Master Thesis:
Choice set Size effect on contracting in Supply chain

Author: **Diego Arinze Uchendu**

221616

Supervisor: **Prof. Dr. Sven Müller**

Chair of Operations Management

OTTO VON GUERICKE UNIVERSITÄT MAGDEBURG

WW

FACULTY OF ECONOMICS AND MANAGEMENT

Master Degree in Operations Research and Business Analytics

OUTLINE

Introduction → WorkFlow → Supply Chain (Data) → Modeling → Results → Demo → Conclusion

INTRODUCTION

How choice set cardinality as an attribute effect contract alternatives.

Reduce Choice Overload.

Aim to Maximize the Expected Profit of the supplier while ensuring that a buyer selects a contract among contract alternatives.

WORK FLOW

Data

- Convert from MATLAB to R
- Automate ´to accept other different values.
- Write to Excel

Modelling

- Write Algebraic Model, both MINLP and MILP.
- Translate to GAMS and Write Results to Excel

Result

- Extract from Excel File.
- Performed Visualization.
- Used Shiny to Make sense of results.

SUPPLY CHAIN(DATA)

- Buyer Types
- Supply Chain between seller and buyer
- Contract Menu (Unique Contracts)
- Capacity Reservation Contract

BUYER TYPES

Supplier has 2 types of buyers

- Low type buyer: demand d is less or equal to the Average market demand $\mu = 2$.
- High type buyer: $d > \mu$.

Buyer has no Incentive to reveal her private forecast information ξ.

- Supplier considers ξ zero mean U~[-1,1].
- Supplier knows her prior probability distribution $\mathrm{p}(\xi) = 0.5$.

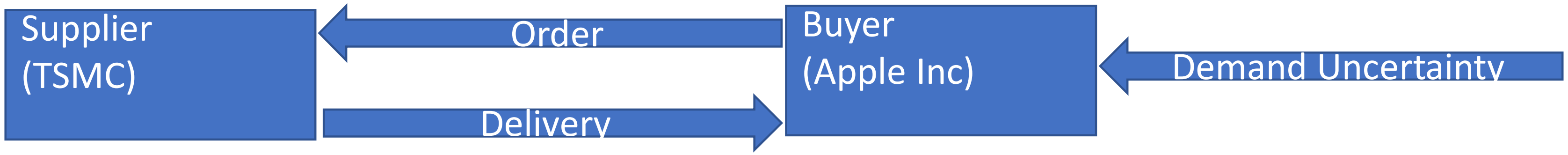
Demand uncertainty ϵ, lying in interval U~[-1,1].

SUPPLY CHAIN BETWEEN A BUYER AND SELLER

C1: unit cost of 5nm chip.
C2: capacity installation cost
C1=0, C2= 0.5

W: Wholesale price per unit
w= 1

r: retail price.
$r > c_1 + c_2$
r= 2.5



Capacity level

$$K_j = \mu + \xi_i + \epsilon$$

$$Z_l = (r - w) \cdot E[\min(\mu + \xi_i + \epsilon, K_j)]$$

Reserve Fees

$$\pi^s(K_j, Z_l, \xi_i) = (w - c_1) \cdot E[\min(\mu + \xi_i + \epsilon, K_j)] - C_2 K_j + Z_l$$

$$\pi^b(K_j, Z_l, \xi_i) = (r - w) \cdot E[\min(\mu + \xi_i + \epsilon, K_j)] - Z_l$$

Supplier Profit: π^s
Buyer Profit π^b

Contract Menu

- Taiwan Semiconductor manufacturing company (TSMC) prepares list of unique contracts.

Average Market Demand

2 — 10

2 3 4 5 6 7 8 9 10

105
unique contracts

Show 10 entries

Search:

	Buyer Type(b)	Reservation Fee(Z)	Capacity(K)	Buyer_Profit(v)	Supplier_Profit(r)
1	b1	0	0	0	0
2	b1	0.25	0	-0.25	0.25
3	b1	0.5	0	-0.5	0.5
4	b1	0.75	0	-0.75	0.75

CAPACITY RESERVATION CONTRACT

- TSMC finds Optimum Menu contracts (K_j, Z_l) from this list that Maximizes his Profit π^s.
  - K_j = Capacity Level $K_j = \{0,1,2,\ldots,4\}$, $|K| = 5$
  - Z_l = Reservation Fee $K_j = \{0,0.25,0.5,..,4.5,4.75,5\}$, $|Z| = 21$
- The Apple Inc chooses one contract from (K_j, Z_l) that Maximizes her Profit π^b. In doing so, she reveals ξ_i her Forecast Information.
- Apple Inc Observes the demand D and places an Order.
- TSMC produces $\min(D, K\ )$.

