Solution Exercise 7 - Dynamic lot sizing

Inventory Management

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Dynamic lotsizing with α service level constraint

1. Calculate the matrices of means, standard deviations and order-up levels S. Derive the total cost matrix.

Table 1: Mean matrix

	1	2	3	4	5	6
1	84	138	168	204	270	360
2		54	84	120	186	276
3			30	66	132	222
4				36	102	192
5					66	156
6						90

Table 2: Standard deviation matrix

	1	2	3	4	5	6
1	16.8	19.97	20.85	22.06	25.71	31.38
2		10.80	12.35	14.30	19.46	26.51
3			6.00	9.37	16.19	24.21
4				7.20	15.04	23.45
5					13.20	22.32
6						18.00

Table 3: Order-up level (S) matrix

	1	2	3	4	5	6
1	123.08	184.46	216.51	255.32	329.81	433.01
2		79.12	112.74	153.27	231.27	337.67
3			43.96	87.80	169.66	278.32
4				52.75	136.98	246.56
5					96.71	207.93
6						131.87

Table 4: Partial cost matrix

	1	2	3	4	5	6
1	10.86	13.23	15.71	19.40	27.27	40.23
2		10.55	11.92	14.44	20.58	31.30
3			10.31	11.75	16.18	24.59
4				10.37	12.99	19.01
5					10.68	14.26
6						10.92

2. Determine the optimal weekly replenishment strategy and calculate expected stock levels as well as order quantities. On which weekdays has retailer to expect potential shortfalls?

Based on the partial cost, the cumulative total cost are as follows:

Table 5: Total cost matrix

	1	2	3	4	5	6
1	10.86	13.23	15.71	19.40	27.27	40.23
2		21.41	22.78	25.30	31.44	42.16
3			23.54	24.98	29.41	37.82
4				26.08	28.70	34.72
5					30.08	33.67
6						38.19

Thus, the optimal policy is to order in periods 1 and 5. Thus, order-up levels are $S_1 = 255.32$ and $S_5 = 207.93$. For expected inventory levels and expected order quantities follows:

	1	2	3	4	5	6
q_t	255.32	0.00	0.00	0.00	156.61	0.00
i_t	171.32	117.32	87.32	51.32	141.93	51.93

Shortfalls may (most likely) occur in period 4 and 6 (on Thursday and Saturday).

3. A consultant suggests that the retail manager should incorporate shortage cost instead of the α service level to determine the optimal ordering policy. The retail manager estimates a shortage cost rate of 0.20 Euro per unit of milk. What happens to the replenishment solution? (no calculation required)

With the given cost rates a critical ratio of $CR = \frac{c^{sf}}{c^{sf}+c^{sh}} = \frac{0.20}{0.20+0.022} \approx 90.1\%$. Thus, the safety multiplier z reduces to 1.29. Consequently, the order-up levels S decrease linearly w.r.t. σ which also holds for the partial cost. As the ratio between means and standard deviations is comparatively small ($c = 0.2$), the structure of partial costs is probably not much affected. Therefore, the solution probably remains the same (same ordering periods), but the total cost decrease.

Dynamic lotsizing with β service level constraint

The following table displays expected demand and standard deviation of a material for the next 7 days.

t	1	2	3	4	5	6	7
μ_t	120	75	82	100	91	65	88
σ_t	30	12	25	10	28	5	10

The demand is assumed to be independently normally distributed. The manager intends to assure a β service level of 98%. Ordering cost are $c^{or} = 150$ Euro and stock-holding cost rate $c^{sh} = 0.5$ Euro per unit and period.

1. Calculate the matrices of means, standard deviations and order-up levels S. Derive the total cost matrix.

Table 8: Mean matrix

	1	2	3	4	5	6	7
1	120	195	277	377	468	533	621
2		75	157	257	348	413	501
3			82	182	273	338	426
4				100	191	256	344
5					91	156	244
6						65	153
7							88

Table 9: Standard deviation matrix

	1	2	3	4	5	6	7
1	30	32.31	40.85	42.06	50.53	50.77	51.75
2		12.00	27.73	29.48	40.66	40.96	42.17
3			25.00	26.93	38.85	39.17	40.42
4				10.00	29.73	30.15	31.76
5					28.00	28.44	30.15
6						5.00	11.18
7							10.00

Table 10: Order-up level (S) matrix

	1	2	3	4	5	6	7
1	150.64	220.77	306.85	400.65	495.37	556.43	640.28
2		84.33	180.12	274.08	372.02	432.97	517.00
3			110.07	201.72	300.57	360.94	444.55
4				104.93	213.65	273.96	358.01
5					122.56	180.22	262.85
6						66.59	156.15
7							93.74

Table 11: Partial cost matrix

	1	2	3	4	5	6	7
1	165.32	213.27	314.27	466.79	669.92	834.29	1095.49
2		154.67	214.12	316.62	475.55	607.44	825.49
3			164.03	219.72	332.35	434.39	610.88
4				152.46	218.15	287.45	420.52
5					165.78	206.72	298.78
6						150.79	197.15
7							152.87

2. Determine the optimal weekly replenishment strategy and calculate expected stock levels as well as order quantities.

Based on the partial cost, the cumulative total cost are as follows:

Table 12: Total cost matrix

	1	2	3	4	5	6	7
1	165.32	213.27	314.27	466.79	669.92	834.29	1095.49
2		319.98	379.44	481.94	640.87	772.75	990.81
3			377.31	433.00	545.62	647.66	824.15
4				466.74	532.43	601.72	734.80
5					598.78	639.71	731.77
6						683.22	729.58
7							754.59

Thus, the optimal policy is to order in periods 1 and 4 and 6. Thus, order-up levels are $S_1 = 306.85$ and $S_4 = 358.01$. For expected inventory levels and expected order quantities follows:

	1	2	3	4	5	6	7
q_t	306.85	0.00	0.00	328.16	0.00	0.00	0.00
i_t	186.85	111.85	29.85	258.01	167.01	102.01	14.01

3. Assume the supplier can ship only 200 units at most. Does the optimal solution change? Try to find an alternative solution by adapting the Wagner-Whithin algorithm.

Yes, the solution needs to be changed as the expected order quantities are larger than the transport capacity. Alternative solution with at least 3 orders?