Fall 2019 - ECON 634 - Advance Macroeconomics - Problem Set 2

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1. Since the Resource constraint (Social Planner Problem) is $c_t = A_t k_t^{\alpha} + (1-\delta)k_t - k_{t+1}$ we can write the budget constraint in recursive form as $c = Ak^{\alpha} + (1-\delta)k - k'$

   - **State variable**: k, A
   - **Control variable**: k'

   Therefore, the Bellman equation:

$$V(k, A) = \underset{k'}{\text{Max}} \left\{ \frac{(Ak^{\alpha} + (1-\delta)k - k')^{1-\sigma}}{1-\sigma} + \beta \sum_{A' \in A} \Pi(A'|A) V(k', A') \right\}$$

   subject to

$$c \in [0, f(k)] \tag{1}$$
$$k' \in [0, f(k)] \tag{2}$$

2. Using the VFI, the Graphs are like follows:

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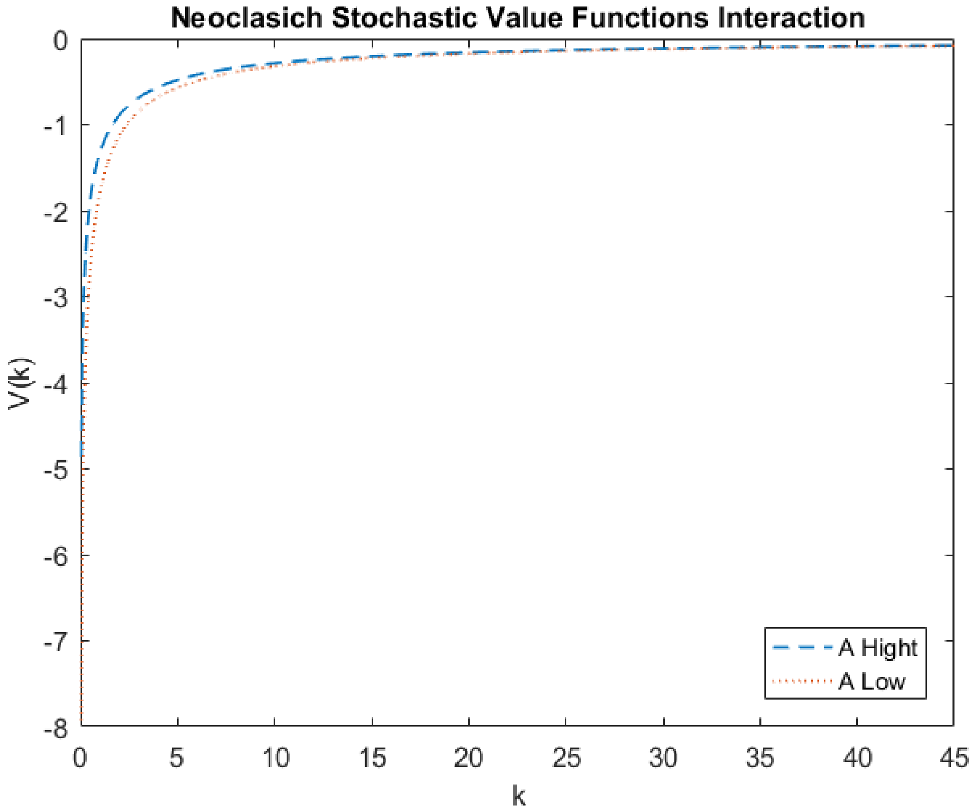
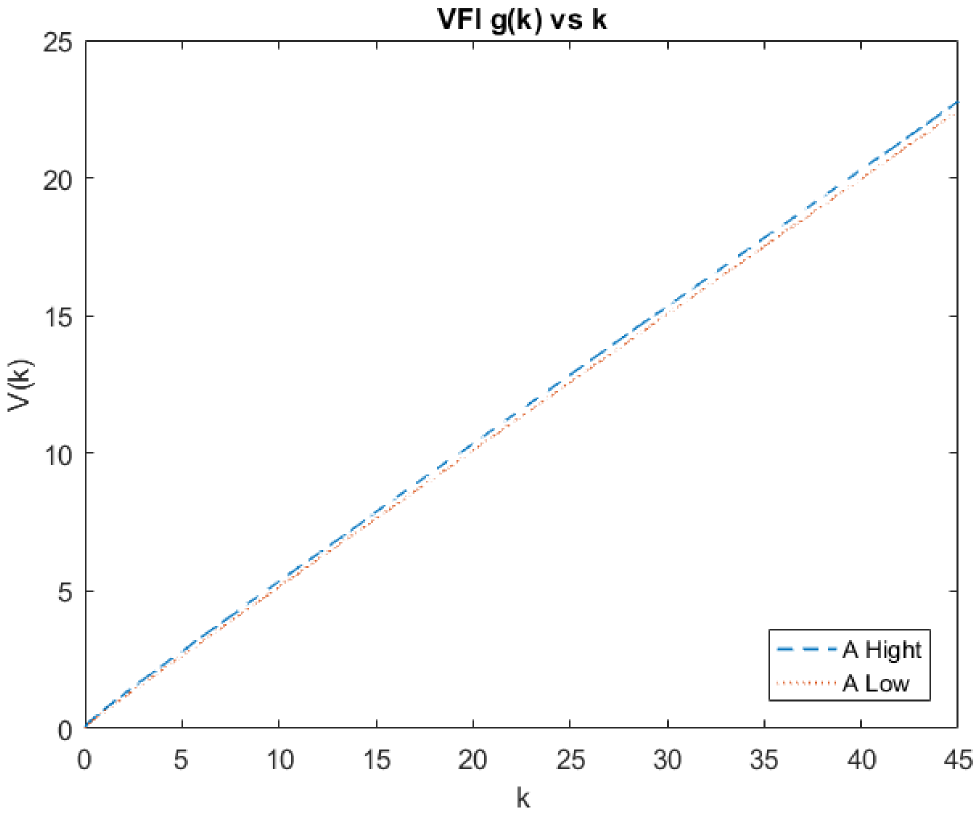


