Solution Exercise 8 - Multi-item lot sizing

Inventory Management

Thomas Kirschstein

Economic lot scheduling

1. Determine the product-specific utilization rates and the total utilization of the machine. Can a feasible schedule be found?

The product-specific utilization rates $\rho_i = \frac{y_i}{p_i}$ are all smaller than 1:

	1	2	3	4	5	6	7	8
ρ_i	0.05	0.04	0.04	0.04	0.05	0.03	0.18	0.17

Additionally, the idle time share $\kappa = 1 - \sum_{i=1}^{n} \rho_i = 0.4$ is greater than 1, too. Thus, a feasible schedule shall exist.

2. Determine the independent and common cycle solution. What are the associated total costs? Is the independent solution feasible?

The independent solution is given by

	1	2	3	4	5	6	7	8
T_i	7.93	14.93	31.73	1.21	7.79	15.83	18.92	3.39
b_i	0.46	0.72	1.44	0.10	0.36	0.72	3.64	0.74

The total cost of the independent solution are

	1	2	3	4	5	6	7	8	$\sum$
$C(T_i)$	117.27	129.38	56.98	46.14	247.39	59.89	71.02	63.15	791.21

As the smallest cycle time of the independent solution is 1.21 but the largest batch time is 3.64, a feasible schedule cannot be found.

For the common cycle solution we obtain the following results

	1	2	3	4	5	6	7	8	$\sum$
b_i	0.50	0.47	0.4	0.40	0.40	0.47	1.78	1.62	6.02
$C(T_i)$	117.76	148.79	111.8	168.45	248.89	70.97	93.62	93.29	1053.58

whereby $T = \max(T^*, \underline{T}) = \max(\ 8.69{,}2.01) = 8.69$ and the total cost are $\sum_{i=1}^{n} C_i(T) = 1053.58$.

3. Try to find a better solution with the power-of-2 heuristic.

- *1st iteration*: We determine the basic cycle time as the minimum cycle time of the indepedent solution $B = 1.21$. Afterwards, we determine the multipliers m_i and round to the closest power of 2 such that

	1	2	3	4	5	6	7	8
m_i	6.53	12.3	26.14	1	6.41	13.04	15.59	2.79
$[m_i]$	8.00	16.0	32.00	1	8.00	16.00	16.00	2.00

Reoptimizing the basic cycle time yields $B = 1.05$ such that the following batch times result

	1	2	3	4	5	6	7	8
$T_i = m_i \cdot B$	8.39	16.78	33.55	1.05	8.39	16.78	16.78	2.10
b_i	0.48	0.79	1.52	0.09	0.39	0.75	3.25	0.52

We observe that again the basic cycle time is smaller than the largest batch time. I.e., this solution cannot be feasible. However, maybe not synchronization feasibility (alone) is the problem, but the capacity constraint. Therefore, we can compute a minimum basic cylce time $\underline{B}$ analogues to $\underline{T}$ in the common cycle solution by rearranging $\sum_{i=1}^{n} \frac{s_i}{m_i \cdot B} \leq \kappa$ for B. Thus, we have $B \geq \frac{1}{\kappa} \cdot \sum_{i=1}^{n} \frac{s_i}{m_i} = \underline{B}$. Here, $\underline{B} = 0.44$such that $B = \max\left(B^*, \underline{B}\right) = \max($ 1.05,0.44 $) = 1.05$. We conclude that the capacity constraint is not violated, but nonetheless no feasible solution can be found.

- *2nd iteration*: We alter the vector of frequencies to cope with the synchronization feasibility. The products with the batch times larger than B are #7 and #3. Thus, we reduce frequencies of thos products:

	1	2	3	4	5	6	7	8
$[m_i]$	8.00	16.00	16.00	1.00	8.00	16.00	8.00	2.00
$T_i = m_i \cdot B$	9.43	18.86	18.86	1.18	9.43	18.86	9.43	2.36
b_i	0.53	0.88	0.86	0.10	0.43	0.82	1.91	0.57

The basic cycle time is $B = 1.18$. Again no feasible solution can be found as B is smaller than the largest batch time.

- *3rd iteration*: Obviously, the frequencies for products #7 and #3 must be reduced more drastically. Thus,

	1	2	3	4	5	6	7	8
$[m_i]$	8.00	16.00	8.00	1.00	8.00	16.00	4.00	2.00
$T_i = m_i \cdot B$	10.93	21.86	10.93	1.37	10.93	21.86	5.46	2.73
b_i	0.60	1.00	0.50	0.10	0.50	0.93	1.19	0.63

The basic cycle time is $B = 1.37$. A feasible solution may exist as B is larger than the largest batch time. Thus, we try to find a feasible schedule:

