HOMEWORK 2

DANQING LAN

Question 1.
The functional equation of this dynamic programming problem is

$$v(K_t,\, A_t) = \max_{0 \le K_{t+1} \le A_t K_t^{\alpha} + (1-\delta)K_t} \left\{ \frac{(A_t K_t^{\alpha} + (1-\delta)K_t - K_{t+1})^{1-\sigma}}{1-\sigma} + \beta \mathbb{E}[v(K_{t+1},\, A_{t+1})] \right\}$$

where $K_t,\, A_t$ are state variables and K_{t+1} is the control variable. Note that $K_{t+1} \in [0,\, A_t K_t^{\alpha} + (1-\delta)K_t]$.

Question 2.

Follow the instructions, I use $A^h = 1.1$, and $A^l = 0.678$ with a transition matrix as $\Pi = \begin{bmatrix} 0.977 & 0.023 \\ 0.074 & 0.926 \end{bmatrix}$. By solving the VFI problem, the value function over K for each state of A are shown in Figure 0.1.

Figure 0.1 clearly shows that under both state of A, the value function over K are both increasing and concave. This comply with the assumption since there should have a diminishing marginal of return in K.

Question 2.

According to Figure 0.2, the policy function over K for each state of A,both policy functions are incresing in K and A. The savings over K for each A can be found in Figure 0.3. In the state with a higher A, saving is increasing in K, while in the state with a lower A, saving is decreasing in K. There is a story can

FIGURE 0.1. Value Function over K at each stage A

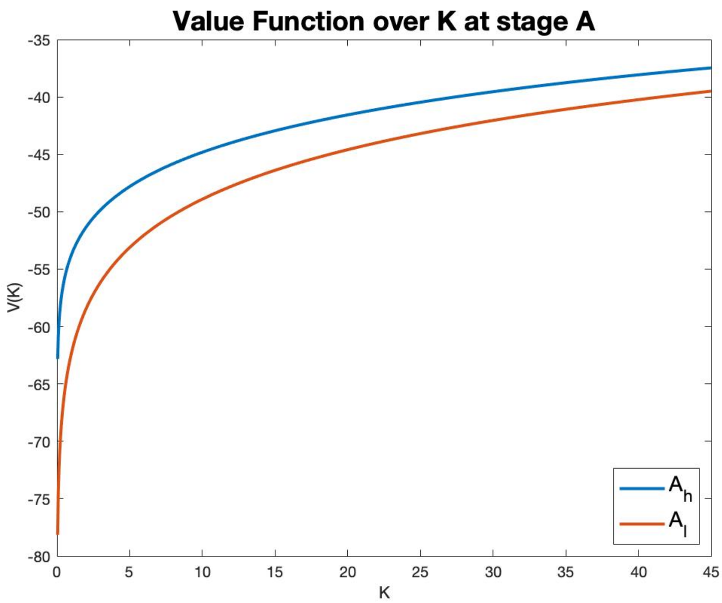
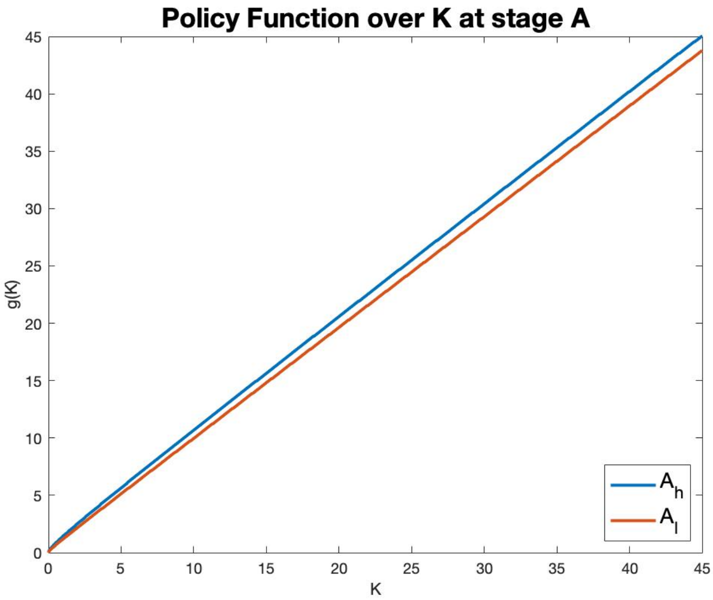


FIGURE 0.2. Policy Function over K at each stage A



expain this result well. When there is a high technology shock which will help to produce more than today, we should save more for tomorrow's production since it can increase our consumption in a larger extent tomorrow. However, when there is a low technology shock, saving will lead to a lower level of production and consumption tomorrow. Then it is naturally to save less and consume more today in order to maximize the lifetime consumption.

Question 4.

The long run probability for A matrix can be generated by the Markov probability matrix. i,e,:

$$\begin{aligned}\bar{\Pi} &= \Pi^{10000} \\ &= \begin{bmatrix} 0.7629 & 0.2371 \\ 0.7629 & 0.2371 \end{bmatrix}\end{aligned}$$

Therefore, $\begin{bmatrix} \bar{\Pi}^h & \bar{\Pi}^l \end{bmatrix} = \begin{bmatrix} 0.7629 & 0.2371 \end{bmatrix}$

In the simulation, I choose to adjust $A^h$and get the correspongding A^l by satisfying the equation $A^h\bar{\Pi}^h + A^l\bar{\Pi}^l = 1$. With an initial endouwment of $K = 5$, I obtained a set of value in A such that the simulated GDP fluctuation is close to 1.8%. That is,

$$A = \begin{bmatrix} A^h & A^l \end{bmatrix} = \begin{bmatrix} 1.2200 & 0.2922 \end{bmatrix}$$

with a simulated GDP fluction of 1.757%.

FIGURE 0.3. Saving over K at each stage A