MODELING

Previous Work

$v_{i,j,l} = \beta_{\pi^b} \cdot \pi^b(K_j, Z_l, \xi_i)$

Profit

MINLP

MILP

$v_{i,j,l} = \beta_{\pi^b} \cdot \pi^b(K_j, Z_l, \xi_i) - \beta_m M$

Buyers' Utility Function

PREVIOUS WORK

- This Thesis is an extension of A Choice Based Optimization Approach for Contracting in Supply Chains by Römer et al (2020).
- In Choosing a contract, Only the buyer's profit was considered.
- Opt-out utility $\overline{v_i} = 0$
- Utility of choosing contract $v_{i,j,l} = \beta_{\pi^b} \cdot \pi^b(K_j, Z_l, \xi_i)$

Profit Estimate

Buyers' Profit

BUYERS'S UTILITY FUNCTION

Choice set Estimate

Choice Set Cardinality

- Opt-out utility $\overline{v_i} = 0$
- Utility of choosing contract $v_{i,j,l} = \beta_{\pi^b} \cdot \pi^b(K_j, Z_l, \xi_i) - \beta_m M$

Obs	Buyer_Profit $\pi^b(K_j, Z_l, \xi_i)$	Choice Card (M)	Contract (K_j, Z_l)
1	3.25	4	(4,1.25)
2	-3.75	3	(4,4.5)
3	-0.50	2	(3,4)

MINLP

Sets, Parameters Variables	
Sets	
$\boldsymbol{I}$ Set of different private information i $\boldsymbol{J}$ Set of different capacity levels j	$\boldsymbol{L}$ Set of different reservation fee levels l
Parameters	
$\boldsymbol{\pi}^s(\boldsymbol{K}_j, \boldsymbol{Z}_l, \xi_i)$ Profit of the supplier if a buyer chooses contract $(\boldsymbol{K}_j, \boldsymbol{Z}_l)$ given the buyer's private forecast information $\boldsymbol{\xi}_i$. $\boldsymbol{\pi}^b(\boldsymbol{K}_j, \boldsymbol{Z}_l, \xi_i)$ Profit of buyer for choosing contract $(\boldsymbol{K}_j, \boldsymbol{Z}_l)$ given the buyer's private forecast information $\boldsymbol{\xi}_i$. **M** Choice set cardinality for buyer b_i $\boldsymbol{\beta}_{\pi^b}$ Profit estimate of for buyer profit $\boldsymbol{\pi}^b(\boldsymbol{K}_j, \boldsymbol{Z}_l, \xi_i)$	$\boldsymbol{\beta}_m$ Choice set cardinality estimate for choice set cardinality $\boldsymbol{M}$ $\boldsymbol{p}(\boldsymbol{\xi}_i)$ A prior probability that buyer b_i has private forecast information $\boldsymbol{\xi}_i$. $\boldsymbol{v}_{i,j,l}$ Deterministic utility for buyer b_i choosing contract $(\boldsymbol{K}_j, \boldsymbol{Z}_l)$. $\bar{\boldsymbol{v}}_i$ Opt-out utility for buyer b_i choosing no contract. $\boldsymbol{Z}_l$ Contract Reservation fee in level l.
Variables	
$\boldsymbol{X}_{i,j,l}$ Choice Probability of a buyer b_i choosing a contract $(\mathrm{K_j}, \mathrm{Z_l})$. $\boldsymbol{Y}_{i,j,l}$ 1 if the contract $(\mathrm{K_j}, \mathrm{Z_l})$ is offered, otherwise 0.	$\boldsymbol{E}[\boldsymbol{\pi}^s]$ Expected profit of the Supplier (Objective function)

MINLP

DU1

$$\boldsymbol{max\ \ E[\pi^s] = \sum_{i=1}^{I} p(\xi_i) \sum_{j=1}^{J} \sum_{l=1}^{L} X_{i,j,l} \cdot, \xi_i)\, \pi^s(K_j, Z_l(1)}$$

(1). Maximize Supplier‘s expected Profit.