	$m=1$			$m=2$			$m=4$			$m=8$			$m=16$		
	i	$\sum b_i$	rest	i	$\sum b_i$	rest	i	$\sum b_i$	rest	i	$\sum b_i$	rest	i	$\sum b_i$	rest
1	4	0.1	1.27	8	0.63	0.64		0.00	0.64	1	0.6	0.04		0.00	0.04
2	4	0.1	1.27		0.00	1.27	7	1.19	0.08		0.0	0.08		0.00	0.08
3	4	0.1	1.27	8	0.63	0.64		0.00	0.64		0.0	0.64		0.00	0.64
4	4	0.1	1.27		0.00	1.27		0.00	1.27	3+5	1.0	0.27		0.00	0.27
5	4	0.1	1.27	8	0.63	0.64		0.00	0.64		0.0	0.64		0.00	0.64
6	4	0.1	1.27		0.00	1.27	7	1.19	0.08		0.0	0.08		0.00	0.08
7	4	0.1	1.27	8	0.63	0.64		0.00	0.64		0.0	0.64		0.00	0.64
8	4	0.1	1.27		0.00	1.27		0.00	1.27		0.0	1.27	2	1.00	0.27
9	4	0.1	1.27	8	0.63	0.64		0.00	0.64	1	0.6	0.04		0.00	0.04
10	4	0.1	1.27		0.00	1.27	7	1.19	0.08		0.0	0.08		0.00	0.08
11	4	0.1	1.27	8	0.63	0.64		0.00	0.64		0.0	0.64		0.00	0.64
12	4	0.1	1.27		0.00	1.27		0.00	1.27	3+5	1.0	0.27		0.00	0.27
13	4	0.1	1.27	8	0.63	0.64		0.00	0.64		0.0	0.64		0.00	0.64
14	4	0.1	1.27		0.00	1.27	7	1.19	0.08		0.0	0.08		0.00	0.08
15	4	0.1	1.27	8	0.63	0.64		0.00	0.64		0.0	0.64		0.00	0.64
16	4	0.1	1.27		0.00	1.27		0.00	1.27		0.0	1.27	6	1.19	0.08

Obviously, the schedule is feasible. The total cost is 923.87.

Joint replenishment problem

1. Calculate the holding cost multipliers and individual optimal cycle times for each material. Order the products increasingly w.r.t. cycle time.

The holding cost multiplier is defined by $H_i = 0.5 \cdot y_i \cdot c_i^{sh}$ such that

	8	6	1	4	5	7	3	2
c_i^{or}	105.00	121.00	329.00	485.00	895.00	328.00	284.00	688.00
H_i	3.51	3.48	9.24	9.24	11.16	2.41	0.18	0.17
T_i	5.47	5.90	5.97	7.24	8.96	11.68	40.28	63.62

2. Use the basic period heuristic to find a solution for the replenishment problem (round to integers). Determine also the order quantities of each product.

First, we determine $T_i^2 = \frac{c_i^{or}}{H_i}$ and $T^C = \frac{\sum_{i=0}^{i'} c_i^{or}}{\sum_{i=0}^{i'} H_i}$.

	8	6	1	4	5	7	3	2
T_i^2	29.91	34.77	35.61	52.49	80.23	136.38	1622.86	4047.06
T^C	115.38	75.25	52.68	52.61	61.02	65.67	72.62	89.78

Thus, the break-even is reached at position 4, i.e., product 4. The basic cycle time is $B = \min(T_i) = 5.47$. Now, the frequencies m_i can be determined (and rounded).

	8	6	1	4	5	7	3	2
$m_i = \frac{T_i}{B}$	0.98	1.05	1.07	1.29	1.6	2.09	7.2	11.37
$[m_i]$	1.00	1.00	1.00	1.00	2.0	2.00	7.0	11.00
$[\tilde{m}_i]$	1.00	1.00	1.00	1.00	2.0	2.00	7.0	12.00

Reoptimizing basic cycle time yields $B = 5.6$. The order quantities are given by multiplying the cycle times T_i with demand rates y_i, i.e. $q_i = T_i \cdot y_i$. The individual cost per material $C_i(q_i)$ are given by $C_i(q_i) = \frac{c_i^{or}}{m_i \cdot B} + H_i \cdot m_i \cdot B$ such that

	8	6	1	4	5	7	3	2
$[\tilde{m}_i]$	1.00	1.00	1.00	1.00	2.00	2.00	7.00	12.00
T_i	5.60	5.60	5.60	5.60	11.19	11.19	39.18	67.16
q_i	218.26	486.89	369.37	313.40	1085.72	414.14	1371.13	1141.68
$C_i(q_i)$	38.41	41.10	110.50	138.37	204.82	56.22	14.11	21.66

The total cost are then given by $C = \frac{c_0^{or}}{B} + \sum_{i=1}^{8} C_i(q_i) = 678.79$.

3. Try to find another solution by rounding ordering frequencies to powers of 2 (instead of integers). Can the solution from 2. be improved?

Rounding to the closest power of 2 results in the following frequencies:

	8	6	1	4	5	7	3	2
$m_i = \frac{T_i}{B}$	1	1.08	1.09	1.32	1.64	2.14	7.37	11.63
$[\tilde{m}_i]$	1	1.00	1.00	1.00	2.00	2.00	7.00	12.00
$[\tilde{m}_i]_{2^{\mathbb{N}}}$	1	1.00	1.00	1.00	2.00	2.00	8.00	16.00

Reoptimizing the basic cycle time yields $B = 5.52$ and the total cost are 680.76 which is slightly higher than for the integer rounding solution.

However, one can improve the solution by altering the integer-rounding solution slightly (see product #5) as follows:

	8	6	1	4	5	7	3	2
$m_i = \frac{T_i}{B}$	1	1.08	1.09	1.32	1.64	2.14	7.37	11.63
$[\tilde{m}_i]$	1	1.00	1.00	1.00	2.00	2.00	7.00	12.00
$[\tilde{m}_i]_{alt.}$	1	1.00	1.00	1.00	1.00	2.00	7.00	12.00

Reoptimizing the basic cycle time yields $B = 7.01$ and the total cost is 669.54.