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Synchronized lot sizing with capacity constraints

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List of Abbreviations

- EOQ: Economic order quantity
- JO: Joint Ordering
- JRP: Joint Replenishment Problem
- SO: Separate Ordering
- TS: Policy parameters where T is order interval and S is order up to level

List of Symbols

- T_i: $cycle\ time\ for\ item\ i$
- bc_i : $box\ capacity\ for\ item\ i$
- c_i^{-or} : $fixed\ or\ overall\ ordering\ cost, c_i^{-or} =$ €1500
- c_i^{or} : $variable\ ordering\ cost\ per\ box$
- c_i^{sh} : $stock\ holding\ cost, c_i^{sh} = p_i.h$
- d_i : $demand\ per\ day\ per\ item$
- p_i : $unit\ price\ per\ material\ i$
- q_i : $order\ quantity\ of material\ i$
- $v_i^{'}$: order share of each item
- $B(i)$: $Backorders$
- B: $Basic\ cycle\ time$
- $b_i =\ box\ sorting$
- $b_i^{-1} =\ box\ not\ sorting$
- ct: $cycle\ time\ in\ days$
- $i = interest\ rate$
- H_i: $holding\ cost\ multiplier$
- I_t: $inventory\ level\ at\ a\ time\ point$
- $l(qi)$: $number\ of\ lanes\ required\ for\ each\ product$
- L: $lead\ time$
- m_i: $order\ frequency$
- n_i: $number\ of\ boxes\ in\ a\ lane\ for\ item\ i$
- $O(t)$: $Outstanding\ orders\ at\ a\ \ time$
- pr_i: $price\ per\ box\ for\ each\ item\ i$
- rl: $rack\ length$
- $S(t)$: $On - hand\ stock$
- S_i: $order\ up\ to\ level\ for\ item\ i$
- t: $time\ period\ \{1, \ldots.,262\}$
- T^c: $cumulative\ sum\ of\ individual\ ordering\ cost$
- v_i: $value\ share\ of\ material$
- $\tilde{v}_i$: $cumulative\ order\ value\ shre\ of\ each\ item$
- yb_i: $demand\ per\ day\ in\ boxes\ for\ item\ i.$
- yd_i: $demand\ per\ day\ for\ item\ i$
- y_t: $demand\ at\ a\ time\ point$
- α : $alpha\ service\ level$

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1. Introduction

Production planning is the process of determining a tentative plan for how much to produce in the next time period, during an interval of time called planning horizon. It is an important challenge for industrial companies because it has a strong impact on their performance in terms of customer service quality and operating costs. Production planning often proves to be a very complex task because of several reasons, some of which are discussed below. One of the reasons is variability of products. A production resource is not only used for the production of one product, but to produce different types of products. The production resources available are limited and inflexible, as they can be used to produce only one type of product at a given time with a given production rate. Though the efficiency of a production resource is aimed to be maximized, it can still be used for a single product at a given time. The production planner thus needs to decide which products should be produced, when and in what quantities, while taking into account all the constraints in the system. Another reason is that a production plan might have to meet several conflicting objectives, for example, minimizing production and inventory costs while simultaneously increasing the productivity and the flexibility of production facilities and guaranteeing excellent customer service levels.

One major planning decision is finding the economic lot size (production or order) which balances the inventory, ordering and set up costs. Deciding which materials are required to realize an aggregate production plan per period as well as determining how to properly bundle production lots in order to save ordering costs and stock up cost constitutes some of the most important operational activities of industrial companies.

Since the late 1940's, inventory theory has been one of the most successful operational research techniques to be applied in business, industry and the public sector (Goyal and Gupta, 1989). The very first lot size model was proposed by Ford Whitman Harris' in 1913 in his seminal paper entitled how many parts to make at once. In his paper, Harris proposed the Economic order Quantity model (EOQ) which enables the determination of optimum order quantities while avoiding excess inventory holding cost and shortage costs. Since its publications many variations have been made to this basic lot sizing problem including but not limited to: lot sizing models with dynamic or stationary demand, single or multiple level problems, single item or multiple items, capacitated or incapacitated, deteriorating items, and deterministic or stochastic demand. Detailed description of these variations and more can be found in the papers by Karimi et al. (2003) and Christoph H. et al. (2014).

This paper aims to determine an optimal approach for coordinating the orders for a multi item capacitated lot sizing problem with deterministic demand. The reasoning here is that although the EOQ is optimal for the buyer, the buyer may stand to benefit from ordering more than the EOQ in situations where this leads to decreased transportation costs, administrative and handling costs as well as increased price discounts from suppliers. In this paper, a case study of a single storage unit of flow rack with multiple levels is explored where the demand for various materials is deterministic. The rest of this paper is structured as follows: a detailed description of the problem is given in chapter 2; data preparation and the separate ordering and joint ordering solution approaches is presented in chapters 3 and 4 respectively. In Chapter 5 an analysis of the overall results will be presented while Chapter 6 contains the concluding remarks.

2. Case Description

The given case describes an inbound warehouse where multiple materials are stored in different storage facilities. We are concerned with the storage facility which is the continuous flow rack. A continuous flow rack is used to store small and medium sized parts. There are a total of 8 racks, each flow rack consists of 4 levels and multiple runways or lanes per level.

2.1 Materials and Boxes

There are 62 materials in total and 9 types of boxes with varying dimensions. The materials are ordered and stored in boxes. There are different types of boxes, but each part is stored in one particular box type. Each box has a specific capacity for the number of items that can be stored in it.

In each runway only one part should be stored. Thereby, the width of a runway can be adapted to the width of the box type which corresponds to the part assigned to the runway. Thus, the total number of runways per level depends on the assigned parts and is limited by the total width of the rack.

2.2 Cost Distribution

All parts are ordered from one supplier and only complete boxes can be ordered. The demand per day is given. Ordering of each box with a particular item incurs an ordering cost which is variable and dependent on the material ordered. A fixed ordering cost €1500 is incurred every time at least one order is placed. The stock holding cost is based on the unit price and the interest rate.

2.3 Task

The aim of the project is to determine a lot sizing model that determines lot sizes for each part such that rack capacity constraints are adhered to and the total cost per period for stock holding and ordering are minimal.

3. Data Preparation

Prior to analyzing the data and formulating the models for the solution approaches, several manipulations were made to convert the data into a form that can be readily and accurately studied.

The following storage information is derived from the data sheet provided.

	Total_racks	levels_per_rack	rack_length	rack_width	rack_height
Rack Information	8	4	6000	1750	300

Table 1: Storage Information

The following modifications were made to the data:

3.1 Product Number

Numbers were assigned to the materials ranging from 1 to 62 in an increasing order. The purpose of this was to make it easier to use and to refer to the products instead of using their material IDs.

3.2 Conversion to Boxes

- From the data given, demand per item d_i, quantity ordered per item q_i, and the unit price per item p_i, are all converted in terms of boxes, using the capacity of the box for part i which is c_i . Thus, bc_i is the capacity of a box for part i.

- The demand per item per day $d_i$is transformed to yearly demand in terms of boxes y_i by multiplying it by 262 days and dividing that by capacity of the box for that part. It was assumed that there are 262 working days per year, according to the Iowa Working Day Payroll Calendar.

  $y_i = demand\_per\_year$

$$y_i = \frac{d_i * 262}{bc_i}$$

- The prices per item p_i is converted into price per box pr_i (box_cost in R file) by multiplying the price of the item by the number of items in each box.