Subject to.

$$X_{i,j,l} = \frac{e^{v_{i,j,l}} \cdot Y_{j,l}}{\sum_{j'=1}^{J} \sum_{l'=1}^{L} e^{v_{i,j',l'}} \cdot Y_{j,l} + e^{\overline{v_i}}} \quad \forall\, i,j,l \qquad (2)$$

(2) Buyer‘s Choice Probability.

$$\sum_{j=1}^{J} Y_{j,l} \leq 1 \quad \forall l \qquad (3)$$

$$\sum_{l=1}^{L} Y_{j,l} \leq 1 \quad \forall j \qquad (4)$$

$$\sum_{j=1}^{J} \sum_{l=1}^{L} Y_{j,l} \leq M \qquad \forall l \quad (5)$$

$$X_{i,j,l} \geq 0 \qquad \forall i,j,l \quad (6)$$

$$Y_{j,l} \in \{0, 1\} \qquad \forall j,l \quad (7)$$

(3) Select a contract from capacity levels.
(4) Select a contract from Reservation fee levels.
(5) Choice set Size constraint.
(6) And (7) domain contraints

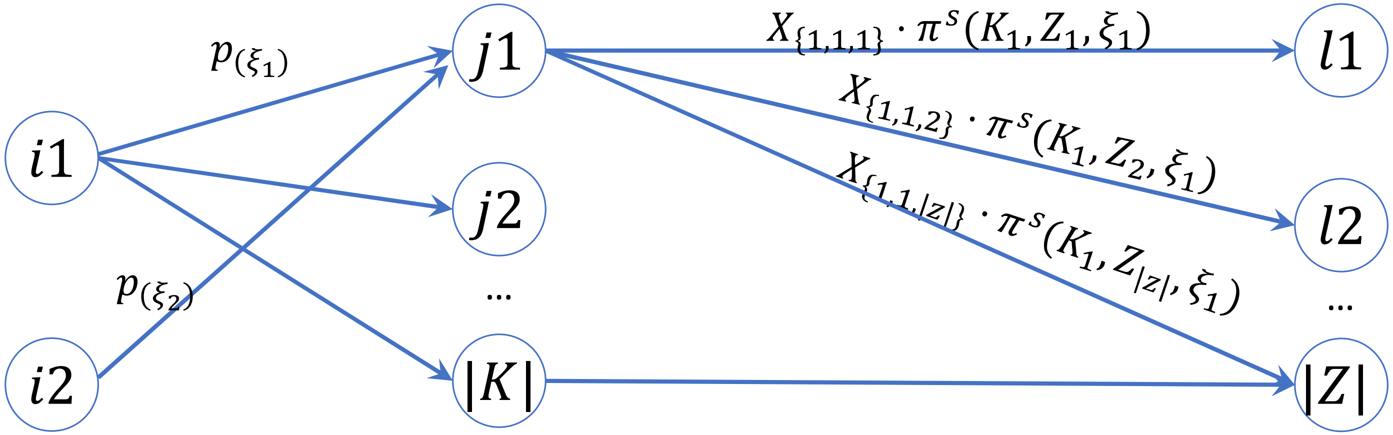
Slide 14

DU1 Diego Uchendu, 06/05/2021

MINLP (Objective Function)

max $E[\pi^s] = \sum_{\{i=1\}}^{I} p_{(\xi_i)} \sum_{\{j=1\}}^{J} \sum_{\{l=1\}}^{l} X_{\{i,j,l\}} \cdot \pi^s(K_j, Z_l, \xi_i)$

$I = \{1,2\}$



Equation (1)

MINLP (OFFERED CONTRACT)

J,L	j=1	j=2	j=3	j=|K|	$\sum_{j=1}^{J} Y_{j,l} \le 1$
l=1	Y_{11}	Y_{21}	Y_{31}	$Y_{\|K\|1}$	$\sum_{j=1}^{J} Y_{j,1} \le 1$
l=2	Y_{12}	Y_{22}	Y_{32}	$Y_{\|K\|2}$	$\sum_{j=1}^{J} Y_{j,2} \le 1$
l=3	Y_{13}	Y_{23}	Y_{33}	$Y_{\|K\|3}$	$\sum_{j=1}^{J} Y_{j,3} \le 1$
l=|Z|	$Y_{1\|Z\|}$	$Y_{2\|Z\|}$	$Y_{3\|Z\|}$	$Y_{\|K\|\|Z\|}$	$\sum_{j=1}^{J} Y_{j,\|Z\|} \le 1$
$\sum_{l=1}^{L} Y_{j,l} \le 1$	$\sum_{l=1}^{L} Y_{1,l} \le 1$	$\sum_{l=1}^{L} Y_{2,l} \le 1$	$\sum_{l=1}^{L} Y_{3,l} \le 1$	$\sum_{l=1}^{L} Y_{\|K\|,l} \le 1$	$\sum_{j=1}^{J}\sum_{l=1}^{L} Y_{j,l} \le M$

Euations (3),(4) and (5)
(3) Select a contract from capacity levels.
(4) Select a contract from Reservation fee levels.
(5) Choice set Size constraint.

