Exercise 6 - Continuous review policy

Inventory Management

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Continuous review with fixed lead time (I)

The weekly demand (assuming 5 workdays per week) of bananas in supermarket is normally distributed with $\mu = 50$ [kg] and $\sigma = 8$. Ordering cost are estimated at 80 Euro and the holding cost rate is $c^{sh} = 0.05$ Euro per kg and week.

1. Calculate the cost-optimal order quantity q and re-order level s assuming an order lead time of 1 week and a minimum β service level of 97.5%.
2. What happens with the re-order level s if the order lead time reduces to 3 days? (no calculations required)

Continuous review with fixed lead time (II)

A warehouse operator is observing the demand of a particular product. The operator estimates an expected daily demand of 10 and a standard deviation of 5. However, she is not sure which underlying demand distribution is appropriate; a normal distribution or a Gamma distribution are considered as likely candidates. The operator pursues a (s, q) policy for the product which shows a reorder time of 1 day.

Determine the re-order point s for a normal and Gamma distribution of demand assuming that an α or a β service level of 99% should by realized. The order quantity is set to $q = 80$ units.

Hint: Use the following tabulated values of the Gamma distribution.

s	$P(Y^{\Gamma(\alpha,\beta)} \leq s)$	$L\left(Y^{\Gamma(\alpha,\beta)}, s\right)$	s	$P(Y^{\Gamma(\alpha,\beta)} \leq s)$	$L\left(Y^{\Gamma(\alpha,\beta)}, s\right)$	s	$P(Y^{\Gamma(\alpha,\beta)} \leq s)$	$L\left(Y^{\Gamma(\alpha,\beta)}, s\right)$
10.0000	0.5665	1.9537	20.1695	0.9595	0.1417	30.3390	0.9979	0.0065
10.5085	0.6052	1.7432	20.6780	0.9648	0.1225	30.8475	0.9982	0.0055
11.0169	0.6417	1.5518	21.1864	0.9694	0.1058	31.3559	0.9985	0.0047
11.5254	0.6759	1.3785	21.6949	0.9734	0.0912	31.8644	0.9987	0.0040
12.0339	0.7078	1.2219	22.2034	0.9769	0.0786	32.3729	0.9989	0.0034
12.5424	0.7373	1.0809	22.7119	0.9800	0.0677	32.8814	0.9991	0.0029
13.0508	0.7646	0.9544	23.2203	0.9827	0.0582	33.3898	0.9992	0.0025
13.5593	0.7895	0.8411	23.7288	0.9850	0.0500	33.8983	0.9993	0.0021
14.0678	0.8123	0.7400	24.2373	0.9871	0.0430	34.4068	0.9994	0.0018
14.5763	0.8330	0.6499	24.7458	0.9889	0.0369	34.9153	0.9995	0.0015
15.0847	0.8518	0.5698	25.2542	0.9904	0.0316	35.4237	0.9996	0.0013
15.5932	0.8687	0.4988	25.7627	0.9917	0.0271	35.9322	0.9996	0.0011
16.1017	0.8840	0.4361	26.2712	0.9929	0.0232	36.4407	0.9997	0.0009
16.6102	0.8977	0.3806	26.7797	0.9939	0.0198	36.9492	0.9997	0.0008
17.1186	0.9099	0.3318	27.2881	0.9948	0.0169	37.4576	0.9998	0.0006

s	$P(Y^{\Gamma(\alpha,\beta)} \leq s)$	$L\left(Y^{\Gamma(\alpha,\beta)}, s\right)$	s	$P(Y^{\Gamma(\alpha,\beta)} \leq s)$	$L\left(Y^{\Gamma(\alpha,\beta)}, s\right)$	s	$P(Y^{\Gamma(\alpha,\beta)} \leq s)$	$L\left(Y^{\Gamma(\alpha,\beta)}, s\right)$
17.6271	0.9208	0.2888	27.7966	0.9955	0.0145	37.9661	0.9998	0.0005
18.1356	0.9306	0.2511	28.3051	0.9961	0.0123	38.4746	0.9998	0.0005
18.6441	0.9392	0.2180	28.8136	0.9967	0.0105	38.9831	0.9999	0.0004
19.1525	0.9468	0.1891	29.3220	0.9972	0.0090	39.4915	0.9999	0.0003
19.6610	0.9536	0.1638	29.8305	0.9976	0.0076	40.0000	0.9999	0.0003

Continuous review with stochastic lead time

Assume that the daily demand of cement on a construction side is Gamma distributed. The expected daily demand is estimated to be 8 tons and the standard deviation is supposed to be 6 tons. When new cement is ordered, the lead time varies. Based on historical records, with 20% probability the delivery arrives after 2 days. A delay of 3 days is observed in 50% of all cases and with 30% probability 4 days expire before the shipment arrives.