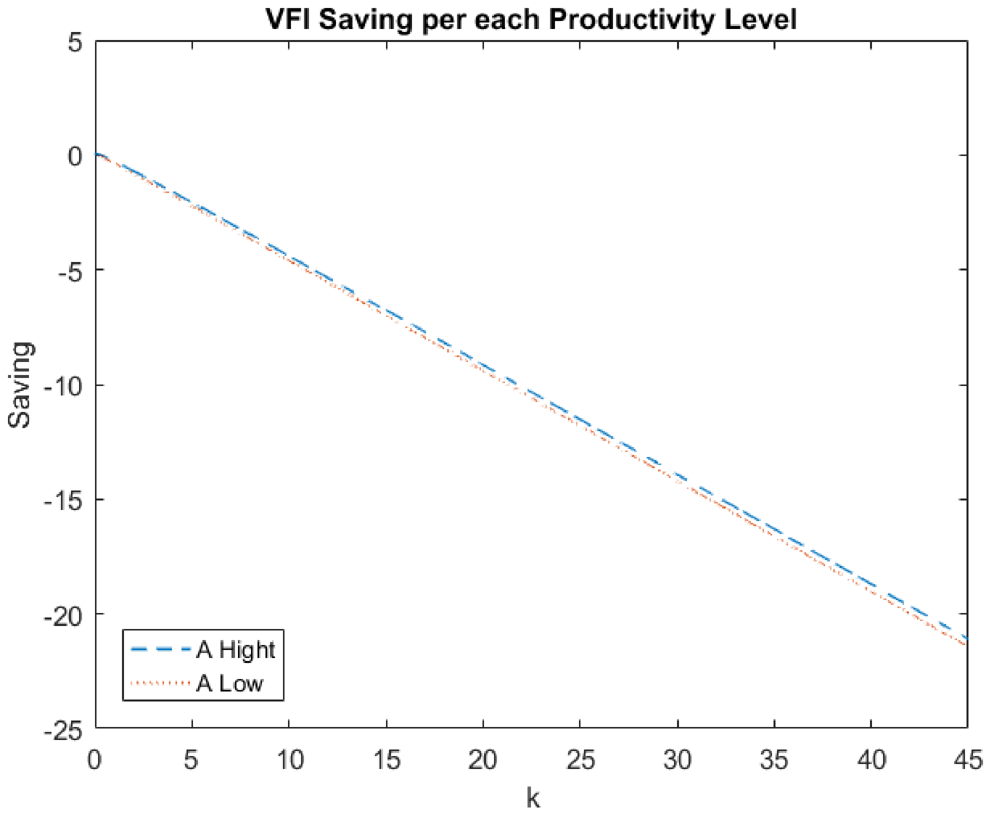
As we can see, both are increasing and concave functions.

