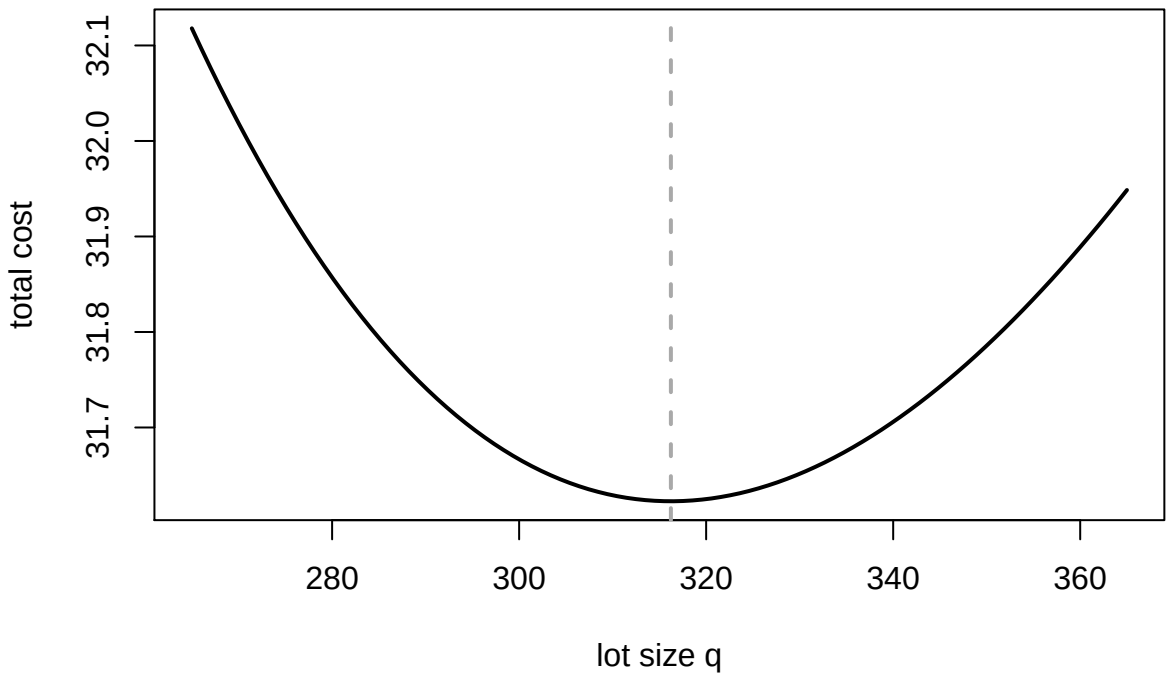
whereby $\epsilon \sim N(0, 0.5)$. Can you estimate a regression function?

```
n <- 100                                  # number of points to plot
x <- runif(n, min=-3, max=3)              # 100 x coordinates
y <- sin(3 * x^2 - 4) * x + 0.5 * x + rnorm(n, mean = 0, sd = sqrt(.5))
plot(x[order(x)], y[order(x)], xlab ="x", ylab = "y", pch = 16)
lin.reg <- lm(y~x)                        # fit a linear regression model
abline(lin.reg, col = "red", lwd =2)      # ... and plot the regression line
```



2. Plot the EOQ model in an appropriate range.

```
q.vec <- seq(265, 365, length.out = 100)
y.vec <- cost_eoq_fun(q = q.vec , d = 100, cl = .1, co = 50)
plot(q.vec, y.vec, xlab ="lot size q", ylab = "total cost", type = "l", lwd =2)
abline(v = eoq_fun(d = 100, cl = .1, co = 50), col="darkgrey", lwd=2, lty=2)
```