  $pr_i = bc_i \cdot p_i$

  A glimpse of the incorporated data manipulations mentioned above can be seen in the table below.

box ID	NO	material ID	demand per day	box_cost	ordering_cost
6203060	1	7305667+74	12	67.95	75
6203059	2	7305669+77	30	114.30	80
6203059	3	7305670+77	30	24.30	80
6203059	4	7305673+76	30	24.16	80
6203059	5	7305674+76	30	24.16	80

Table 2: Merged data with yearly demand

3.3 Lane Occupation

The data pertaining to the boxes includes information regarding the way the boxes are sorted in the lanes. Two notations are derived from this information and added to the table product_data, which are briefly discussed below.

- b_i= box_sorting: This represents how the box is sorted. Some boxes are sorted by width and others are sorted by length. If a box is sorted by width, this means the width of the runway is adapted to the width of the box type, when the box is assigned to the runway/lane. If a box is sorted by length, this means the width of the runway/lane is adapted to the length of the box type, when the box is assigned to the runway.

- b_i^{-1} = box_not_sorting: This represents the inverse of box sorting b_i. If a box is sorted by width, then the box_not_sorting b_i^{-1} is the length of the box. This value is used for the rack length calculations and to determine the number of boxes of a type and item that can fit in one lane. In the same way, if a box is sorted by length, then the inverse is the width which is used in the rack length calculations.

- The rack length is then used in combination with the box_sorting and box_not_sorting to determine the following;
    - Number of boxes in a lane
      $$n_i = \frac{rack\_length}{b_i^{-1}}$$
    - Number of lanes part i will occupy if a certain number of boxes q_i is ordered.
      $$lane_i = \frac{q_i}{n_i}$$

For example, using box_ID 6203060, sorted by length $b_i = 396$ and $b_i^{-1} = 297$ for material ID 7305667+74 and assuming $q_i = 214$, to calculate the number of this particular boxes in a lane with $rack_{length} = 6000\ mm$,

$$n_i = \frac{rack\_length}{b_i^{-1}} = \frac{6000}{297} = 20.20\ boxes \approx 20\ boxes$$

and, $lane_i = q_i . \frac{b_i^{-1}}{rack\_length} = 180. \frac{297}{6000} = 8.91\ lanes \approx 9\ lanes$

In the given example, the number of boxes is rounded down to the nearest integer because fraction of a box cannot be stored in a lane. On the other hand, the number of lanes that would be occupied by the boxes is rounded up to the nearest integer because rounding down would imply unfulfilled demand, which might lead to back orders.

3.4 ABC Analysis

Inventory systems are used to supply processes with materials which are required to carry out these processes. These processes could be a part of production, distribution or storage systems. Materials are therefore essential and as such need to be kept and managed in order to enable other processes. The cost-effective goal of every organization is to implement a material provisioning concept, which strives to minimize inventory while keeping up with high service levels.

The characteristics of the materials determines the most appropriate provisioning concepts to be implemented. The case in this study uses a storage warehouse provisioning concept. To better understand the characteristics of the materials in this study, the products were categorized by value using the ABC analysis.

Since the value of the material is one of the most important drivers in terms of inventory system and stock levels, ABC analysis was deemed suitable for data preparation. If a material has a higher value, inventory control needs to be tighter because more costs are implied due to the opportunity costs involved in stock holding and maintenance. Thus, the focus should be on material minimization especially on higher value products,

As enumerated in the lecture slides for the course "Inventory Management" for summer term 2020 by Prof. Dr. T. Kirschstein, the following steps are used to categorize the material by value.

1. Calculate value share of material

   $$v_i = \frac{d_i \,.\, p_i}{\sum_{j \in I} d_j \,.\, p_j}$$

   where, d_i is the consumption of the material and p_i is the price of the material
2. Order materials in a descending order according to value share.

   $v'_1 \geq v'_2 \geq \cdots \geq v'_{[I]}$
3. Calculate cumulative ordered value share of each material.

   $$\tilde{v} = \sum_{j=1}^{i} v'_j$$
4. Categorize materials according to $\tilde{v}_i$ into classes A, B, and C by class limits, e.g. 80,95,100.

The results of the categorization are summarized in the table below, for the first 3 materials of each class. The full table can be found in the appended R script.

Ordered Material ID	**Price**	**Demand**	**Material values**	**Cummulative Material values**	**Relative Cumulative material Shares**	**class**
60	256.80	1	0.07	0.07	6.7	A
36	252.98	2	0.07	0.13	13.2	A
50	215.68	1	0.06	0.19	18.8	A
35	45.60	0	0.01	0.80	80.2	B

11	45.00	1	0.01	0.81	81.4	B
15	44.10	2	0.01	0.83	82.6	B
4	24.16	2	0.01	0.95	95.4	C
5	24.16	2	0.01	0.96	95.0	C
38	22.56	4	0.01	0.97	96.6	C

Table 3: ABC analysis results for the first 3 materials of each class (values are classified by boxes)

A visual representation of the results is shown in the graph below.

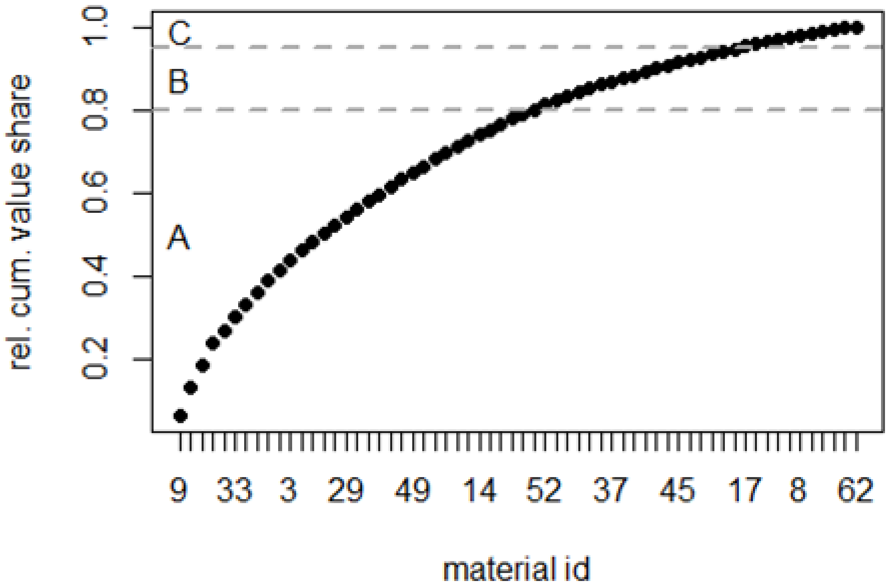

Figure 1: ABC Analysis results

From the graph above, it can be seen that although there are lots of materials with low values (Class A), the number of high valued materials (Classes B and C) is also quite significant. Given that these materials are all kept in inventory, there is a need to develop an optimal inventory planning system, such that the costs involved for stock holding will be minimized while ensuring that all demand is fulfilled. The next section of this report gives an overview of such a model.

4. Solution Approaches

4.1 Notation

The following notations were used while devising the solution approaches.

- $i \in I$ denote the set of parts considered, $\mathrm{i} = \{1 \ldots .62\}$
- d_i : *demand per day*
- y_i : *demand per year*
- q_i : *order quantity of material i*
- p_i : *unit price per material i*
- bc : *box capacity*
- c_i^{-or} : *fixed or overall ordering cost*, $c_i^{-or} = €1500$
- c_i^{or} : *variable ordering cost per box*
- c_i^{sh} : *stock holding cost*, $c_i^{sh} = p_i.h$, *where* $h =$ *interest rate*

4.2 Assumptions

- Every demand is fulfilled.
- Demand is deterministic.
- Lead time is zero.
- Waiting time is zero.
- There is limited storage capacity.
- Only complete boxes can be stored.
- In each runway, only one type of boxes is stored.

