Solutions Exercise 4 - Newsvendor

Inventory Management

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## Warning: package 'DT' was built under R version 4.0.2

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Newsvendor (I)

Data: $c_u = 1.95 - 0.80 = 1.15$ Euro and $c_o = 0.80 - 0.10 = 0.70$ Euro with $X \sim N(22, 6^2)$

1. critical ratio $CR = \frac{c_u}{c_u+c_o} = \frac{1.15}{1.15+0.70} = 0.6216216 \Rightarrow q^* = 22 + 6 \cdot 0.30974 = 23.858 \approx 24$ loafs is optimal
2. expected cost: $E(Z) = (c_u + c_o) \cdot \varphi(z^*) \cdot \sigma = 1.85 \cdot 6 \cdot \varphi(0.30974) = 4.22$ Euro expected profit: $G(Z) = c_u \cdot \mu - (c_u + c_o) \cdot \varphi(z^*) \cdot \sigma = 1.15 \cdot 22 - 1.85 \cdot 6 \cdot \varphi(0.30974) = 21.07$ Euro
3. α SL equals the critical ratio, i.e. 62%. β SL is $1 - \frac{L(Y,q)}{\mu} = 1 - \sigma \cdot \frac{L(Z,q')}{\mu}$. Thus, with $q' = 0.30974$ and $\sigma = 6$ it follows $\beta = 1 - 6 \cdot \frac{0.26}{22} = 92.9\%$.

Newsvendor (II)

$$
\begin{aligned}
Z(q) =& c_o \cdot \int_0^q (q-y) \cdot f_y(y)dy + c_u \cdot \int_q^\infty (y-q) \cdot f_y(y)dy && \text{where } y \sim U(75, 125) \\
=& c_o \cdot \int_{75}^q (q-y) \cdot 0.02dy + c_u \cdot \int_q^{125} (y-q) \cdot 0.02dy \\
=& c_o \cdot \left[0.02 \cdot q \cdot y - 0.01 \cdot y^2\right]_{75}^q + c_u \cdot \left[0.01 \cdot y^2 - 0.02 \cdot q \cdot y\right]_q^{125} \\
=& 0.3 \cdot q^2 - 45 \cdot q + 1687.5 + 9375 - 150 \cdot q + 0.6 \cdot q^2 && \text{with } c_u = 60 \text{ and } c_o = 30 \\
=& 0.9 \cdot q^2 - 195 \cdot q + 11062.5
\end{aligned}
$$

The derivative of $Z(q)$ is $Z'(q) = 1.8 \cdot q - 195$ such that $q^* \approx 108$. The expected profit is $100 \cdot 60 - Z(108) = 6,000 - 500.1 \approx 5,500$ Euro.

Discrete newsvendor

The spare part demand Y for solar aggregates is binomially distributed with $n = 20 \cdot 5 = 100$ and $p = 0.04$, i.e. $Y \sim B(n,p)$. Thus, the density values can be calculated as $P(Y = y) = \binom{n}{y} \cdot p^y \cdot (1-p)^{n-y}$.

Total cost are calculated by $E(C(q)) = c^u \cdot (\mu - q) + (c^u + c^o) \cdot E(S^+(q))$. Undershooting cost and overshooting cost are calculated as $E(C^{us}(q)) = c^u \cdot E(S^-(q)) = c^u \cdot \sum_{y=q}^{15}(y - q) \cdot f(y)$ and $E(C^{os}(q)) = c^o \cdot E(S^+(q)) = c^o \cdot \sum_{y=0}^{q}(q - y) \cdot f(y)$.

y.q	density	cost.os	cost.us	total.cost
0	0.0169	0.00	19.99	20.00
1	0.0703	0.02	15.08	15.10
2	0.1450	0.10	10.51	10.62
3	0.1973	0.34	6.68	7.02
4	0.1994	0.77	3.82	4.59
5	0.1595	1.39	1.97	3.37
6	0.1052	2.18	0.91	3.10
7	0.0589	3.08	0.38	3.46
8	0.0285	4.03	0.14	4.18
9	0.0121	5.01	0.05	5.06
10	0.0046	6.00	0.01	6.02
11	0.0016	7.00	0.00	7.01
12	0.0005	8.00	0.00	8.00
13	0.0001	9.00	0.00	9.00
14	0.0000	10.00	0.00	10.00
15	0.0000	11.00	0.00	11.00

Cost optimal is a buffer stock of 6 with associated total cost per month of 3.1 Euro.

Note that there are small differences due to rounding betweenthe values of column “total.cost” and the sum of the columns “cost.os” and “cost.us”.