HOMEWORK 4

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Question 1.

The firm's problem can be descrided as:

$$\underset{\{K_{t+1}^d,\, N_t^d\}}{max} \sum_{t=0}^{\infty} (\frac{1}{\Pi_{i=0}^t r_i})(F(K_t, N_t) - w_t N_t - r_t K_t + (1-\delta)K_t)$$

subject to $F(K_t, N_t) = K_t^\alpha N_t^{1-\alpha}$.

The first-order conditions for the firm with respect to K_{t+1} is

$$\frac{1}{\Pi_{i=0}^t r_i}(\alpha K_{t+1}^{\alpha-1} N_{t+1}^{1-\alpha} - r_{t+1} + (1-\delta)).$$

The first-order conditions for the firm with respect to N_t is

$$\frac{1}{\Pi_{i=0}^t r_i}((1-\alpha) K_t^a N_t^{-\alpha} - w_t).$$

Set both equations equal to zero we can get the factor prices given K_{t+1} and N_t, which are

$$r_{t+1} = \alpha K_{t+1}^{\alpha-1} N_{t+1}^{1-\alpha} + (1-\delta)$$

$$w_t = (1-\alpha) K_t^a N_t^{-\alpha}.$$

Question 2.

The household's recursive problem is

$$v(z,a) = \underset{a' \in [\underline{a}, \bar{a}]}{max} \left\{ \frac{(zwl + ra - a')^{1-\sigma}}{1-\sigma} + \beta \mathbb{E}_{z,a|z',a'}[v(z', a')] \right\}$$

where the labor unit each worker has is $l_t = \bar{l} = 1$, a is the amount of assets and z is the worker's labor efficiency.

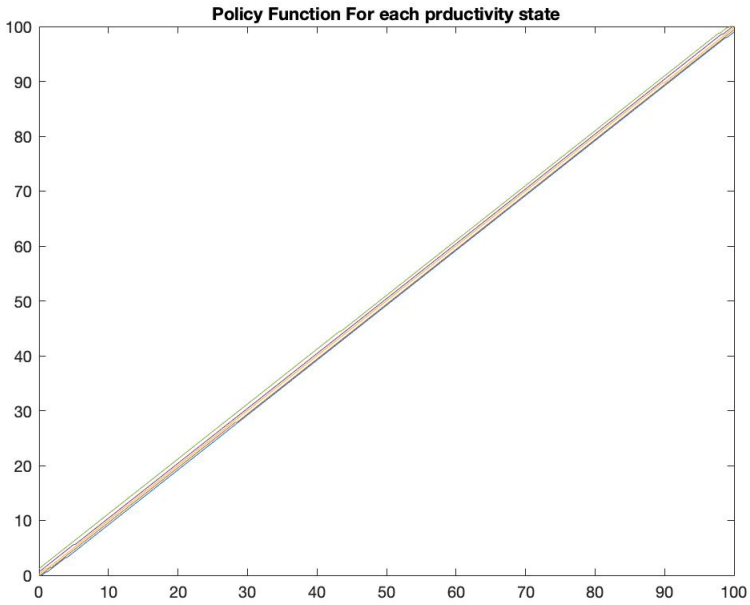
Question 3.

Let $m = 5$, with Rouwenhorst's method, the discrete set of possible values for z is

$$z = \begin{bmatrix} 0.6301 & 0.79381.5871 & 1 & 1.12598 & 1.587 \end{bmatrix}$$

with transition matrix $\Pi = \begin{bmatrix} 0.3164 & 0.4219 & 0.2109 & 0.0469 & 0.0039 \\ 0.1055 & 0.4219 & 0.3516 & 0.1094 & 0.0117 \\ 0.0352 & 0.2344 & 0.4609 & 0.2344 & 0.0352 \\ 0.0117 & 0.1094 & 0.3516 & 0.4219 & 0.1055 \\ 0.0039 & 0.0469 & 0.2109 & 0.4219 & 0.3164 \end{bmatrix}$. The invariant distribution of the implied Markov process over productivity states $\Pi^{inv}(z) = \Pi^{1000} = \begin{bmatrix} 0.0625 & 0.2500 & 0.3750 & 0.2500 & 0.0625 \end{bmatrix}$.

FIGURE 0.1. Policy Function for m=5 Productivity States



Given $\bar{l} = 1$,the aggregate effectie labor supply I obtained is $N^s = 1.0270$.

Question 4.

with $n = 500$, the lower and upper bounds for the grid I set in this practice are $a_{min} = 0$ and $a_{max} = 100$.

Question 6.

Accroding to my results, the steady state interest rate in this model $r = 1.0098$, which is slightly lower it in competitive market, i.e. $r^{CM} = \frac{1}{\beta} = 1.0101$.

Comparing to the Hugget model, the wealth distribution of Aiyagari model has a thinner and longer right tail, which is more similar to the empirical wealth distribution.

Question 7

. I am still working on the policy function iteration with $k = 30$ to solve for the same value function.

FIGURE 0.2. Lorenz Curv

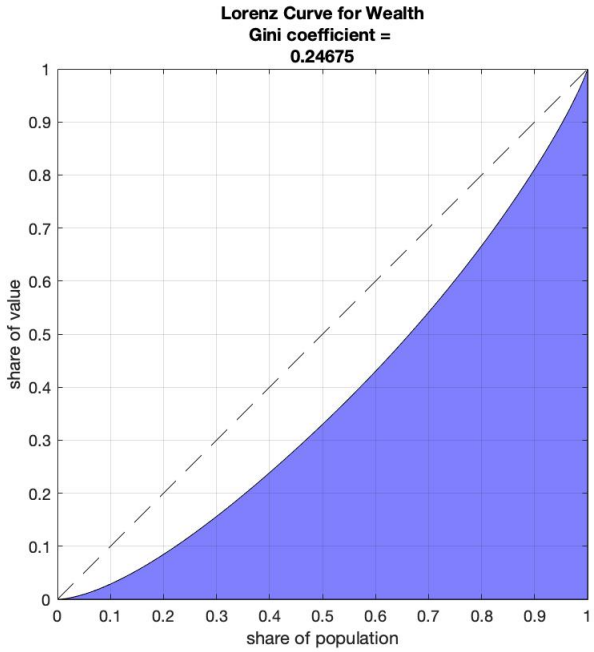


FIGURE 0.3. Wealth distribution