3. The Policy function over k looks as follows:



This relationship is decreasing in k but increasing in A.

Assuming that by saving, it means what is left from production after consumption: $s = k' - (1-\delta)k$, the Saving over k looks as follows:



where this relationship is decreasing in k and increasing in A. This make sense because

4. For this, I will use the code for the part (5), since it seems more reasonable results. Using the results, I get a $sd(y) = 0.1755$, whci does not match the results from the data. Doing the markov change process to find stationary probabilities:

$$\begin{pmatrix} \bar{\pi}_h & \bar{\pi}_l \end{pmatrix} = \begin{pmatrix} \bar{\pi}_h & \bar{\pi}_l \end{pmatrix} P$$

This means

$$\begin{pmatrix} \bar{\pi}_h & \bar{\pi}_l \end{pmatrix} = \begin{pmatrix} \bar{\pi}_h & \bar{\pi}_l \end{pmatrix} \begin{bmatrix} 0.977 & 0.023 \\ 0.074 & 0.926 \end{bmatrix},$$

$$\begin{pmatrix} \bar{\pi}_h & \bar{\pi}_l \end{pmatrix} = \begin{pmatrix} 0.977\bar{\pi}_h + 0.074\bar{\pi}_l & 0.023\bar{\pi}_h + 0.926\bar{\pi}_l \end{pmatrix},$$

Therefore,

$$\bar{\pi}_h = 3.22\bar{\pi}_l \tag{3}$$
$$\bar{\pi}_l = 0.31\bar{\pi}_h \tag{4}$$

Since $\bar{\pi}_h + \bar{\pi}_l = 1$

$$4.22\bar{\pi}_l = 1 \tag{5}$$
$$\bar{\pi}_l = 0.24 \tag{6}$$
$$\bar{\pi}_h = 0.76 \tag{7}$$

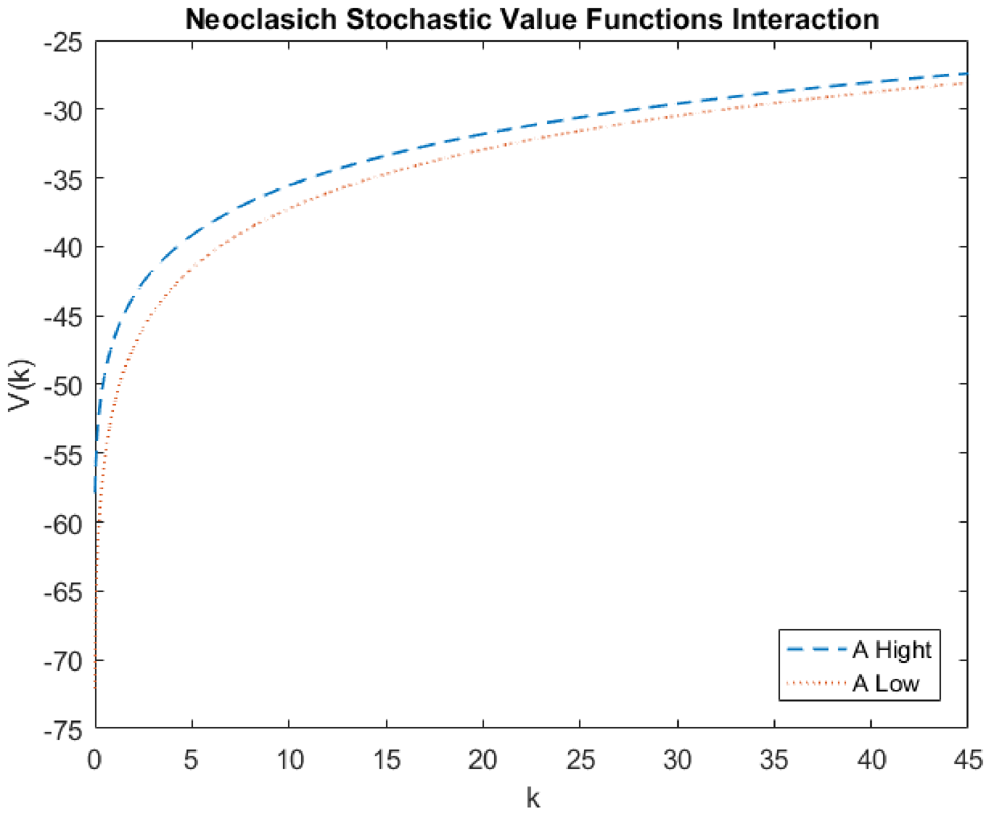
Therefore, I will try first, $A_h = 1.05$, which implies sung the Long-run probabilities $A_l = \frac{1-\bar{\pi}_h A_h}{\bar{\pi}_l} = 0.839$. With this results, the standard deviation is $sd(y) = 0.1445$, which means I need to keep trying for a smaller value of A_h. Using $A_h = 1.00005$, $A_l = 0.99984$, then