4.3 Separate Ordering

In separate ordering, each box containing the unique material is ordered separately. Contrary to joint ordering, the orders are not batched together. Thus, each order incurs the fixed ordering cost of €1500 in addition to the holding and variable ordering cost.

The given case contains 62 different materials so in case of separate ordering, each of the materials are ordered separately. Thus, there will be 62 different order frequencies.

The data about materials and their expected demand per day has been provided in the excel sheet “Product data.xlsx”

Firstly, to determine how much to order (order quantity) and when to place order (the reorder point), we turned to the EOQ model. EOQ Model can be used in this case because the problem description matches with the assumptions in an EOQ model that are given by Muckstadt and Sapra (2010). The information for the case with respect to assumptions in the EOQ model is listed below.

1. Demand arrives continuously at a constant and known rate per year. In this case, the expected demands are given.
2. Whenever an order is placed, a fixed cost is incurred. For the case, the fixed cost is denoted by c_i^{-or} which equals €1500.
3. Ordering cost for each material is given separately. In this case, ordering cost for part i is denoted by c_i^{or}.
4. The stock holding cost c_i^{sh} is based on the unit price p_i and interest rate h
5. Since, a material's unit cost is p_i , it will cost $h \,.\, p_i$ to stock one unit of that material.
6. The order arrives immediately after the placement of the order. This implies zero lead time
7. All the model parameters are unchanging over time.
8. All the demand is satisfied on time.

4.3.1 Unconstrained EOQ Optimization

EOQ is calculated for each part i using the given formula which is denoted by $q_i^* \ or \ q_i^{max}$ (q_i^{max} notation is used to differentiate it from q_i^{min} notation used in the later part) and minimum EOQ is calculated by omitting the fixed ordering cost c_i^{-or} of €1500. This is done to find the starting feasible solution.

$$q_i^{max} = \sqrt{\frac{2 \,.\, y_i \,.(c_i^{or} + \ c_i^{-or})}{pr_i \,.\, h}}$$

$$q_i^{min} = \sqrt{\frac{2\ .\ y_i\ .c_i^{or}}{pr_i\ .\ h}}$$

- **Checking capacity constraints:**

After obtaining the order quantities for each material/product, the next step is to verify if the available storage capacity can contain the derived quantities. To do this, let us revisit the concepts of b_i ($box\_sorting$) and b_i^{-1} ($box\_not\_sorting$) again. b_i represents the way that the box is sorted. b_i^{-1} represents the complement of b_i , i.e., if the box is sorted by length, b_i^{-1} represents the width. b_i^{-1} is used in the calculation of rack length.

Adding both EOQs, and b_i ($box\ sorting$) and b_i^{-1} ($box\ not\ sorting$) to the product_data table, the following result can be obtained.

N0	material ID	ordering_cost	eoq.min	eoq.max	b_sorting	b_not_sorting
1	7305667+74	75	39	180	396	297
2	7305669+77	80	86	381	594	396
3	7305670+77	80	186	825	594	396
4	7305673+76	80	181	802	594	396
5	7305674+76	80	181	802	594	396

Table 4: Adding Both EOQs to the table

From the example in the lane occupation section of this report, the number of boxes in a lane (n_i) is calculated as 20 and the number of lanes that part i occupies is 9 $lanes$. From this information, the number of lanes that boxes of each material occupy can be calculated. An extract of this for 12 products is tabulated below. The Min lane is derived from q^{min} and the Max lane is derived from q^{max}.

	1	2	3	4	5	6	7	8	9	10	11	12
Min lane	2	6	13	12	12	8	12	24	4	5	4	14
Max lane	9	26	55	53	53	33	52	103	18	22	22	61

Table 5:number of lanes per items (in boxes)

Next, the viability of the capacity constraint is checked to see if the capacity constraints are met. That is verifying the overflow. For this, all the levels in the flow racks are collapsed to form one level. Since there are 8 available racks, each with 4 levels, it can be seen that there are $8 * 4 = 32$ levels in total. Thus, the total rack width available can be calculated by multiplying the rack width by 32.

$rack\_total\_width\ = rack\_width$ * 32 = 1750 * 32 = 56,000mm

- **Overflow**

Overflow represent how much we are lacking in capacity assuming we considered optimum lanes thus optimum quantities for all our items when placing an order.

- **Overflow in mm**

Expressing it in terms of mm:

$$[\sum_{i=1}^{n} l\,ane_i \cdot b_i] - rack_{total_{width}}$$

From this expression, sorting dimension is multiplied by the number of lanes required by each product as depicted in Table 5. This is then subtracted from the total rack width in order determine the overflow. The overflow obtained can be expressed in relative percentage terms by using the formula below:

$$\frac{[\sum_{i=1}^{n} l\,ane_i \cdot b_i] - rack_{total_{width}}}{rack_{total_{width}}} \cdot 100$$

The table below shows the Rack Capacity overflow in mm and percentage.

	mm_lane_min	mm_lane_max	% lane_min	% lane.max
Overflow	218694	1173092	427.1367	2291.195

Table 6: Rack capacity overflow in mm and percentage

As seen above, taking q^{min} ,for example (mm_lane_min) the capacity is exceeded by 427%. For q^{max} the margin is exceeded by 2291%. Therefore, capacity constraints need to be incorporated into the model.

The total cost values and capacities are summarized below:

	Cost Function (€)	Capacity in (mm)
Eoq Max	172279.7	1224292
Eoq Min	124501.5	269894

Table 7: Cost values and capacity for unconstraint EOQ

4.3.2 EOQ Optimization with Constraints

A need for optimization is necessary, since the capacity is limited. As seen above, constraint was violated, therefore q_i needs to be optimized to fit our capacity.

- **Introducing capacity constraint**

$$\sum_{i=1}^{n=62} l\,ane_i \cdot b_i \leq rack_{Total_{width}}$$

Substituting for lane, the expression above becomes the following:

$$\sum_{i=1}^{n=62} q_i \cdot \frac{b_i^{-1} \cdot b_i}{rack_{length}} \leq rack_{Total_{width}}$$

From the given information the following objective function is obtained.

$$Min \left[\left(62 \,.\, c_i^{-or} + \sum_{i=1}^{n=62} \frac{y_i}{q_i} \,.\, c_i^{or} \right) + \left(h \,.\, \sum_{i=1}^{n=62} \frac{q_i}{2} \,.\, pr_i \right) \right]$$

Subject to:

$$\sum_{i=1}^{n=62} q_i \,. \frac{b_i^{-1} \,.\, b_i}{rack\_length} \leq rack\_total\_width$$

Where,

$\frac{y_i}{q_i}$ = number of orders for part i

$\frac{q_i}{2}$ = average inventory for part i

Solving this optimization problem by ROI using the default solver yielded the following cost values in table 8 below, which is compared to the values obtained for unconstraint EOQ:

	Cost Function (€)	**Capacity**
Optimized Q Constraint	205079.8	50340
Eoq Max Unconstraint	172279.7	1224292

Table 8: Cost values and capacity for constraint EOQ vs Unconstraint EOQ

Optimum costs with no capacity constraint total cost is €172279.7, with capacity constraint, it costs €205079.8 This cost is expected to be higher when capacity constraint is included, as limited capacity of the rack will not allow to take advantage of cost savings.

Next, the lot sizes of each of the items is plotted as seen in figure 2 below. Here, the total costs per item when capacity is included with their respective total costs is compared to that without capacity constraint. This is done so that the results obtained so far can be visualized.