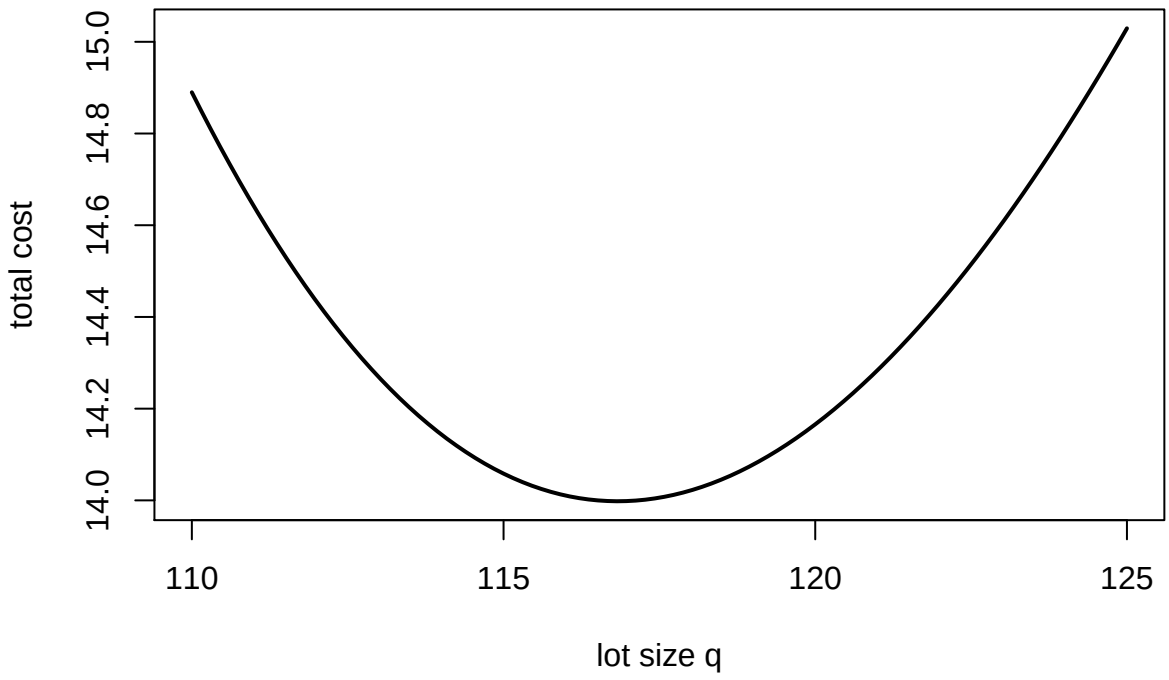
3. Plot the newsvendor model in an appropriate range.

The total cost in the normal newsvendor model is given by

$$Z(q) = (c_u + c_o) \cdot \sigma \cdot (\varphi(q') + q' \cdot \Phi(q')) - c_u \cdot (q - \mu)$$