MILP

Sets, parameters and variables	
Sets:	
M Set of different Choice set cardinality m	
Parameters:	
$v_{i,j,l,m}$ utility component of buyer type b_i choosing contract $(K_{j,} Z_l)$ with a choice set cardinality m.	C Maximum Choice set cardinality of contracts offered, $C = \|M\|$ $Ratio_{i,j,l,m}$ Helps ensure IIA property is implemented
Variables:	
$X_{i,j,l,m}$ Choice probability of buyer type b_i choosing contract (K_j, Z_l) given a choice set cardinality m. $\overline{X}_i$ Opt-out choice probability for buyer type b_i	$Y_{j,l,m}$ 1 if contract (K_j, Z_l) is chosen at choice set cardinality m . $E[\pi^s]$ Expected profit of the supplier.

MILP

$$\boldsymbol{max\ \ E[\pi^s] = \sum_{i=1}^{I} p(\xi_i) \sum_{j=1}^{J} \sum_{l=1}^{L} \pi^s(K_j, Z_l, \xi_i) \ \sum_{m=1}^{M} X_{i,j,l,m}} \qquad (\mathbf{8})$$

Subject to.

$$\bar{X}_i + \sum_{j=1}^{J} \sum_{l=1}^{L} \sum_{m=1}^{M} X_{i,j,l,m} \le 1 \qquad \forall i \qquad (9)$$

$$X_{i,j,l,m} \le Y_{j,l,m} \qquad \forall i,j,l,m \qquad (10)$$

$$X_{i,j,l,m} \le \frac{a_{i,j,l,m}}{\bar{a}_i}\ \bar{X}_i \qquad \forall i,j,l,m \qquad (11)$$

$$\sum_{j=1}^{J} \sum_{m=1}^{M} Y_{j,l,m} \le 1 \qquad \forall l \qquad (12)$$

$$\sum_{l=1}^{L} \sum_{m=1}^{M} Y_{j,l,m} \le 1 \qquad \forall j \qquad (13)$$

(8). Maximize Supplier's expected Profit.

(9) Sum of Probabilities contraint

(10) Choice prob, less or equ selected contract.
(11) Ratio between choice and opt-out probs is obeyed.
(12)(13) Ensures a contract is established in one cardinality or not at all.

MILP

$$\sum_{j=1}^{J}\sum_{l=1}^{L}\sum_{m=1}^{M} Y_{j,l,m} = C \qquad 14)$$

$$\sum_{m=1}^{M} m \cdot Y_{j,l,m} = C \qquad \forall j,l \quad (15)$$

$$X_{i,j,l,m} \geq 0 \qquad \forall i,j,l,m \quad (16)$$

$$C \geq 0 \qquad \forall i \quad (17)$$

$$Y_{j,l,m} \in \{0,1\} \qquad \forall j,l,m \quad (18)$$

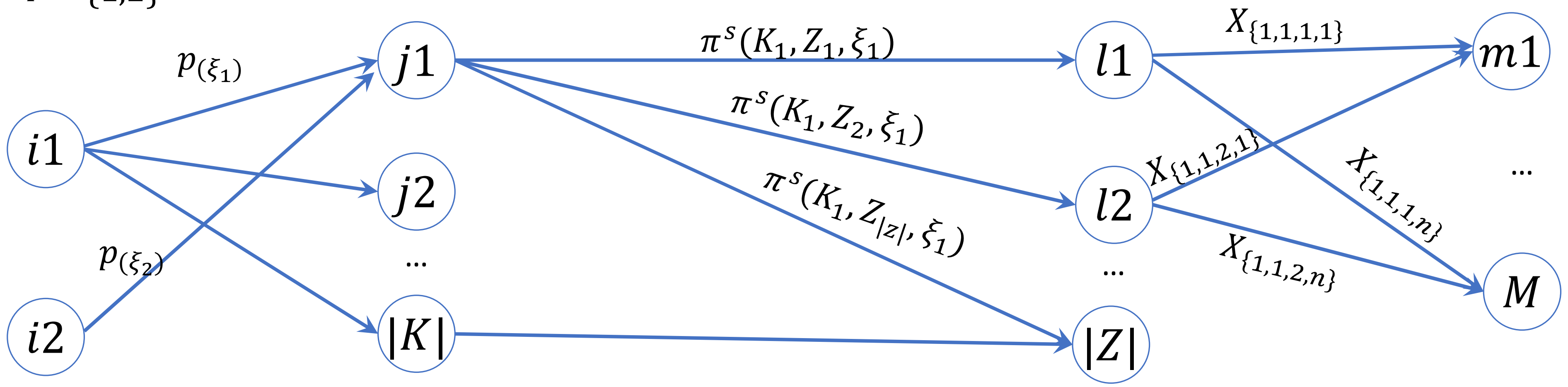
(14) And (15) ensures that a number of contracts C are established.