The cost function per item is given by:

$$C\ (q_i) \rightarrow \left(c_i^{-or} + \frac{y_i}{q_i}\ .\ c_i^{or}\right) + \left(h\ .\frac{q_i}{2}\ .\ pr_i\ \right)$$

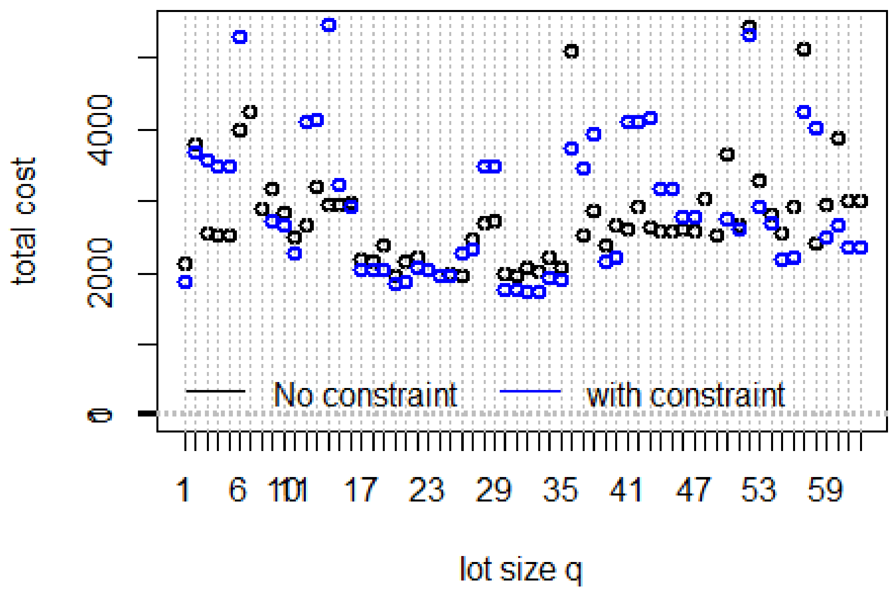

Figure 2: Comparison of costs with and without capacity constraints

The blue dots represent the more expensive items when capacity is accounted for. The variance between the costs with constraints (Blue dots) and costs without constraints is very large.

4.4 Joint Ordering

Joint replenishment problem focuses on how to coordinate the order of multiple items, from the same supplier. This problem is one which is similar to the general lot sizing problem, the difference is this case is that rather than having the same machine, we have the same supplier. Here, a group of items which should be replenished jointly as much as possible shall be considered. (Axsäter, 2006).

When placing a purchase order for multiple items, the cost component can be divided into fixed cost, which is independent of the products being purchased and variable costs, which depends on the purchased items (Goyal 1974). If multiple products are ordered at the same time, the general ordering cost (fixed cost) will have to be paid once, which leads to a reduction in costs. The buyer could also benefit from quantity discounts offered by the supplier when large batch sizes are ordered. Through coordination, transportation cost could be reduced as well.

Because of these huge costs incurred when ordering, using group replenishment may lead to substantial cost savings. The savings from group replenishment are more significant the higher the major ordering cost (Khouja and Goyal, 2008). Joint replenishment therefore aims to determine the optimum ordering frequency which will minimize ordering and holding costs. So, the focus here is on the cycle time; ordering as many products as possible in a cycle.

- **Mathematical Formulation**

The following assumptions were made:

- There is one supplier with outbound storage.
- $i = \{1, \dots, n\}\ products$
- de_i : $demand\ for\ part\ i\ (constant)$
- c_i^{sh}: $stock\ holding\ cost\ rates\ for\ part\ i$
- c_i^{or}: $specific\ ordering\ costs\ for\ part\ i$
- c_i^{-or}: $general\ ordering\ cost$
- T_i: $cycle\ time\ for\ item\ i$

We also assume that no back orders are allowed, and the lead time is zero. Bearing in mind that the joint replenishment problem is basically a quite standard EOQ problem which uses the substitution $T_i = q_i / y_i$, we optimize to determine the optimal cycle times for individual items. To do this, some concepts are introduced.

Basic Cycle time (B): Cycle time is the time between the placing of two successive orders. And the basic cycle is the minimum cycle time. The aim is to determine a basic period B and restrict any cycle time to be a multiple of the basic period m_i. This is done to make it easier and simpler to find a feasible solution.

H_i **=** This is the shortcut for the holding stock rate which is calculated as follows;

$H_i = 0.5.\ y_i\ .\ c_i^{sh}$

$H_o = 0$. Holding cost multiplier are H_o and H_i

4.4.1 Joint Replenishment with no Capacity Constraints

The objective function of the Joint replenishment problem is as follows:

$$C(m_iB) = \sum_{i=0}^{n}(\frac{c_i^{or}}{m_i \cdot B} + H_i \cdot m_i \cdot B) == \frac{c_o^{or}}{B} + \sum_{i=1}^{n}(\frac{c_i^{or}}{m_i \cdot B} + H_i \cdot m_i \cdot B)$$

- Subject to:

$$m_i \geq m_0 \quad \forall i > 0$$

$$m_i \in \mathbb{N} \qquad \forall i$$

The aim of the function above is to find the optimum solution by determining the optimum basic period and the most suitable multiples (m_i) of the basic period for each of the products, such that costs are minimized. In this model, we restrict mi to natural numbers. Note that T_i is substituted in order to add the common cycle (that is T_i. = $m_i \cdot B$)

4.4.1.1 Basic Period Heuristic

A feasible solution is determined by implementing the steps of the Basic period heuristics, as enumerated in the OVGU Inventory management Lecture slides for the ST 2020 by Prof Dr. T. Kirschstein.. The steps are as follows;

Step 1: We consider an ordered set of products. This is done by calculating for each item, the ratio between their individual ordering cost and stock holding cost. The products are then sorted in ascending order of their individual ratios. That is;

$$\frac{C_{(1)}^{0r}}{H_{(1)}} \leq \frac{C_{(2)}^{or}}{H_{(2)}} \leq \cdots \frac{C_{(n)}^{0r}}{H(n)}$$

Step 2: The optimal individual cycle time T_i. is then calculated by taking the square root of the values obtained in step 1.

$$T_i = \sqrt{(\frac{c_i^{or}}{H_i})}$$

$$B = \min(T_i)$$

Next the basic cycle is determined. This is the minimum of the individual cycle times. Since the values are ordered in ascending order, we know that the first item in the ordered list has the smallest cycle time and nth has the highest. An extract of the results is shown in the table 9 below. See appended R script for the full table.

Product number (sorted)	**52** (Material ID 7346592+72)	**57** (Material ID 7455633+73)	**………………**	**24** (Material ID 7306013+73)	**26** Material ID 7306017+79)
C_i^{or}	50.0000000	65.0000000	………………	75.0000000	80.0000000
H_i	9386.4320000	7724.8080000	………………	121.1760000	118.9125000
T_i	0.0729852	0.0917303	………………	0.7867239	0.8202217

Table 9:Optimal Cycle time for each product, sorted in ascending order

From the table above. We see that material with product number 52 has the minimum cycle time while product number 26 has the highest cycle time. The basic period is therefore 0.0729852

Step 3: In this step, we calculate the cumulative sums of the individual ordering cost which also includes general set up costs divided by corresponding holding cost term. This is derived by the formula:

$$T^C = \frac{\sum_{j=0}^{i'} c_j^{or}}{\sum_{j=0}^{i'} H_j}$$

We then sum up the items which belong to one batch and identify at which point we switch from a common procurement period to an individual procurement period. This is done by comparing the cumulative sums of the individual ordering cost with the individual ratios as indicated in the function below.

$$\frac{\sum_{j=0}^{i'} c_j^{or}}{\sum_{j=0}^{i'} H_j} \geq \frac{C_{(i)}^{0r}}{H_{(i)}}$$

The m_i value is set to 1 for all items where the above equation does not hold true At the product index where the cumulative ratio is larger than the individual ratio as in the expression above, we set the ordering frequency mi, to a higher value by rounding to individual $\frac{T_i}{B}$, to the nearest integer value. An extract of the results is shown in the table below.