1. Determine the distributional parameters for all three different lead time realizations.
2. Assume an α service level of 99% should be achieved. Which reorder level s should be chosen?

Hint: Choose the correct values from the following tabulated values of some Gamma distributions.

s	$P(Y^{\Gamma(\alpha\cdot 1,\beta)} \leq s)$	s	$P(Y^{\Gamma(\alpha\cdot 2,\beta)} \leq s)$	s	$P(Y^{\Gamma(\alpha\cdot 3,\beta)} \leq s)$	s	$P(Y^{\Gamma(\alpha\cdot 4,\beta)} \leq s)$	s	$P(Y^{\Gamma(\alpha\cdot 5,\beta)} \leq s)$
8	0.5996	16.0000	0.5706	24.0000	0.5576	32.0000	0.5499	40.0000	0.5446
9	0.6592	17.4499	0.6340	25.7734	0.6220	34.0265	0.6142	42.2320	0.6083
10	0.7112	18.8997	0.6909	27.5469	0.6805	36.0529	0.6731	44.4640	0.6672
11	0.7562	20.3496	0.7410	29.3203	0.7326	38.0794	0.7262	46.6961	0.7205
12	0.7949	21.7995	0.7847	31.0938	0.7784	40.1058	0.7730	48.9281	0.7678
13	0.8280	23.2493	0.8221	32.8672	0.8179	42.1323	0.8136	51.1601	0.8091
14	0.8561	24.6992	0.8540	34.6407	0.8515	44.1587	0.8483	53.3921	0.8447
15	0.8799	26.1491	0.8808	36.4141	0.8798	46.1852	0.8777	55.6242	0.8747
16	0.9000	27.5989	0.9032	38.1876	0.9034	48.2116	0.9021	57.8562	0.8999
17	0.9169	29.0488	0.9217	39.9610	0.9229	50.2381	0.9223	60.0882	0.9207
18	0.9310	30.4987	0.9370	41.7345	0.9388	52.2646	0.9388	62.3202	0.9376
19	0.9429	31.9485	0.9495	43.5079	0.9517	54.2910	0.9520	64.5523	0.9513
20	0.9527	33.3984	0.9597	45.2813	0.9621	56.3175	0.9627	66.7843	0.9623
21	0.9609	34.8483	0.9679	47.0548	0.9704	58.3439	0.9711	69.0163	0.9710
22	0.9678	36.2982	0.9745	48.8282	0.9770	60.3704	0.9778	71.2483	0.9778
23	0.9734	37.7480	0.9799	50.6017	0.9822	62.3968	0.9830	73.4804	0.9831
24	0.9781	39.1979	0.9841	52.3751	0.9863	64.4233	0.9871	75.7124	0.9872
25	0.9820	40.6478	0.9875	54.1486	0.9895	66.4498	0.9902	77.9444	0.9904
26	0.9852	42.0976	0.9902	55.9220	0.9919	68.4762	0.9926	80.1764	0.9928
27	0.9879	43.5475	0.9923	57.6955	0.9939	70.5027	0.9945	82.4084	0.9947
28	0.9900	44.9974	0.9940	59.4689	0.9953	72.5291	0.9959	84.6405	0.9961
29	0.9918	46.4472	0.9953	61.2423	0.9965	74.5556	0.9969	86.8725	0.9971
30	0.9933	47.8971	0.9964	63.0158	0.9973	76.5820	0.9977	89.1045	0.9979
31	0.9945	49.3470	0.9972	64.7892	0.9980	78.6085	0.9983	91.3365	0.9984
32	0.9955	50.7968	0.9978	66.5627	0.9985	80.6349	0.9988	93.5686	0.9989