(16), (17) and (18) Domain Constraints.

MILP(OBJECTIVE FUNCTION)

max $E[\pi^s] = \sum_{\{i=1\}}^{I} p_{(\xi_i)} \sum_{\{j=1\}}^{J} \sum_{\{l=1\}}^{l} \pi^s(K_j, Z_l, \xi_i) \sum_{\{m=1\}}^{m} X_{\{i,j,l,m\}}$ Eq.(8)

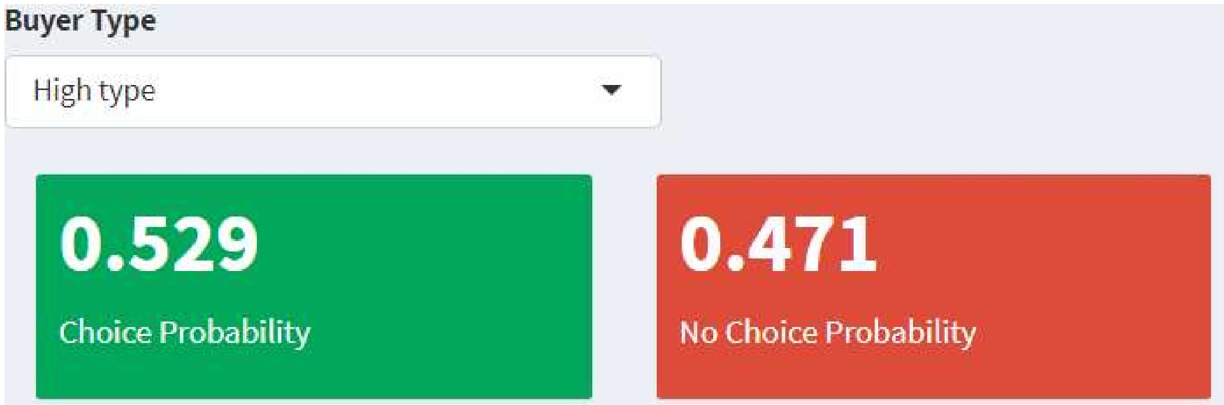
$I = \{1,2\}$



MILP (Probabilities)

- $\bar{X}_i + \sum_{\{j=1\}}^{J} \sum_{\_\{l=1\}}^{L} \sum_{\{m=1\}}^{M} X_{i,j,l,m} \leq 1 \qquad \forall\, i$
- $M = 4 \quad \beta_m = 1$

(9) Choice probability and Opt-out probability.



MILP (CONTRACT RATIO CONSTRAINT)

- $X_{i,j,l,m} \leq \frac{a_{ij,l,m}}{\overline{a_i}} \; \overline{X}_i \qquad \forall i,j,l,m \qquad (11)$
- Ensures that ratio of contract probability are obeyed
- $a_{ij,l,m} = e^{v_{i,j,l,m}}$
- $\overline{a_i} = e^{\overline{v_i}}$
- therefore $Ratio_{i,j,l,m} = \frac{a_{i,j,l,m}}{\overline{a_i}} = \frac{e^{v_{i,j,l,m}}}{e^{\overline{v_i}}}.$
- This exploits the IIA(Independence from irrelevant Alternative) property

MILP (CONTRACT ESTABLISHMENT CONTRAINTS)

eqn (12)	J=1, m=1	J=1,m=2	J=2,m=1	J=2,m=2	$\sum_{j=1}^{J}\sum_{m=1}^{M} Y_{j,l,m} \leq 1$
l=1	$Y_{1,1,1}$	$Y_{1,1,2}$	$Y_{2,1,1}$	$Y_{2,1,2}$	$\sum_{j=1}^{J}\sum_{m=1}^{M} Y_{j,1,m} \leq 1$
l=2	$Y_{1,2,1}$	$Y_{1,2,2}$	$Y_{2,2,1}$	$Y_{2,2,2}$	$\sum_{j=1}^{J}\sum_{m=1}^{M} Y_{j,2,m} \leq 1$
eqn(13)	l=1, m=1	l=1,m=2	l=2,m=1	l=2,m=2	$\sum_{l=1}^{L}\sum_{m=1}^{M} Y_{j,l,m} \leq 1$
j=1	$Y_{1,1,1}$	$Y_{1,1,2}$	$Y_{1,2,1}$	$Y_{1,2,2}$	$\sum_{l=1}^{L}\sum_{m=1}^{M} Y_{1,l,m} \leq 1$
j=2	$Y_{2,1,1}$	$Y_{2,1,2}$	$Y_{2,2,1}$	$Y_{2,2,2}$	$\sum_{l=1}^{L}\sum_{m=1}^{M} Y_{2,l,m} \leq 1$

Equation (12) and (13) Ensures that a contract is exactly in one cardinality or not at all

MILP (MAXIMUM CONTRACT CONSTRAINTS)

- $\sum_{j=1}^{J} \sum_{l=1}^{L} \sum_{m=1}^{M} Y_{j,l,m} = C \qquad \forall j$
- $\sum_{m=1}^{M} m \cdot Y_{j,l,m} = C \qquad \forall j, l$
- Ensures that not more than number of contracts C are established.