Product number (sorted)	**52** (Material ID 7346592+72)	**57** (Material ID 7455633+73)	…………	**13** Material ID 7305834+73)	…………	**24** (Material ID 7306013+73)	**26** Material ID 7306017+79)
T_i^2	0.0053268	0.0084144	………..	0.0476727	…………	0.6189344	0.6727636
T^2	0.1651320	0.0943824	………..	0.0472033	…………	0.0708066	0.0716999

Table 10: Cumulative sums vs individual ratios

From the table above, materials from index 1 up to index 13 are allocated a value of 1. All items from index 14 (Product number 13-Material ID 7305834+73) up to index 62 (product number 26 Material ID 7306017+79) will have higher frequencies. The table below shows an extracted part of the results obtained. The full illustration can be found in the appended R script file.

	52	57	36	7	60	6	53	2	50	61	62	56	38	13	…….	20	25	24	26
$m_i = \frac{T_i}{B}$	1	1.26	1.26	1.85	1.9	1.98	2.21	2.24	2.38	2.47	2.47	2.78	2.91	2.99	…….	10.15	10.23	10.78	11.24
$\lceil m_i \rceil$	1	1.00	1.00	2.00	2.0	2.00	2.00	2.00	2.00	2.00	2.00	3.00	3.00	3.00	…….	10.00	10.00	11.00	11.00
$\lceil \bar{m}_i \rceil$	1	1.00	1.00	1.00	1.0	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	3.00	…….	10.00	10.00	11.00	11.00

Table 11:Optimal m_i values for materials

The table illustrates the individual $\boldsymbol{m_i}$, for all the products. The optimal values for $\boldsymbol{m_i}$ are found in the last row of the table. The break occurs after the first 13 ordered elements. This means every item below that is allocated a basic period multiplier of 1. The highest value rounded to the nearest integer is 11.

Step 4: Update B by solving the corresponding problem below for fixed m_i.

$$B = \sqrt{\frac{\sum_{i=0}^{n} \frac{c_i^{or}}{m_i}}{\sum_{i=0}^{n} m_i \cdot H_i}}$$

The idea is to summarise and sum up the most frequently acquired products into one general set up so that the general set up cost is distributed to as many products as possible. The reoptimized basic cycle time yields B = 0.09.

4.4.1.2 Determining EOQs and Total Cost

From the values above, the optimal cycle times is determined by $\boldsymbol{m}_i$* B = $\boldsymbol{T}_i$. The order quantities are given by multiplying the cycle times that $\boldsymbol{T}_i$ with the demand rates $\boldsymbol{y}_i$. (that is $\boldsymbol{q}_i = \boldsymbol{T}_i * \boldsymbol{y}_i$,) as seen in the table below.

Product number	**52** (Material ID 7346592+72**)**	**57** (Material ID 7455633+73)	**7** Material ID 7305819+79)	**…………..**	**24** (Material ID 7306013+73)	**26** Material ID 7306017+79)
$[\tilde{\boldsymbol{m}}_i]$	1.00	1.00	1.00	…………….	11.00	11.00
$\boldsymbol{T}_i$	0.09	0.09	0.09	……………..	1.0	1.00
$\boldsymbol{q}_i$	133.87	71.38	54.95	………………	98.9	174.83

Table 12: Optimal cycle time for each material sorted in ascending order

The Total costs obtained for the joint replenishment problem (JRP) without capacity constraint is **49943 Euros**. Now the question is whether this solution is feasible or not? The capacity required by the above quantities is **281834 mm**, however the available capacity (Rack Total width) is 51200 which is insufficient. This Solution is not feasible.

Another approach is to try to optimize the Joint Replenishment problem, without introducing the substitution $\boldsymbol{T}_i$. = $\boldsymbol{m}_i$ * B. In order to this, the JRP function is updated as follows;

$$C(T) = \sum_{i=0}^{n}(\frac{c_i^{or}}{T} + H_i \cdot T) == \sum_{i=1}^{n}\frac{c_o^{or}}{T} + \sum_{i=1}^{n}(\frac{c_i^{or}}{T} + H_i \cdot T)$$

The function is optimized with the R software and yields the following results

Total Cost in Euros 151,952

Capacity in mm 1,104,014

Compared to the previous solution this is far more inefficient. As it costs more and requires more capacity. There is therefore a need to re optimize the Objective cost function, taking capacity into consideration. This is done in the next section of the report.

4.4.2 Joint Replenishment including Capacity Constraints

As seen from the previous section, the optimal order quantities for the joint replenishment exceed the available capacity. In this section, we integrate the shelf capacity constraint while using the basic period approach as it so far has the lowest cost when not constrained.

All the assumptions and formulations for joint replenishment without constraint holds true here. In addition, we formally define the capacity constraint as follows (Same as in separate ordering with capacity constraint)

$$\sum_{i=1}^{n} b_i \cdot \left\lceil \frac{b_i^{-1} \cdot q_i}{rl} \right\rceil \leq \text{tot\_rack\_length}$$

Because the basic period approach will be used some adjustments are made to the left-hand side of the function as follows; q_i is substituted with $T_i * y_i$. T_i is in turn substituted with m_i * B as shown below;

$$\sum_{i=1}^{n} b_i \cdot \left\lceil \frac{b_i^{-1} \cdot T_i \cdot y_i}{rl} \right\rceil = \sum_{i=1}^{n} b_i \cdot \left\lceil \frac{b_i^{-1} \cdot m_i \cdot B \cdot y_i}{rl} \right\rceil$$

After optimisation, the following results are obtained:

Total Cost in Euros 268,254

Capacity in mm 51,132

The resulting cost is higher in this case but the capacity constraint is met. The resulting order quantities are determined in the same manner as the JRP without constraint and are sorted based on the product numbers as explained in the data preparation section. The table below shows the results (Also see results in R script):

Product number No	**material ID**	**Order quantity q_i**	**Product number No**	**material ID**	**Order quantity q_i**
1	7305667+74	16	**32**	7306033+77	16
2	7305669+77	20	**33**	7306034+77	16

3	7305670+77	20	**34**	7306037+79	17
4	7305673+76	19	**35**	7306038+30	17
5	7305674+76	19	**36**	7308013+79	18
6	7305817+74	19	**37**	7308014+79	18
7	7305819+79	13	**38**	7313474+73	19
8	7305820+79	13	**39**	7313981+31	18
9	7305823+73	18	**40**	7313982+31	18
10	7305824+73	18	**41**	7326393+74	19
11	7305832+73	18	**42**	7326394+74	19
12	7305833+73	19	**43**	7326409+74	14
13	7305834+73	19	**44**	7326825+72	17
14	7305863+75	19	**45**	7326826+72	17
15	7305865+77	16	**46**	7328209+73	19
16	7305867+76	19	**47**	7328210+73	19
17	7305971+75	17	**48**	7332421+76	15
18	7305972+75	17	**49**	7332422+76	15
19	7305977+74	17	**50**	7343137+72	17
20	7306005+73	17	**51**	7343138+74	19
21	7306006+73	16	**52**	7346592+72	14
22	7306009+75	18	**53**	7346813+73	15
23	7306010+75	18	**54**	7348808+73	17
24	7306013+73	17	**55**	7355005+72	17
25	7306014+73	17	**56**	7355006+72	17
26	7306017+79	19	**57**	7455633+73	15
27	7306018+79	19	**58**	7455634+73	15
28	7306023+78	20	**59**	7455687+72	18
29	7306024+78	20	**60**	7455688+72	18
30	7306025+75	16	**61**	7455691+71	18
31	7306026+75	16	**62**	7455692+71	18

Table 13:Optimal Order quantities of materials for JRP with capacity constraint

After obtaining the optimal quantities of each of the materials the next step in this modelling process is to assign the orders to the rack. The next section of this report tackles that issue.