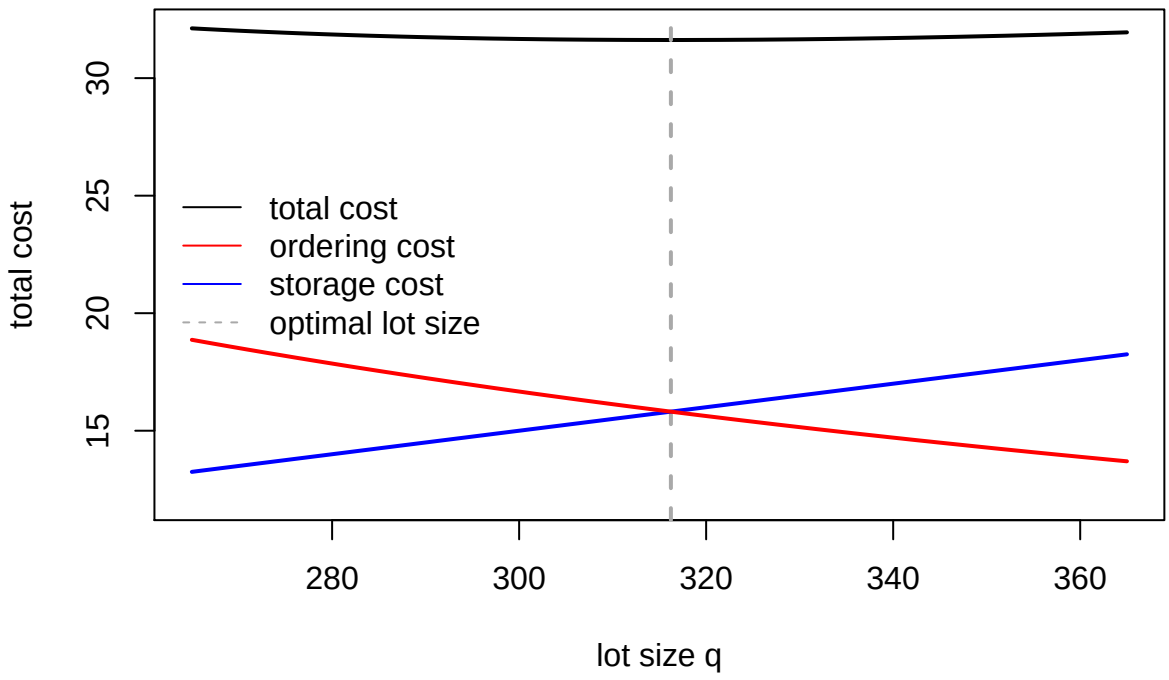
with $q' = \frac{q-\mu}{\sigma}$

```
newsven_obj_fun <- function(x, cu, co, mu, sigma){
      q.prime <- (x-mu)/sigma
         (cu + co) * sigma * (dnorm(q.prime) + q.prime * pnorm(q.prime)) - cu * sigma * q.prime
}
q.vec <- seq(110, 125, length.out = 100)
cost.vec <- newsven_obj_fun(x = q.vec , mu = 100, cu = 2, co = 0.5, sigma = 20)
plot(q.vec, cost.vec, xlab ="lot size q", ylab = "total cost", type = "l", lwd = 2)
```



4. Plot the EOQ model in an appropriate range and add holding cost function and ordering cost function in different colors. Add a legend to the plot.

```
q.vec <- seq(265, 365, length.out = 100)
y.vec <- cost_eoq_fun(q = q.vec , d = 100, cl = .1, co = 50)
plot(q.vec, y.vec, xlab ="lot size q", ylab = "total cost", type = "l", lwd =2, ylim=c(12,max(y.vec)))
abline(v = eoq_fun(d = 100, cl = .1, co = 50), col="darkgrey", lwd=2, lty=2)
lines(x=q.vec, y=(1/2)*.1*q.vec, lwd=2, col="blue")
lines(x=q.vec, y=(100/q.vec)*50, lwd=2, col="red")
legend("left", lty = c(1,1,1,2), col = c("black","red","blue","darkgrey"), legend = c("total cost", "or
```



5. Consider the joint economic lot size model. Supplier and customer face setup costs ($c_{set}^{sup.}$,$c_{set}^{cust.}$) as well as stock-holding cost rates ($c_{sh}^{sup.}$, $c_{sh}^{sup.}$). The demand rate of the customers is λ and production rate of the supplier is μ (with $\lambda < \mu$). The cost functions of supplier and customer are

$$C^{cust.}(q) = \frac{\lambda}{q} \cdot c_{set}^{cust.} + c_{sh}^{cust.} \cdot \frac{q}{2}$$

and

$$C^{supp.}(q) = \frac{\lambda}{q} \cdot c_{set}^{sup.} + c_{sh}^{sup.} \cdot \frac{q}{2} \cdot \frac{\lambda}{\mu}.$$