Equations (14) and (15)

RESULTS

Model Comparison

Contracts Offered

Expected Supplier Profit

Probability

Supply Chain Profit

Current vs Previous Model

RESULTS (MODEL COMPARISON)

- When both MINLP and MILP are tested, it yields approximately same results, with MILP faster than MINLP.
- Increase in Choice set Size M, results in more delayed time in computation.
- $\beta_{\pi^b} = 1$ and $\beta_{\pi^b} = 1$

M	MINLP (CPU(s))	MILP (CPU(s))	MINLP $E[\pi^s]$	MILP $E[\pi^s]$
1	7.1	3.6	1.369	1.369
5	18.0	6.16	0.341,	0.343
10	35.27	6.7	0.004	0.004
15	44.7	9.7	0.000031	0.000027

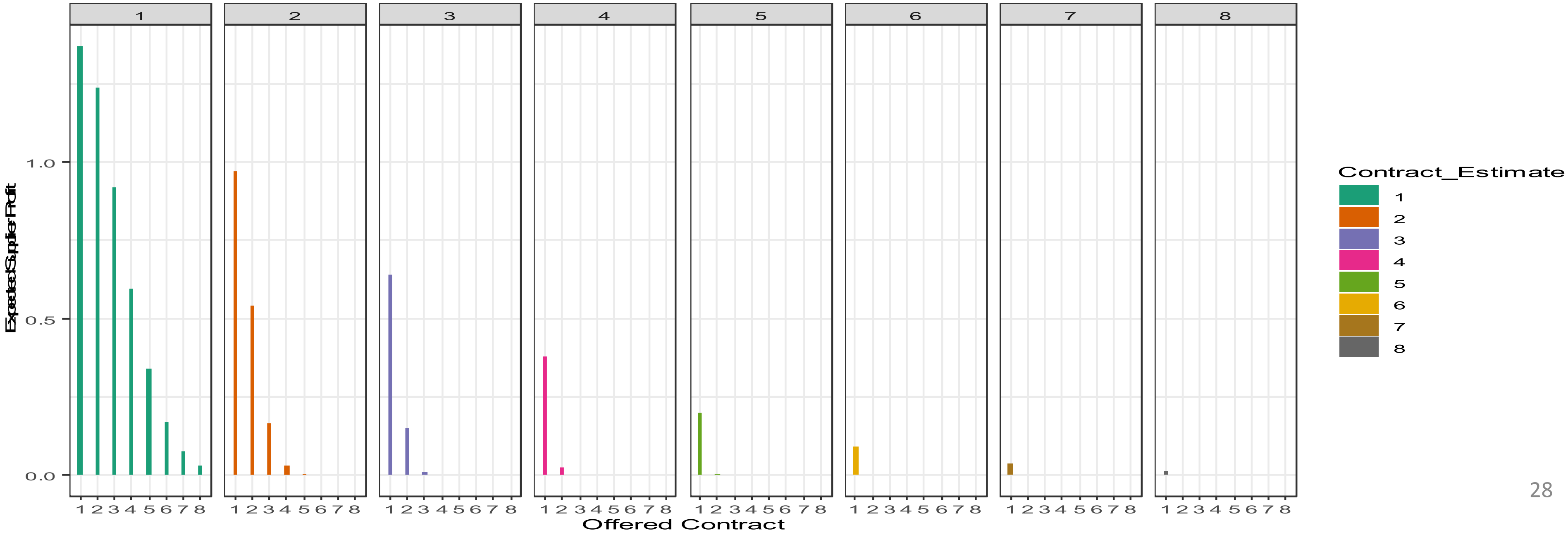
RESULTS (CONTRACTS OFFERED)

- Assuming that Choice Set cardinality M=4 and it's estimate is $\beta_m = 1,\ \beta_{\pi^b} = 1$
- Assuming TSMC Does not Know the Buyer types, he offers the buyer all the contract below.
- But lets say that Apple Inc, is high type buyer, so the last 2 contract is offered.