4.5 Cutting Stock Problem

The problem here is assigning material boxes into the lanes for each level. Since we have limited space, we have to make sure we optimise the space or better still minimise waste when assigning our box materials into lanes at each level. To solve this issue a concept known as a cutting stock problem should be understood.

4.5.1 Cutting Stock solution Approach

Cutting stock problem consists of cutting a set of boxes or rectangles from a sheet of material in order to minimize the trim loss. The problem arises in various production processes in the glass, steel, wood, paper and textile industries (K. K. LAI and JIMMY W. M. CHAN 1996).

For our case the box shapes were already given ,we are concerned in assigning those boxes such that at each level a minimum waste of space should be made. Our maximum rack width per level is 1750mm. For each of these levels, we have to create a pattern for the boxes to fit in, such that wastage is minimised. This is done in the following steps.

Step 1: Determine the Patterns for each of the boxes

Each box has a sorting pattern as explained in the lane occupation part of data preparation. We have to determine the width sizes our lanes would be made up of. The lanes are just described by their width. Due to the safety margins we round the lane width to full millimetres. These widths are 200mm, 400mm and 600mm which represents the lanes width sizes. Luckily, there is only a small number of useful patterns of lanes per level. To be precise there are 9 efficient patterns to arrange these 3 lane types (assuming we use each level exhaustively). The patterns are shown in the table below:

Pattern No	**200**	**400**	**600**
1	8	0	0
2	0	4	0
3	0	1	2

4	2	0	2
5	6	1	0
6	4	2	0
7	2	3	0
8	3	1	1
9	1	2	1

Table 14:Pattern assignment of boxes at each level

From our above table, the nine patterns don't mean all the patterns will be used since we have optimal quantities for each material type. Thus, the lane box quantities gotten will rightly determine our pattern schedule.

Step 2: Determine number of lanes per Product

We determine the number of lanes **$l(q_i)$** required by each of the products, by using the lot size quantities q_i (integer number of boxes) obtained from the joint replenishment optimization function (see table 13 *Optimal Order quantities of materials for JRP with capacity constraint*.):

$$l(q_i) = \left\lceil \frac{q_i}{n_i} \right\rceil$$

$$n_i = \left\lfloor \frac{rl}{b_i^{-1}} \right\rfloor$$

whereby n_i is the number of boxes per lanes for product i.(as seen in the separate ordering solution approach above.) i.e. for item ID 7305667+74 **q_i**= 16 boxes, n_i = 20boxes. This implies

$l(q_i)$=16/20= 0.8 $\sqsupseteq$ 1.

Thus, a sample solution of the number of lanes is shown below. The full table is shown in the R script attached to this report:

Items	1	2	3	4	5	------	60	61	62
Lanes number	1	2	2	2	2	------	3	1	1

Table 15: Sample number of lanes for each item

NB: The full table also contains the adjusted values of some of the products. The reason for the adjustment is given in Step 5 below.

Step 3: Determine the Total number of lanes per lane width type

In step one, we rounded up the lane widths to get 3 distinct lane widths of 200mm, 400mm and 600mm. Here, we calculate the total number of lanes of each lane width type that is needed by all the products. Let $p_{k,j}$ indicate the number of lanes of type j associated to pattern k. Now we need to assess the number of lanes per required of each type. As outlined above, we know the number of lanes per product $l(q_i)$, thus we can deduce the demand for lane type j (ld_j) by summing up the $l(q_i)$ for each lane type, i.e., $ld_j = \sum_{i \in P | b_i = j} l\,(q_i)$:

Summing the lane number according to its type will yield to a total of:

Lane type	**200**	**400**	**600**
Total lane number	14	15	70

Table 16: Total number of lanes for each lane type

The table above shows the total number of lanes which have to be assigned to the various rack levels.

Step 4: Choose the patterns to use

We now have the 9 possible patterns as well as the total lane numbers per lane width type. The next step is to choose from the 9 possible patterns, the best pattern which will enable us to assign all the materials to the available rack levels.

From the table labelled pattern assignment of boxes at each level the patterns which best fit the assignment of our optimum quantities are pattern number 3 and 4 shown on the table

J / **Pattern p_k**	**200**	**400**	**600**
1	0	1	2
2	2	0	2

Table 17: Optimal pattern table

These patterns are chosen because they contain the highest assignments of boxes with box sorting Dimension 600.

Step 5: Assignment of Products to Rack Level

We know from the data preparation that the Total number of available levels is 32. In this step the products are assigned unto the racks depending on the number of lanes they each require, following the optimal patterns from step 4.

During the assignment, we realised that for lane type 600mm, we need 70 lanes but due to capacity limitations, we can only assign 64 lanes. That is, we only have 32 levels and a maximum of 2 lanes of type 600mm can fit at each level. Thus, we need a total of 64lanes for type 600mm.This misfit is due to the fact that, rather than using fraction values in the lane assignments, we made approximations by using the ceiling values instead.

To rectify this problem, we needed to reduce lane assignment values for some products, while fulfilling demand. To do this, rather than using ceiling values for all materials, we changed some values to their floor values. Care was taken to ensure that these adjustments did not cause back orders for any of the adjusted materials. The Products whose lane numbers were rounded down to their floor values are 15,17,18,50, 57 and 58. Below is a table for their values in fraction before and after the floor rounding.

Items	15	17	18	50	57	58
Lane number before adjustment	1.066667	1.133333	1.133333	1.133333	2.142857	2.142857
Lane number after rounding down	1	1	1	1	2	2

Table 18: summary calculation for adjusted lanes

From our optimal pattern table, we will now assign the various materials into the rack levels. We should have in mind that we have 8 racks of 4 levels each which gives a total of 32 levels into which our materials have to be partitioned. Also due to the various adjustment made our total level width which was 56000mm (32*1750) has been reduced to 51200mm since we observe a waste of 150mm from all the potential patterns obtained (56000mm - (150mm*32)).

This new total rack width is the one which has been used in our JRP with capacity constraint and the new rack width for a level will be 1600mm following our optimal patterns. The below table shows material assignment for each item following our optimal pattern.