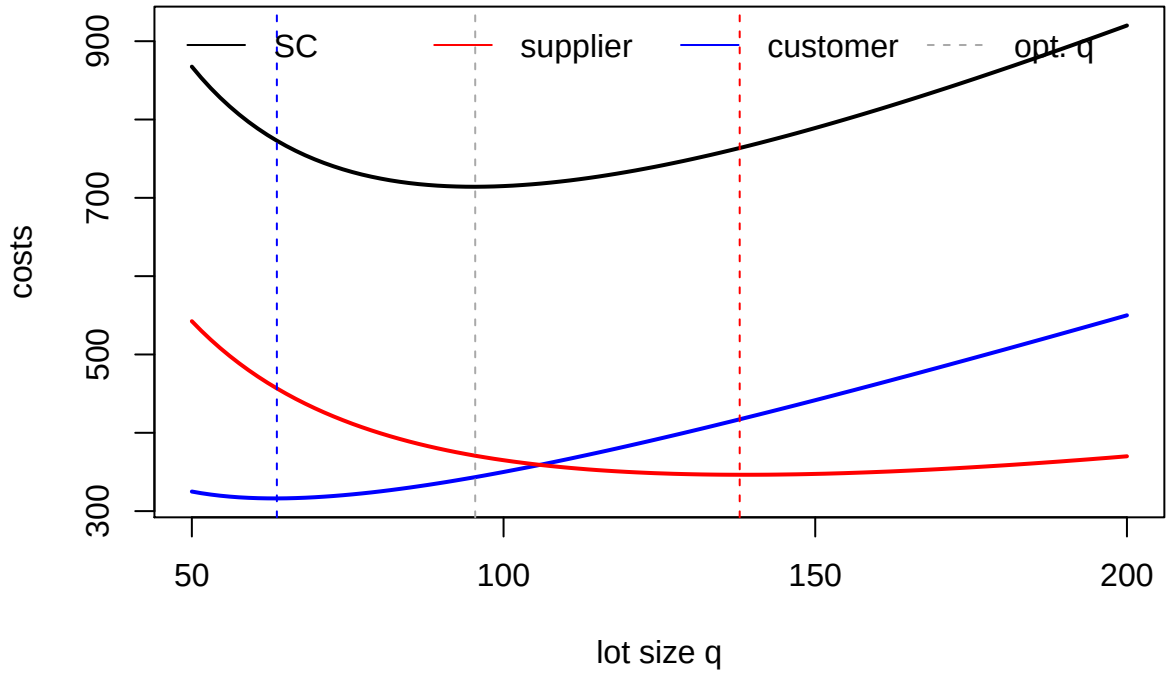
Plot the cost functions of supplier, customer, and the SC (i.e., total cost function) in different colors. Add a legend to the plot.

```
jel_func <- function(q, c.supp.set, c.supp.sh, c.cust.set, c.cust.sh, lambda, mu){
  # costs in joint economic lot sizing model
  if(lambda > mu) stop("infeasible (lambda > mu)") # check feasibility
  c.cust <- lambda/q * c.cust.set  + c.cust.sh *  q/2
  c.supp <-  lambda/q * c.supp.set  + c.supp.sh * q/2 * lambda/mu
  c.sc <- c.supp + c.cust
  return(data.frame(cost.customer = c.cust, cost.supplier = c.supp, cost.sc = c.sc))
}

q.vec <- seq(50, 200, length.out = 100)
y.df <- jel_func(q = q.vec, c.supp.set = 240, c.supp.sh = 4, c.cust.set = 100, c.cust.sh = 5, lambda =

plot(q.vec, y.df$cost.sc, xlab ="lot size q", ylab = "costs", type = "l", lwd = 2, ylim=range(y.df))
```

```r
lines(x = q.vec, y = y.df$cost.customer, lwd = 2, col = "blue")
lines(x = q.vec, y = y.df$cost.supplier, lwd = 2, col = "red")
abline(v = q.vec[which.min(y.df$cost.customer)], col = "blue", lwd = 1, lty = 2)
abline(v = q.vec[which.min(y.df$cost.supplier)], col = "red", lwd = 1, lty = 2)
abline(v = q.vec[which.min(y.df$cost.sc)], col = "darkgrey", lwd = 1, lty = 2)
legend("top", lty = c(1,1,1,2), col = c("black","red","blue","darkgrey"), legend = c("SC", "supplier",
```