K_j	Z_l	*Buyer Profit(ξ = − 1)*	*Buyer Profit(ξ = 1)*	*Supplier Profit(ξ = − 1)*	*Supplier Profit(ξ = 1)*
1	1.5	-0.375	0.000	1.75	2.00
2	1	0.500	2.000	1.00	2.00
3	0.75	0.750	3.375	0.25	2.00
4	1.25	0.250	3.250	0.25	2.25

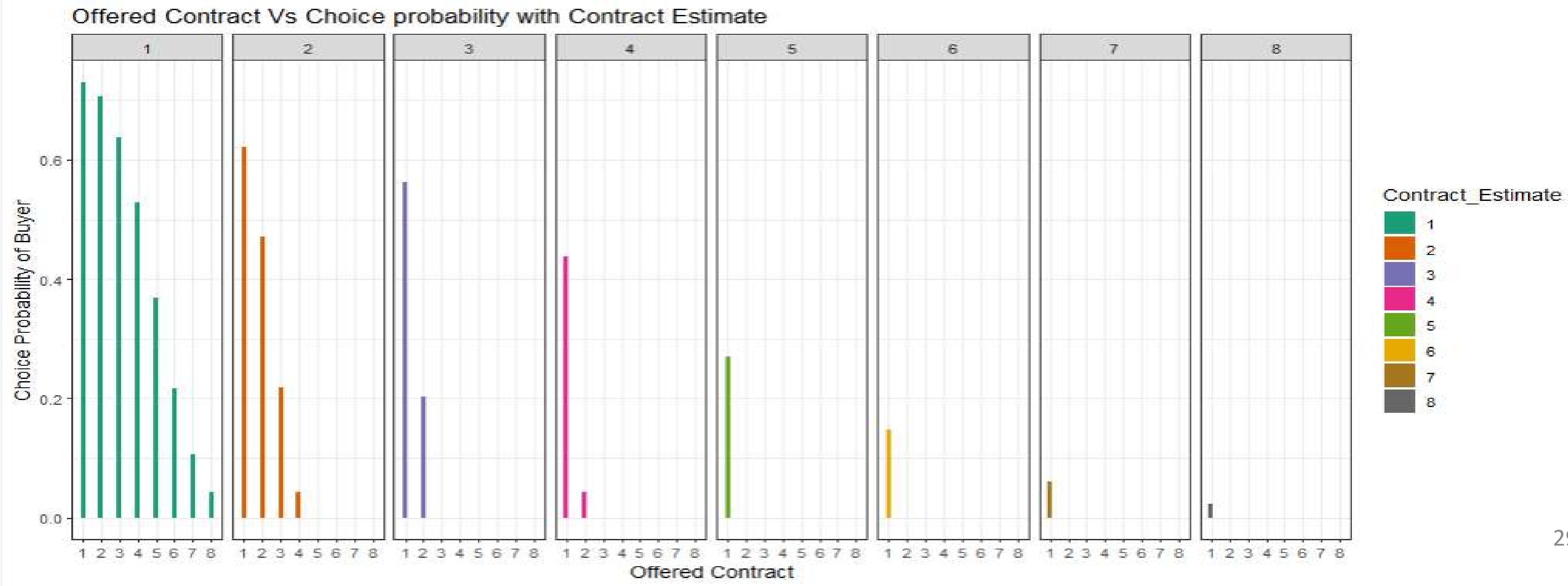
RESULT (EXPECTED SUPPLIER PROFIT)

- The inclusion of Choice set size M as an attribute, reduces the choice probability of buyer $\boldsymbol{X_{i,j,l}}$, this effects the $\boldsymbol{E}[\boldsymbol{\pi}^{s}]$ as seen in equation (1).



RESULT (PROBABILITY)

- Thus, Higher the offered Contract M, lowers $X_{i,j,l}$.



RESULT (SUPPLY CHAIN PROFIT)

- Due to this effect, the mathematical model (MINLP or MILP) tries to maximize $\boldsymbol{E}[\boldsymbol{\pi}^{\boldsymbol{s}}]$ by selecting contracts $(\boldsymbol{K}, \boldsymbol{Z})$ with higher buyers' profit $\pi^b(K_j, Z_l, \xi_i)$ to increase $\boldsymbol{X}_{\boldsymbol{i},\boldsymbol{j},\boldsymbol{l}}$ at the same time maximizing $\boldsymbol{E}[\boldsymbol{\pi}^{\boldsymbol{s}}]$.
- As the supply chain profit $\boldsymbol{E}[\boldsymbol{\pi}^{\boldsymbol{sc}}] = \boldsymbol{E}[\boldsymbol{\pi}^{\boldsymbol{b}}]+ \boldsymbol{E}[\boldsymbol{\pi}^{\boldsymbol{s}}]$ remains constant, TSMC will show a more balanced contract to Apple Inc.