Material Assignment to Rack levels

LEVEL	200	400	600
1		Material ID 7305667+74	Material ID 7305669+77 Material ID 7305669+77
2		Material ID 7305817+74	Material ID 7305670+77 Material ID 7305670+77
3		Material ID 7306009+75	Material ID 7305673+76 Material ID 7305673+76
4		Material ID 7306010+75	Material ID 7305674+76 Material ID 7305674+76
5		Material ID 7306013+73	Material ID 7305819+79 Material ID 7305820+79
6		Material ID 7306014+73	Material ID 7305823+73 Material ID 7305823+73
7		Material ID 7306023+78	Material ID 7305824+73 Material ID 7305824+73
8		Material ID 7306024+78	Material ID 7305833+73 Material ID 7305833+73
9		Material ID 7306037+79	Material ID 7305834+73 Material ID 7305834+73
10		Material ID 7306038+30	Material ID 7305863+75 Material ID 7305863+75
11		Material ID 7326409+74	Material ID 7305865+77 Material ID 7305971+75
12		Material ID 7326825+72	Material ID 7305867+76 Material ID 7305867+76
13		Material ID 7326826+72	Material ID 7305972+75 Material ID 7306005+73
14		Material ID 7455691+71	Material ID 7306005+73 Material ID 7306005+73
15		Material ID 7455692+71	Material ID 7306006+73 Material ID 7306006+73
16	Material ID 7305832+73 Material ID 7305977+74		Material ID 7306006+73 Material ID 7332421+76
17	Material ID 7306025+75 Material ID 7306026+75		Material ID 7306017+79 Material ID 7306017+79
18	Material ID 7306033+77 Material ID 7306034+77		Material ID 7306018+79 Material ID 7306018+79
19	Material ID 7313474+73 Material ID 7328209+73		Material ID 7308013+79 Material ID 7308013+79

20	Material ID 7328210+73 Material ID 7346592+72		Material ID 7308013+79 Material ID 7308014+79
21	Material ID 7346813+73 Material ID 7348808+73		Material ID 7308014+79 Material ID 7308014+79
22	Material ID 7355005+72 Material ID 7355006+72		Material ID 7313981+31 Material ID 7313981+31
23			Material ID 7313982+31 Material ID 7313982+31
24			Material ID 7326393+74 Material ID 7326393+74
25			Material ID 7326394+74 Material ID 7326394+74
26			Material ID 7332422+76 Material ID 7343137+72
27			Material ID 7343138+74 Material ID 7343138+74
28			Material ID 7455633+73 Material ID 7455633+73
29			Material ID 7455634+73 Material ID 7455634+73
30			Material ID 7455687+72 Material ID 7455687+72
31			Material ID 7455687+72 Material ID 7455688+72
32			Material ID 7455688+72 Material ID 7455688+72

Table 19: Material assignment to rack levels

The above table shows a sample combination of materials at each level for the 32 levels. Thus, materials are packed following the two patterns earlier explained. A full pattern partitioning of the above table is attached in the appendix. We see that materials with lane type 600mm appear in all levels due to the numerous spaces it occupies. It should be clear that our table is not a definite way of packing the materials. Any product in lane say 200 can appear at any level only that once it occupies the lane it means all the boxes in that lane belong to the said material.

After the assignment of the materials, we now have to do some comparisons and visualisations to ensure that our coordinated ordering solution approach actually leads to fulfilment of demand in all cases. This breakdown and comparison are seen in the illustration section of the report.

4.5.2 Demand vs Quantity

In order to ease the visualisation and interpretation process, we will convert cycle times to days since periods could be expressed in days, months or years. Thus, the necessity to specify it. Our cycle time will be converted thus in days since we are dealing with inventory and our demands are made daily. In order to do this, the following notations are introduced;

bc_i: *the capacity of a box for item i*

y_i: *demand in years for item i*

yb_i: *demand per day in boxes for item i*

yd_i: *demand per day for item i*

ct: *cycle time, the number of days it takes before a new order*

Example item 4 which is represented by material id 7305673+76 has a lot size quantity of 19 boxes and a daily demand in boxes of 1.875. calculating its ct will be;

$$ct = \frac{q}{yb_i} = \frac{m_i \cdot B \cdot y_i}{yb_i}$$

ct=19/1.875=9.9 approx. 9days. Thus item 4 is ordered every 9days. This is done for all materials. It is important to note that all items with the same order frequency (m_i) have same cycle time. That is, their order intervals in days is same, this is summarised in the table below.

m_i	1	2	3	4	6	7	8	10	11	14	15	17	18	21	22	24	31
days	2	4	7	9	14	17	19	24	27	34	37	42	44	51	54	59	76

Table 20: order frequency in days

The above table shows for every order frequency the number of days it takes for every order to be made. That is the cycle time in days(ct).

Below is a sample table showing the order cycle of each item. That is the occurrence of each material and their order periods. This table permits to know which materials are ordered in which interval and looking on the table above we know exactly the interval number of days for each item. The full table is attached in the RMD file

	Mi=1	Mi=2	Mi=3
1	2,15,21,61,62	10,11,18,39,41,59	5,7,8,14,24,25,34,58
2	2,15,21,61,62	0	0
3	2,15,21,61,62	10,11,18,39,41,59	0
4	2,15,21,61,62	0	5,7,8,14,24,25,34,58
5	2,15,21,61,62	10,11,18,39,41,59	0
6	2,15,21,61,62	0	0
7	2,15,21,61,62	10,11,18,39,41,59	5,7,8,14,24,25,34,58

Table 21: Illustration table of the ordering frequencies for materials

4.5.3 Proof that demand is always met

The (T, S) policy has been used to prove that our demand is always met. It is adapted from lecture notes of Inventory Management by Prof. T. Kirschstein. By T, S policy we mean T for order interval and S for order up to level. For this to be realized we made some assumptions.

- y_t :Assuming demand is uniformly distributed thus the same quantity of demand repeats each time (number of lanes for each item)
- S: Initial stock level for the 62 items are given based on lanes assigned
- T: Each Item has some form of order frequency mm which we derived cycle time ct from, going over a time period of 262 days. This represents order interval
- L: Lead time is zero
- At each order interval, we order up to S which is the stock level based on lane assigned for each item meaning, lanes are filled
- Backorder is allowed but customers are willing to wait
- Stock is filled to the capacity in period

Notations and Formulae

On-hand stock S(t)

Backorders B(t)

Inventory level $I_t = I_{t-1} - y_t$ available units

The reason why TS policy is been used is because capacity cannot be exceeded, thus we order up to lane capacity of each item. Also, each item has its cycle time in days which was derived from order frequency m.

$$S_i = \left\lfloor \frac{lanes_i \times rl}{b_i^{-1}} \right\rfloor$$

S is the stock level, gotten from lane assigned to each item, the floor of stock level ensures that complete boxes are assigned to a lane. Example material id 7305673+76 i.e. item 4

$$s_4 = \frac{2 \times 6000}{396} = 30.30 \approx 30\ boxes$$

The initial stock for item 4 is 30 boxes since its lot size quantity requires that it should be assigned to two lanes. S for other items will be display when running the code. Below is a sample table for the first ten (10) items.

items	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
Stock level	20	30	30	30	30	20	15	15	30	30

Table 22: Stock level derived from lanes assigned per item

The stock level on the table above represents our order up to level S for the various items which is also used as our initial stock level. The outstanding orders here are zero because our lead time is zero thus once an order is placed the item reaches the warehouse immediately. Our risk period (T+L) will be the order interval T plus the lead time L. Since our lead time is zero (0) the risk period is T. The demands in pieces have been converted to boxes for each item. A summary table for item 4 will be shown below for a sample period of time(days). This has to run for 262 days but a reduce number of days will be shown:

Days	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**
y_t	1.875	1.875	1.875	1.875	1.875	1.875	1.875	1.875	1.875	1.875	1.875
q_t	30	0	0	0	0	0	0	0	0	19	0
I_t	28.125	26.250	24.375	22.500	20.625	18.750	16.875	15	13.125	30	28.125

Table 23: Periodic review calculation for item 4

Above is a summary table result of item 4. This is computed for each item in the code. Its graphical representation can be seen below:

