Solutions Exercise 4 - Newsvendor

Inventory Management

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Newsvendor (I)

Data: $c_u = 1.95 - 0.80 = 1.15$ Euro and $c_o = 0.80 - 0.10 = 0.70$ Euro with $X \sim N(22, 6^2)$

1. critical ratio $CR = \frac{c_u}{c_u + c_o} = \frac{1.15}{1.15+0.70} = 0.6216216 \Rightarrow q^* = 22 + 6 \cdot 0.30974 = 23.858 \approx 24$ loafs is optimal
2. expected cost: $E(Z) = (c_u + c_o) \cdot \varphi(z^*) \cdot \sigma = 1.85 \cdot 6 \cdot \varphi(0.30974) = 4.22$ Euro expected profit: $G(Z) = c_u \cdot \mu - (c_u + c_o) \cdot \varphi(z^*) \cdot \sigma = 1.15 \cdot 22 - 1.85 \cdot 6 \cdot \varphi(0.30974) = 21.07$ Euro
3. α SL equals the critical ratio, i.e. 62%. β SL is $1 - \frac{L(Y,q)}{\mu} = 1 - \sigma \cdot \frac{L(Z,q')}{\mu}$. Thus, with $q' = 0.30974$ and $\sigma = 6$ it follows $\beta = 1 - 6 \cdot \frac{0.26}{22} = 92.9\%$.

Newsvendor (II)

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\begin{aligned}
Z(q) =& c_o \cdot \int_0^q (q-y) \cdot f_y(y)dy + c_u \cdot \int_q^\infty (y-q) \cdot f_y(y)dy && \text{where } y \sim U(75, 125) \\
=& c_o \cdot \int_{75}^q (q-y) \cdot 0.02dy + c_u \cdot \int_q^{125} (y-q) \cdot 0.02dy \\
=& c_o \cdot \left[0.02 \cdot q \cdot y - 0.01 \cdot y^2\right]_{75}^q + c_u \cdot \left[0.01 \cdot y^2 - 0.02 \cdot q \cdot y\right]_q^{125} \\
=& 0.3 \cdot q^2 - 45 \cdot q + 1687.5 + 9375 - 150 \cdot q + 0.6 \cdot q^2 && \text{with } c_u = 60 \text{ and } c_o = 30 \\
=& 0.9 \cdot q^2 - 195 \cdot q + 11062.5
\end{aligned}
$$

The derivative of $Z(q)$ is $Z'(q) = 1.8 \cdot q - 195$ such that $q^* \approx 108$. The expected profit is $100 \cdot 60 - Z(108) = 600 - 500.1 \approx 100$ Euro.

Discrete newsvendor

The spare part demand Y for solar aggregates is binomially distributed with $n = 20 \cdot 5 = 100$ and $p = 0.04$, i.e. $Y \sim B(n, p)$. Thus, the density values can be calculated as $P(Y = y) = \binom{n}{y} \cdot p^y \cdot (1-p)^{n-y}$.

y	density	cost.os	cost.us	total.cost
0	0.02	0.00	4.00	20.00
1	0.07	0.02	3.02	15.10
2	0.14	0.10	2.10	10.62

y	density	cost.os	cost.us	total.cost
3	0.20	0.34	1.34	7.02
4	0.20	0.77	0.77	4.59
5	0.16	1.39	0.39	3.37
6	0.11	2.18	0.18	3.10
7	0.06	3.08	0.08	3.46
8	0.03	4.03	0.03	4.17
9	0.01	5.01	0.01	5.06
10	0.00	6.00	0.00	6.02
11	0.00	7.00	0.00	7.01
12	0.00	8.00	0.00	8.00
13	0.00	9.00	0.00	9.00
14	0.00	10.00	0.00	10.00
15	0.00	11.00	0.00	11.00

Cost optimal is a buffer stock of 6 with associated total cost per month of 3.1 Euro.