RESULT (Current vs Previous model)

0.595

Expected Supplier Profit

$$v_{i,j,l} = \beta_{\pi^b} \cdot \pi^b(K_j, Z_l, \xi_i) - \beta_m M$$

Show 10 entries

Search:

	K	Z	Buyer profit(Low Forecast)	Buyer profit(high Forecast)	Supplier profit(Low Forecast)	Supplier profit(high Forecast)	Supply Chain profit(Low)	Supply Chain profit(High)
1	1	1.5	-0.375	0	1.75	2	1.375	2
2	2	1	0.5	2	1	2	1.5	4
3	3	0.75	0.75	3.375	0.25	2	1	5.375
4	4	1.25	0.25	3.25	0.25	2.25	0.5	5.5

3.95

Expected Supplier Profit

$$v_{i,j,l} = \beta_{\pi^b} \cdot \pi^b(K_j, Z_l, \xi_i)$$

	K	Z	Buyer profit(Low Forecast)	Buyer profit(high Forecast)	Supplier profit(Low Forecast)	Supplier profit(high Forecast)	Supply Chain profit(Low)	Supply Chain profit(High)
1	1	4.25	-3.125	-2.75	4.5	4.75	1.375	2
2	2	5	-3.5	-2	5	6	1.5	4
3	3	4.5	-3	-0.375	4	5.75	1	5.375
4	4	4.75	-3.25	-0.25	3.75	5.75	0.5	5.5

DEMO

Contract Menu generation in R Shiny.

And Results in R shiny.

https://diegouchendu.shinyapps.io/contract_selection_app/

CONCLUSION

Buyers‘ choice overload can be reduced by this model if buyers‘ choice set size effect is captured.

it is also worth finding how other attributes of a contracts and characteristics of a buyer affects the buyer’s choice probability.

THANKS YOU

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Explaining Equation 5 slide 14

- $\sum_{j=1}^{J} Y_{j,l} \leq 1 \quad \forall l$ $\qquad (3)$
- $\sum_{l=1}^{L} Y_{j,l} \leq 1 \quad \forall j$ $\qquad (4)$
- $\sum_{j=1}^{J} \sum_{l=1}^{L} Y_{j,l} \leq M$ $\qquad \forall l$ (5)
- Equation 5 takes into account what was selected in equations (3) and (4).
- M was used as a Parameter in my model, assuming M is a variable, GAMS will always give you $M = 1$, since this will yield the maximum value of $E[\pi^s]$, this is because of $v_{i,j,l} = \beta_{\pi^b} \cdot \pi^b(K_j, Z_l, \xi_i) - \beta_m M$.
- If $M = 0$, No contract is selected.

Equation 14 Table (Selected Contracts in Red)

J,L	j=1	j=2	j=3	j=|K|	$\sum_{j=1}^{J} \boldsymbol{Y_{j,l}} \leq \mathbf{1}$
l=1	Y_{11}	Y_{21}	Y_{31}	$Y_{\|K\|1}$	$\sum_{j=1}^{J} Y_{j,1} \leq 1$
l=2	Y_{12}	Y_{22}	Y_{32}	$Y_{\|K\|2}$	$\sum_{j=1}^{J} Y_{j,2} \leq 1$
l=3	Y_{13}	Y_{23}	Y_{33}	$Y_{\|K\|3}$	$\sum_{j=1}^{J} Y_{j,3} \leq 1$
l=|Z|	$Y_{1\|Z\|}$	$Y_{2\|Z\|}$	$Y_{3\|Z\|}$	$Y_{\|K\|\|Z\|}$	$\sum_{j=1}^{J} Y_{j,\|Z\|} \leq 1$
$\sum_{l=1}^{L} \boldsymbol{Y_{j,l}} \leq \mathbf{1}$	$\sum_{l=1}^{L} Y_{1,l} \leq 1$	$\sum_{l=1}^{L} Y_{2,l} \leq 1$	$\sum_{l=1}^{L} Y_{3,l} \leq 1$	$\sum_{l=1}^{L} Y_{\|K\|,l} \leq 1$	$\sum_{j=1}^{J} \sum_{l=1}^{L} Y_{j,l} \leq M$

- Assuming $y_{\{1,2\}}, y_{\{21\}}, y_{\{3,3\}}, y_{\{|K|,|Z|\}}$ where selected.
- $y_{\{1,2\}} + y_{\{21\}} + y_{\{3,3\}} + y_{\{|K|,|Z|\}} = 4$
- This is equivalent to choosing contracts {(0, 0.25), (1,0), (2,0.5), (5,21)} using the step side explained in slide 9.