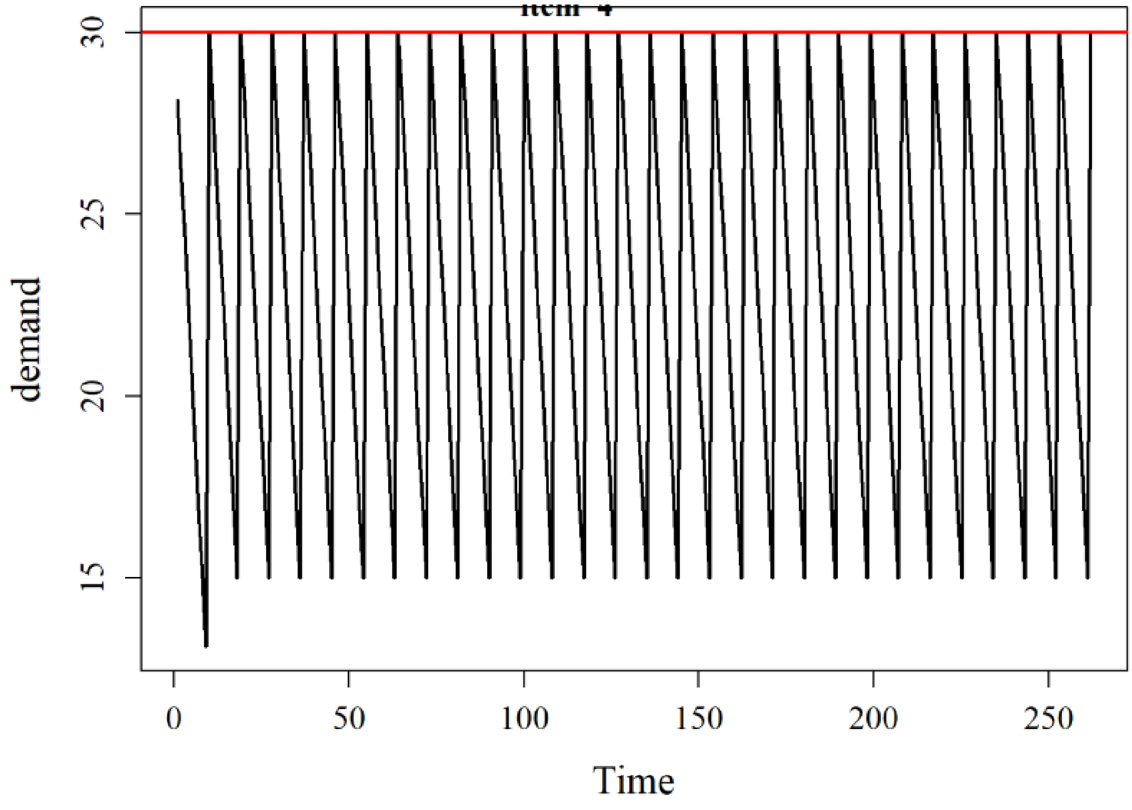

Figure 3: Depicting inventory movement for item 4

The above graph shows the behaviour of our inventory over the 262 days. The order up to level S = 30 which is shown by the red line. we can compute our alpha service level by summing for each item the inventory level over the periods divided by the total periods

$$\alpha \ service \ level = \frac{\sum_{t=1}^{262} i_t \ \times 100}{262}$$

Where $i_t \ \geq 0$

Running our codes gives for almost all items 100% alpha service level. A summary table for some of the items especially those with less than 100%

items	**1**	**17**	**18**	**50**
Average inventory	12.64326	7.060305	7.060305	7.567987
α service	100.0000	90.458015	90.458015	99.618321

Table 24: summary computation of alpha service levels

As seen from the result, only Items 17 and 18 have an α service level of 90.458% and item 57 has α service level of 93.9% and the rest of the items has 100% alpha service level. we need our alpha service levels to be 100% for all items. Thus, item 17,18 and 50 will be adjusted by reducing their ordering interval. That is from 42 days to 37 days for items 17 and 18 and 17 days to 15 days for item 50. This certainly brought changes in the inventory level as it is reflected on the changes in the average inventory of these three items as below while the other items do not change:

Items	**1**	**17**	**18**	**50**
Average inventory	12.64326	7.821374	7.821374	8.48017
α service	100.0000	100.0000	100.0000	100.0000

Table 25: new computation of alpha service level

These 100% alpha service levels show that demand is always made thus no stock out situation observed for all items. This was our objective here, since we have a lead time of zero which means every time, we place an order the items reach us at that same time. We also could observe that the ordering frequency is important to avoid stock out. Though its disadvantage is well known that is no discount gain from bulk buying and also more costly as we incur more ordering cost over increasing cycle times. These open a breach for us to analyse our cost from the various methods used. That is the separate ordering and the joint replenishment problem.

5. Cost Analysis

In this section of the report we compare the Total costs and total quantities for each of the solution approaches explored so far. Because the number of materials which is 62 is too large to show all the results here, we display the figure for the first 7 products. The full display can be found in the appended R script.

The first table shows the comparison of the Total cost obtained when Separate ordering and JRP models are implemented, for both constrained and unconstraint scenarios.

	Unconstrained	**Constrained**
Separate ordering	172,279.7	205,079.8
Joint replenishment (Coordinated ordering)	49,943	268,254

Table 26: Total Cost Matrix

The second table compares the EOQs of 7 products under Separate ordering and JRP models, for both constrained and unconstraint scenarios.

Product No	**1**	**2**	**3**	**4**	**5**	**6**	**7**
Separate ordering without capacity constraint	180	381	825	802	802	652	787
Separate ordering with capacity constraint	16	20	20	20	20	19	15
JRP without capacity constraint	51	48	238	223	223	89	119
JRP with capacity constraint	16	20	20	19	19	19	13

Table 27: EOQ Matrix

There is a reduction in EOQs for each of the products at any ordering time is reflected in the EOQ matrix where one can see that the Economic Order Quantity for each product under capacitated separate and Joint replenishment models is smaller when compared to the uncapacitated separate and joint ordering respectively. This is because of the limited space available to store the ordered products.

From the total cost matrix, total cost is high for both separate and joint replenishments models, when capacity constraints are incorporated. This cost is expected to be higher when capacity constraint is included, as limited capacity of the rack will not allow to take advantage of cost savings such as discounts from buying in large quantities. This can also be attributed to the fact that with limited capacity, orders can only be placed in smaller quantities and multiple times. Every time an order is placed, costs such as administrative cost and transportation cost (both fixed and variable components) are incurred. These costs when incurred multiple times will only lead to an increase in the total cost.

6. Conclusion

This case study ‘Synchronized lot sizing with capacity constraints’ was aimed at coordinating orders for a given set of materials in a way that determines lot sizes for each part such that, rack capacity constraints are adhered to and the total cost per period for stock holding and ordering are minimal. The EOQ model was used as the basis for Separate and Joint Replenishment planning methods. The methods discussed are used widely for inventory planning and control. The EOQ for both optimisation models was first calculated without taking capacity constraints into consideration, this permitted us to check whether our EOQ obtained would give quantities which are less than or equal to our capacity limits. This unfortunately gave us very high values which could not fit the available capacity. Consequently, capacity constraints were taken into consideration and a recalculation of EOQ made. The capacity constraints were investigated using the cutting stock problem, where results obtained were subject to data manipulation and adjustment to arrive at feasible lot size quantities

The service levels were checked using (T, S) policy and this proved that using the proposed assignment, demand for all items will be met when all assumptions and conditions are considered. From the cost perspective, using the cutting stock problem, given the current rack capacity, there are slight cost savings when choosing separate ordering compared to joint replenishment problem. More analysis needs to be performed such as cost of ordering over time in order to ascertain which method is better. However, it is clear that without capacity constraint, the need for JRP arises.

The validity of ordering plans is restricted to the stated constraints and the planning horizon. When reaching the end of the planning horizon, a new plan must be designed that reflects the current status of the system, which might be different to the status before. Moreover, reality might deviate from the plan and if the discrepancy between the plan and reality is high, other methods need to be explored.

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