6. Consider the contract design problem with buy-back option (Thonemann, 2010, p. 479). Given are unit cost per product unit c of the supplier, the sales price of the supplier w, the sales price of retailer r, the scrap revenue v of the SC as well as normally distributed demand with $y \sim N(\mu, \sigma^2)$. To be determined is the buy-back price b such that the optimal profit of the SC is generated. The optimal SC profit is obtained if the retailer orders $q = \mu + \sigma \cdot \Phi^{-1}(CR)$ units, whereby $CR = \frac{r-c}{r-v}$ is the SC's critical ratio. For the optimizing buy-back price b holds:

$$b(w) = -r \cdot \frac{c-v}{r-c} + w \cdot \frac{r-v}{r-c}$$

. The profits of retailer and supplier in case of optimal combinations of b and w holds

$$G^{ret.}(w) = (r-w) \cdot \mu - (r-b) \cdot \sigma \cdot \Phi^{-1}(CR)$$

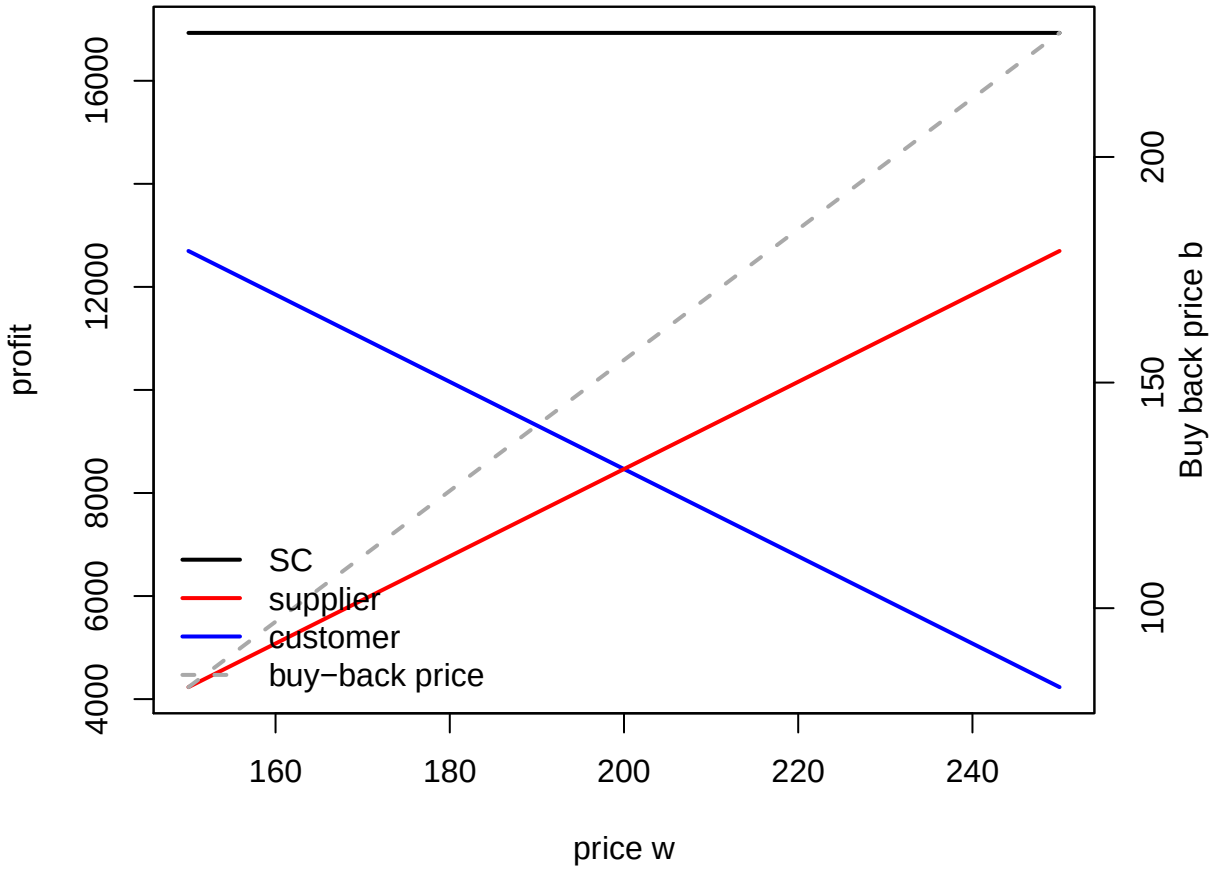
and

$$G^{sup.}(w) = (w-c) \cdot \mu - (b-v) \cdot \sigma \cdot \Phi^{-1}(CR)\,.$$

Plot the profit functions of retailer and supplier as well as the buy-back price function depending on w.

```r
buy_back_func <- function(w, r, c, v, mu, sigma){
  # costs & buy-back price in buy-back model
  crit.rat.sc <- (r-c)/(r-v) # critical ratio SC
  z.sc <- qnorm(crit.rat.sc)
  b <- -(c-v)/(r-c)*r + (r-v)/(r-c)*w
  profit.cust <- (r-w)*mu - (r-b)*dnorm(z.sc)* sigma
  profit.supp <- (w-c)*mu - (b-v)*dnorm(z.sc)* sigma
  return(data.frame(buy.back.price = b, profit.supplier = profit.supp, profit.cust = profit.cust, profi
}

w.vec <- seq(150, 250, length.out = 100)
y.df <- buy_back_func(w = w.vec, c = 100, r = 300, v = 10, mu = 100, sigma = 30)
par(mar = c(4, 4, .1, 4) )
