Solution Exercise 6 - Continuous review policy

Inventory Management

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Continuous review with fixed lead time (I)

1. Calculate the cost-optimal order quantity q and re-order level s assuming an order lead time of 1 week and a minimum β service level of 97.5%.

We use the algorithm presented in the lecture. In the first iteration we start with $q^0 = \sqrt{\frac{2 \cdot 50 \cdot 80}{0.05}} = 400$, $l^0 = 0.025 \cdot 400 = 10$, $s = 40.45$, and $\lambda^0 = 22.64$ as follows:

```
##      iter         q       ef        s    lambda
##   1.00000 400.00000 10.00000 40.45521  22.63507
##      iter         q       ef        s    lambda
##   2.00000 411.16180 10.27905 40.14075  23.07043
##      iter         q       ef        s    lambda
##   3.00000 411.68638 10.29216 40.12603  23.09098
##      iter         q       ef        s    lambda
##   4.00000 411.71136 10.29278 40.12533  23.09195
##      iter         q       ef        s    lambda
##   5.00000 411.71254 10.29281 40.12530  23.09200
```

```
## [1]  40.1253 411.7126
```

2. What happens with the re-order level s if the order lead time reduces to 3 days? (no calculations required)

The reorder level decreases as the expected demand in the lead time decreases. However, s decreases non-linearly as the standard deviation scales by $\sqrt{\frac{3}{5}} > \frac{3}{5}$.

Continuous review with fixed lead time (II)

A warehouse operator is observing the demand of a particular product. The operator estimates an expected daily demand of 10 and a standard deviation of 5. However, she is not sure which underlying demand distribution is appropriate; a normal distribution or a Gamma distribution are considered as likely candidates. The operator pursues a (s, q) policy for the product which shows a reorder time of 1 day.

Determine the re-order point s for a normal and Gamma distribution of demand assuming that an α or a β service level of 99% should by realized. The order quantity is set to $q = 80$ units.

For the normal distribution, we have $s^\alpha = 10 + 2.3263 \cdot 5 = 21.63$ and $s^\beta = 10 + 0.63 \cdot 5 = 13.15$ whereby $L^{-1}(Z, 0.16) \approx 0.63$.

For the gamma distribution, we have $s^\alpha = 25.2542$ and $s^\beta = L^{-1}\left(\Gamma\left(\frac{10^2}{5^2}, \frac{10}{5^2}\right), 0.8\right) \approx 13.7$.

Continuous review with stochastic lead time

Assume that the daily demand of cement on a construction side is Gamma distributed. The expected daily demand is estimated to be 8 tons and the standard deviation is supposed to be 6 tons. When new cement is ordered, the lead time varies. Based on historical records, with 20% probability the delivery arrives after 2 days. A delay of 3 days is observed in 50% of all cases and with 30% probability 4 days expire before the shipment arrives.

1. Determine the distributional parameters for all three different lead time realizations.

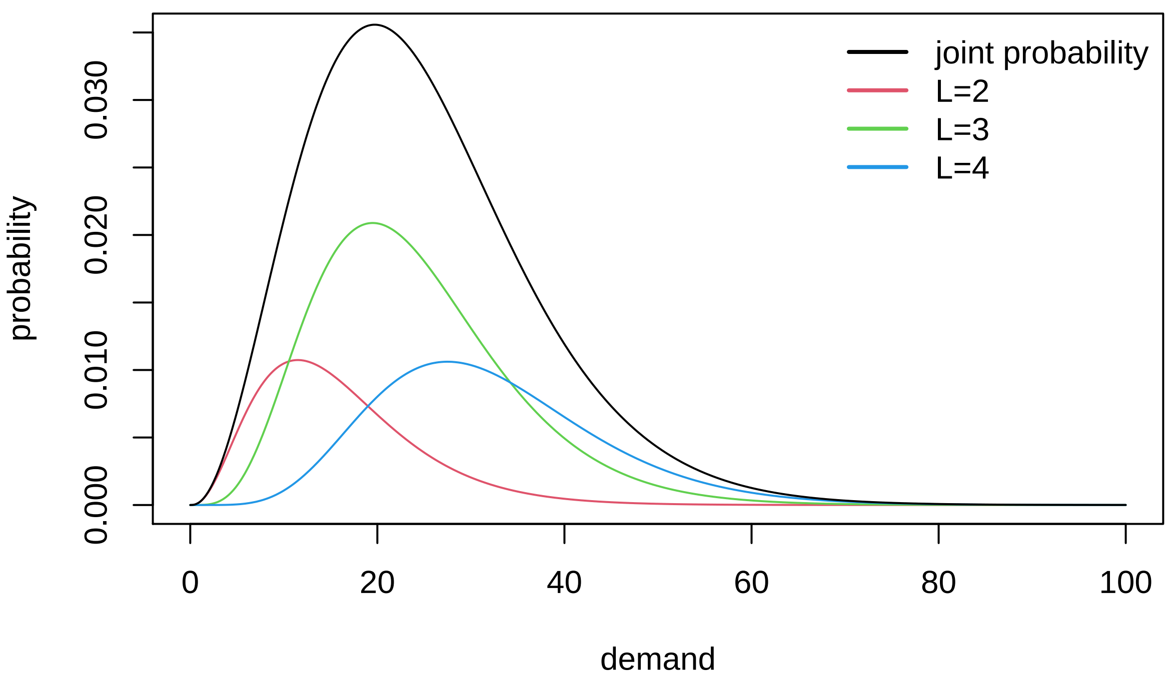
The distribution parameters for daily demand are $\alpha = \frac{\mu^2}{\sigma^2} = 1.778$ and $\beta = \frac{\mu}{\sigma^2} = 0.222$. If the demand is i.i.d., it follows $Y^l \sim \Gamma\left(\sum_{i=1}^{l} \alpha_i, \beta\right)$

l	2	3	4
$\sum_{i=1}^{l} \alpha_i$	3.556	5.333	7.111

Thus, $Y^l \sim \Gamma(l \cdot 1.778, 0.222)$.

2. Assume an α service level of 99% should be achieved. Which reorder level s should be chosen?

The (probability weighted) demand distributions for all realizations of the lead time as well as joint demand dis-



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Based on the tables, we estimate s^α for each realization of lead time:

l	2	3	4
$P(L=l)$	20	50	30
s_l^α	42.0	55.9	66.6

We conclude that $s^\alpha < s_4^\alpha = 66.6$. Furthermore, we have $P(Y^{L=2} > 66.6) \approx 0$, $P(Y^{L=3} > 66.6) < 0.0015$ and $P(Y^{L=4} > 66.6) = 0.01$, we can look for x such that $P(Y^{L=4} > s) = 1 - \frac{0.01}{0.3}$ such that $s^\alpha \approx 57.3$.