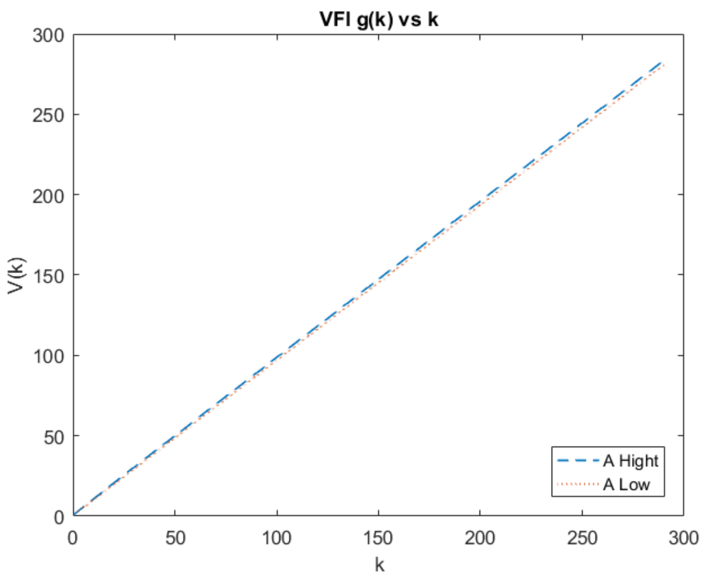
Ideally, I should do a while loop and try for a low tolerance between my simulated standard deviation and the actual standard deviation. Because the code **VFIP5** is so slow, I decide against it and just try randomly picking a number.

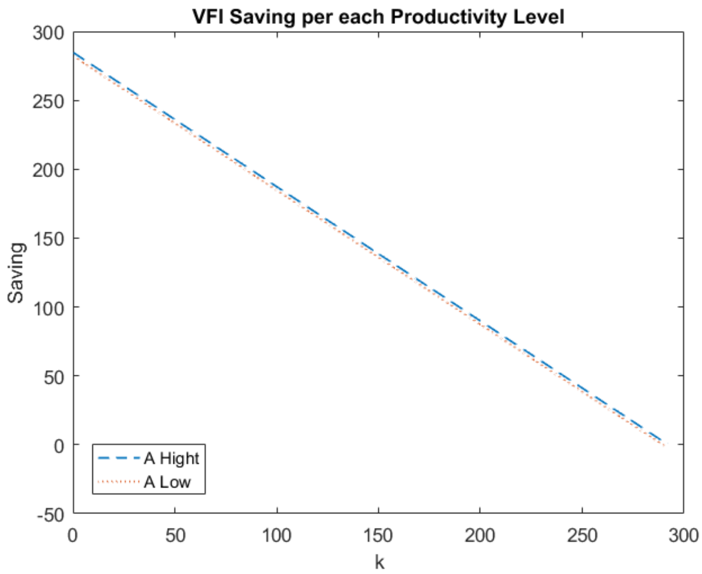
5. See Code **VFIP5**. Using two loops over all K is quite slow and does spend a long time to find a solution.

   The time on the previous program **Elapsed time is 0.326278 seconds.** For the second one, limiting to the **Elapsed time is 1049.453223 seconds.** Therefore, time is significantly higher for this one.

   Since, to me, this second version is correct, but I get differents results, I will add the graphs for this. The results for the VFI is:


Neoclasich Stochastic Value Functions Interaction
V(k)
k
A Hight
A Low


VFI g(k) vs k
V(k)
k
A Hight
A Low


VFI Saving per each Productivity Level
Saving
k
300
250
200
150
100
50
0
-50
0
50
100
150
200
250
300
A Hight
A Low