plot(w.vec, y.df$profit.sc, xlab ="price w", ylab = "profit", type = "l", lwd = 2, ylim=range(c(y.df$pr
lines(x = w.vec, y = y.df$profit.cust, lwd = 2, col = "blue")
lines(x = w.vec, y = y.df$profit.supplier, lwd = 2, col = "red")
# second axis
par(new=T)
# data without axis
plot(w.vec, y.df$buy.back.price, xaxt = "n", yaxt = "n", ylab="", xlab = "", type = "l", lwd = 2, col =
# add axis
axis(4)
# add label
mtext( "Buy back price b", side  = 4, line = 2)
legend("bottomleft", col = c("black","red","blue", "darkgrey"), lty=c(1,1,1,2), lwd=2, legend = c("SC",
```



7. Build a tidy data frame with `tibble()` containing two groups of observations: group A consisting of 100 observations with $x_i^A, y_i^A \sim \mathcal{N}(0, 0.5)$ and group B consisting of 50 observations with $x_i^B, y_i^B \sim \mathcal{N}(3, 1.5)$. Plot all points as a scatterplot and color the points groupwise.

8. The data set above is untidy. Create the same data set in a tidy way and recreate the plots above.

9. Alter the structured plot by connecting points by arrows. Test other smoothing methods offered by `geom_smooth`. Add three different smoothing functions to the plot.

10. Create a random sample $S = S_1 \cup S_2$ consisting of 50 obersations with $S_1 \sim \mathcal{E}(0.5)$ and 20 observations with $S_2 \sim \mathcal{N}(3, 1)$. Plot a histogram of the sample overlaid with the theoretical exponential and normal distribution. (Hint: Use `rexp()` to sample from the exponential distribution.)

11. Plot the standard exponential distribution, mark 95%-quantile and highlight the area under the density curve beyond the 95%-quantile.

12. Try to recreate these violine plots from an untidy data set.